GROWTH AND EXCHANGE RATES IN THE GLOBAL ECONOMY

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Abstract

Analytical results from a dynamic optimisation model for a global economy show how exchange rates are determined by relative prices of trading countries. Prices depend on preferences on domestic and foreign goods, marginal productivities of capital and labour as well as the relative rates of taxes and tariffs across two countries. Dynamic model is solved for numerical simulation and scenario analyses. GMM estimation of dynamic panel is used to find determinants of growth of per capita output and the exchange rates across eleven countries representing the global economy. Estimates support the standard neoclassical theory of economic growth and uncovered interest parity theory of exchange rate though country specific factors also can have significant influence estimation in each model.

Key words: growth, exchange rate, global economy

JEL Classification: C6, D9, E6, F41

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Global economy is interdependent. Economic activities in one country influence those in other countries. First part of this paper presents a dynamic model in which growth of output, their prices and the exchange rates ate determined endogenously on the basis of intertemporal optimisation decisions by households who consume both domestic and foreign goods. A small general equilibrium model of the global economy is solved analytically and numerically for a set of parameters and endowments and results examined Then it assesses the determinants of growth and the exchange rates from panel series of eleven small, medium and large and advanced and less developed economies representing the global economy using the a dynamic generalised methods of moment routines for a dynamic panel data model available in the Econometric package in PcGive(2003).

I. Microfounded Macro Model of Two country Global Economy

Global economy consist of N number of countries, *i* is index for home country and *j* for foreign countries. Household utility function in country *i* contains goods produced at home, imported from abroad and leisure. Government uses taxes on consumption, imported goods and labour income to provide for public consumption. With the Cobb-Douglas production function, the household problem can be stated as:

$$\text{Max } U_0^i = \sum_{t=0}^{\infty} \theta^t \left(C_t^{\alpha} M_t^{\beta} l_t^{\gamma} \right) \quad \text{where } \alpha + \beta + \gamma = 1;\, 0 < \alpha, \beta, \gamma < 1;\, 0 < \theta < 1 \quad (1)$$

Subject to life time budget constraint

$$\sum_{t=0}^{\infty} \left[P_{i,t}(1+tc_i)C_{i,t} + P_{j,t}(1+tm_i)M_{i,t} + w_{i,t}(1-tw_i)l_{i,t} \right] \leq \sum_{t=0}^{\infty} \left[w_{i,t}(1-tw_i)\overline{L}_{i,t} + r_{i,t}(1-tk_i)K_{i,t} \right] \quad (2)$$

Firms maximise profit in each period

Max $\Pi_{i,t} = P_{i,t}Y_{i,t} - r_{i,t}K_{i,t} - w_{i,t}LS_{i,t}$ (3)

Subject to

$$Y_{i,t} = K_{i,t}^{\eta_i} L_{i,t}^{(1-\eta_i)} \quad (4)$$

$$I_{i,t} = K_{i,t} - (1-\delta)K_{i,t-1} \quad (5)$$

Government Sector

$$R_{i,t} = P_{i,t}tc_i C_{i,t} + tm_i P_{j,t} M_{i,t} + tw_i LS_{i,t} + tr_i K_{i,t} \leq G_{i,t} \quad (6)$$

Market clearing

$$Y_{i,t} = C_{i,t} + X_{i,t} + G_{i,t} \quad (7)$$

There can be two different ways of trade balance

Period by period trade balance: $M_{i,t} = X_{i,t}$ (8)

$$\sum_{t=0}^{\infty} \theta^t \left(M_{i,t} - X_{i,t}\right) = \sum_{t=0}^{\infty} \theta^t \left(TB_{i,t}\right) \quad (9)$$

$$\left(S_{i,t} - I_{i,t}\right) + \left(X_{i,t} - M_{i,t}\right) = 0 \quad (10)$$

$$\left(\bar{L}_{i,t} - I_{i,t}\right) = LS_{i,t} \quad (11)$$

Prices from the inter temporal arbitrage condition

$$P_{i,t} = \frac{P_{i,t+1}}{1 + r_{i,t}} \quad (12)$$

Bilateral exchange rate between economies *i* and j is given by:

$$E_{i,t} = \frac{P_{i,t}}{P_{j,t}} \quad (13)$$

A competitive economy is the sequence of prices $P_{i,t}\ P_{j,t}\ r_{i,t}\ r_{j,t}\ w_{i,t}\ w_{j,t}\ E_{i,t}\ E_{j,t}$ and public policy $tc_{i,t}\ tc_{j,t}\ tm_{i,t}\ tm_{j,t}\ tw_{i,t}\ tw_{j,t}\ tr_{i,t}\ tr_{j,t}$ in which allocation of $C_{i,t}$, $M_{j,t}$, $l_{i,t}, C_{j,t}, M_{i,t}$, $l_{j,t}$ maximise the lifetime utility of households U_0^i and U_0^j and choices $LS_{i,t}\ K_{i,t}, LS_{j,t}\ K_{j,t}$ maximise firms profit and government expenditures are $G_{i,t}$, $G_{j,t}$ are compatible with the government revenue $R_{i,t}$, $R_{j,t}$ and exports $X_{i,t}\ X_{j,t}$ are compatible with imports $M_{i,t}$ and $M_{i,t}$. Global market mechanism influences allocations of resources among countries through relative prices.

For empirical implementation the infinite horizon problem is reduced to finite horizon by fixing the terminal period to be some T in the far distance in the future.

Similarly the labour endowment in each period $\overline{L}_{i,t}$ and $\overline{L}_{j,t}$ are taken as given as are the model parameters α , β , γ and θ and the policy parameters $tc_{i,t}$, $tc_{j,t}$, $tm_{i,t}$, $tm_{j,t}$, $tw_{i,t}$, $tw_{j,t}$, $tr_{i,t}$ and $tr_{j,t}$.

II. Analytical Results from Optimisation

The infinite horizon problem is analytically intractable. Such problems are solved using the first order inter temporal optimisation for any two time intervals with generalisation that solutions that satisfy any two periods can be extended to any other periods. First order conditions for households for two periods are:

$$C_t: \qquad \alpha_i\theta^t\left(C_t^{\alpha_i-1}M_t^{\beta_i}l_t^{\gamma_i}\right)=\lambda_t P_{i,t}\left(1+tc_i\right) \tag{14}$$

$$C_{t+1}: \qquad \alpha_i\theta^{t+1}\left(C_{t+1}^{\alpha_i-1}M_{t+1}^{\beta_i}l_{t+1}^{\gamma_i}\right)=\lambda_t P_{i,t+1}\left(1+tc_i\right) \tag{15}$$

$$M_t: \qquad \beta_i\theta^t\left(C_t^{\alpha_i}M_t^{\beta_i-1}l_t^{\gamma_i}\right)=\lambda_t P_{j,t}\left(1+tm_i\right) \tag{16}$$

$$M_{t+1}: \qquad \beta_i\theta^{t+1}\left(C_{t+1}^{\alpha_i}M_{t+1}^{\beta_i-1}l_{t+1}^{\gamma_i}\right)=\lambda_t P_{j,t+1}\left(1+tm_i\right) \tag{17}$$

$$l_t: \qquad \gamma_i\theta^t\left(C_t^{\alpha_i}M_t^{\beta_i}l_t^{\gamma_i-1}\right)=\lambda_t w_{i,t}\left(1-tw_i\right) \tag{18}$$

$$l_{t+1}: \qquad \gamma_i\theta^{t+1}\left(C_{t+1}^{\alpha_i}M_{t+1}^{\beta_i}l_{t+1}^{\gamma_i-1}\right)=\lambda_t w_{i,t+1}\left(1+tw_i\right) \tag{19}$$

$$\lambda_t: \; P_{i,t}\left(1+tc_i\right)C_{i,t}+P_{j,t}\left(1+tm_i\right)M_{i,t}+w_{i,t}\left(1-tw_i\right)l_{i,t}=w_{i,t}\left(1-tw_i\right)\overline{L}_{i,t}+r_{i,t}\left(1-tk_i\right)K_{i,t} \tag{20}$$

$$\lambda_{t+1}: \; P_{i,t+1}\left(1+tc_i\right)C_{i,t+1}+P_{j,t+1}\left(1+tm_i\right)M_{i,t+1}+w_{i,t+1}\left(1-tw_i\right)l_{i,t+1}=w_{i,t+1}\left(1-tw_i\right)\overline{L}_{i,t+1}+r_{i,t+1}\left(1-tk_i\right)K_{i,t+1} \tag{21}$$

Above first order conditions can be simplified in terms of Euler equations as:

$$\frac{C_{i,t}}{C_{i,t+1}}: \qquad \frac{1}{\theta}\left(\frac{C_{i,t}}{C_{i,t+1}}\right)^{(\alpha_i-1)}\left(\frac{M_{i,t}}{M_{i,t+1}}\right)^{\beta_i}\left(\frac{l_{i,t}}{l_{i,t+1}}\right)^{\gamma_i}=\frac{P_{i,t}}{P_{j,t}} \tag{22}$$

$$\frac{M_{i,t}}{M_{i,t+1}}: \qquad \frac{1}{\theta}\left(\frac{C_{i,t}}{C_{i,t+1}}\right)^{\alpha_i}\left(\frac{M_{i,t}}{M_{i,t+1}}\right)^{(\beta_i-1)}\left(\frac{l_{i,t}}{l_{i,t+1}}\right)^{\gamma_i}=\frac{P_{j,t}}{P_{j,t+1}} \tag{23}$$

$$\frac{M_{i,t}}{M_{i,t+1}}: \qquad \frac{1}{\theta}\left(\frac{C_{i,t}}{C_{i,t+1}}\right)^{\alpha_i}\left(\frac{M_{i,t}}{M_{i,t+1}}\right)^{(\beta_i-1)}\left(\frac{l_{i,t}}{l_{i,t+1}}\right)^{(\gamma_i-1)} = \frac{w_{i,t}}{w_{i,t+1}} \tag{24}$$

$$\frac{C_{i,t+1}}{M_{i,t+1}}: \qquad \frac{\alpha_i}{\beta_i}\left(\frac{M_{i,t+1}}{C_{i,t+1}}\right) = \frac{P_{i,t+1}(1+tc_i)}{P_{j,t+1}(1+tm_i)} \tag{25}$$

$$\frac{l_{i,t+1}}{M_{i,t+1}}: \qquad \frac{\alpha_i}{\gamma_i}\left(\frac{l_{i,t+1}}{C_{i,t+1}}\right) = \frac{P_{i,t+1}(1+tc_i)}{w_{i,t+1}(1+tw_i)} \tag{26}$$

$$\frac{M_{i,t+1}}{l_{i,t+1}}: \qquad \frac{\beta_i}{\gamma_i}\left(\frac{l_{i,t+1}}{M_{i,t+1}}\right) = \frac{P_{j,t+1}(1+tm_i)}{w_{i,t+1}(1+tw_i)} \tag{27}$$

Similarly the first order conditions for firms are:

$$\Pi_{i,t} = P_{i,t}K_{i,t}^{\eta_i}L_{i,t}^{(1-\eta_i)} - r_{i,t}K_{i,t} - w_{i,t}LS_{i,t} \tag{28}$$

$$K_{i,t}: \ \eta_{i,t}P_{i,t}K_{i,t}^{\eta_i-1}L_{i,t}^{(1-\eta_i)} = r_{i,t} \ \text{ or } \ \frac{\eta_{i,t}P_{i,t}Y_{i,t}}{K_{i,t}} = r_{i,t} \tag{29}$$

$$K_{j,t}: \ \eta_{j,t}P_{j,t}K_{j,t}^{\eta_i-1}L_{j,t}^{(1-\eta_i)} = r_{j,t} \ \text{ or } \ \frac{\eta_{j,t}P_{j,t}Y_{j,t}}{K_{j,t}} = r_{j,t} \tag{30}$$

$$L_{i,t}: \ (1-\eta_{i,t})P_{i,t}K_{i,t}^{\eta_i-1}L_{i,t}^{-\eta_i} = w_{i,t} \qquad \text{or} \qquad \frac{(1-\eta_{i,t})P_{i,t}Y_{i,t}}{L_{i,t}} = w_{i,t} \tag{31}$$

$$L_{j,t}: \ (1-\eta_{j,t})P_{j,t}K_{j,t}^{\eta_i-1}L_{j,t}^{-\eta_i} = w_{j,t} \ \text{ or } \ \frac{(1-\eta_{j,t})P_{j,t}Y_{j,t}}{L_{j,t}} = w_{j,t} \tag{32}$$

Initial condition $K_{i,0}$ $K_{j,0}$ and (33)

Terminal conditions $I_{i,T} = (g+\delta)K_{i,T-1}; I_{j,T} = (g+\delta)K_{j,T-1}$. (34)

Whether the wages rates and the interest rates are same or differ from one country to another depends partly upon the mobility and productivity of factors and partly to the tariff rates across countries. If labour and capital are perfectly mobile then the ratios of use of labour and capital across two countries depend on ratios of production.

$$\frac{\eta_{j,t}P_{j,t}Y_{j,t}}{\eta_{i,t}P_{i,t}Y_{i,t}}\frac{K_{i,t}}{K_{j,t}} = \frac{r_{j,t}}{r_{i,t}} \tag{35}$$

$$\frac{(1-\eta_{j,t})P_{j,t}Y_{j,t}}{(1-\eta_{i,t})P_{i,t}Y_{i,t}}\frac{L_{i,t}}{L_{j,t}}=\frac{w_{j,t}}{w_{i,t}} \tag{36}$$

The exchange rate between two countries should be compatible with goods, labour and capital markets.

$$E_{j,t}=\frac{P_{j,t}}{P_{i,t}}=\frac{r_{j,t}}{r_{i,t}}\frac{K_{j,t}}{K_{i,t}}\frac{\eta_{i,t}Y_{i,t}}{\eta_{j,t}Y_{j,t}}=\frac{(1-\eta_{i,t})Y_{i,t}}{(1-\eta_{j,t})Y_{j,t}}\frac{L_{j,t}}{L_{i,t}}\frac{w_{j,t}}{w_{i,t}}=\frac{\alpha_i}{\beta_i}\frac{M_{i,t}}{C_{i,,t}}\frac{(1+tm_i)}{(1+tc_i)} \tag{37}$$

These analytical results show intedepedency of exchange rates and growth rates. The model solutions differ in autarky, when two countries do not cooperate each other from when they cooperate. Both analytical and numerical solutions are significantly different than found in the literature (Dornbusch (1976), Miller and Spencer (1977), Taylor (1995)).

III. **Numerical Results**

I implement a simple version of the above model with the following parameters.

Table 1
Parametric specification of micro-founded macro model of the global economy

Parameters	Country 1	Country 2
Alpha	0.6	0.8
Beta	0.4	0.2
gm	0.2	0.2
K0	2000	1000
Th	0.95	0.95
Tc	0.15	0.2
Tm	0.05	0.1
Tw	0.25	0.15
Tr	0.01	0.15
Gr	0.03	0.04
D	0.01	0.02
Lbar	4000	2000
R	0.05	0.03
Nu	0.4	0.3
Q0	2000	1000
P0	1	1

Table 2
Time series of macro economic variables in micro founded dynamic general equilibrium model of the real exchange rate.

Model Generated Solutions for Economy 1												
Years	consum	govex	exports	imports	utility	incom	Exp	taxrev	Intrest	price	mprice	rexrate
5	2185	1171	2185	2185	2185	2444	3916	1171	0.05	0.815	0.815	1.000
10	3800	1182	2534	2534	3231	4412	4897	1182	0.05	0.697	0.697	1.000
15	2937	1368	2937	2937	2937	4352	4831	1368	0.05	0.889	0.593	1.500
20	3405	919	3405	3405	3405	3212	3565	919	0.05	0.566	0.377	1.500
25	3947	755	3947	3947	3947	2881	3198	755	0.05	0.438	0.292	1.500
Model Generated Solutions for Economy 2												
5	1360	904	1125	1125	1309	2391	3391	904	0.03	1.406	0.885	1.589
10	1369	1039	1369	1369	1369	2772	3822	1039	0.03	1.63	0.76	2.145
15	1665	1305	1665	1665	1665	3213	4972	1305	0.03	1.89	0.653	2.895
20	2032	1662	2026	2026	2031	5678	6594	1662	0.03	2.191	0.561	3.908
25	2465	2171	2465	2465	2465	7825	9234	2171	0.03	2.54	0.635	4.000

Higher level of utility of the households in country 1 than that in country 2 reflects not only the larger amount of capital and labour endowments in country 1 but also the good policy measures in that country. Price level in country two almost doubles and real exchange rate increases by three times in country 2 while the price level keeps falling in country 1. Inflationary pressure in country 2 is due to expansionary nature of public and private expenditure pushed up by lower interest rate than in country1. As in any dynamic general equilibrium models these results, are sensitive to a set of parameters.

IV. Dynamic Panel Data Regression: Fixed and Random Effects Models

Fixed effect, random effect and dynamic panel data models have been applied for estimation of parameters in models of growth regression arranging the panel data on growth rates and their determinants for $i = 1,\dots N$ countries and $t = 1,\dots,T$ years as:

$$\begin{bmatrix} y_{1,1} & x_{1,1} & e_{1,1} \\ . & . & . \\ y_{1,T} & x_{1,T} & e_{1,T} \\ y_{2,1} & x_{2,1} & e_{2,1} \\ . & . & . \\ y_{2,T} & x_{2,T} & e_{2,T} \\ . & . & . \\ y_{N,1} & x_{N,1} & e_{N,1} \\ . & . & . \\ y_{N,T} & x_{N,T} & e_{N,T} \end{bmatrix}.$$

where $\mathrm{y}_{i,t}$ is growth rate of country *i* in period *t*, $\mathrm{x}_{i,t}$ denotes the vectors of determinants of growth rates of country *i* in period *t*, and $\mathrm{e}_{i,t}$ are corresponding random factors that cannot be easily ascertained. Then specify the behavioural equation for a fixed effect growth regression model for country *i* with *T* year as:

$$\mathrm{y}_{i,t} = \alpha_i + \mathrm{x}_{i,t}\beta + \varepsilon_{i,t} \qquad \varepsilon_{i,t} \sim IID\left(0, \sigma^2_{æ}\right) \tag{38}$$

where parameter α_i picks up the fixed effects that differ among individuals, β is the vector of growth coefficients. These parameters can be estimated by OLS when *N* is small but not when that is large but the model need to be transformed to the least square dummy variable method when N is too large.

$$\bar{\mathrm{y}}_i = \alpha_i + \bar{\mathrm{x}}_i\beta + \varepsilon_i \quad \text{where } \bar{\mathrm{y}}_i = T^{-1}\sum_i y_{i,t}$$

$$\mathrm{y}_{i,t} - \bar{\mathrm{y}}_i = \left(\mathrm{x}_{i,t} - \bar{\mathrm{x}}_i\right)\beta + \left(\varepsilon_{i,t} - \varepsilon_i\right)$$

fixed effect least square dummy variable estimator of β is

$$\beta_{FE} = \left(\sum_t^T\sum_i^N \left(\mathrm{x}_{i,t} - \bar{\mathrm{x}}_i\right)\left(\mathrm{x}_{i,t} - \bar{\mathrm{x}}_i\right)'\right)^{-1} \sum_t^T\sum_i^N \left(\mathrm{x}_{i,t} - \bar{\mathrm{x}}_i\right)\left(\mathrm{y}_{i,t} - \bar{\mathrm{y}}_i\right) \tag{39}$$

$$\alpha_i = \bar{\mathrm{y}}_i - \bar{\mathrm{x}}_i\beta_{FE} \tag{40}$$

These estimators are unbiased, consistent and efficient with corresponding covariance matrix given by:

$$\mathrm{cov}(\beta_{FE}) = \sigma_\varepsilon^2 \left(\sum_t^T \sum_i^N (\mathrm{x}_{i,t} - \bar{\mathrm{x}}_i)(\mathrm{x}_{i,t} - \bar{\mathrm{x}}_i)' \right)^{-1} \qquad (41)$$

$$\sigma_\varepsilon^2 = \frac{1}{\mathrm{N}(\mathrm{T}-1)} \sum_i^N \sum_t^T (\mathrm{y}_{i,t} - \alpha_i - \mathrm{x}_{i,t}\beta_{FE})$$

This fixed effect model is more succinctly presented in matrix notation as:

$$\mathrm{y}_i = i\alpha_i + \mathrm{X}_i\beta + \varepsilon_i$$

$$\begin{bmatrix} Y_1 \\ Y_1 \\ . \\ . \\ Y_n \end{bmatrix} = \begin{bmatrix} I & 0 & . & . & 0 \\ 0 & I & 0 & . & 0 \\ 0 & 0 & I & . & 0 \\ . & . & . & . & . \\ 0 & 0 & . & . & I \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \\ \\ \alpha_n \end{bmatrix} + \begin{bmatrix} X_1 \\ X_2 \\ \\ \\ X_n \end{bmatrix} \beta + \begin{bmatrix} e_1 \\ e_2 \\ \\ \\ e_n \end{bmatrix} \qquad (42)$$

$$Y = [d_1 \quad d_2 \quad . \quad . \quad . \quad d_n \quad X] \begin{bmatrix} \alpha \\ \beta \end{bmatrix} + \varepsilon$$

$$Y = D\alpha + X\beta + \varepsilon \qquad (43)$$

This can be easily estimated by the OLS when number of cross section units are small but many panel data studies have much larger cross section observations. It results in over parameterisation and loss of degree of freedom. For this model is transformed by a projection matrix

$$M_d = I - D(D'D)D'$$

$$M_d Y = M_d D\alpha + M_d X\beta + M_d \varepsilon$$

$$b = [X'M_d X]^{-1}[X'M_d Y] \qquad (16)$$

$$M_d = \begin{bmatrix} M^0 & 0 & 0 & . & . & 0 \\ 0 & M^0 & . & . & . & . \\ 0 & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & M^0 \end{bmatrix} \text{ where } M^0 = I_T - \frac{1}{T} ii' \qquad (44)$$

Multiplying any variable by M^0 is equivalent to taking deviation from the mean i.e.

$$M^0 X_i = X_i - \overline{X}i$$

$$Y = D\alpha + X\beta + \varepsilon$$

$$a = (D'D)^{-1} D(Y - Xb)$$

$$\text{var}(b) = s^2\left[X'M_d X\right]^{-1} \text{ and } s^2 = \frac{\sum_t \sum_i \left(y_{it} - a_i - x_{i,t}b\right)}{nT - n - k} \tag{45}$$

Whether country specific effects are fixed is an arguable issue. These effects are more likely to be random. Therefore random effect models are more appropriate for analysing determinants of growth as

$$\mathrm{y}_{i,t} = \mu + \mathrm{x}_{i,t}\beta + \alpha_i + \varepsilon_{i,t} \tag{46}$$

where $\alpha_{i,} \sim IID\left(0, \sigma_\alpha^2\right)$ are individual specific random errors and $\varepsilon_{i,t} \sim IID\left(0, \sigma_{æ}^2\right)$ are remaining random errors.

$$\alpha_i \iota_T + \varepsilon_i \quad \iota_T = (1,1,.....1)'$$

$$Var\left(\alpha_i \iota_T + \varepsilon_i\right) = \Omega = \sigma_\alpha^2 \iota_T \iota_T' + \sigma_\varepsilon^2 I_T$$

Errors are correlated therefore this requires estimation by the Generalised Least Square estimator. Transform the model by pre-multiplying by Ω^{-1} where

$$\Omega^{-1} = \sigma_\varepsilon^2\left[I_T - \frac{\sigma_\alpha^2}{\sigma_\varepsilon^2 + T\sigma_\alpha^2}\iota_T \iota_T'\right]$$

$$\beta_{GLS} = \left(\sum_t^T \sum_i^N \left(\mathrm{x}_{i,t} - \bar{\mathrm{x}}_i\right)\left(\mathrm{x}_{i,t} - \bar{\mathrm{x}}_i\right)' + \psi T \sum_i^N \left(\bar{\mathrm{x}}_i - \bar{\mathrm{x}}\right)\left(\bar{\mathrm{x}}_i - \bar{\mathrm{x}}\right)'\right)^{-1}$$
$$\left(\sum_t^T \sum_i^N \left(\mathrm{x}_{i,t} - \bar{\mathrm{x}}_i\right)\left(\mathrm{y}_{i,t} - \bar{\mathrm{y}}_i\right) + \psi T \sum_i^N \left(\bar{\mathrm{x}}_i - \bar{\mathrm{x}}\right)\left(\bar{\mathrm{y}}_i - \bar{\mathrm{y}}\right)'\right) \tag{47}$$

$$\Omega = \begin{bmatrix} \sigma_\alpha^2 + \sigma_\varepsilon^2 & \sigma_\alpha^2 & \sigma_\alpha^2 & . & . & \sigma_\alpha^2 \\ 0 & \sigma_\alpha^2 + \sigma_\varepsilon^2 & . & . & . & . \\ 0 & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \sigma_\alpha^2 & \sigma_\alpha^2 & \sigma_\alpha^2 & . & . & \sigma_\alpha^2 + \sigma_\varepsilon^2 \end{bmatrix} \tag{48}$$

$$\Omega^{-\frac{1}{2}} = \frac{1}{\sigma_\varepsilon}\left[I_T - 1 - \frac{\sigma_\varepsilon}{\sqrt{T\sigma_u^2 + \sigma_\varepsilon^2}}\right]$$

$$\beta_{GLS} = \sum_i \left(X'\Omega^{-1}X\right)^{-1} \sum_i \left(X'\Omega^{-1}Y\right) \tag{49}$$

Both random and fixed effect models are essentially static models which are not adequate to capture the dynamics of economic growth among countries. This requires a dynamic panel data model which can be efficiently estimated by a generalised method of moments (GMM) as proposed by Hansen (1982).

$$\mathrm{y}_{i,t} = \gamma \mathrm{y}_{i,t-1}\beta + \alpha_i + \varepsilon_{i,t} \quad \text{with} \quad \gamma < 1 \tag{50}$$

which generates the following estimator

$$\gamma_{FE} = \frac{\sum_t^T \sum_i^N \left(\mathrm{y}_{i,t} - \bar{\mathrm{y}}_i\right)\left(\mathrm{y}_{i,t} - \bar{\mathrm{y}}_{i,t-1}\right)}{\sum_t^T \sum_i^N \left(\mathrm{y}_{i,t-1} - \bar{\mathrm{y}}_{i,-1},\right)^2} \; ; \; \bar{\mathrm{y}}_i = T^{-1}\sum_i y_{i,t} \text{ and } \bar{\mathrm{y}}_{i,-1} = T^{-1}\sum_i y_{i,t-1}$$

This is not asymptotically unbiased estimator:

$$\gamma_{FE} = \gamma + \frac{(1/NT)\sum_t^T \sum_i^N \left(\varepsilon_{i,t} - \bar{\varepsilon}_i\right)\left(\mathrm{y}_{i,t} - \bar{\mathrm{y}}_{i,t-1}\right)}{(1/NT)\sum_t^T \sum_i^N \left(\mathrm{y}_{i,t-1} - \bar{\mathrm{y}}_{i,-1},\right)^2}$$

$$\underset{N\to\infty}{P\lim} \quad (1/NT)\sum_t^T \sum_i^N \left(\varepsilon_{i,t} - \bar{\varepsilon}_i\right)\left(\mathrm{y}_{i,t} - \bar{\mathrm{y}}_{i,t-1}\right) = -\frac{\sigma_\varepsilon^2}{T^2}\frac{(T-1) - T\gamma + \gamma^T}{(1-\gamma)^2} \neq 0$$

Instrumental variable methods have been suggested to solve this inconsistency

$$\hat{\gamma}_{IV} = \frac{\sum_t^T \sum_i^N \mathrm{y}_{i,t-2}\left(\mathrm{y}_{i,t-1} - \bar{\mathrm{y}}_{i,t-2}\right)}{\sum_t^T \sum_i^N \mathrm{y}_{i,t-2}\left(\mathrm{y}_{i,t-1} - \mathrm{y}_{i,t-2},\right)} \quad \text{and}$$

where y_{i-2} is used as instrument of $\left(\mathrm{y}_{i,t-1} - \mathrm{y}_{i-2}\right)$

It is asymptotically

$$\underset{N\to\infty}{P\lim} \quad (1/NT)\sum_t^T \sum_i^N \left(\varepsilon_{i,t} - \bar{\varepsilon}_i\right)\mathrm{y}_{i,t-2} = 0$$

Moment conditions with vector of transformed error terms

$$\Delta\varepsilon_i = \begin{pmatrix} \varepsilon_{i,2} - \varepsilon_{i,1} \\ \varepsilon_{i,3} - \varepsilon_{i,2} \\ . \\ - \\ \varepsilon_{i,T} - \varepsilon_{i,T-1} \end{pmatrix}$$

$$Z_i = \begin{bmatrix} [y_{i0}] & 0 & 0 & . & . & 0 \\ 0 & [y_{i0}, y_{i1}] & . & . & . & . \\ 0 & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & [y_{i0}, y_{i,T-2}] \end{bmatrix}$$

$$E\{Z_i^{'}\Delta\varepsilon_i\} = 0$$

Or for moment estimator write the transformed errors as

$$E\{Z_i^{'}(\Delta \mathrm{y}_{i,t} - \gamma\Delta \mathrm{y}_{i,t})\} = 0$$

$$\min_{\gamma}\left(\frac{1}{N}\sum_{i=1}^{N} Z_i^{'}(\Delta \mathrm{y}_{i,t} - \gamma\Delta \mathrm{y}_{i,t})\right)^{'} W_N\left(\sum_{i}^{n} Z_i{}'(\Delta \mathrm{y}_{i,t} - \gamma\Delta \mathrm{y}_{i,t})\right)$$

GMM method includes the most efficient instrument

$$\hat{\gamma}_{GMM} = \left(\left(\sum_{i=1}^{N}\Delta y_{i,-1}^{'} Z_i\right) W_N\left(\sum_{i=1}^{N} Z_i{}'\Delta y_{i,-1}^{'}\right)\right)^{-1} \times \left(\sum_{i=1}^{N}\Delta y_{i,-1}^{'} Z_i\right) W_N\left(\sum_{i=1}^{N} Z_i{}'\Delta y_i^{'}\right) \quad (51)$$

Blundell and Smith (1989) and Verbeek (2004), Wooldridge (2002) among others have more extensive exposure in GMM estimation. The essence of the GMM estimation remains in finding a weighting matrix W_N that can guarantee the most efficient estimator. This should be inversely proportional to transformed covariance matrix.

$$\hat{W}_N^{opt} = \left(\frac{1}{N}\sum_{i=1}^{N} Z_i^{'}\Delta\varepsilon_i\Delta\varepsilon_i^{'} Z_i\right)^{-1}$$

Doornik and Hendry (2001, chap. 7-10) provide a procedure on how to estimate coefficients β using fixed effect, random effect and the GMM methods including a lagged terms of dependent variable among explanatory variables for a dynamic panel data model:

$y_{i,t} = \sum_{s=1}^{p} a_k y_{i,t-s} + \beta^t(L)x_{i,t} + \lambda_t + \alpha_i + e_{i,t}$ or in short, $y_i = W_i\delta + \iota_i a_i + e_i$.

The GMM estimator with instrument Z_i (levels, first differences, orthogonal deviations, deviations from individual means, combination of first differences and levels) used in PcGive is : $\hat{\delta} = \left[\left(\sum_i W_i^{*'} Z_i\right) A_N \left(\sum_i Z_i^{'} W_i\right)\right]^{-1} \left(\sum_i W_i^{*'} Z_i\right) A_N \left(\sum_i Z_i^{'} y_i^{*}\right)$,

with $A_N = \left(\frac{1}{N}\sum_i Z_i^{'} H_i Z_i\right)^{-1}$, where H_i is the individual specific weighting matrix.

V. Empirical Estimation

Propositions of growth models are tested on time series data for eleven countries to represent the global economy. China, Nepal, India, Pakistan and Bangladesh and Japan are taken from Asia; South Africa, Brazil are from Africa and Latin America to represent middle income countries; Australia, Germany, UK and USA represent advanced economies. Raw data used for analysis is presented in Figures 1 to 7 in the Appendix which show very interesting patterns on growth rate of per capita income (in PPP) and populations (Figure 1 and 2); exchange rates, real interest rates and inflation (Figures 3, 4 and 5); export and investment ratios.

Table 3

Determinants of growth rate of per capita income across countries

	Coefficient	Std.Error	t-value	t-prob
Investment ratio	0.1820	0.0654	2.7800	0.0060
Export ratio	0.0257	0.0294	0.8740	0.3830
Popgrowth	-0.8849	0.6186	-1.4300	0.1540
Constant	3.0116	2.2300	1.3500	0.1780
Nepal	-3.0341	0.7938	-3.8200	0.0000
India	-2.0244	0.5427	-3.7300	0.0000
Bangladesh	-2.6448	0.7241	-3.6500	0.0000
Pakistan	-1.6057	0.9276	-1.7300	0.0840
South Africa	-5.1070	0.9100	-5.6100	0.0000
Brazil	-4.5529	0.6078	-7.4900	0.0000
UK	-4.5630	1.4740	-3.1000	0.0020
Japan	-5.9846	0.5678	-10.5000	0.0000
USA	-3.7902	0.7793	-4.8600	0.0000
Australia	-4.8613	0.4238	-11.5000	0.0000
Germany	-5.6408	1.2720	-4.4300	0.0000
	N	324	R-Square	0.46

Results from the GMM panel estimation for economic growth rates are presented in Table3. Investment ratio and population growth rates represent capital and labour inputs in the process of growth and both are very significant. In addition export ratio represents the international linkage to the domestic economy and is also statistically significant. In addition growth vary across countries significantly for many other country specific reasons which are captured by coefficients attached to the specific coutry. Growth rates of all other countries were lower compared to China, which was used as the base country for this estimation. These estimates were robust even if the individual series were nonstationary as checked by the unit root and cointegration tests presented in Table 4.

Table 4

Stationarity and Cointegration				
Augmented Dickey Fuller Test**(T=321, Constant; 5%=-2.87 1%=-3.45)**				
Investment ratio		-4.449**		Stationary
Population growth rate		-1.9000		non-stationary
Growth rate of per capita income		-6.171**		Stationary
Residual		-10.62**		Stationary
Cointegrated equation.				

Similar estimates were carried on the determinants of exchange rate for which the results from the GMM (in levels) two step estimation are presented in Table 5. It was important to adopt an autoregressive structure to explain significant part of variation in the exchange rate. Results support the usual uncovered interest parity hypothesis as the coefficients in the real interest rates were found to be negative and significant. Positive influence of population growth on exchange rate may be explained from the demand side of the foreign exchange market. Nevertheless, the lagged exchange rates seem very significant determinant of the exchange rate as is often explained in the random walk theory of exchange rate. As in growth rates the country specific factors matter and are significant, mostly positively except for Brazil, Germany and Japan which have experienced appreciation in their currencies during the study period relative to the US dollar which has been anchored to the Chinese currency.

Table 5

Determinants of exchange rate of per capita income across countries				
	Coefficient	Std.Error	t-value	t-prob
Exrate(-1)	0.9701	0.0030	326	0
rl_intrest	-0.0290	0.0033	-8.69	0
popgrowth	0.7917	0.0481	16.5	0
Constant	0.3400	0.0204	16.7	0
Nepal	0.0662	0.0040	16.6	0
India	0.0496	0.0030	16.5	0
Bangladesh	0.0568	0.0034	16.6	0
Pakistan	0.0735	0.0045	16.4	0
South Africa	0.0709	0.0044	16.2	0
Brazil	-0.0324	0.0020	-16.4	0
UK	0.0031	0.0002	16.2	0
Japan	-0.0422	0.0029	-14.5	0
USA	0.0295	0.0018	16.2	0
Australia	0.0351	0.0022	16.2	0
Germany	-0.0074	0.0005	-16.2	0
	N	312	R-Square	0.9857

Robustness of the exchange rate estimates are again tested by using the cointegration relation among these variables. Residuals of the regression were stationary despite non-stationarity of exchange rate, population growth rate and the real interest rate as shown in Table 6.

Table 6

Stationarity and Cointegration In Exchange Rate Estimation				
Augmented Dickey Fuller Test**(T=321, Constant; 5%=-2.87 1%=-3.45)**				
Exrate(-1)		-1.51		Stationary
rl_intrest		-2.59		non-stationary
Popgrowth		-0.02		Stationary
Residual		-4.96**		Stationary
Cointegrated equation.				

VI. **Conclusion**

Analytical results from a dynamic optimisation model for a global economy show how exchange rates are determined by relative prices of trading countries. Prices depend on preferences on domestic and foreign goods, marginal productivities of capital and labour as well as the relative rates of taxes and tariffs across two countries. Dynamic model is solved for numerical simulation and scenario analyses. GMM estimation of dynamic panel is used to find determinants of growth of per capita output and the exchange rates across eleven countries representing the global economy. Estimates support the standard neoclassical theory of economic growth and uncovered interest parity theory of exchange rate though country specific factors also can have significant influence estimation in each model.

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Appendix

Plots of Time Series Used for Empirical Analyses

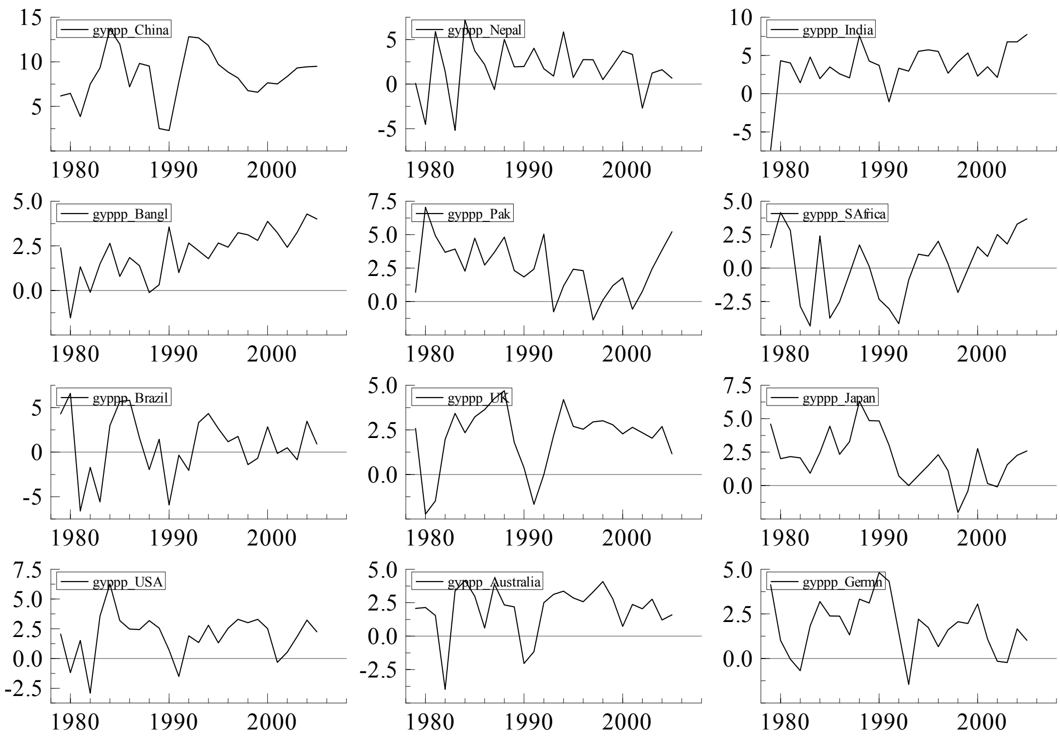

Figure 1: Growth rates of output

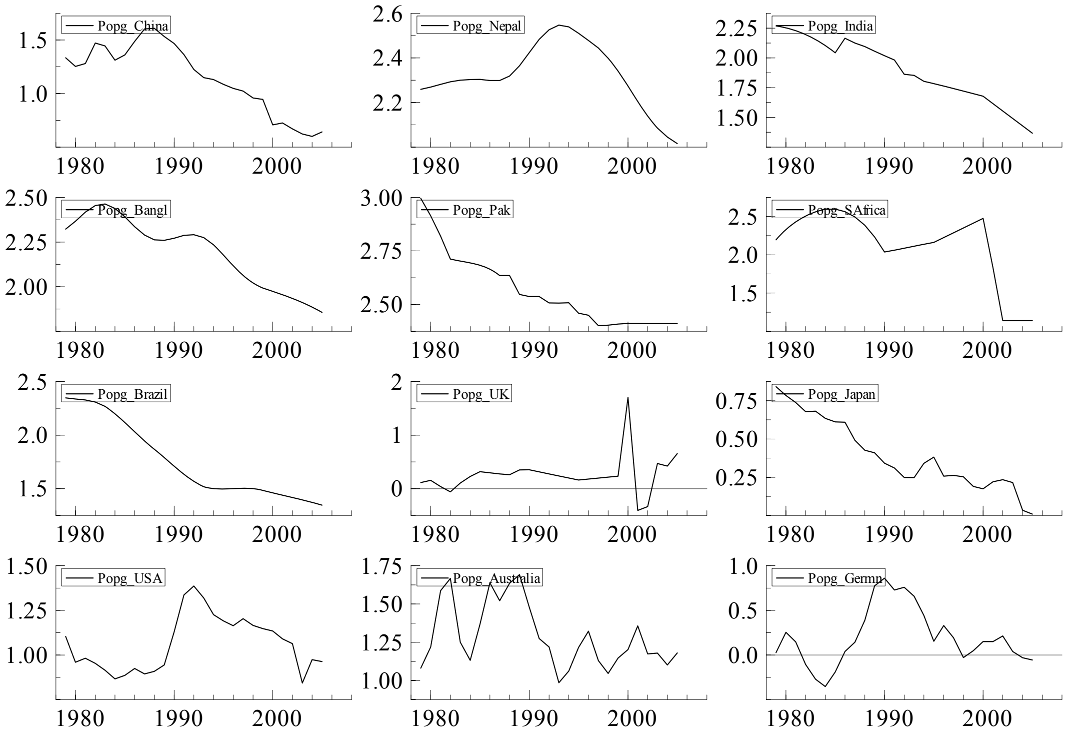

Figure 2: Growth rates of output

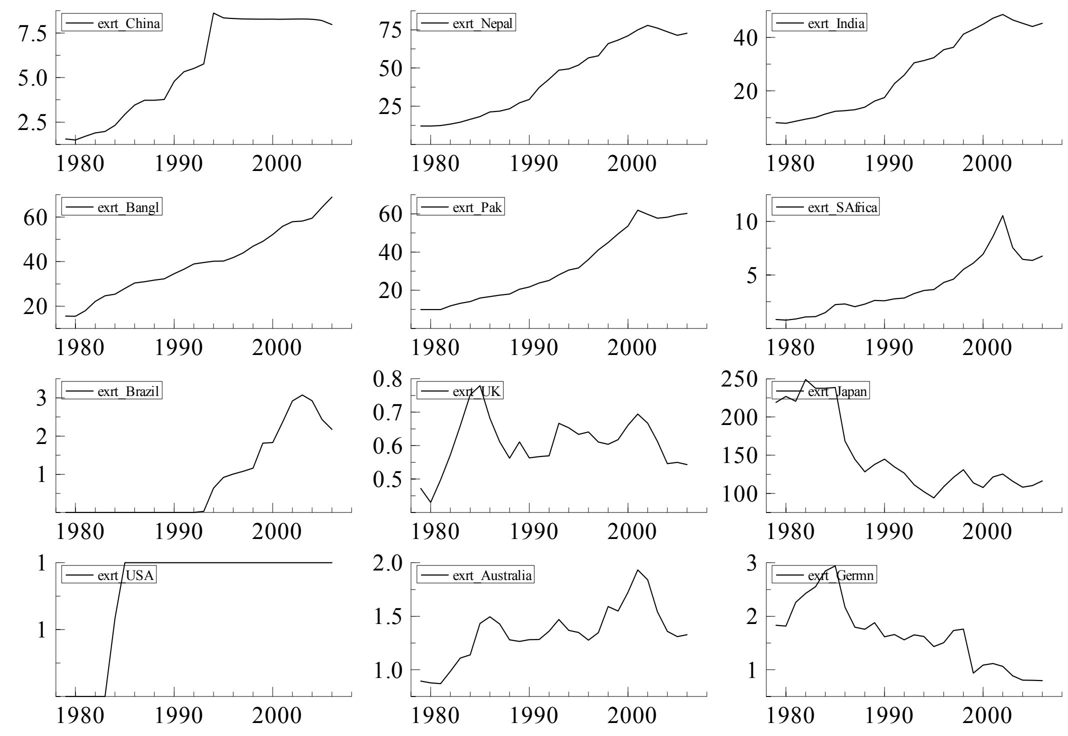

Figure 3: Growth rates of output

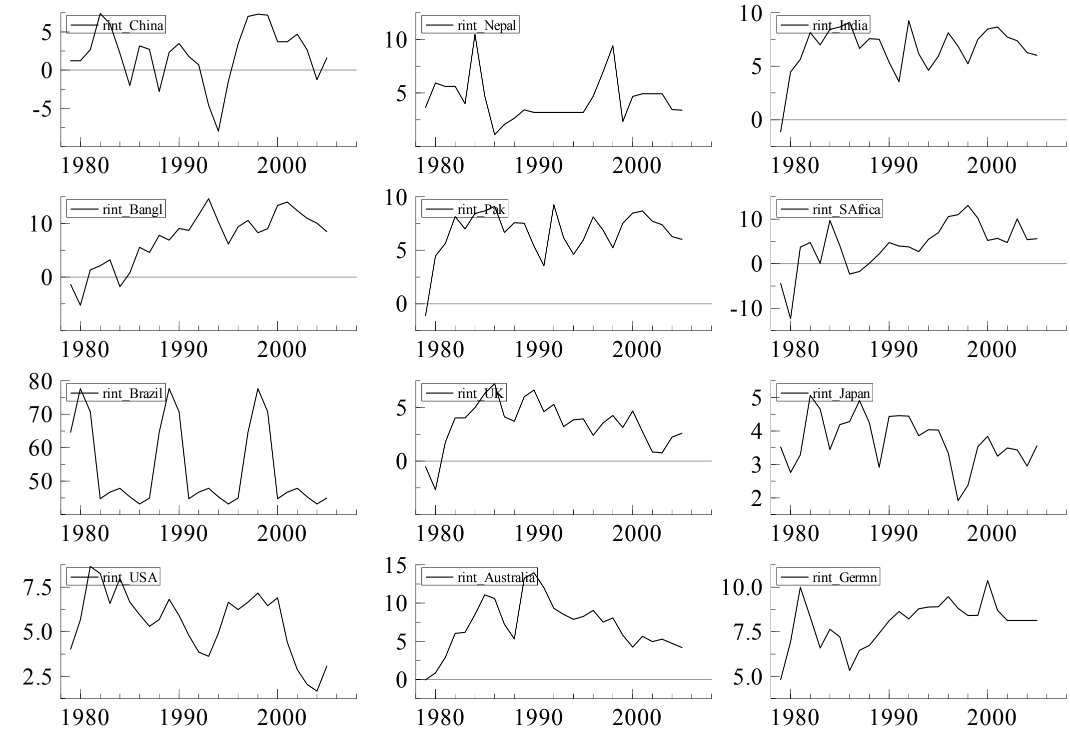

Figure 4: Growth rates of output

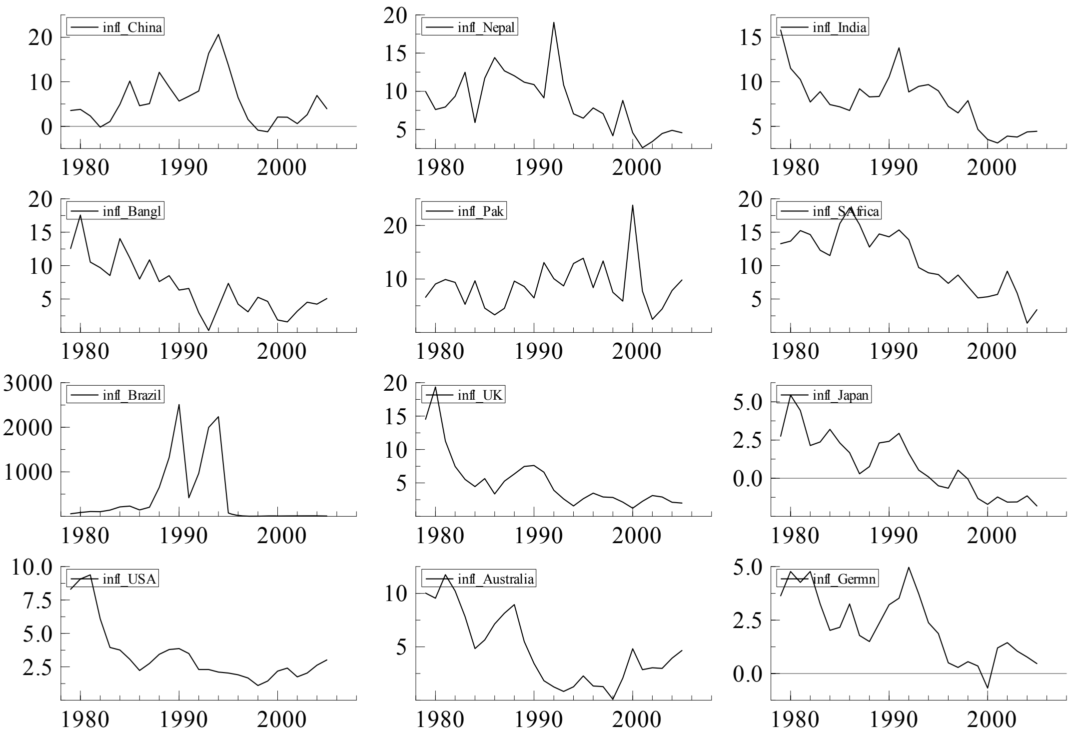

Figure 5: Growth rates of output

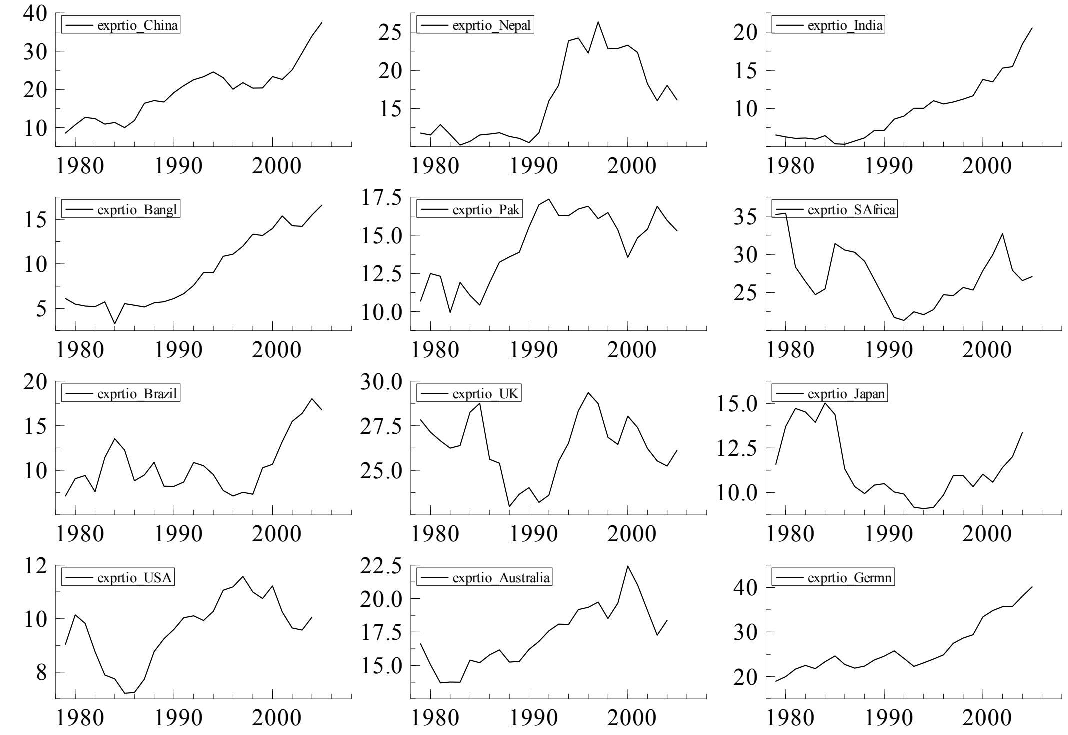

Figure 6: Growth rates of output

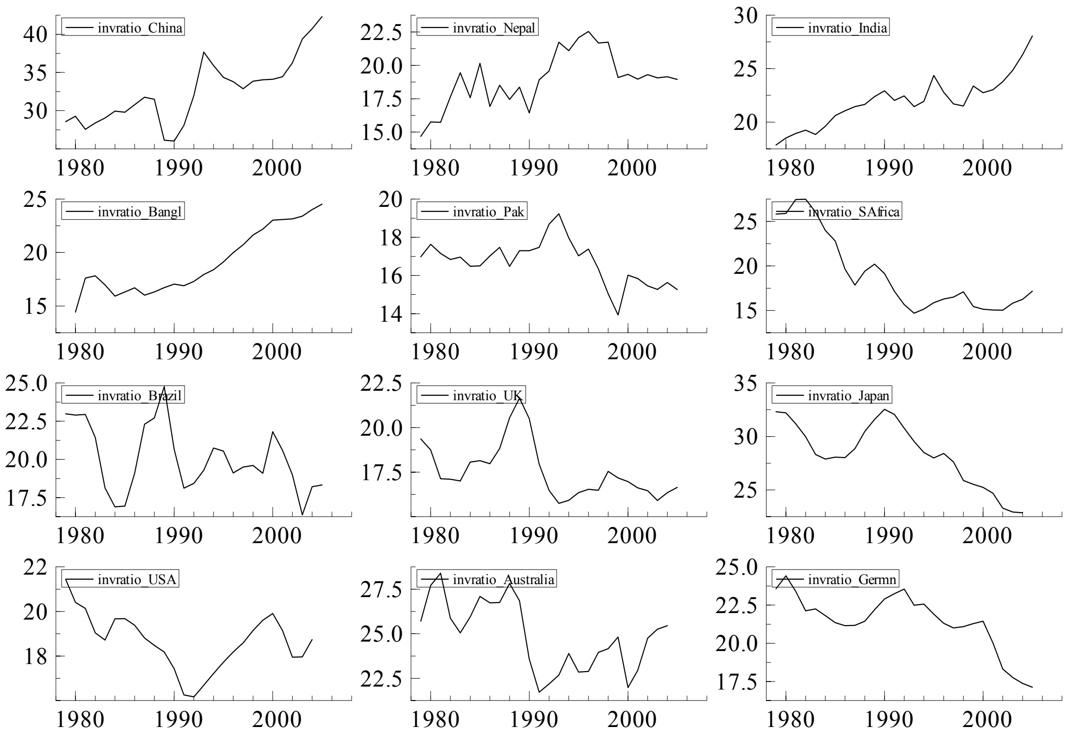

Figure 7: Growth rates of output