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Consumption, investment and financial intermediation in a Ramsey model

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The productivity of capital and the discount rate for future consumption are more important for economic growth than the cost of financial intermediation. Using analytical and numerical illustrations from a version of the Ramsey model this study illustrates how parameters of preferences in consumption, productivity and depreciation rates of capital and costs of financial intermediation interact and determine the levels of output, consumption, investment and capital stock in a growing economy.

I. Introduction

The Ramsey (1928) model of optimal consumption and saving in a growing economy, extended by Goodwin (1961) and Cass (1965) and others, has been applied frequently to analyse consumption, saving, investment and capital accumulation over time. It focuses on the optimal rate of saving and the long run growth in contrast to a model with a constant and exogenously fixed rate of saving that preoccupies itself with the fluctuations in the short run as in Keynes (1936) and Hicks (1937) and their followers. The Ramsey model has become prominent more recently and remained a benchmark model for sophisticated analyses of consumption and saving in Caballero (1991), Deaton (1992), Pemberton (1997) and Blundell and Preston (1999), Gul and Pesendorfer (2004) and Meghir (2004). Despite this it is hard to find a study that links the productivity of capital stock and the cost of financial intermediation and impacts of these on consumption, saving, investment and capital accumulation and economic growth under a simple version of Ramsey model. Using analytical solutions and numerical examples this study shows how output, capital stocks, consumption and saving can be sensitive to the parameters of preferences, production and financial intermediation in a growing economy. Analyses contained here complement various other studies found in applied economics literature in recent years that use either a VAR model for impulse response analyses or macroeconometric model for simulations or panel or multiple regression models to test economic hypotheses as in Dawson (2003), Kim (2003), Ataullah *et al.* (2004), Liu and Shu (2004), Mallick (2004), Park and Lim (2004).

II. Infinite Horizon Ramsey Model of Consumption and Saving

In Ramsey (1928), a benevolent social planner or a representative household optimises the lifetime utility from consumption in each period and his saving equals investment in equilibrium. Investment generates additional capital stock and enhances the productive capacity of the economy. The basic Ramsey model can be expressed in five functions expressing the utility of a representative household, production of a firm, the process of capital

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accumulation, conditions for market clearing and the initial state of the economy:

Preference:

$$\text{Max}\, U = \sum_{t=0}^{\infty} \beta^t \ln(C_t) \quad 0 < \beta < 1 \tag{1}$$

Subject to:
Output and the technology constraint:

$$Y_t = AK_t^{\alpha} \quad 0 < \alpha < 1 \tag{2}$$

Market clearing condition:

$$C_t + I_t = Y_t \tag{3}$$

Capital formation:

$$K_{t+1} = K_t(1-\delta) + I_t \quad 0 < \delta < 1 \tag{4}$$

Boundary (initial) condition

$$K_0 = K_0 \tag{5}$$

The solutions for optimal consumption from this infinite horizon problem can be obtained by substituting the consumption term from the market clearing condition in the utility function and using the standard first order conditions for utility maximization and by analysing optimal conditions for any two periods in terms of control and state variables that apply to all other periods in the model as illustrated below.

$$C_t = Y_t - I_t \rightarrow C_t = AK_t^{\alpha} - K_{t+1} - K_t(1-\delta) \tag{6}$$

$$U_t = \sum_{t=0}^{\infty} \beta^t \ln\left(AK_t^{\alpha} - K_{t+1} + K_t(1-\delta)\right) \tag{7}$$

Two periods of this infinite sum actually can be sliced as:

$$U_t = +\beta^t \ln\left(AK_t^{\alpha} - K_{t+1} + K_t(1-\delta)\right) + \beta^{t+1} \ln\left(AK_{t+1}^{\alpha} - K_{t+2} + K_{t+1}(1-\delta)\right) + \cdots + \tag{8}$$

For optimal consumption and savings, the loss in utility by not consuming at period t should equal gain from production (consumption) next period, $t+1$. This can be ascertained by comparing the first order conditions or the Euler equations taking consumption and investment as control and capital stock as the state variables.

$$\frac{\partial U_t}{\partial C_t}\frac{\partial C_t}{\partial C_{t+1}}\frac{\partial C_{t+1}}{\partial K_{t+1}} = +\frac{\beta^t}{C_t} + \frac{\beta^{t+1}}{C_{t+1}}\left(\alpha AK_{t+1}^{\alpha-1} + (1-\delta)\right) + = 0 \tag{9}$$

$$\rightarrow \frac{C_{t+1}}{C_t} = \beta\left(\alpha AK_{t+1}^{\alpha-1} + (1-\delta)\right) \tag{10}$$

This equation implies that the ratio of consumption between two periods should equal discounted value of the marginal product of capital in the next period, and in a competitive equilibrium this should equal the gross interest rate.

The level of consumption and capital stock are constant in the steady state; $\cdots = K_{t-1} = K_t = K_{t+1} = \cdots = \overline{K}$ and $\cdots = C_{t-1} = C_t = C_{t+1} = \cdots = \overline{C}$. When this steady state value of capital stock is substituted in the parentheses of Equation 7 the lifetime utility becomes a constant number and the $\overline{K}$ term can be taken out of the summation sign as $U_t = \ln(A\overline{K}^{\alpha} - \overline{K} + \overline{K}(1-\delta)) \sum_{t=0}^{\infty} \beta^t$. The steady state capital stock $\overline{K}$ can be found using steady state values of consumption in Equation 10.

$$\frac{C_{t+1}}{C_t} = \frac{\overline{C}}{\overline{C}} = \beta\left(\alpha A\overline{K}^{\alpha-1} + (1-\delta)\right) \rightarrow \left(\alpha A\overline{K}^{\alpha-1} + (1-\delta)\right) = \left(\frac{1}{\beta}\right) \rightarrow \left(\overline{K}^{\alpha-1}\right) = \frac{1}{\alpha A}\left(\frac{1}{\beta} - (1-\delta)\right) \times \left(\overline{K}^{\alpha-1}\right) = \frac{1}{\alpha A}\left(\frac{1-\beta(1-\delta)}{\beta}\right) \rightarrow \overline{K} = \left(\frac{1-\beta(1-\delta)}{\alpha A\beta}\right)^{1/(\alpha-1)} \rightarrow \overline{K} = \left(\frac{\alpha A\beta}{1-\beta(1-\delta)}\right)^{1/(1-\alpha)} \tag{11}$$

Output in the steady state is obtained by substituting the steady state capital stock in the production function:

$$\overline{Y} = A\overline{K}^{\alpha} \rightarrow \overline{Y} = A^{(2-\alpha)/(1-\alpha)}\left(\frac{\alpha\beta}{1-\beta(1-\delta)}\right)^{\alpha/(1-\alpha)} \tag{12}$$

Investment and consumptions in the steady state are:

$$\bar{I} = \overline{K} - (1-\delta)\overline{K} \rightarrow \bar{I} = \delta\overline{K} \rightarrow \bar{I} = \delta\overline{K} = \delta\left(\frac{\alpha A\beta}{1-\beta(1-\delta)}\right)^{1/(1-\alpha)} \tag{13}$$

$$\overline{C} = \overline{Y} - \bar{I} \rightarrow \overline{C} = \left(\frac{\alpha A\beta}{1-\beta(1-\delta)}\right)^{\alpha/(1-\alpha)} - \delta\left(\frac{\alpha A\beta}{1-\beta(1-\delta)}\right)^{1/(1-\alpha)}. \tag{14}$$

Thus the output, capital stock and consumption in the steady state of this economy are ultimately functions of preference of the social planner or the representative consumer (β), technological factor (A), the elasticity of output to the capital stock (α) and the rate of depreciation (δ). The implications to higher or lower depreciation rate on long run growth of the economy is clear from above derivations as the steady state values for consumptions, investment and capital stock significantly differ when $0 < \delta < 1$ than when $\delta = 1$.

III. Impacts of Cost of Financial Intermediation

The above two scenarios assume an ideal benevolent social planner and a smooth and efficient financial market. The financial markets are incomplete in the real world and the investments are not equal to saving because of intermediation cost. When θ represents a charge imposed by financial institutions in the intermediation process with a higher value of θ representing more inefficiency in the financial system then $\phi = (1+\theta)$ of saving is required for one unit of investment. Higher intermediation cost, (higher values of ϕ) reduces capital, output, consumption and investment and modifies the macroeconomic balance to:

$$C_t = Y_t - I_t \rightarrow C_t = AK_t^{\alpha} - \phi\{K_{t+1} - K_t(1-\delta)\} \tag{15}$$

Now the financial system deviates from the standard Arrow-Debreau competitive equilibrium; only $1/(1+\theta)$ fraction of saving is channelled to investment; thus the investment equals savings net of intermediation cost $S_t = \phi I_t = (1+\theta)I_t$ as in Pagano (1993); $\theta \cdot I_t$ amount of savings is wasted in process of financial intermediation. As such a higher value ϕ represents more inefficiency in the financial system. Solutions of models with $0 < \delta < 1$ and $\phi \geq 1$ are further modified as:

$$C_t = Y_t - I_t \rightarrow C_t = AK_t^{\alpha} - \phi\{K_{t+1} - K_t(1-\delta)\} \tag{16}$$

$$U_t = \sum_{t=0}^{\infty} \beta^t \ln\left[AK_t^{\alpha} - \phi\{K_{t+1} - K_t(1-\delta)\}\right] \tag{17}$$

As before part of this infinite sum actually can be written as:

$$U_t = +\beta^t \ln\left[AK_t^{\alpha} - \phi\{K_{t+1} - K_t(1-\delta)\}\right] + \beta^{t+1} \ln\left[AK_{t+1}^{\alpha} - \phi\{K_{t+2} - K_{t+1}(1-\delta)\}\right] + \cdots +$$

First order conditions or the Euler equation in terms of consumption as a control and capital stock as a state variable changes to

$$\frac{\partial U_t}{\partial C_t} = +\frac{\phi\beta^t}{C_t} + \frac{\beta^{t+1}}{C_{t+1}}\left(\alpha AK_{t+1}^{\alpha-1} + \phi(1-\delta)\right) + = 0 \rightarrow \frac{C_{t+1}}{C_t} = \frac{\beta}{\phi}\left(\alpha AK_{t+1}^{\alpha-1} + \phi(1-\delta)\right) \tag{18}$$

As above this equation implies that the ratio of consumption between two periods should equal the discounted value of the marginal product of capital in the next period, in a competitive equilibrium this should equal the gross interest rate. Again for an optimal allocation between consumption and savings, loss in utility by not consuming now should equal gain from production (consumption) next period.

As above in a steady state both capital and consumption remain constant; $\cdots = K_{t-1} = K_t = K_{t+1} = \cdots = \overline{K}$ and also $\cdots = C_{t-1} = C_t = C_{t+1} = \cdots = \overline{C}$. Like in Equation 7 above the term in the parentheses of Equation 17 becomes a constant number and can be taken out of the summation sign $U_t = \ln(A\overline{K}^{\alpha} - \phi\overline{K} + \overline{K}\phi(1-\delta)) \sum_{t=0}^{\infty} \beta^t$. This again gives a constant utility in the steady state. The steady state capital stock $\overline{K}$ can be found using steady state values of consumption in Equation 18.

$$\begin{aligned}
\frac{C_{t+1}}{C_t} &= \frac{\overline{C}}{\overline{C}} = \frac{\beta}{\phi}\left(\alpha A\overline{K}^{\alpha-1} + \phi(1-\delta)\right) \\
&\rightarrow \left(\alpha A\overline{K}^{\alpha-1} + \phi(1-\delta)\right) = \left(\frac{\phi}{\beta}\right) \rightarrow \left(\overline{K}^{\alpha-1}\right) \\
&= \frac{1}{\alpha A}\left(\frac{\phi}{\beta} - \phi(1-\delta)\right) \rightarrow \left(\overline{K}^{\alpha-1}\right) \\
&= \frac{1}{\alpha A}\left(\frac{\phi - \beta\phi(1-\delta)}{\beta}\right) \rightarrow \overline{K} \\
&= \left(\frac{\phi - \beta\phi(1-\delta)}{\alpha A\beta}\right)^{1/(\alpha-1)} \rightarrow \overline{K} \\
&= \left(\frac{\alpha A\beta}{\phi - \beta\phi(1-\delta)}\right)^{1/(1-\alpha)}
\end{aligned} \tag{19}$$

Thus in case of financial intermediation the output, investment and consumption in the steady state are:

$$\overline{Y} = A\overline{K}^{\alpha} \rightarrow \overline{Y} = \left(\frac{\alpha A\beta}{\phi - \beta\phi(1-\delta)}\right)^{\alpha/(1-\alpha)} \tag{20}$$

$$\begin{aligned}
\bar{I} &= \overline{K} - (1-\delta)\overline{K} \rightarrow \bar{I} = \delta\overline{K} \rightarrow \bar{I} = \delta\overline{K} \\
&= \delta\left(\frac{\alpha A\beta}{\phi - \beta\phi(1-\delta)}\right)^{1/(1-\alpha)}
\end{aligned} \tag{21}$$

$$\overline{C} = \overline{Y} - \bar{I} \rightarrow \overline{C} = \left(\frac{\alpha A\beta}{\phi - \beta\phi(1-\delta)}\right)^{\alpha/(1-\alpha)} - \delta\left(\frac{\alpha A\beta}{\phi - \beta\phi(1-\delta)}\right)^{1/(1-\alpha)} \tag{22}$$

In addition to β, A, α and δ the steady state consumption, investment and capital stock also depend on the cost of intermediation $\phi = (1+\theta)$.

IV. Numerical Example

The numerical solutions of three versions of the infinite horizon model are summarized in Table 1. The upper part of the table lists the set of parameters used in each scenario and the corresponding model solutions for the steady state are given at the lower part of the table.

Scenarios I to IV are solutions without intermediation cost as given in Equations 11 to 14; scenarios I and II compare role of depreciation, scenarios I–IV demonstrate the role of productivity of capital. The scenarios IV–VI illustrate distortions due to intermediation costs as presented in Equations 19 to 22 and scenarios VII to IX show impact of technical progress for varying rate of depreciation and intermediation costs.

The major points emerging from analyses of results in this table are following:

1. Greater the productivity of capital larger is the level of capital stock, output, consumption and investment in the steady state (I–IV). This implies that countries with higher capital share or productivity of capital (α) have higher level of output and consumption.
2. Higher financial intermediation cost (ϕ) reduces the level of output in the steady state but it is less important than the productivity of capital (V–VI). Higher productivity of capital can compensate for higher cost of financial intermediation. Economies that can employ capital more efficiently can grow faster despite higher transaction costs of financial intermediation.
3. Higher rate of technological progress (A) leads to higher level of income in the steady state (VII–IX). Countries that are able to complement technical progress with larger amount of capital have better prospect of economic growth.
4. Countries with the higher rate of depreciation of the capital (δ) experience lower rate of growth of output (I and II).
5. When the subjective discount factor (β) was changed to 0.8 from 0.9 in scenario IX the levels of capital stock, output, consumption and investments were 500 728, 262 882, 237 846 and 25 036 respectively. Countries that discount the future consumption heavily have lower level of output in the steady state.

Analytical and numerical results from a simple Ramsey model presented above shows the role of productivity and depreciation rates of capital, level of technology, the cost of financial intermediation and discount factors for future relative to current consumption and how the real factors of financial sector determine the rate and space of economic growth in an economy. These results should be studied together with the results from the other models as mentioned in Section I considering the fact that a greater space of financial deepening brings a larger proportion of the real physical assets into financial assets making expectations of investors and consumers more important and complicated than simply productivity based arbitrage conditions used here in allocation of resources between consumption and investment. More complex issues like this are beyond the scope of this paper.

V. Conclusion

The optimal allocations of income between consumption and investment over time in a growing economy is shown with analytical solutions as well as numerical examples from a simple version of the

Table 1. Capital stock, output, consumption and investment in the steady state

	I	II	III	IV	V	VI	VII	VIII	IX
Parameters of the infinite horizon model									
Technology (A)	44	44	44	44	44	44	100	100	100
Productivity of capital (α)	0.4	0.4	0.2	0.6	0.6	0.6	0.4	0.4	0.6
Discount rate (β)	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
Initial capital (K_0)	100	100	100	100	100	100	100	100	100
Depreciation rate (δ)	1	0.05	0.05	0.05	0.05	0.05	1	0.05	0.05
Intermediation cost (ϕ)	1	1	1	1	1.2	1.05	1	1	1.05
Infinite horizon economy in the steady state									
Capital stock (K)	100	2499	149	344 202	218 202	304 677	392	9807	2 369 142
Output (Y)	278	1006	120	92 424	70 310	85 902	1090	3950	667 967
Consumption (C)	178	882	112	75 214	59 400	70 668	698	3460	549 509
Investment (I)	100	125	7	17 210	10 910	15 234	392	490	118 457

Ramsey model with production and cost of financial intermediation in an infinite horizon setting. On the consumption side, economies with strong preferences for future consumption (that have a lower discount rate and higher saving rate) grow faster and have higher level of capital stock, output and consumption in the steady state than economies that discount future consumption at a higher rate. On the production side, economies with a higher rate of productivity of capital will have a higher level of standard of living. Similarly a lower rate of depreciation and a lower cost of financial intermediation raises the level of output and capital stock in the steady state. Model results show that productivity of capital and discount rate for future consumption are found to be more important than the cost of financial intermediation.

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