The Business School

26214

Economic Modelling
(2004)

Level: 5
Semester: 1
Credits: 20

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Economic Modelling

Additional Notes

Economic Modelling **(26214)**

This module builds on the foundations laid in Microeconomics and Macroeconomics and provides training for those students who wish to have a career in economics related areas.

Macroeconomics part of the model contains theories and applications required for analysis of economic growth and fluctuations in output, employment, income and prices in an economy. It focuses on the role of accumulation of human and physical capital, research and technological advancement in the growth rates of output under the neoclassical and endogenous theories of economic growth as well as on the Keynesian, New Keynesian, monetarists and new classical approaches to stabilisation of macroeconomic fluctuations in output, employment, price level, interest rate and the exchange rates. It includes simple models of goods, labour, financial and foreign exchange that macroeconomists use to assess the effectiveness of prudent policy measures introduced to achieve a target rate of inflation with low natural rates of unemployment and higher rates of economic growth.

Microeconomics part relates to quantitative analysis of optimal behaviour of consumers and producers who are constrained by their budgets or the technology of production. These microeconomic models will be used to provide micro-foundation to macroeconomics taking a general equilibrium approach that has become increasingly popular in macroeconomics in recent years. Demands for products and factors will be derived from the utility maximising behaviour of a representative household and profit maximising behaviour of representative firms. Inter-temporal and strategic optimisation models will be used to evaluate optimal decisions of households and firms who take account of current and future prospects in goods, financial and factor markets. These models will be used to evaluate price system as an outcome of the macroeconomic impacts of fiscal, monetary, exchange rate, labour and financial market policies at the macro level. It will study macroeconomic impacts of consumption, income taxes or transfer and subsidy programs at micro level. Strategic models used to evaluate the economic policy game between private, public and the rest of the world sectors will be covered at the final part of the module.

Objectives of the module:

1) Provide a solid foundation and confidence among students in building simple economic models that can be used to evaluate impacts of economic policies relating to economic growth, macroeconomic stability and microeconomic efficiency in a real world situation.

2) Transmit the numerical and quantitative skills required to build and apply economic model taking account of the structure of an economy under investigation.

3) Develop skills in strategic thinking using policy games between the public and private sector.

4) Create awareness among students on how results of a certain model are sensitive to behavioural assumptions, structure and analytical form an economic model.

Expected Learning outcomes: Students are expected to be able to solve

(i) the popular versions of the open economy IS-LM and AS AD models often used for analysis of fiscal and monetary policy issues.

(ii) neoclassical and endogenous growth models used for analysing the importance of capital accumulation, technology as well as human capital in enhancing economic growth.

(iii) constrained optimisation for consumption-saving and investment decision taking account of expectations in the financial markets.

(iv) rules for fiscal, monetary, exchange rate and economic growth policies

(v) simple models with strategic moves to analyse policy games between the public and the private sector.

Module requirement:

- Activities: 20 lectures; 10 problem solving tutorials, homework and mock tests.
- Weights: one hour in-class **mid-term exam 25%; one essay 25%; final exam 50%.**
- The mid-term exam is on February 16, 2004 and essay is due on March 30, 2004.
- Students should read the Handbook regarding standard academic practice in the Business School.

Main Text:

Blanchard Oliver (2000) Macroeconomics, 2nd edition, Prentice Hall.

Other Good Texts:

David Miles and Andrew Scott (2002) Macroeconomics: Understanding the Wealth of Nations, John Wiley and Sons, Inc. ISBN 0-47084288-1.
Gartner Manfred, Macroeconomics, Printice Hall.
N. Gregory Mankiw (2002) Macroeconomics 5th edition ,New York Worth.
Burda and Wyplosz (2002) Macroeconomics: A European Text, 3rd edition Oxford Univ. Press.
Varian Hall R, Intermediate Microeconomics: A Modern Approach, W.W. Norton.
Romp Graham (1997) Game Theory, Oxford University Press.

Assessment methods:

1). **Midterm exam** will cover all the lectures and tutorials up to the day of the mid-term test.
2) Write **an essay** in any one of the given topics in about 1500 words.

3) **Final exam** is by a two hours written test. It will be comprehensive and will cover all the materials taught in the module. **Reassessment** will be by two hours written exam.

Economic Modelling

Lecture Schedule Day Time Room

Lecture	Week	Lecture topics	Chapters in Blanchard
1	1	Basics on Economic Modelling	1,2, A1
2	1	Modelling economic growth: facts	10
3	1	Neoclassical (Solow) Growth Model: Steady State	11
4	2	Steady State and golden rule of saving	12
5	2	Optimal consumption and saving (micro foundation)	12, 16
6	2	Optimal investment (micro-foundation)	12, 16
7	3	Human capital, technology & economic growth	13
8	3	Convergence	13
9	3	Trade, capital flow, growth, wage & unemployment	13
10	4	Review of IS-LM and AS-AD model	3-7, 27.1
11	4	Phillips curve and natural rate of unemployment	8, 22
12	4	Stabilisation programme: Monetarist's approach	9, 23
13	5	Fiscal policy: tax and spending	5.3, 7.4
14	5	Deficit financing and national debt	26
15	5	Ricardian equivalence	26
16	6	Transmission mechanism of monetary policy	5.4, 25
17	6	Interest rate rule and central bank independence	7.4, 25
18	6	Adaptive and rational expectation	14, 27.3
19	7	Financial market: stocks and bonds	15
20	7	Financial market and economy	16, 17
21	7 Feb 23	**Midterm exam**	
22	8	Small open economy model	18, 19
23	8	PPP and UIP theory of Exchange rate	20
24	8	Balance of payment crisis	18.2 21
25	9	Model for macroeconomic forecasting	24.2
26	9	Application of game theory in macroeconomics	24.3
27	9	Fiscal-Monetary Policy Co-ordination	22
28	10	Credibility and time inconsistency	24.2
29	10	Policy game in the global economy	21.4
30	10	Strategic trade policy	19.5
31	11	Micro-foundation to macro: Households and firms	Notes
32	11	Micro-foundation to macro: VAT and income taxes	Notes
33	11	New Classical: political business cycle theory	24.3
34	12	New Keynesian: nominal rigidity and menu cost	27
35	12	Review 1	
36	12	Review 2	

Students need follow instructions in the business school handbook about academic honesty.

Tutorial schedule and topics Schedule Day Time Room

Calss #	Weeks	Class topics
1	2	Neoclassical Solow Model
2	3	Human Capital, Technology and Growth
3	4	Stabilisation programme
4	5	Fiscal policy
5	6	Monetary policy
6	7	Financial Market
7	8	Open economy
8	9	Policy game and time inconsistency
9	10	Micro foundation to macro
10	11	Macroeconomic forecasting

Topics for the Essay

Write **an essay** in any one of the following topics in about 1500 words.
For each topic you need to provide a model and its application to the real world situation, be critical, systematic, concise, organised and original in presentation of your arguments. Essay is due on

a. Can poor countries grow faster and catch up to rich countries by adopting an export oriented growth strategy by 2100? Analyse this issue using both exogenous and endogenous theories of economic growth.

b. Analyse the role of fiscal, monetary and exchange rate policy rules for economic growth, internal and external stability. Assess how the breakdown of stability and growth pact would impact the course of European economies in the near future. Use an open economy IS-LM/AS-AD and/or policy co-ordination model to analyse this issue.

c. Evaluate the impacts of top up fees in accumulation of human capital and economic growth in the UK using theories of the financial markets, the life cycle model of consumption using a micro-foundation based model of the macro economy.

d. Examine the importance of flexibility in the labour market for macroeconomic stability, higher rate of economic growth and redistribution of income. How do your results vary when workers have rational expectation or when they have adaptive expectation?

e. Assess critically how unexpected moves in tax and trade policies can harm confidence of consumers and investors and cause fluctuations and harm the growth prospects of an economy.

f. What is the connection between the interest rate and the house prices? Should the central bank increase the interest rate to control the house prices? Analyse this questions explaining factors behind the supply and demand of houses and transmission mechanism of monetary policy.

Problem 1
Neoclassical Solow Growth Model

1 Consider a version of the Solow Model

Production function with capital and labour as its inputs: $Y_t = A_t K_t^{\alpha} L_t^{\beta}$

Market clearing: $Y_t = C_t + I_t$

Saving: $S_t = sY_t$

Investment requirement: $I_t = (n+\delta)K_t$

Saving investment identity: $S_t = I_t$

Capital accumulation: $K_t = (1-\delta)K_{t-1} + I_t$

where Y = output A = technology, L = Labour input, K = Capital input ; α share of capital in output ; β share of labour in output, s is the saving rate, I is the investment requirement, n is the population growth rate, (δ) is the depreciation rate. Assume that $\alpha + \beta = 1$ and drop the time subscript for simplicity.

(a) Write the production function in intensive form. Represent the above model using appropriate diagrams.

(b) Find the steady state value of per capita capital and per capita output in terms of the saving rate (s), the depreciation rate (δ) and population growth rate (n).

(c) What will be value of per capita capital and per-capita output if the savings rate was 32%, the depreciation rate was 8%, the population was growing at 2% per annum and $\alpha = 0.3$?

(d) Now assume that the computer hackers and terrorists destroy computing system and other infrastructure. The effective capital stock is virtually reduced to half of its previous stock. How does this affect the steady state obtained in (c) above?

(e) Global warming causes floods that contaminates all food stocks and brings wide spread diseases. Many people die. As a result the labour force reduces by one quarter. How does this affect the income and capital stock in the steady state?

(f) Show how the growth rate in output can be decomposed into the growth rate of technology, and the growth rates of labour and capital inputs. Also show how the growth rate only depends on technical progress but not in the saving rate on the balanced growth path for this economy.

(g) What is the golden rule of savings and consumption in the Solow model? How does it differ from the steady state? What prevents a higher saving rate from leading to a higher growth rate in the model?

2. Show possibility of an unbounded growth with higher rate of savings when an economy has "AK" production technology: Y = AK. How may such technology develop?

Application: study a spreadsheet on how to calculate the steady state.

Problem 2
Human Capital, Technology and Economic Growth

1. Modify the production function used in the previous tutorial by including human capital as following: $Y = AK^{\alpha}L^{\beta}H^{\gamma}$ where Y = output, A = technology, L = Labour input, K = physical capital, H = human capital and $0 < \alpha < 1$, $0 < \beta < 1$ and $0 < \gamma < 1$; assume that $\alpha + \beta + \gamma = 1$. Show how the growth rate of output can be decomposed into the growth rate of technology and the growth rates of labour, capital and human capital inputs in this case. Also show that a higher saving rate can generate a higher growth rate of output in the balanced growth path if human capital enters into the production process as described in this production function.

2. Stock of human and physical capital and labour in country 1 is twice of that in country 2. Assume a generic production function of the following form: $Y_i = \sqrt{A_i H_i K_i L_i}$ where subscript i represent country 1 or 2 and H_i is stock of human capital, K_i is stock of physical capital and L_i is labour in country i;. For simplicity assume that level of technology A_i is the same in both countries.

a. Does this production function represent a constant, increasing or decreasing returns to scale relative human, physical and labour inputs? Prove your claim.
b. State how much is production in country 1 relative that in country 2.
c. If H_i K_i L_i grow by 3%, 1% and 2% respectively what would be the growth rate of output in country 1? How much would it be different than in country 2?

2. One stylised fact according to Robert Lucas is that both skilled and unskilled labour tends migrate from a low-income countries to high-income countries. Suppose country-1 represents high-income economies and country-2 represents low income economies. Country-1 has more capital stock than that in country-2. Again take a generic production function $Y_i = A_i K_{ii}^{\alpha} L_i^{\beta}$. What would happen to stock of capital, labour, interest rate, real wage rate and output across these two countries if (a) capital is mobile but labour is not (b) labour is mobile but capital is not in each of the following three parametric specifications. Demonstrate results using diagrams and equations.

I. $\alpha + \beta = 1$ and $0 < \alpha < 1$ and $0 < \beta < 1$
II. $\alpha > 1$ and $0 < \beta < 1$
III. $0 < \alpha < 1$ and $\beta > 1$

Application: study the spreadsheet about the role of human capital in economic growth.

Problem 3
Consumption and Investment

Consider a two period economy in which preferences of households, endowments and government policy is given by the following set of equations.

Preference: $U(C_1, C_2) = \ln C_1 + \beta \ln C_2$

Endowments: $\{w_1, w_2\}$ Government policy: $\{G_1, G_2, \tau_1, \tau_2, B\}$

where C_1 and C_2 are consumption in period 1 and period 2; G_1 and G_2 are government expenditure in period 1 and period 2; w_1 and w_2 are endowments in period 1 and period 2; τ_1 and τ_2 are tax rates in period 1 and period 2 and B is the borrowing by government in period 1.

a. Provide a graphical explanation of inter-temporal optimisation process for a representative household in this economy. Indicate choices between current and future consumption for this representative consumer who faces inter-temporal budget constraint using correctly labelled diagrams.
b. What is the role of parameter β in the utility function and how would the point of inter-temporal choice that you showed in (a) change if rate of interest rises?
c. Construct inter-temporal budget constraints for a representative household and the government in real terms for this economy. Explain in simple word what do two sides of this budget constraint represent for the household and for the government?
d. Given government policy $\{G_1, G_2, \tau_1, \tau_2, B\}$ and endowments $\{w_1, w_2\}$ and preference $U(C_1, C_2) = \ln C_1 + \beta \ln C_2$, what are the conditions for optimal choices for the representative household? Solve for optimal choices available to this representative consumer in period 1 and period 2 in terms of endowment, interest rate and model parameters.
e. Find the equilibrium rate of interest and consumption choices C_1 and C_2 using the market clearing condition if $\{w_1, w_2\} = \{100,150\}$ and $\{\tau_1, \tau_2\} = \{0.2, 0.2\}$ and $\{G_1, G_2\} = \{20,30\}$, β is 0.95.and there is no borrowing by the government, B =0.
f. Using the same equilibrium interest rate find the value of equilibrium choices if $\{w_1, w_2\} = \{100,150\}$ and $\{\tau_1, \tau_2\} = \{0, \tau_2\}$ and the government finances all its expenditure $\{G_1, G_2\} = \{20,30\}$ by borrowing in period 1 to be paid by taxes, τ_2, in period 2. What is value of government revenue in terms of tomorrow's income and what is the tax rate τ_2?
g. Prove that consumption choices of households are not affected whether the government pursues a (i) tax only policy as in (e) or (ii) borrowing only policy as in (f) above. What is the popular name given to this result in macroeconomics?
h. How would your result in (d) change if the representative household faced a borrowing constraint such that she will not be able to borrow more than 10 in period 1?

2. A car manufacturer sells each car at 8000 and pays 2000 for capital equipment per car. The nominal interest rate is 6%, appreciation of value of capital stock (capital gain) is 3% and the depreciation of capital stock is 3% per year. The production function for this company is given by $Y = K^{\alpha}$ with $\alpha = 0.75$. What is the optimal capital stock for this manufacturer? (hint $P.F'(K_1) = P_1^k(r + \delta)$). Show how the investment tax credit affect the optimal capital stock employed by a firm using an appropriate diagram.

Application: study the spreadsheet about the intertemporal model of consumption and investment.

Problem 4
Stabilisation Programme

An economy is experiencing a relatively high inflation, which is reported to be 11 percent per year. It is creating uncertainty and is signalling towards a serious macroeconomic instability if it remains unchecked. Government is keen to bring a programme to assure macroeconomic stability by reducing inflation at 2% annual rate until it comes down to 3% annual rate. Once inflation reaches 3% annual rate the government would like to stabilise it at 3% rate for all following years. The current unemployment rate is at 6 percent per year. Econometric estimates suggest that this is also the natural rate of unemployment, u_n for this economy. Productivity is growing by 2% annually and labour force is growing at 1% per year. The basic economic relations between unemployment rate, inflation and growth rates of output and money supply that describe this economy are given by the following set of equations.

$$u_t - u_{t-1} = -0.4\left(g_{yt} - \overline{g}_{yt}\right) \qquad \text{Okun's law}$$

$$\pi_t - \pi_{t-1} = -\left(u_t - u_n\right) \qquad \text{Phillips curve}$$

$$g_{yt} = g_{mt} - \pi_t \qquad \text{aggregate demand}$$

where g_{yt} is an actual growth rate of output, $\overline{g}_{yt}$ is a trend growth rate of output, u_t is unemployment rate in period t, π_t is inflation in period t, g_{mt} is growth rate of money supply in period *t*.

a. Provide a brief graphical illustration on how inflation, unemployment rate and growth rates of output and money supply are related to each other in this economy in the short run and in the medium run.

b. From the information given above find the inflation rate before one year, π_{t-1}. Also find the growth rates of output and money supply given this inflation for the current year (the benchmark year of the inflation reduction programme). How far was the inflation last year away from the target inflation of 3%?

c. Calculate u_t, g_{yt}, π_t and g_{mt} for all following years until the economy reaches 3% inflation rate if government is strict in its policy of reducing inflation by 2% annually. What are the values of u_t, g_{yt}, π_t and g_{mt} when the economy becomes stabilised after a successful implementation of the inflation reduction programme. What is the growth rate of money supply consistent with the target inflation and the trend growth rate of output in the long run?

d. What is the meaning of sacrifice ratio in macroeconomics? What is its value for different years of the inflation reduction programme in (c)?

e. Justify the required path of money growth in the inflation reduction programme that you have outlined in (c).

Application: study the spreadsheet about the role stabilisation programme.

Problem 5
Fiscal policy

Consider a simple Keynesian model as following:

National income identity: $Y = C + I + G + X - M$
Consumption function: $C = 100 + 0.8(Y - T)$
Where Y is national income, C consumption, I investment, G government spending, X exports, M imports and T lump sum tax. Here Y and C are endogenous variables and I, G , X and M are exogenous variables. Assume that I =100, T= 100; G= 100. Also consider that the trade is balanced, exports equal imports, X =M.
(a) What is the equilibrium national income in this model? What is the size of government budget multiplier or investment multiplier?
(b) What is the multiplier effect of a 10 percent increase in government spending, holding the amount of tax as constant at 100?
(c) What would happen to equilibrium income and consumption in (3) above if both tax and government spending increase at the same rate? What is the meaning of balance budget multiplier in this economy?
(d) Now suppose that the tax rates are proportional to income as $T = tY$, where the t is the tax rate, $0 < t < 1$. How would the multiplier in this case?
(e) Why is this called a automatic stabiliser for an economy?
(f) G remains fixed. Represent how the budget deficit or surplus arise when the output is below or above its natural rate in a diagram.

(c) Now consider a simple dynamic version of the above model as:

$$Y_t = C_t + I_t + G_t + X_t - M_t$$
$$C_t = 100 + 0.8(Y_{t-1} - T_{t-1})$$

Assuming the initial income to be 500 $Y_0 = 500$ $I_0 = 0$ and the tax revenue and government spending to be the same for every period $T_t = G_t = 100$ and trade is balanced for in each period $X_t = M_t$ show how income and consumption change over time?
What would happen in this model if investment changes occur according the acceleration principle as: $I_t = 100 + 0.3(Y_{t-1} - Y_{t-2})$ or $I_t = 100 + 0.8(Y_{t-1} - Y_{t-2})$?

2. You are asked to construct a Keynesian model for an economy to analyse the impacts of fiscal and monetary policy measures on output and interest rates. This economy is in recession and has plenty of unused resources, i.e. the aggregate supply curve is infinitely elastic (perfectly horizontal). [100]
Specification of the model

Consumption:	$C = a_0 + a_1(Y - T)$	$a_0 > 0 \quad 0 < a_1 < 1$
Tax collection:	$T = tY$	$0 < t < 1$
Investment:	$I = b_0 - b_1 r$	$b_o > 0 \;\; b_1 > 0$
Imports:	$IM = m_0 + m_1 Y$	$m_0 > 0 \quad 0 < m_1 < 1$
Goods market equilibrium:	$Y = C + I + G + X - IM$	
Money demand:	$\frac{M}{P} = kY - nr$	$0 < k < 1 \;\; n > 0$

Consumption (C) , taxes (T), investment (I), interest rate (r), imports (IM) and national income (Y) are endogenous variables. Both government spending (G) and money supply (M) as well as the external demand for goods, exports (X), are assumed to be exogenous. Among parameters a_0 is autonomous consumption, a_1 the marginal propensity to consume, t the tax rate, b_o the autonomous investment, b_1 is the response of investment to change in the interest rate, m_0 autonomous imports, m_1 the marginal propensity to import, k the coefficient for demand for money for transaction purposes and n the response of money demand to the interest rate. Prices are assumed to remain constant at 1. This means there is no difference between real and nominal interest rate. [**note: Each of the following sections is worth 20 marks]**.

a. What is the meaning of goods market equilibrium in this model? Determine the combination of the output and the interest rate that guarantees such equilibrium (the IS curve) using above equations. Represent it in a diagram. Why is an IS curve downward sloping?
b. Find the level of output and the rate of interest that satisfies equilibrium conditions in both goods and money markets. Represent your solutions in a diagram in the (Y, i) space. Explain why either the output or the interest rate has to adjust towards this equilibrium point if it is not there in the beginning.
c. What at are the impacts of a fiscal monetary policy mix that raises government spending (G) and reduces money supply (M), in output and interest rate? Illustrate these new equilibrium points in another diagram in the (Y, i) space.
d. Modify the imports and export functions in the above model as

$$IM = m_0 + m_1 Y + m_2 ER \qquad m_0 > 0;\ 0 < m_1 < 1;\ m_2 > 0$$

$$X = \eta_1 ER + \eta_2 Y^* \qquad \eta_1 > 0\ \ \eta_2 > 0$$

where ER is the exchange rate and Y^* is the foreign income; both ER and Y* are exogenous. Show how the equilibrium income derived in (b) above changes when you take account of influence of the foreign income and the exchange rate in this way.
e. You have following information on parameters and policy variables of an economy for model outline in (d) above: $a_o = 200$, $a_1 = 0.8$, t =0.2, $b_o = 30$, $b_1 = 50$, k =0.4 and n =1000, P=1 $m_o = 25$, $m_1 = 0.2$, $m_2 = 20$, $\eta_1 = 20$, $\eta_2 = 0.1$, G =100, X = 50 and M=100, ER =2 and Y^* =500. What are the values of Y, r, T, C, I, IM that are consistent with equilibrium in both goods and money markets? Does this economy have capital inflow or outflow? Do your solutions satisfy the equilibrium conditions of the model?

3. Derive conditions for sustainable debt.

Application: study the spreadsheet about the IS-LM model with closed and open economy.

Problem 5
Monetary policy

A central bank operates its monetary policy through open market operations. It changes the discount rate (base interest rate) at which commercial banks and other financial institutions can borrow from the central bank if their deposits are not enough to satisfy demand for loans.

(a) Discuss the transmission mechanism, explaining how a change in the interest rate affects the income, output, employment, price level and exchange rates in an economy. What do monetarists, such as Milton Friedman, regard as the strengths and weaknesses of a monetary policy like this?

(b) Consider a simple model for the interest rate policy rule which includes the following three equations

Output gap: $$y_t - y^* = d\left(i_{t-1} - i_t^*\right) \quad (1)$$

Supply function: $$\pi_t = \pi_t^* + c\left(y_{t-1} - y_{t-1}^*\right) \quad (2)$$

Interest rate rule: $$i_t = i_t^* + a\left(y_t - y_t^*\right) + b\left(\pi_t - \pi_t^*\right) \quad (3)$$

where $\left(y_t - y^*\right)$ is the current output gap, $\left(i_{t-1} - i^*\right)$ the deviation of the interest rate one period earlier from the target interest rate of the monetary authority, y_t and y_t^* are the actual and natural level of output, i_t is the actual rate of interest in period *t*, i_t^* is the interest target of the monetary authority. π_t and π_t^* are actual and target inflation rates. How can such a rule be implemented by the monetary authority? How does such a rule help to stabilise the economy?

(c) Why has inflation targeting and central bank independence been more popular among the policy makers in recent years, when a money supply rule or exchange rate targeting has been popular in the past?

(d) What sort of monetary policy do the International Monetary Fund and the World Bank suggest for an economy which has experienced both budgetary and trade deficits? What sort of the exchange rate regime is necessary for monetary policy to be effective?

Application: study the spreadsheet about the interest rate determination model.

2. The government and the central banks have two different policy objective (loss) functions as given by equation (1) and equation (2) below. Both of them face the same supply constraint as given in equation (3).

Governments objective function: $\underset{\pi_t}{Min} \quad L^G = \frac{1}{2}\pi_t^2 + \frac{b}{2}\left(y_t - y_t^*\right)^2$ (1)

Central banks' objective function: $\underset{\pi_t}{Min} \quad L^{CB} = \frac{1+\varepsilon}{2}\pi_t^2 + \frac{b}{2}\left(y_t - y_t^*\right)^2$ (2)

Constraints: $y_t = \left(\pi_t - \pi_t^e\right) + u_t$ (3)

where L^G is the loss function of the government and the L^{CB} is the loss function of the central bank, π_t is the inflation rate, π_t^e is the expected inflation rate, y_t is the actual output, u_t is a supply shock (such as a rise in the oil price) y_t^* is the target output, b is a positive parameter. The parameter ε is an inflation aversion factor of the central bank.

a) what would the target inflation rate if the government sets the inflation target independently minimising its loss function (1) subject to the constraint (3)?
b) What is the inflation target if the central bank can set it on its own minimising its loss function (2) subject to the constraint (3)?
c) Now assume that both central bank and the government co-operate to each other as given expressed in equation (4) .

$$M_t = \gamma L^{CB} + (1-\gamma)L^G \qquad (4)$$

where γ is 1 if the central bank is fully independent and 0 if it is fully under the spell of the government and between 0 and 1 in the intermediate case.
What will be the inflation target if the monetary policy decision is taken with due account of government and the central bank as in (4), when $0 < \gamma < 1$?

d) discuss the importance of the central bank independence for economic stability with particular reference to recent policy developments in the Bank of England and the ECB.

Problem 6
Financial Market and Economy

What is the relation between the real and nominal interest rate according to the Fisher equation? If the gross nominal return next year on P amount invested today is $(1+i)P_t$ and expected price next period is $P_{t+1}^e = (1+\pi^e)P_t$ then show that the gross real interest is given by $1+r = \frac{1+i}{1+\pi^e}$ or $r \approx i - \pi^e$. Illustrate how this approximation is valid for lower rates of inflation and interest rates $(i, \pi^e) \approx (0.04, 0.02)$ (as in the UK and Western countries in recent years) but not for trouble prone countries with $(i, \pi^e) \approx (0.84, 0.56)$ (as in Turkey or Zimbabwe in recent years)?

Value of bonds

What is the value of a console that pays 100 forever at the interest rates of (a) 5 percent and (b) 1 percent?

What is the yield to maturity of a £100 bond maturing in two periods if this bond is selling at 90 $(P_{2t}^B = 90)$ in the market ?

Show how the long run interest rate is approximated by the short term interest rates in the financial markets?

You borrow 20,000 from a bank today promising to pay back the principle in 12 instalments and the interest rate accrued for that month. The bank lends at APR of 8.9 percent. What is the amount of instalment for the seventh month?

Value of stocks

Use the arbitrage condition between the bonds and stock market to derive the value of stock from the discounted values of expected stream of dividend payments and resale value of stock in the financial market. What will be the value of stock of a company that promises to pay 1000 every year if the growth rate of dividend is 3 percent, market interest rate is 5 percent and the risk premium on this stock equal 2 percent?

Role of financial sector in the economy

How is the efficiency of financial markets affected with a higher transaction costs of funds charged by the financial intermediaries in the market? How does it affect the steady state of an economy in the Solow model is θ amount is wasted away in the intermediation process?

What are the conditions on the inter-temporal budget of households, firms, government and the economy as a whole for a sound financial system?

Application: study the spreadsheet about the discount rate and impact of financial market in economic growth.

4. A person is 21 years old now and starts working from 22 with initial income of 20,000 and retires at 65 years and dies when he/she is 90.
    a. If his/her income grows continuously at 5 percent per year what would be her the life time income?
    b. What would be the level of consumption implied by the life cycle model for this consumer?
    c. Study the spread sheet and construct the debt-profile of this person over the life period.
    d. How can this model be used to show how increasing the retirement age could solve the expected deficit in the pension system in the majority of Western economies in near future?

Application: study the spreadsheet about the life cycle model of consumption.

Problem 7
Open economy

Consider a standard Keynesian macroeconomic model for an open economy

National income identity: $Y = C(Y-T) + I(i-\pi) + G + NX(Y, Y^f, \lambda)$ (1)

Consumption: $C = 200 + 0.8(Y-T)$ (2)

Investment: $I = 50 - 20(i-\pi)$ (3)

Tax and spending: $T = 100 \quad G = 100$ (4)

Net exports: $NX = -10 + 0.3Y^f + 0.1Y + 20\lambda$ (5)

Real exchange rate: $\lambda = \dfrac{EP^*}{P}$ (6)

Money Demand: $\dfrac{M}{P} = 200 - 50i + 0.5Y$ (7)

Financial integration condition: $i = i^* = 5$ (9)

where Y is output, C consumption, I investment, G government spending, NX net exports, T tax revenue, i nominal interest rate, π rate of inflation, Y^f foreign income, E nominal exchange rate, λ real exchange rate P^* foreign price level and P domestic price.

(a) What will be the value of national income (Y), consumption (C), net exports (NX), net national saving (S-I) in this model if foreign income (Y^f) is 500, foreign price level (P^*) is 2, nominal exchange rate (E) equals 1 and the inflation rate is 2. Show whether your solutions satisfy the national income identity as given by equation (1).

(b) How will be the new values of national income (Y), consumption (C), net exports (NX), net national saving (S-I) if the foreign income (Y^f) rises to 1000 and foreign price (P^*) reduces to 1?

Country 1 and country 2 are interdependent economies. Analyse the impact of expansionary fiscal and monetary policies of country 1 in output, interest rate and foreign asset reserves of both countries using two country Mundell-Fleming model under the flexible exchange rate system when the capital is perfectly mobile and when it is immobile. Do the same question under the fixed exchange rate system.

Application: study the spreadsheet about the exchange rate.

Problem 8

Policy game and time inconsistency

1. Explain how a dynamic inconsistency may arise when policy makers prefer low unemployment rate rather than low inflation rate and public prefers low inflation rate rather than low unemployment rate using the following pay-off matrix of inflationary expectation (Miles and Scott has similar matrix on pp. 421-423).

$$\begin{array}{cc} \text{.....} & \text{Private Sector} \\ \text{Government Sector} & \begin{bmatrix} & H & L \\ H & -3,0 & 3,-3 \\ L & -5,-3 & 1,1 \end{bmatrix} \end{array}$$

a. Write this game in an extensive form.
b. Find the Nash equilibrium for this game
c. What is the value of this game if it is plaid infinite number of times?
d. What is value of game is it is plaid finite number of times?

2. Policy makers like stabilise the economy by managing aggregate demand and letting the economy grow at its natural rate as far as possible. Let their objective function take the following form

$$S(\pi) = b(y - y^*) - a\pi^2 \quad (1)$$

where y is actual output y* is the natural level of output and $(y - y^*)$ is the output gap and π is the actual inflation rate. Further suppose a simple aggregate supply function of the following form in which that output responds to higher level of inflation as below

$$y = y^* + c(\pi - E(\pi)) \quad (2)$$

Given the is inflationary expectation of people, $E(\pi)$, policy makers may choose inflation π either using a discretionary policy or by committing themselves to a policy rule (by fixing a target rate of inflation).

(a) Using both of above equations calculate the optimal inflation rate under the discretionary policy regime.
(b) If policy makers were committed to a target rate of inflation and people knew that they would stick to that target (such as 2% rate by the ECB and 2.5 rate by the MPC of Bank of England) what would be the optimal rate of inflation?
(c) Which one of the above regimes is Pareto efficient outcome?

3. If the loss function of the monetary policy committee is given by $L(u,\pi) = u + \gamma\pi^2$, where u is unemployment and π is inflation. Suppose that the Phillips curve describes the relation between the unemployment rate and inflation. With u^n to be natural rate of unemployment and π^e the expected rate of inflation the trade off between inflation and unemployment is given by $u = u^n - \alpha(\pi - \pi^e)$.

(a) What will be the unemployment rate if the monetary authority operates under a fixed rule regime?
(b) Under discretionary policy private agents first form expectations about inflation π^e, monetary authority set actual inflation afterwards by minimising its loss function. What will be the unemployment rate if people are rational about their expected inflation π^e?
(c) Based on your answers to (a) and (b) which one the policy rules would you choose?

4. Suppose you are writing a policy memo about an economy which had inflation of 3% and the unemployment rate equal to the natural rate during the last year . The Phillips curve took the following form $\pi_t = \pi_{t-1} - \alpha(u_t - u_n)$. If this government can use either monetary or fiscal policy or both to achieve the desired unemployment what inflation and unemployment rate should it try to achieve? How would your memo change if the Phillips curve is given by $\pi_t = \pi_t^e - \alpha(u_t - u_n)$ and there is evidence that people form a rational expectation from the government policy.

5. An economy is experiencing a higher rate of inflation. Government introduces a demand reduction policy by contractionary fiscal or monetary policy.

(a) Demonstrate using the Phillips and Okun curves how the reduction in the aggregate demand causes an increase in the unemployment rate (or lowers output) in the short run.

(b) Show how the economy returns to its natural rate of unemployment in the long run even if the government persistently tries to reduce it below that rate. Explain how expectations of a higher rate of inflation due to the expansionary policy affect the position of the Phillips curve in the economy.

(c) Draw the aggregate demand and aggregate supply diagram in $(u - u_n)$ or $(y - y_n)$ and $(\pi - \pi^e)$ space. Explain how the actual rate of unemployment (u) compares to the natural rate of output (y_n) and the gap between actual and expected inflation $(\pi - \pi^e)$ using this diagram. What would be the impact of an expansion in demand (i) under adaptive expectations and (ii) under rational expectations.

(d) Policy makers want to minimise the loss $L = u + a\pi^2$ subject to constraints imposed by a Phillips curve $u = u_n - b(\pi - \pi^e)$ where L is loss, u the actual unemployment rate, u_n is the natural rate of unemployment, π the actual inflation rate, π^e the rate of expected inflation, a denotes the weight of inflation in the loss function and b is the slope of the Phillips curve. Assume that a=15 and b =2. What would be the inflation rate and loss under (i) the optimal policy rule and (ii) the optimal discretion?

(e) What are the major supply side policies on goods, labour and financial markets that the government can adopt to fight a recession? Provide recent examples from European or other economies as far as practicable.

Application: study the spreadsheet dynamic policy game.

Problem 9 -10
Micro foundation to macro

1. There are two people living in an economy. For simplicity assume that a fixed amount of output of 200 is produced each year. Entire output is consumed in the same year. Utility of individual 1 and 2 is represented by $U_1 = \sqrt{Y_1}$ and $U_2 = \frac{1}{2}\sqrt{Y_2}$.

(a) What is the utility received by each individual if the output is divided equally between these two people? What is the output received by each if it is distributed so that each of them gets the same amount of the utility?
(b) What is the distribution of output that maximises the total utility for the whole economy?
(c) If person 2 needs utility 5 in order to survive how should the output be distributed?
(d) Suppose that the authorities like to maximise the social welfare function $W = U_1^{\frac{1}{2}} U_2^{\frac{1}{2}}$, how should the output distributed between them?

2. Consider an economy consisting of a representative household and a representative firm. A representative household tries to maximise utility by consuming goods and services and from enjoying leisure subject to his budget constraints. The producer wants to maximise profit by selling goods produced using the labour supplied by the household.

The household maximisation problem can be stated as the following:

$$Max \quad U = c^{\phi} l^{1-\phi}$$

Subject to:

i. $l + h^s = 1$ time constraint

ii. $pc = wh^s + \pi$ budget constraint

iii. $c \geq 0; l \geq 0; h^s \geq 0$ non-negativity constraint

where c is consumption, l is leisure and h^s is labour supply, p is the price of the commodity, w is the wage rate π .is the profit from owning the firm.
Maximisation problem for the representative firm can be states as

$$Max \quad \pi = py - wh^d$$

subject to :

i. $y \leq \left(h^d\right)^{\alpha}$ technology constraint

ii. $y \geq 0; h^d \geq 0$ non negativity constraint

where y is the output supplied by the firm and h^d is its demand for labour.
Form a Lagrangian maximisation problem for the household and the firm.

a. Derive its demand for the consumption goods and leisure and labour.
b. Define a competetive equilibrium for this economy and compute the real wage rate that brings goods and labour market in equilibrium.
c. What are the quantities of c, y, l and h^d in equilibrium? If the parameters and endowments were what are the values of c, y, l and h^d? Do they satisfy the equilibrium conditions?
d. Reformulate the model with consumption and income taxes where the government transfers all revenues to the household.

Application: study the spreadsheet about the general equilibrium model.

Consider an economy consisting of two periods and two types of agents

Preferences for each type of agent i is given by

$U\left(C_1^i, C_2^i\right) = \ln C_1^i + \beta \ln C_2^i$ i = A and B

where C_1^i and C_2^i are the consumption for period 1 and period two of a type i agent and β is a subjective discount factor with its value between 0 and 1. Endowment of agents is given by ω_t^i. This means endowments for agent A and B for period 1 and period 2 are $\left\{\omega_1^A, \omega_2^A, \omega_1^B, \omega_2^B\right\}$. Each agent is allowed to borrow and lend. Market activities allows these agents to exchange goods to maximise their utility.

a) Write the first and second period budget constraints for each of these two agents.
b) Formulate an intertemporal budget constraint for both agents A and B.
c) Derive the first order conditions of maximisation of utility subject to these budget constraints for each type of these two agents.
d) Define a competitive equilibrium for this economy.
e) Solve for equilibrium quantities and prices for economy.
f) Now assume that the endowments of individuals were $\left\{\omega_1^A, \omega_2^A, \omega_1^B, \omega_2^B\right\} = \{100,50,200,150\}$; $\beta = 0.9$ and the real interest rate is 5 percent. What are the consumption and saving of each of these individuals? Are these quantities consistent with a general equilibrium in the product market?

Application: study the spreadsheet with the general equilibrium with two goods.