Economic Modelling Workshop
Business School, University of Hull

Economists use models to consolidate their reasoning and to justify their arguments. These models can be small or big, static or dynamic, deterministic or stochastic, strategic or optimising or analytic, mathematical or econometric. They may analyse aggregate behaviour at macro level or more details in the micro level. They may aim to explain local, regional, national or global markets or economic policies aimed for stability, efficiency, growth and redistribution.

This workshop is on the main analytical technique and numerical methods required to solve these models. Particularly it will focus on non-linear and dynamic programming, applied general equilibrium and econometric techniques required for economic models. It aims to transfer skills required for formulating, analysing, programming, solving and using the results for these models for economic policy analysis.

In each day first session will be on general equilibrium analysis and the second session will be on econometric exercises. Sufficient exercises will be provided for both types of exercises.

Learning objectives:

1) Ability to solve non-linear and dynamic programming problems.
2) Ability to specify, solve and compute static and dynamic general equilibrium models for policy analysis for economic growth, efficiency and redistribution.
3) Ability to construct cross section, time series and panel data models for econometric analysis and forecasting.
4) Ability to build models required for journal articles on specific economic issues.
5) Ability to write programs required for economic models using General Equilibrium Modeling software such as GAMS/MPSGE and econometric software such as GiveWin PcGive, STAMP, OX and (Shazam,Limdep and Microfit if required)

Contents:

- What is a general equilibrium model: two sector general equilibrium model for analysis to tax policy
- Optimal growth model to analyse the impact of taxes on investment, saving and economic growth
- Central planners problem and problem for the decentralised market Ramsey model
- Ramsey dynamic effect of capital income tax
- Multisectoral and multihousehold tax model with labour leisure choice
- Three period model of consumption and saving
- Global economy model with taxes and tariffs
- Macroeconometric simultaneous equation model
- Trends, seasons,. cycles, random factors in a time series: business cycle analysis
- Static and dynamic energy general equilibrium model with energy sector
- Multisectoral static General Equilibrium tax model for the UK economy
- Multisectoral dynamic general equilibrium model to analyse economic growth
- Multisectoral and multi-regional model for global trade.
- Cross-section analysis

Target participants: Ph.D. and M.Sc. students.

Some relevant materials and readings are kept in http://www.hull.ac.uk/php/ecskrb

Software:

Brook, A K., D. Kendtrick, A. Meeraus (1992) *GAMS: User's Guide,* Release 2.25, The Scientific Press, San Francisco, CA.

Doornik J A and Hendry D.F. (2001) Econometric Modelling Using PcGive vol. I-III. Timberlake Consultants, London.

GAMS User Manual, GAMS Development Corporation, 1217 Potomac Street, Washington D.C.

Greene W.H. (1998) LIMDEP Version 7: User Manual, Econometric Software Inc.

Koopman SJ, AC Harvey, JA Doornik and N. Shephard (2000) Structural Time Series Analyser, Modeller and Predictor (STAMP), Timberlake Consultants Ltd.

Pesaran and Pesaran (1987) Microfit 4.0, Oxford University Press.

Shazam (1997) User's Reference Manual, Version 8.0. http://shazam.econ.ubc.ca/

Programming Activities in GAMS/MPSGE and PcGive and Give Win

I. Schedule for the General Equilibrium workshop

Session	Topic
Session 1	Introduction to the Dynamic and non-linear programming
Session 2	Tax policy model (labour leisure choice, simple multi-sectoral models) Basics of General Equilibrium Models, Introduction of GAMS/MPSGE programming Language (declaration of parameters, variables, equations, solution of the model, reporting the results); A few exercises in general equilibrium analysis using input-output tables; calibration of GE models
Session 3	Multisectoral tax and trade policy models
Session 4	Dynamic single or multiple sector Ramsey models for analysing economic growth
Session 5	Redistribution and household labour supply Strategic models in GAMS

Programming Activities in GAMS/MPSGE and PcGive and Give Win

II. Schedule for the Econometric Analysis workshop

Session	Topic
Session 1	Review on basics of simple and multiple regression analysis
Session 2	Cross section, time series , static and dynamic panel data modelling using GiveWin, PcGive Time series volatility and business cycle analysis using STAMP
Session 3	Analysis of limited dependent variable models using Shazam
Session 4	Interpretation of results: elasticities and long run and short run multipliers, prediction, forecasts, judging the reliability of estimates

Assessment

This assessment consists of projects in general equilibrium analysis and in econometric modelling. The general equilibrium models are used mainly for analysing efficiency, reallocation and redistribution impacts of tax and trade policies in an economy. Tax reform may consist of switching from direct to indirect taxes or altering the weight attached for various taxes or changing the structure of public spending. Trade reform involves removing trade and non-trade barriers through bilateral and multilateral negotiations and harmonising the tariffs or co-operation among a number of economies. Dynamic general equilibrium models aim to analyse the impact of capital income taxes on investment and capital accumulation on growth and investment.

The macroeconometric models are used to study regularities based on properties time series data of an economy or the interdependent global economy. Though the structure and precise formulation of a macroeconomic model vary significantly in the literature it is agreed that these models are used for prediction and forecasting the short run macroeconomic variables of interest in day to day policy making. The results of a macroeconomic model are often used to analyse the impacts of monetary, fiscal or exchange rate policies for economic growth. These models are used to study trend, seasonal, cyclical and random factors behind an economic time series.
The assessment here aims to develop these two types of modelling skills among the participants. You should complete small scale modelling exercises 1-10 before starting these modelling projects. These modelling project require skills contained on those exercises.

Write up Motivation/discussion/ illustration/
comparison/criticism/synthesis/conclusion.Drafts and redrafts and styles. (e.g. Macloskey)
Due date: July 31, 2004.

Project on the General Equilibrium Model of the UK Economy

Consider the structure of consumption, production, trade, tax and spending as included in the input output table for the UK economy. The most recent input-output table can be obtained from the economy section of the http://ww.statistics.gov.uk .

1. Reduce this table to ten production sectors agriculture, energy, manufacturing, construction, distribution, transport, business services and other sector (see one for 1995 in the appendix).
2. Households receive income from factor inputs and transfers from the government Disaggregate households to ten different categories as in the family expenditure survey or income distribution categories in economic trends.
3. Insure the micro-consistency of the data set in this table
    a. Zero profit conditions among production sectors
    b. Income, consumption and saving
    c. Government revenue and taxes
    d. Exports imports and balance of payment condition
    e. Backward and forward linkages of change in final demand in the engineering sector by 10 percent
4. Disaggregate government revenue as obtained from the input-output table, i.e. from the value added tax on final demand, labour and capital income taxes and tariffs and revenue balances with government spending.
5. Specify a static general equilibrium model with:
    a. Cobb-Douglas (or CES) production function for each production sectors
    b. Cobb-Douglas (or, CES) preference for the households
    c. Armington specification for international trade
    d. Market clearing conditions in goods and factors markets
6. Define a competitive equilibrium for the static economy. Calibrate the key parameters required in the model from the benchmark dataset.
7. Write a GAMS/MPSGE program to compute the general equilibrium in the static model. Use the model to evaluate the impacts of replacing taxes on consumption by taxes in labour income.

8. Assume that household follows a Ramsey model in determining consumption and saving over an infinite horizon and producers maximise profit in each period considering a planning horizon of next 50 years.
    a. Specify a dynamic general equilibrium for this economy.
    b. Calibrate the key benchmark parameters to compute the general equilibrium in this economy.
    c. Write a GAMS/MPSGE programme to compute the dynamic general equilibrium for this economy.
    d. What are the impacts of replacing capital income taxes by labour income on investment and capital accumulation in the various sectors of the economy in the presence of equal-yield constraint.
    e. What are the impacts of taxes when people anticipate the tax changes five years before they are implemented.
    f. How does the change in the steady state interest rate (by the central bank) affect the growth and capital accumulation over time?
    g. Illustrate the role of intertemporal elasticity of substitution between labour and leisure in the economy.
    h. How do different tax rates affect the distribution of the household income over time?

9. Study possibility of constructing a regional general equilibrium model for the UK economy at NUTS level North, Yorkshire and Humberside,East Midlands,East Anglia,Greater London, Rest of the South East, South West,West Midland, North West,Wales,Scotland and ,Northern Ireland as specific regions making up the national economy. NUTS stands for the European Nomenclature for Units of Territorial Statistics. Construct a micro-consistent data set. Formulate and solve the model.

Econometrics Project

Econometric Model for a National Economy

The main objective of this project is to construct a small macro-econometric model and to implement it using time series data of the UK economy. This exercise emphasises on policies relating to public finance, monetary aggregates and international competitiveness and trade flows within Mundell-Fleming type open economy model. Policy scenarios should be analysed using ex-post forecasts form 1990 to 2003 and ex-ante forecasts for 2003-2015 period. Bear in mind the fact that an ex-ante forecasts generated by the model are more creditworthy the model tracks more closely the economy in ex-post forecasts.

1. Specify structural equations of a simple macro econometric model keeping in mind the data set that you have constructed. Keep the model structure very simple, not more that six equations. Retrieving structural coefficients becomes more difficult as the order of the determinant of the coefficients becomes bigger. You may consider some of the following relations while formulating your model.

a) Consumption as a function of current and past values of incomes, lagged consumption, and prices.
b) Investment function that relates the level of investment to the interest rate, changes in the GDP (accelerator principle), depreciation rates, inflow and outflow of foreign capital.
c) Import function showing relation between the domestic output, nominal exchange rate, foreign prices and domestic prices.
d) Export as a function of foreign GDP, prices of exportable goods and the exchange rate.
e) Exchange rate that follows purchasing power parity condition between domestic and foreign price levels.
f) Tax function showing the amount of revenue collection as a function of past income, a given income tax rate and part level of government revenue.
g) Exogenous Government revenue.
h) Interest rate as a function of money demand and money supply and foreign prices.
i) Price level settings according to labour and non-labour costs.
j) Wage settings that is based on core inflation and productivity and cyclical component.
k) Unemployment and aggregate supply that relate with price levels.
l) Money demand as a function of income, interest rate, and exchange rates.
m) Exogenous money supply rule.

Identities:

n) GDP in terms of GDP components.
o) Investment-savings gap plus the Budget gap = equals trade gap = capital inflow or outflows.

2. Write down the reduced form of the model that you have specified above. Consider whether you will be able to retrieve the structural coefficients of the above model if you have estimates on the reduced form coefficients. Check the rank and order conditions of identification.

3. Using the data macrotimeseries.xls (you may need more data) estimate the parameters of the reduced form equations. You may consider using an appropriate method (ILS, 2SLS, SURE or GLS) to estimate the model equations as you had done for simultaneous equation or a VAR model in the exercises. Use GiveWin PcGive Shazam or LINDEP 7 to generate estimates of parameters, and plot actual and simulated series.

4. Retrieve structural coefficients using your estimates for the reduced form coefficients. Write down key equations listed in question (1) above in terms of numerical values of the parameters. It may be quite involved task. Reducing the number of equations to basic minimum is very helpful.

5. Do ex-post simulation for any six major macro macroeconomic series (output, net exports, government budget balance, exchange rates, interest rate). Compare the simulated model with the actual path of these variables. Take initial values for the simulated series from the actual data set to generate model variables in the ex post simulation. Comment on how well your model tracks the events of the UK economy for last ten years.

6. Now is time to use the model for the policy analysis. You may take the model constructed so far to study effects of policy on the growth path of the economy. You may have to turn to various economic theories in order explain model results in a more sensible manner.

Fiscal Policy

(a) Fiscal policy: consider a growth in the revenue by a 2, 4, and 6 percent annually over 2004-2015 period. How does it affect growth path of forecasted series that you have generated.
(b) Consider a cut in government expenditure by a 2, 3 and 4 percent annually over the period to generate simulated series. How do these reduction affect the economy?

Monetary Policy:

(a) In order to study the effects of inflation targeting policy of the Bank of England, provide a low, medium and high inflation scenarios of model simulation.
(b) Check the effect of money supply rule according to the growth rate of GNP.

Exchange rate and Trade Policy Simulation:

(a) The competitiveness of the UK economy depends upon the real exchange rates between the sterling pounds and against foreign currencies. Provide scenarios for 10, 20 and 30 percent appreciation of pounds in the early three years of the ex-ante simulation period.

(b) Provide model scenarios if the pound depreciates by 10, 20 and 30 percent annually in the 4^{th}, 5^{th} and 6^{th} years of the model simulation.

Econometric Model for the Global Economy

Global economy is interdependent. Growth rates of output in one country affects growth rates of output in other countries. Policy makers like to implement economic policies to achieve a higher rate of growth at home country. This requires a good analysis of country specific and the international factors of economic growth. Data on growth rate of output, ratios of savings and investment to the GDP, real interest rate, growth rates of population and per capita output and ratio of trade to GDP is included in the Growth5.xls. You are free to add any other variable if required by economic reasoning and analysis.

a. Specify a simple (one equation) econometric model to explain growth rate of output of each individual country. Determine the expected signs of the parameters based on economic theory. Estimate this model using GiveWin or Shazam and interpret your results.

b. Specify a interdependent global multi-country model in which growth in one country not only depends on internal factors but also external factors. Estimate this multi-country growth model to explain growth rates of each of five major industrial economies, UK, Germany, France, Japan and USA separately.

c. Specify a model to examine determinants of aggregate growth rate among these countries. Estimate this model pooling country specific data for all these five countries over time. Apply two stage least square (2SLS) or instrumental variable (IV) technique to estimate the relation in the pooled data.

d. Now consider simultaneous equation model in which growth rate of output in one country depends on growth rate of output in other four countries. Estimate such model using SURE or simultaneous equation technique. Explain briefly results for each country.

e. Write a brief report about the growth experience of these five countries based on theoretical arguments as well as empirical estimates that you obtained following above steps.

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Key reference articles for cross section analysis:

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Bhattarai KR (2004) Unemployment-inflation Trade-offs in OECD Countries: Lessons from Panel data and Theories of Unemployment , University of Hull, Manuscript.
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II. Resources

Get the PFE editor to write programs. OX-metrics, Givewin, PcGive, STAMP, GAMS, Shazam, Microfit and Limdep are available in the University Network. Student version of Consol OX is freely downloadable form www.oxmetrics.net. Similarly a student version of GAMS software is freely downloadable from GAMS site www.gams.com or ftp://allnew4u@ftp.gams.com. Global data: www.imf.org, worldbank CD or www.worldbank.org andd http://www.gtap.agecon.purdue.edu/.

Following three manuals are available at the short loan section in the library

- GiveWin, PcGive, Ox ; Shazam User's Guide
- GAMS user guide and solver manuals the GAMS Corporation
- Economic equilibrium modelling with GAMS and Introduction of GAMS/MCP and GAMS/MPSGE

Brief Instructions on softwares

Get a good editor such as pfe32.exe or epsilon to write programs.

1) **GAMS** is good in solving linear, non-linear or discrete programming models that involve constrained optimisation. Choices of consumers and producers are explained well by solving a model for a decentralised (micro-founded) markets using constrained optimisation technique. This software can solve very large scale models using detailed structure of consumption, production and trade for sigle, multiple or global economy. It is very easy to formulate and solve a model in GAMS. The basic steps writing a GAMS programme involve (study ISLM.gms for example). MPSGE program is good for large scale standard general equilibrium models. Basic steps in GAMS/MPSGE involve

a. declaration of model, set, parameters, variables, equations and model

b. solving the model using linear or non-linear or discrete programming techniques. Various solvers are available in GAMS

c. Solving the model and analysing the solution.

To run GAMS in the Hull university network make a directory called models in G:drive

*G:\> md Models

* G:\> cd Models

*copy a GAMS in G:\models\islm.gms

Type following to execute the programme in the network

* N:\special\ec\gams\gams islm.gms

Results and scenarios of the programme are given in the list file ISLM.LST

*check this list file and analyse the result.

*Are the results consistent with the macroeconomic theory as analysed using a standard ISLM-ASAD.

*diagram for expansionary fiscal and monetary policies. If not you need to adjust the programme

* and execute again. Do the same for the Solow.gms. An example of the ISLM model in GAMS.

2) **OX-GiveWin-PcGive-STAMP** based econometric model on time series and cross section data

This software is available in all labs (Start/applications/economics/givewin). They can be used for estimation, forecasting and graphics either using menu driven commands or by writing a batch file.

a. save the data in a standard excel file.

b. start give win at start/applications/economics/givewin.

c. open the data file using file/open datafile command.

d. choose PcGive package among the modules for econometric analysis.

e. select the package such as econometric modelling.

d. choose dependent and independent variables as asked by the menu.

e. do the estimation and analyse the results.

3) **Excel** Spreadsheets are very user friendly and could be used to calculate many of small and simple models required for economic modelling. First it is important to solve a model analytically by hand. Then Excel formula/optimiser like goal seek or excel solver, e.g. Keynesian1.xls.

Join the network

Email to econometric-research-request@jiscmail.ac.uk if you like to join the econometrics network.

Email to GAMS-L@listserv.gmd.de if you like to join the GAMS network

Information on Shazam is available from http://shazam.econ.ubc.ca/

Good information about conferences is available at http://www.inomics.com/cgi/conference.

Let me know if you come across other good web pages.

Exercise 1

Two Sector General Equilibrium Tax Model

Consider an economy with two production sectors in which a representative household supplies labour and capital inputs to firms and receives wages and interest rates in return. Government imposes taxes on labour and capital income and transfers it back to the household.

Household's problem

$$\underset{X_1,X_2}{Max} \quad U_1 = X_1^{\alpha} X_2^{\beta} \qquad 0<\alpha<1,\ 0<\beta<1,\ \alpha+\beta=1$$

subject to

$$I = P_1(1+t_1)X_1 + P_2(1+t_2)X_2$$
$$I = (1-t_k)r(K_1+K_2) + (1-t_w)w(L_1+L_2)$$

Production takes places in two levels of nests. At the bottom producers use capital, labour input in producing value added . At the top level of the nest gross output is given by value added and intermediate inputs.

$$\underset{L_1,K_1}{Max} \quad \Pi_1 = P_1Y_1 - wL_1 - rK_1 \text{ Subject to: } Y_1 = K_1^{\gamma_1} L_1^{(1-\gamma_1)} \quad GY_1 = F(Y_1, X_{i,1}),\ 0<\gamma_1<1$$

$$\underset{L_2,K_2}{Max} \quad \Pi_2 = P_2Y_2 - wL_2 - rK_2 \text{ subj. to: } Y_2 = K_2^{\gamma_2} L_2^{(1-\gamma_2)} \quad GY_2 = F(Y_2, X_{i,2}),\ 0<\gamma_2<1$$

Revenue of the government equals its spending

$$R = t_k r(K_1+K_2) + t_w w(L_1+L_2) + t_1P_1X_1 + t_2P_2X_2 = G$$

$$X_1 \ge 0\ X_2 \ge 0\ Y_1 \ge 0\ Y_2 \ge 0\ I \ge 0\ P_1 \ge 0\ P_2 \ge 0\ w>0\ r>0\ \Pi_1 \ge 0\ \Pi_2 \ge 0$$

A simple input-output table for this economy summarises all these transactions.

	Primary	***Secondary***	***C***	***I***	***G***	***X***	***M***	***Total***
Primary	x_{11}	x_{12}	C_1	I_1	G_1	X_1	M_1	GY_1
Secondary	x_{21}	x_{22}	C_2	I_2	G_2	X_2	M_2	GY_2
Labour	L_1	L_2						***wL***
Lab-tax	$T_{l,k}$	$T_{l,k}$						***TL***
Capital	K_1	K_2						***rK***
Cap taxes	$T_{1,k}$	$T_{2,k}$						***TK***
Total	GY_1	GY_2	***C***	***I***	***G***	***X***	***M***	

The base year data set for this economy us given in the following table.

Benchmark Data Set for a Two Sector Economy

	Sector 1	Sector 2	C	I	G	GY
Sector 1	15	15	40	10	20	100
Sector 2	10	20	30	10	30	100
Labour	30	24				54
Labour income tax	20	16				36
Capital	15	20				35
Capital income tax	10	5				15
Total	100	100	70	20	50	

a. Define precisely the competitive equilibrium for this economy.

b. Provide an analytical solution of this model.
c. Check the micro-consistency of the data set on both production and consumption sides of the economy.

d. Write a MPSGE program to replicate the benchmark equilibrium by calibrating the model to this benchmark data set. (Optional verify MPSGE solution either by writing and solving the model either in Excel or in GAMS Algebra).

e. Test the sensitivity of the model to five different values (1, 2, 3, 4 and 5) of the elasticity of substitution production and consumption.

f. Evaluate the welfare gains from eliminating the labour income taxes without equal yield constraint.

g. Evaluate the welfare impacts of eliminating labour income taxes with equal yield constraint.

h. Take the ten sector static model of the UK economy (as given by ptax.gms and uk10h1.gms files). Report the change in output, employment, capital stock and consumers' welfare when labour income taxes are replaced by value added taxes.

(Reference: Rutherford (1998) An Introduction to GAMS/MCP and GAMS/MPSGE; GAMS Corporation: http://www.gams.com; Shoven and Whalley (1992), Bhattarai and Whalley (2000))

2. Consider Brock-Mirman dynamic programming problem

$$Max \quad U = \sum_{t}^{\infty} \beta^t \ln C_t \qquad 0 < \beta < 1$$

subject to

$$K_{t+1} + C_t = AK_t^{\alpha} \qquad 0 < \alpha < 1$$

a. What are control and state variables in this model and why?

b. Explain the meaning of the value function (Bellman equation) and policy functions of this problem.

$$V_1(K) = \ln C + \beta V_0(K)$$

c. Assume $V_0(K)$=0. Demonstrate a recursive method of solving this problem by solving for policy and value functions for four iterations.

d. Use limit theorem to find the explicit solution of the value function.

e. Introduce a stochastic disturbance term for the state variable and show how it can be solved.

(Reference: Sargent TJ (1987) Dynamic Macroeconomic Theory, Chapter 1; Ljungqvist L and T.J. Sargent (2000), *Recursive Macroeconomic theory*, MIT Press).

3. Consider an optimal growth model with 50 years to time horizon as given by the following set of equations

Capacity constraint: $Y_t = K_t^{\beta} L_t^{(1-\beta)} = C_t + I_t$ growth rate of labour: $L_t = (1+g)^{t-1} L_0$

Capital accumulation: $K_{t+1} = (1-\delta)K_t + I_t$ initial capital: $K_0 = K_0$

Terminal condition $I_T = (g+\delta)K_T$

First order conditions for investment: $P_t = PK_{t+1}$ and $P_T = PTC_T$

First order condition for investment: $P_t C_t = C_0 \alpha^t = C_0 \left(\frac{1+g}{1+r}\right)^t$

First order conditions of capital market $P_t^k = RK_t + (1-\delta)P_{t+1}^k$ and $PTC_T^k = RK_t + (1-\delta)P_{t+1}^k$

The marginal revenue product of capital: $RK_t = P_t \beta K_t^{\beta} L_t^{(1-\beta)} / K_t$

Find the reference growth path for this economy when the labour force growth rate is 2 percent, the real interest is 5 percent, capital output ratio in the base is 3, initial capital stock is 1 and the rate of depreciation is7 percent. The capital value share in the benchmark is r*k0. How would a capital income tax of 20 percent affect the growth path of this economy?

Exercise 2
Growth in Centralised and Decentralised Economies

1. Consider a growing economy in which a representative consumer makes consumption and saving choices to maximise the lifetime utility. With the instantaneous utility function, $U_t = \ln C_t$, this consumers life time problem can be given by

$$Max \quad U = \sum_t^T \alpha^t \ln C_t \quad \text{where } \alpha^t = \left(\frac{1+g}{1+r}\right)^t$$

subject to:

Technology constraint: $Y_t = K_t^\beta L_t^{(1-\beta)}$ or $Y_t = \left(\beta \cdot K_t^\rho + (1-\beta)L_t^\rho\right)^{\frac{1}{\rho}}$

Capital accumulation: $K_{t+1} = (1-\delta)K_t + I_t$

Terminal investment: $I_T = (g+\delta)K_T$

Market clearing condition: $Y_t = C_t + I_t$

Labour force: $L_t = (1+g)^{t-1} L_0$

Parameters and benchmark values of variables are given as:

$L_0 = 100, K_0 = 3, Y_0 = 1,\ C_0 = 1-(g+\delta)K_0$, r =0.05, g = 0.02, $\delta = 0.07$, $\rho = 0.5$

a. calculate the optimal values of consumption (C_t), capital stock (K_t), output (Y_t), instantaneous utility (U_t) and life time utility (U) of consumer considering a 50 years time horizon.

b. Do ten step sensitivity analysis with respect to
   i. growth rate, g (0.0, 0.01, 0.02,., , …………., 0.1)
   ii. benchmark interest rate, r (0.0, 0.01, 0.02,., , …………., 0.1)
   iii. rate of depreciation of the capital stock δ (0.0, 0.01, 0.02,., , …………., 0.1)

2. Consider a growth model for a decentralised economy similar to 1.

$$Max \quad U = \sum_t^T (1+\beta)^{-t} \ln C_t$$

subject to:

Technology: $Y_t = \Phi K_t^{(1-\alpha)} L_t^\alpha$

Capital accumulation: $K_{t+1} = (1-\delta)K_t + I_t$

Terminal investment: $I_T = (g+\delta)K_T$

Euler equation: $C_t(1+R_{t+1}) = C_{t+1}(1+\beta)$

Capital market equilibrium: $R_{t+1} + \delta = (1-\alpha)\Phi K_t^{-\alpha} L_t^\alpha$

Market clearing condition: $Y_t = C_t + I_t$

Labour force: $L_t = (1+g)^{t-1} L_0$

Parameters Φ =1, α =0.75, β =0.02, g = 0.02 and δ = 0.05.

a. Calculate the market interest rate and optimal values of consumption (C_t), capital stock (K_t), output (Y_t), instantaneous utility (U_t) and life time utility (U) of consumer considering a 50 years time horizon.

b. Analyse the impact of a 20 percent capital income tax in the growth path of output and capital stock this model.

Exercise 3

Econometric Forecasting and Business Cycle Analysis

1.Consider a model consisting of **m** number of endogenous ***y*** variables and **k** number of exogenous ***x*** variables as following:

$$a_{11}y_{1i} + a_{12}y_{2i} + .. + a_{1m}y_{mi} + b_{11}x_{1i} + b_{12}x_{2i} + .. + b_{1k}x_{ki} = e_{1i}$$
$$a_{21}y_{1i} + a_{22}y_{2i} + .. + a_{2m}y_{mi} + b_{21}x_{1i} + b_{22}x_{2i} + .. + b_{2k}x_{ki} = e_{2i}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$a_{m1}y_{1i} + a_{m2}y_{2i} + .. + a_{mm}y_{mi} + b_{m1}x_{1i} + b_{m2}x_{2i} + .. + b_{mk}x_{ki} = e_{mi} \qquad (1)$$

a. Find the values of **m** endogenous variables in terms of k exogenous variables and the error term.
b. Determine the structure of impulse response functions in this model.
c. Take GDP, consumption, investment, interest rate, inflation, tax, imports as endogenous variables and exports, government spending and money supply as exogenous variables. Write a batch file in PC-Give to estimate the above model.
d. Determine the order-cointegration among the variables included in this model.
e. Use data in macrotimeseries.xls to estimate the above model. Report the estimated model along with plots of actual and predicted series.
f. Provide an impulse response analysis for a unit shock in consumption and investment.
g. Provide an eight period ahead forecast of revenue, growth rates and trade balance for this economy. Comment on the reliability of those forecasts (provide confidence bands).
h. Estimate of VAR of order 3 for GDP, consumption, investment, interest rate, inflation, tax, and imports in line with Sim's atheoretical approach.

(Reference: Doornik and Hendry (2001) Pc-Give and GiveWin, Timberlake).

2. Consider a univariate time series model as given by (refer STAMP manual p.140 ; Manuals and instructions are in (http://www.oxmetrics.net))

$$y_t = \mu_t + \gamma_t + \psi_t + \nu_t + \varepsilon_t \text{ with } \varepsilon_t \sim NID\left(0, \sigma_\varepsilon^2\right)$$

where the trend component

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t \quad \varepsilon_t \sim NID\left(0, \sigma_\eta^2\right)$$
$$\beta_t = \beta_{t-1} + \zeta_t \quad \zeta_t \sim NID\left(0, \sigma_\zeta^2\right)$$

the seasonal component is given either in a dummy or trigonometric form:

$$\gamma_t = -\gamma_{t-1} - .. - \gamma_{t-s+1} + \omega_t \quad \omega_t \sim NID\left(0, \sigma_w^2\right)$$

Trigonometric form:

$$\begin{bmatrix} \gamma_{j,t} \\ \gamma_{j,t}^* \end{bmatrix} = \begin{bmatrix} \cos\lambda_j & \sin\lambda_j \\ -\sin\lambda_j & \cos\lambda_j \end{bmatrix} \begin{bmatrix} \lambda_{j,t-1} \\ \lambda_{j,t-1}^* \end{bmatrix} + \begin{bmatrix} \omega_{j,t} \\ \omega_{j,t}^* \end{bmatrix} \quad \begin{matrix} j = 1, \ldots, [s/2] \\ t = 1, \ldots., T \end{matrix}$$

The cycles are specified as

$$\begin{bmatrix} \psi_t \\ \psi_t^* \end{bmatrix} = \rho_\psi \begin{bmatrix} \cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c \end{bmatrix} \begin{bmatrix} \psi_{j,t-1} \\ \psi_{t-1}^* \end{bmatrix} + \begin{bmatrix} \kappa_t \\ \kappa_t^* \end{bmatrix} \quad t = 1, \ldots., T \quad 0 < \rho_\psi < 1$$

The irregular component follows an AR(1) process as

$$\nu_t = \rho_\nu \nu_{t-1} + \xi_t \quad \xi_t \sim NID\left(0, \sigma_\zeta^2\right)$$

(a) Estimate ARMA(2,2) mode for unemployment and inflation. Plot predicted and fitted values (b) Estimate plot trend, cycle, seasonal and irregular components of unemployment and inflation in univariate and bivariate time series models using STAMP. (c) Provide a 10 period ahead forecast for unemployment and inflation using above models. (d) Detrend the data with Hodrik-Prescott filter and conduct stochastic volatility tests.

3. Take micro level data set in wages and earnings among the cross-sections of the UK households by age, education, regions, occupational categories and many other personal characteristics as contained in wage1.xls (you may get more up to date household level data from the http://www.archive.ac.uk. Analyse the determinants of wage rates in terms of qualification, age, training, labour market areas and other relevant variables. Remember to correct for specification by using Heckman procedure. PcGive, Shazam or Limdep are good for such analysis.

Exercise 4

Optimal consumption, saving and investment in a two period two person economy

1. Consider a pure exchange economy consisting of two periods and two individuals. Preferences of individual i is given by

$$U\left(C_1^i, C_2^i\right) = \ln C_1^i + \beta \ln C_2^i \qquad \text{i = A and B}$$

where C_1^i and C_2^i are the consumption for period 1 and period 2 of a type i agent and β is a subjective discount factor with its value between 0 and 1.

Endowment of agents is given by ω_t^i. This means endowments for agent A and B for period 1 and period 2 are $\left\{\omega_1^A, \omega_2^A, \omega_1^B, \omega_2^B\right\}$. Each agent is allowed to borrow and lend at the interest rate *r*. Market activities allow these agents to exchange goods to maximise their utility subject to their life time budget constraint.

a) Write budget constraints for the first and the second period for each individual and form a single intertemporal budget constraint for each individual. Represent it using a diagram along with their utility functions.

b) Derive the demand function for consumption in the first and second periods in terms of present value of life time wealth and the subjective discount factor using constrained optimisation technique for maximisation of life time utility subject to the intertemporal budget constraint for each individual.

c) Find the gross interest rate that is consistent with a competitive equilibrium for this economy if endowments were $\left\{\omega_1^A, \omega_2^A, \omega_1^B, \omega_2^B\right\} = \left\{50, 100, 150, 200\right\}$; $\beta = 0.9$. What are the consumption and saving for each individual ? Check whether these quantities are consistent with a general equilibrium in the product market?

What would have been the equilibrium interest rate if the endowments were $\left\{\omega_1^A, \omega_2^A, \omega_1^B, \omega_2^B\right\} = \left\{100, 50, 200, 150\right\}$ instead? Explain.

Exercise 5

Impact of Taxes on Consumption and Saving and Income Distribution

An economy is inhibited by low, middle and high income people. Each live for three periods. Their economic problem involves maximising utility given their life time budget constraint as:

$$Max \quad U\left(C_1^i, C_2^i, C_3^i\right) = \ln C_1^i + \beta_2^i \ln C_2^i + \beta_3^i \ln C_3^i \qquad i = A,B,C$$

subject to:

$$C_1^i + b_1^i = \omega_1^i \qquad \text{while young}$$

$$C_2^i + b_2^i = b_1^i(1+r) + \omega_2^i \qquad \text{while adult}$$

$$C_3^i = b_2^i(1+r) + \omega_3^i \qquad \text{while old}$$

where C_1^i, C_2^i and C_3^i are consumptions for periods 1, 2 and 3 for type i agent and β_2^i and β_3^i are subjective discount factors for period 2 and 3 consumptions with their values between 0 and 1. Endowment of agent *i* is given by ω_t^i. This means endowments for agent A and B for periods 1, 2 and 3 are $\left\{\omega_1^A, \omega_2^A, \omega_3^A, \omega_1^B, \omega_2^B, \omega_3^B, \omega_1^C, \omega_2^C, \omega_3^C\right\}$. Each agent is allowed to borrow and lend at the interest rate *r*.

(a) Find the rate of interest that guarantees equilibrium in the financial and goods markets in this economy.

(b) Find equilibrium allocations between consumption and saving for periods 1,2 and 3 if $\left\{\beta_2^A, \beta_3^A, \beta_2^B, \beta_3^B, \beta_2^C, \beta_3^C,\right\}$ = (0.95, 0.9; 0.90,0.85;0.85,0.8) and $\left\{\omega_1^A, \omega_2^A, \omega_3^A, \omega_1^B, \omega_2^B, \omega_3^B, \omega_1^C, \omega_2^C, \omega_3^C\right\}$=(100, 2000,300;300, 2000,1000;7000,20000,10000).

(c) Consider the income distribution by categories of households as given in the following table.

Income Distribution for Young, Adult and Old

Households	Young	Adult	Old	Beta2	Beta3
h1	7091	17728	8864	0.95	0.90
h2	7780	19451	9726	0.90	0.85
h3	7958	19894	9947	0.85	0.80
h4	8910	22274	11137	0.80	0.75
h5	10743	26857	13429	0.75	0.70
h6	13312	33281	16641	0.70	0.65
h7	17803	44508	22254	0.65	0.60
h8	22494	56234	28117	0.60	0.55
h9	25417	63543	31772	0.55	0.50
H10	42585	106462	53231	0.50	0.45

Income for adults are taken from Economic Trends (ONS, 2002). Young and old people are assumed to have 40 percent and 50 percent of adults' income respectively for each categories of households (need further investigation for the actual data).

What is the interest rate that is consistent with intertemporal optimisation by all households in their life time in this economy? What are the levels of consumptions and savings in equilibrium? What are their life time utilities?

(d) What are the impacts on consumption and savings of households in this economy if the interest income is taxed (i) at 20 percent (ii) 30 percent and (iii) 40 percent under two scenarios (1) when tax revenues are given to the low income households (2) when the tax revenues are distributed equally among these three types of households.

Exercise 6
Global Economy: Pure Exchange

Consider a global economy consisting of two economies. Each economy has a representative household and producer and their problems as following.

Household's problem for country 1

$$\underset{C_1, M_1}{Max} \quad U_1 = C_1^{\alpha_1} M_1^{\beta_1}$$

subject to

$$I_1 = P_1 C_1 + PM_1 M_1$$
$$I_1 = w_1 LB_1$$
$$C_1 \geq 0 \; M_1 \geq 0 \; I_1 \geq 0 \; P_1 \geq 0 \; PM_1 \geq 0 \; w_1 > 0$$
$$0 < \alpha_1 < 1; \; 0 < \beta_1 < 1; \; \alpha_1 + \beta_1 = 1$$

LB_1 given endowment of labour.

Firm's problem for country 1

$$\underset{L_1}{Max} \quad \Pi_1 = P_1 Y_1 - w_1 L_1$$

Subject to

$$Y_1 = L_1^{\gamma_1}$$
$$\Pi_1 \geq 0 \; P_1 \geq 0 \; Y_1 \geq 0 \; L_1 \geq 0 \; w_1 > 0 \; 0 < \gamma_1 < 1$$

Household's problem for country 2

$$\underset{C_2, M_2}{Max} \quad U_2 = C_2^{\alpha_2} M_2^{\beta_2}$$

subject to

$$I_2 = P_2 C_2 + PM_2 M_2$$
$$I_2 = w_2 LB_2$$
$$C_2 \geq 0 \; M_2 \geq 0 \; I_2 \geq 0 \; P_2 \geq 0 \; PM_2 \geq 0 \; w_2 > 0$$
$$0 < \alpha_2 < 1; \; 0 < \beta_2 < 1; \; \alpha_2 + \beta_2 = 1$$

LB_2 given endowment of labour.

Firm's problem for country 2

$$\underset{L_2}{Max} \quad \Pi_2 = P_2 Y_2 - w_2 L_2$$

Subject to

$$Y_2 = L_2^{\gamma_2}$$
$$\Pi_2 \geq 0 \; P_2 \geq 0 \; Y_2 \geq 0 \; L_2 \geq 0 \; w_2 > 0 \; 0 < \gamma_2 < 1$$

domestic market: $Y_1 = C_1 + M_1$; $Y_2 = C_2 + M_2$; $L_1 = LB_1$; $L_2 = LB_2$

Global market: $X_1 = M_1$ and $X_2 = M_2$

For a certain year set of preference, technology and labour market parameters were as following:

$\alpha_1 = 0.7$ $\beta_1 = 0.3$ $\gamma_1 = 0.8$ $LB_1 = 400$ $\alpha_2 = 0.6$ $\beta_2 = 0.4$ $\gamma_2 = 0.7$ $LB_2 = 600$

a. define the general equilibrium for this global economy.

b. find allocations C_1, M_1, C_2, M_2 and prices $P_1, PM_1, P_2, PM_2, w_1$ and w_2 **consistent with general equilibrium in both economies.**

Exercise 7

Global Economy: Impact of Tax and Tariffs

Consider a global economy consisting of two economies. Each economy has a representative household and producer and a government. Households pay taxes on consumption, imports, labour income and exports. The revenue is used in public spending. Households and firms do not take it into account.

Household's problem for country 1

$$\underset{C_1,M_1}{Max} \quad U_1 = C_1^{\alpha_1} M_1^{\beta_1}$$

subject to

$$I_1 = P_1(1+tc_1)C_1 + PM_1(1+tm_1)M_1$$

$$I_1 = w_1(1-tl_1)LB_1$$

$$C_1 \geq 0 \; M_1 \geq 0 \; I_1 \geq 0 \; P_1 \geq 0 \; PM_1 \geq 0 \; w_1 > 0$$

$$0 < \alpha_1 < 1;\; 0 < \beta_1 < 1;\; \alpha_1 + \beta_1 = 1$$

LB_1 is given endowment of labour.

Firm's problem for country 1

$$\underset{L_1}{Max} \quad \Pi_1 = P_1 Y_1 - w_1 L_1$$

Subject to

$$Y_1 = L_1^{\gamma_1}$$

$$\Pi_1 \geq 0 \; P_1 \geq 0 \; Y_1 \geq 0 \; L_1 \geq 0 \; w_1 > 0 \; 0 < \gamma_1 < 1$$

Government: $RV_1 = (tc_1 \times P_1 \times C_1) + (tm_1 \times PM_1 \times M_1) + (tl_1 \times w_1 \times L_1) = GV_1$

Trade balance: $P_1(1+tx_1)X_1 = PM_1(1+tm_1)M_1$

Household's problem for country 2

$$\underset{C_2,M_2}{Max} \quad U_2 = C_2^{\alpha_2} M_2^{\beta_2}$$

subject to

$$I_2 = P_2(1+tc_2)C_2 + PM_2(1+tm_2)M_2$$

$$I_2 = w_2(1-tl_2)LB_2$$

$$C_2 \geq 0 \; M_2 \geq 0 \; I_2 \geq 0 \; P_2 \geq 0 \; PM_2 \geq 0 \; w_2 > 0$$

$$0 < \alpha_2 < 1;\; 0 < \beta_2 < 1;\; \alpha_2 + \beta_2 = 1$$

LB_2 given endowment of labour.

Firm's problem for country 2

$$\underset{L_2}{Max} \quad \Pi_2 = P_2 Y_2 - w_2 L_2$$

Subject to

$$Y_2 = L_2^{\gamma_2}$$

$$\Pi_2 \geq 0 \; P_2 \geq 0 \; Y_2 \geq 0 \; L_2 \geq 0 \; w_2 > 0 \; 0 < \gamma_2 < 1$$

Government: $RV_2 = (tc_2 \times P_2 \times C_2) + (tm_2 \times PM_2 \times M_2) + (tl_2 \times w_2 \times L_2) = GV_2$

Trade balance: $P_2(1+tx_2)X_2 = PM_2(1+tm_2)M_2$

Preference, technology and labour market parameters were as following:

$\alpha_1 = 0.7$ $\beta_1 = 0.3$ $\gamma_1 = 0.8$ $LB_1 = 1000$ $\alpha_2 = 0.6$ $\beta_2 = 0.4$ $\gamma_2 = 0.7$ $LB_2 = 600$

$tc_1 = 0.1$ $tm_1 = 0.2$ $tl_1 = 0.2$ $tx_1 = 0.05$ $tc_2 = 0.2$ $tm_2 = 0.05$ $tl_2 = 0.3$ $tx_2 = 0.1$

a. find allocations C_1, M_1, RV_1, C_2, M_2 RV_2 and prices $P_1, PM_1, P_2, PM_2, w_1$ and w_2 **consistent with general equilibrium in both economies.**

Exercise 8
Dynamic Model for a Global Economy and Dynamic Programming

1. Consider a two country global economy model

$$Max \quad U_i = \sum_{t}^{T} (1+\beta_i)^{-t} \ln C_{i,t}$$

subject to:

Technology: $Y_{i,t} = \Phi_i K_{i,t}^{(1-\alpha_i)} L_{i,t}^{\alpha_i}$

Capital accumulation: $K_{i,t+1} = (1-\delta_i) K_{i,t} + I_{i,t}$

Terminal investment: $I_{i,T} = (g_i + \delta_i) K_{i,T}$

Market clearing condition: $Y_{i,t} = C_{i,t} + I_{i,t}$

α_1 =0.4 α_2=0.6 β_1=0.9 β_2=0.95 δ_1=0.05 δ_2=0.03 g_1=0.05 g_2 =0.03 L_1 =100 L_2 =500. Time horizon 30 years.

a. What is the time path of consumption $(C_{i,t})$, capital stock $(K_{i,t})$, output $(Y_{i,t})$, instantaneous utility $(U_{i,t})$ and life time utility (U_i) if policy makers agree on maximisation of the global welfare?
b. How would they be affected if only one country maximises its utility ignoring the other.
c. Solve this model decentralising the capital market.
d. First country provides investment tax credit up to 20 percent. How would it affect the growth path in another economy.

Exercise 9
Tariff Game in the two country global economy model

Consider a global economy consisting of two countries. Country 1 has two households and country 2 has five households. Each economy has fixed endowment of resources which it used to supply goods, type and type two services.

Supply $Y_n = \left(\eta_n^1 G_n^{\frac{\sigma_n - 1}{\sigma_n}} + \eta_n^2 S_n^{1\,\frac{\sigma_n - 1}{\sigma_n}} + \eta_n^3 S_n^{2\,\frac{\sigma_n - 1}{\sigma_n}} \right)^{\frac{\sigma_n}{\sigma_n - 1}}$

Resource constraint: $W_n = P^g G_n + P_n^{s_1} S_n^1 + P_n^{s_2} S_n^2$

Demand: Cobb-Douglas or CES

Income and budget constraint: $I_n^h = \theta_n^h W_n \;\; I_n^h = P^g G_n^h + P_n^{S_1} S_n^{1,h} + P_n^{S_2} S_n^{2,h}$

Tariff structure: $t_2^1 = t_2^1\left(t_1^1, t_1^2, t_2^2\right)$, $t_1^1 = t_1^1\left(t_2^1, t_1^2, t_2^2\right)$, $t_1^2 = t_2^1\left(t_2^1, t_1^1, t_2^2\right)$, $t_2^2 = t_2^1\left(t_2^1, t_1^1, t_1^2\right)$

a) Find the consumption, supply of goods and services and public revenue in equilibrium in autarky.
b) Analyse equilibrium in the presence of tariffs.
c) Consider a tariff game between these two economies. What are the optimal tariff rates if both economies cooperate? What are the tariff rates if these economies play a non-cooperative game?
d) Apply this model in case of the transport, telecommunication or financial services networks in the global economy.

(reference: Bhattarai and Whalley (1998) The Division and Size of Gains from Liberalization in Service Networks, NBER working paper no.6712).

Exercise 10

Energy and Economy: A Three Sector Model for Policy Evaluation

Consider an economy with manufacturing, services and energy production sectors with a representative household as following:

Household's problem

$$\underset{X_1, X_2}{Max} \quad U_1 = X_1^{\alpha} X_2^{\beta}$$

subject to

$$I = P_1 X_1 + P_2 X_2$$

$$I = rK_1 + rK_2 + rK_e + wL_1 + wL_2 + wL_e$$

Producers use capital, labour and energy input in production

$$\underset{L_1, K_1}{Max} \quad \Pi_1 = P_1 Y_1 - wL_1 - rK_1 - PE \cdot E_{y1}$$

Subject to:

$$Y_1 = K_1^{\gamma_1} L_1^{(1-\gamma_2)} E_{y1}$$

$$\underset{L_2, K_2}{Max} \quad \Pi_2 = P_2 Y_2 - wL_2 - rK_2 - PE \cdot E_{y2}$$

subject to:

$$Y_2 = K_2^{\gamma_2} L_2^{(1-\gamma_2)} E_{y2}$$

$$\underset{L_2, K_2}{Max} \quad \Pi_3 = PE \cdot E - wL_e - rK_e - P_1 XE_1 - P_2 XE_2$$

subject to

$$E = XE_1 \cdot XE_2 \cdot L_e K_e$$

$$X_1 \geq 0 \;\; X_2 \geq 0 \;\; Y_1 \geq 0 \;\; Y_2 \geq 0 \;\; E \geq 0 \;\; I \geq 0 \;\; P_1 \geq 0 \;\; P_2 \geq 0 \;\; PE \geq 0 \;\; w > 0 \;\; r > 0$$

$$\Pi_1 \geq 0 \;\; \Pi_2 \geq 0 \;\; \Pi_3 \geq 0 \;\; XE_1 \geq 0,\; XE_2 \geq 0 \;\; E_{y1} \geq 0,\; E_{y2} \geq 0 \;\; K_e \geq 0,\; L_e \geq 0$$

$$0 < \alpha < 1;\; 0 < \beta < 1;\; 0 < \gamma_1 < 1; 0 < \gamma_2 < 1; \alpha + \beta = 1$$

$$\alpha = 0.4;\; \beta = 0.6;\quad \gamma_1 = 0.3;\quad \gamma_2 = 0.4;\ \text{Kb} = 1000;\ \text{Lb} = 1000$$

(a) Solve this model the prices and allocations that guarantee equilibrium in this economy.
(b) Test the sensitivity of the model solutions for the parametric specification of the model
(c) Solve the model when household also consumes the energy directly.
(d) Modify the model introducing taxes on goods X_1 X_2 and E as well as on labour and capital income *r* and *w* . Solve for two scenarios (1) when the revenue is recycled to the households as transfer (2) when all revenue goes to meet government spending
(e) Find the mix of optimal tax fixing the revenue at ten percent of GDP for this economy.
(f) Use recursive dynamics to solve the model assuming that labour endowment grow by 3 percent and the capital endowment grows by 5 percent.
(g) Rewrite the model in the form of Ramsey model with and without taxes. Solve for the steady state assuming growth rate of 2 percent, real interest rate of 3 percent and the depreciation rate of 2 percent for the benchmark the economy.

Table A2

A 16 Sector Industry by Industry Input-Output Table of the United Kingdom 1995

I x I Domestic Use Matrix	Agriculture	Extraction	Other Mining	Chemicals	Metals	Engineering	Food, drink	Other Manuf.	Utilities	Construction	Distribution	Transport	Financial	Public Admin	Educ. Health,	Housing	Total intermediate	Consumers' expenditure	GGFC	GDFCF	Stocks	Exports	Total final demand	Total
Agriculture	2,096	0	14	27	7	5	12,132	435	0	4	564	48	15	0	148	0	**15,495**	6,730	42	0	0	1,942	8,713	**24,208**
Extraction	0	2,439	0	4,697	3	0	0	0	3,622	0	0	0	0	0	0	0	**10,762**	0	0	0	0	6,942	6,942	**17,704**
Other Mining	20	0	353	218	846	26	45	130	1,897	401	105	17	8	0	57	0	**4,124**	339	47	0	0	983	1,369	**5,493**
Chemicals	1,433	10	37	3,899	433	546	571	1,484	466	737	1,299	1,254	913	0	3,204	19	**16,304**	3,764	3,116	0	261	28,663	35,804	**52,108**
Metals	110	162	192	1,225	7,249	6,320	1,831	5,197	50	7,074	503	389	5	0	84	0	**30,392**	346	588	7,158	779	10,230	19,101	**49,493**
Engineering	0	576	317	682	1,254	5,705	528	2,432	634	788	848	1,808	1,018	0	1,567	36	**18,192**	0	1,589	2,613	332	50,923	55,457	**73,649**
Food, drink	2,797	52	25	356	82	120	6,382	350	64	51	6,589	650	1,058	0	1,796	4	**20,377**	25,904	411	0	153	10,270	36,737	**57,114**
Other Manuf.	583	80	134	1,781	1,839	3,005	2,816	16,404	474	4,242	6,702	4,139	8,242	0	3,340	283	**54,064**	18,082	3,872	8,933	1,185	39,858	71,928	**125,992**
Utilities	279	0	160	1,330	1,596	1,189	931	1,980	12,273	272	1,201	857	1,184	0	705	23	**23,981**	16,353	1,323	0	0	62	17,738	**41,719**
Construction	172	0	122	109	32	56	0	31	0	21,085	603	151	1,985	0	146	3,929	**28,420**	3,521	4,414	47,764	285	0	55,983	**84,404**
Distribution	1,005	200	206	1,479	2,489	4,115	1,647	3,724	355	1,371	4,164	2,470	2,276	0	790	0	**26,289**	111,181	1,229	2,586	0	13,701	128,698	**154,987**
Transport	245	704	335	1,232	2,047	1,415	1,583	3,614	183	887	14,871	15,642	17,082	0	3,175	198	**63,216**	19,715	2,637	779	0	12,194	35,324	**98,540**
Financial	1,949	671	471	4,070	2,781	6,194	4,205	9,177	1,884	10,483	22,425	12,387	50,836	0	13,435	15,221	**156,189**	25,373	8,458	8,483	0	12,545	54,859	**211,047**
Public Admin	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**0**	0	63,843	0	0	0	63,843	**63,843**
Educ. Health,	378	1	41	520	253	581	496	2,618	179	242	1,001	1,369	4,031	0	7,756	67	**19,535**	43,653	46,265	0	0	4,504	94,422	**113,957**
Housing	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**0**	53,269	0	0	0	0	53,269	**53,269**
Total intermediate	**11,067**	**4,895**	**2,410**	**21,626**	**20,912**	**29,276**	**33,168**	**47,576**	**22,081**	**47,638**	**60,876**	**41,182**	**88,652**	**0**	**36,201**	**19,781**	**487,339**	**328,229**	**137,832**	**78,316**	**2,995**	**192,816**	**740,188**	**1,227,527**
Imports	1,630	989	425	10,639	7,613	15,965	8,827	30,336	3,612	5,151	3,532	4,895	3,949	0	2,960	19	**100,541**	52,021	9,995	28,174	1,563	2,494	**94,248**	**194,789**
Duty on imports	34	6	5	136	101	214	171	405	48	66	51	26	2	0	9	0	**1,273**	547	91	382	20	32	**1,073**	**2,346**
VAT	0	0	0	0	0	0	0	0	0	0	0	218	3,259	0	1,181	0	**4,658**	33,257	3,915	3,731	0	0	**40,902**	**45,561**
Duties and levies	211	2	103	1,175	344	176	460	331	1,378	130	1,275	2,026	896	0	344	36	**8,887**	22,713	434	0	0	0	**23,147**	**32,034**
Other taxes and subsidies	-265	-25	-10	-50	-53	-46	-1,454	-212	-10	-34	-443	-404	-409	0	-186	-6	**-3,607**	4,559	-577	-45	4	-556	**3,384**	**-223**
Value added – Labour	7,143	1,409	1,822	10,151	15,790	18,529	9,691	36,483	5,492	29,947	61,877	35,191	70,149	60,316	69,067	0	**433,059**	0	0	0	0	0	**0**	**433,059**
Value added – Gross profits etc	4,388	10,428	738	8,432	4,786	9,536	6,250	11,074	9,118	1,505	27,820	15,406	44,549	3,527	4,381	33,440	**195,376**	0	0	0	0	0	**0**	**195,376**
Total inputs	**24,208**	**17,704**	**5,493**	**52,108**	**49,493**	**73,649**	**57,114**	**125,992**	**41,719**	**84,404**	**154,987**	**98,540**	**211,047**	**63,843**	**113,957**	**53,269**	**1,227,526**	**441,325**	**151,691**	**110,558**	**4,582**	**194,786**	**902,942**	**2,130,468**

Source: ONS, Input-Output Tables of the United Kingdom, 1995; Siddorn (1999).