Triposland

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To live in the world of Part II Mathematics is to live in a world full of symmetry and simplicity; for it is the home of the smooth sphere, the ideal gas, the perfect fluid, the simple harmonic motion, and the regular function (together with all its derivatives). Even complex numbers are not really complicated.

Almost all the objects in this world are particles, spheres, or infinite circular cylinders. Not all spheres and cylinders are allowed, however, but only those with radii a, b, c, r, R or ρ. A few other bodies exist, but they are always axially symmetric. It is found that whenever a particle and a body with an axis of symmetry occur together, the particle lies on the axis of symmetry — this position no doubt being one of highly stable equilibrium. Occasionally we may find more complex configurations, such as a particle and two spheres, but then, as we should expect, the spheres are of equal radius and the particle is at the mid-point of the line joining their centres.

Part II Mathematics is not without its poetry. Who can remain unmoved at the spectacle of an infinite, inviscid, incompressible, homogeneous liquid in irrotational motion under no body forces and at rest at infinity, and at the thought of a vacuous spherical bubble of initial radius R in the midst of this solitude?

In complete contrast to the above "nature poetry" there is a style of poetry which imposes a stern discipline of form on its writers and which, to the beginner, may appear harsh and inhospitable; but many are the masterpieces written in this style, starting:

> Given ε greater than 0, however small,
> there exist $\delta = \delta(\varepsilon)$ greater than 0
> such that $|f(x') - f(x)|$ is less than
> ε
> for all x, x' in $[a, b]$ and satisfying $|x - x'|$ less than δ ...

A development of the above style begins slightly differently:

> Given ε greater than 0, however small,
> there exist $\delta = \delta(\varepsilon)$ greater than 0
> such that $|f(x') - f(x)|$ is less than

[1] *Eureka* is the journal of the Archimedeans, the Mathematics Society of the University of Cambridge (http://www.cam.ac.uk/societies/archim/index.html).
Contents pages of all back issues of *Eureka* can be found online at: http://www.srcf.ucam.org/archim/eureka/backissues.html.

fifteen-seventeenths of the maximum for x in I_n of $|g(x)|$ divided by
one plus the greater of N_1 and N_2, multiplied by the least upper
bound of the sequence $\{a_r\}$ divided by $N(a, b)$, times ε
for all x, x' in $[a, b]$ and satisfying $|x - x'|$ less than δ ...

but we need not be alarmed, for the poem concludes with touching simplicity:

... therefore $|g(x) - g(x')|$ is less than ε for $|x - x'|$ less than δ.

The reader himself is probably steeped in poetry of the above style, and no doubt he will have his favourite lines. Amongst mine is one from a poem by Peono, which opens with the confident assertion:

There exists a positive integer 1,

and I am always moved by the glorious surge of

A continuous function
of a continuous function
is a continuous function.

If Part II Mathematics has its poetry, it has also its bad poetry. Consider the following examples. The first proceeds clumsily and unconvincingly to its disheartening conclusion, and one is left with a feeling of dissatisfaction:

It is clear we need not consider cases (a) and (b),
While case (c) is almost trivial.
The problem is then easily seen to be equivalent to ...
And the reader will readily convince himself that ...
Now Drobycheff has shown that ...
While it is well known that ...
Also by a simple extension of a theorem due to Funfledinger, we see
that ...
It is now quite obvious that ...
Hence the result, as the reader
Will easily verify.

On the other hand the second poem, written in the modern style, proceeds with crystal-clear logic to its uplifting termination, and one is left with a clear, enlightened mind, free from all doubt. (If not, the reader is probably unaccustomed to the modern style, and, after drinking two or three cups of very black coffee, he should read it through several times.)

In order to prove that the complement of the intersection of
the sets A
is equal to the union of the complements of
the sets A,
we first suppose that a belongs to the complement of the intersection
of
the sets A
then a does not belong to the intersection of

the sets A
and hence a does not belong to A for some A
thus a belongs to the complement of A for some A
hence a belongs to the union of the complements of
the sets A;

secondly we suppose that a belongs to the union of the complements
of
the sets A
then a belongs to the complement of A for some A
hence a does not belong to A for some A
thus a does not belong to the intersection of
the sets A
that is a belongs to the complement of the intersection of
the sets A
hence we have proved that the complement of the intersection of
the sets A
is equal to the union of the complements of
the sets A.

In Part II Mathematics there is also a mystical poetry. Who does not experience a thrill when he reads:

$$\text{curl curl} \equiv \text{grad div} - \nabla^2$$

and perhaps alarm when he sees:

$$\text{If div } F = 0, \quad \text{then} \quad \text{curl curl curl curl } F = \nabla^2\nabla^2 F),$$

even though at first he does not understand? But later, the understanding comes — before the end of May, one hopes!