GENERAL EQUILIBRIUM MODELLING OF UK TAX POLICY[1]

Keshab Bhattarai
University of Warwick

John Whalley
Universities of Warwick and Western Ontario
and NBER

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1. INTRODUCTION

This paper summarises activity thus far in the general equilibrium tax modelling area undertaken at Warwick University as part of a wider project on "General Equilibrium Analysis of UK Policy Issues" supported by the ESRC Macromodelling Consortium. Some of this work has been undertaken jointly with the Research Division of the Inland Revenue, to whom we are indebted for collaborative support on the data front.

We set out the basic approach, summarise a standard general equilibrium tax model for the UK providing some initial results, and indicate some extensions to the approach which have been undertaken in stand alone research papers.

2. GENERAL EQUILIBRIUM TAX MODELS[2]

Broad Approach

In a traditional general equilibrium model of either the tax or non tax variety, a number of consumers are identified, each with an initial endowment of commodities and preferences. Under optimization, the latter yield household demand functions for each commodity, with market demands given by the sum of the individual consumer's demands. Market demands depend on all prices, are continuous, nonnegative, homogeneous of degree zero (i.e., there is no money illusion), and satisfy Walras's law (i.e., that at any set of prices the total value of consumer expenditures equals consumer incomes). The production side of such models incorporates technology described by either constant-returns-to-scale activities or nonincreasing returns-to-scale production functions, and has producers maximise profits.

An equilibrium is characterised by a set of relative prices and levels of production by each industry such that market demand equals supply for all commodities (including

[2] The discussion in this section draws on Whalley (1985).

disposals if any commodity is a free good). Since producers are assumed to maximise profits, this implies that in the constant-returns-to-scale case no one does any better than break even at the equilibrium prices. The zero homogeneity of demand functions and linear homogeneity of profits in prices (i.e., doubling all prices doubles money profits) implies that only relative prices are of any significance in this model; the absolute price level has no impact on the equilibrium outcome.

Taxes typically appear in these models in *ad valorem* form (see Shoven and Whalley (1973); Shoven (1974)), either as producer taxes on inputs or consumer taxes on incomes or expenditures, revenues either being redistributed to consumers or used to finance publicly provided goods and services. The taxes that characterise modern tax systems (personal, corporate, sales, excise, property, social security, resource, and other taxes) are usually represented in model-equivalent *ad valorem* form. The equilibrium behaviour of the model can be investigated as taxes change and, on that basis, policy evaluations made.

A Numerical Example

A numerical example is a good way to illustrate this approach; although, in representing actual economies, much more specificity is required than the general form of the model usually admits. Particular functional forms for production and demand functions need to be chosen and parameter values selected. The tax policy instruments to be analysed need to be incorporated, and the treatment (or model closure) of the various items such as foreign trade, savings, and public goods all need to be settled.

A simple numerical example of a tax-policy-oriented general equilibrium model is presented by Shoven and Whalley (1984). In this example, there are two final goods (manufacturing and nonmanufacturing), two factors of production (capital and labour), and two classes of consumers (a “rich” consumer group that owns all the capital and a “poor” group that owns all the labour). There are no consumer demands for factors (i.e., there is no labour-leisure choice). Each consumer group generates demands by maximising a constant elasticity of substitution (CES) utility function subject to its budget constraint. CES production functions are assumed.

The CES utility functions are

$$U^c = \left[\sum_{i=1}^{2} \left(a_i^c\right)^{1/\sigma_c} \left(X_i^c\right)^{\left(\sigma_c - 1\right)/\sigma_c} \right]^{\sigma_c/(\sigma_c - 1)} \tag{2.1}$$

where X_1^c is the quantity of good *i* demanded by the *c*th consumer, a_1^c are share parameters, and σ_c is the substitution elasticity in consumer c's CES utility function. The consumer's budget constraint is $P_1X_1^c + P_2X_2^c \le P_LW_L^c + P_K^cW_K^c = I^c$, where P_1 and P_2 are the consumer prices for the two goods, W_L^c and W_K^c are consumer *c*'s endowment of labour and capital, and I^c is the income of consumer *c*.

Maximising this utility function subject to the budget constraint yields the commodity demands

$$X_i^c = \frac{a_i^c I^c}{P_i^{\sigma_c}\left(a_i^c P_1^{(1-\sigma_c)} + a_2^c P_2^{(1-\sigma_c)}\right)} \qquad i = 1,2; c = 1,2 \tag{2.2}$$

The production functions are

$$Q_i = \phi_i \left[\delta_i L_i^{(\sigma_i - 1)/\sigma_i} + \left(1 - \delta_i\right) K_i^{(\sigma_i - 1)/\sigma_i}\right]^{\sigma_i/(\sigma_i - 1)} i = 1,2 \tag{2.3}$$

where Q_i denotes output of the *i*th industry, ϕ_i is a scale or unit parameter, δ_i is a distribution parameter, K_i and L_i are capital and labour factor inputs, and σ_i is the elasticity of factor substitution in industry *i*.

This example has six production function parameters (i.e., ϕ_i, δ_i, and σ_i for i = 1, 2), six utility function parameters (i.e., $a_1^1, a_2^1, a_1^2, a_2^2, \sigma_1$ and σ_2, and four exogenous variables (the endowment of labour (W_L) and capital (W_K) for each of the two consumers).

Table 1

Production and demand parameters and endowments used by Shoven and Whalley (1984) for their two-sector general equilibrium numerical example

			ϕ_i	δ_i	σ_i
Production parameters					
Manufacturing			1.5	0.6	2.0
Nonmanufacturing			2.0	0.7	0.5
Demand parameters					
Rich consumers			Poor consumers		
α_1^1	α_2^1	σ^1	α_1^2	α_2^2	σ^2
0.5	0.5	1.5	0.3	0.7	0.75
Endowments					
			K	L	
Rich households			25	0	
Poor households			0	60	

An equilibrium for the model is given by the four prices P_1, P_2, P_L, and P_K and eight quantities $x_1^1, x_1^2, x_2^1, x_2^2$ and K_1, K_2, L_1, and L_2, which meet the equilibrium conditions that market demand equals market supply for all inputs and outputs and that zero profits apply in each industry. Once the parameters are specified and the factor endowments are known, a complete general equilibrium model is available. Tax and other policy variables can then be added as desired.

Table 1 presents the values for the parameters and the exogenous variables used by Shoven and Whalley in their example. The equilibrium solution is reported in Table 2 for a case where a 50 percent input tax applies to the use of capital in manufacturing. Only relative prices are relevant in this model and, somewhat arbitrarily, labour is chosen as the numeraire. At the equilibrium prices, total demand for each output equals production, and producer revenues equal costs. Labour and capital endowments are fully employed, and consumer incomes from sale of factors plus transfers equal consumer expenditures. Because of the assumption of constant returns to scale, the per-unit costs in each industry equal producer selling prices, meaning that economic profits are zero. Expenditures by each household exhaust their income. Shoven and Whalley illustrate how this general equilibrium model can be adapted for policy evaluation work by comparing the with-tax equilibrium solution reported in Table 2 to the no-tax equilibrium solution to obtain measures of the welfare costs of such a tax.

Empirical Implementation and Model Structure

The differences between the model actually used to analyse tax policy proposals and the numerical example above lie in their dimensionality (i.e., the number of sectors and consumer types modelled.) their parameter specification procedures and use of data, and their inclusion of more complex policy regimes than a simple tax on one factor in one sector.

Although the appropriate general equilibrium model for any particular policy analysis varies with the issue, most tax models are variants of static, two-factor models, that have long

Table 2

Equilibrium solution for Shoven and Whalley's example with a 50% input tax on capital in manufacturing

Equilibrium prices

Manufacturing output	1.47
Nonmanufacturing output	1.01
Capital	1.13
Labour	1.00

Production side

	Outputs	
	Quantity	Revenue
Manufacturing	22.39	32.83
Nonmanufacturing	57.31	57.64

	Inputs					
	Capital	Capital cost (including tax)	Labour	Labour cost	Total cost	Cost per unit output
Manufacturing	4.04	6.83	1.00	26.00	32.83	1.47
Nonmanufacturing	20.96	23.64	34.00	34.00	57.60	1.00

Demand side

	Manufacturing	Nonmanufacturing	Expenditure
Rich households	8.94	15.83	29.10
Poor households	13.40	41.48	61.37

	Labour income	Capital income	Transfers	Total income
Rich households	0	28.19	0.91	29.10
Poor households	60.00	0	1.37	61.37

been employed in public finance and international trade. Most involve more than two goods even though factors of production are classified into two broad categories of capital and labour services. In some models these are further disaggregated into subgroups (for instance, labour may be identified as skilled or unskilled). Intermediate transactions are usually incorporated either through fixed or flexible input-output coefficients.

The rationale for proceeding this way is that tax and policy issues are frequently analysed using a general equilibrium theoretical framework, and it is natural to retain the same basic structure in applied work. This is especially the case if the major contribution of the numerical work is to advance from qualitative to quantitative analysis. Also, most data on which the numerical specifications of tax models are based come in a form consistent with the two-sector approach. National accounts data, for instance, identify wages and salaries and operating surplus as major cost components. Input-output data provide intermediate transaction data, with value added broken down in a similar way. This all suggests a model in which capital and labour are identified as the principal factor inputs.

The partition between goods and factors in two-sector models is used in tax models to simplify computation. By using factor prices to generate cost-covering goods prices, consumer demands can be calculated and the derived demands for factors that meet consumer demands evaluated. Thus, even a model with a large number of goods can be solved by working only with the implicit system of excess factor demands.

There are a range of more specific model design issues that are also encountered with tax models, including the treatment of investment, foreign trade, and government expenditures. Where there are no international capital flows, the level of investment in the model reflects household saving decisions (broadly defined to include corporate retentions). These are based on constant-savings propensities in static models and on explicit intertemporal utility maximisation in dynamic models. Government expenditures usually reflect transfers and real expenditures, with the latter frequently determined from assumed utility-maximising behaviour for the government. In this approach, the government is treated as a separate consumption-side agent that buys public goods and services. In a few cases (such as Piggott and Whalley 1985), tax models have been used with public goods explicitly appearing in household utility functions, although this complicates the basic approach.

As regards the treatment of time, some of the static equilibrium tax models have been sequenced through time to reflect changes in the economy's capital stock due to net saving. Models such as those due to Summers (1981), Auerbach, Kotlikoff, and Skinner (1983), and Fullerton, Shoven, and Whalley (1983) have been used to analyse intertemporal issues in tax policy, such as whether a move from an income tax to a consumption tax (under which saving is less heavily taxed) is desirable. This approach links a series of single-period equilibria through saving decisions that change the capital stock of the economy. Saving, in turn, is based on maximisation of a utility function defined over current and expected future consumption. Myopic expectations (i.e., expected future rates of return on assets are assumed equal to current rates of return) are often used to simplify computation. Saving in the period augments the capital stock in all future periods. The general equilibrium computed for each period has all markets clearing, including that for newly produced capital goods. The economy passes through a sequence of single-period equilibria in which the capital stock grows. Tax changes that encourage higher savings typically cause lowered consumption in initial years and eventually higher consumption due to the larger capital stock.

Functional Forms

When building a tax model to represent an actual economy, in addition to selecting the model structure one also has to choose particular functional forms. Typically the major constraint on the choice of demand and production functions is that they be consistent with the theory and are analytically tractable. This involves choosing functions that satisfy the usual demand- and production-side restrictions assumed in general equilibrium models, such as Walras's law. It also requires that excess-demand responses be easy to evaluate for any price vector considered as a candidate equilibrium solution for the model.

The choice of a specific functional form by any given modeller usually depends on how elasticities are to be used in the model. The general approach is one of selecting a functional form that best allows key parameter values (e.g., income and price elasticities) to be incorporated while retaining tractability. This largely explains why the functional forms used in general equilibrium tax models are so often drawn from the family of "convenient" forms [Cobb-Douglas, CES, linear expenditure system (LES), and translog, generalised Leontief, or other flexible functional forms].

Demands from Cobb-Douglas utility functions are easy to work with but have unitary income and uncompensated own-price elasticities and zero cross-price elasticities; restrictions are typically implausible. If all expenditure shares for CES functions are small, compensated own-price elasticities equal the elasticity of substitution in preferences. It may thus be unacceptable to model all commodities as having essentially the same compensated own-price elasticities. One alternative is to use hierarchical or nested CES functions, although here the issues that arise are the global properties of these more flexible functional forms, such as concavity. Unitary income elasticities implied by Cobb-Douglas or CES functions can be relaxed by using LES functions with a displaced origin, but the origin displacements need to be specified.

On the production side, where only two primary factor enter the model, CES value-added functions are usually assumed. If more than two factors are used, hierarchical CES functions or translog cost functions are again used. Intermediate requirements functions may be modelled as fixed coefficients, or intermediate substitutablity may be introduced.

Choice of parameter values

Parameter values for the functions used in tax models are crucial in determining the results of simulations for various tax policies. The procedure most commonly used in these models has come to be labelled "calibration" (Mansur and Whalley (1984)). Under this approach, the economy under consideration is assumed to be in equilibrium in the presence of existing tax policies, that is, at a so-called benchmark equilibrium. Parameters for the model are then calculated such that the model can reproduce the equilibrium data as a model solution.

A feature of this calibration procedure that has both attracted interest and raised concerns is that there is no statistical test of the resulting model specification implied by calibration. The procedure for calibrating parameter values from a constructed equilibrium observation is deterministic. This typically involves the key assumption that the benchmark data represent an equilibrium for the economy under investigation, and required parameter values are then calculated using the model equilibrium conditions. If the equilibrium conditions are not sufficient to identify the model, additional parameter values (typically elasticities) are exogenously specified until the model is identified. These are usually based on a literature search or, less frequently, on separate estimation. In contrast to econometric work that often simplifies the structure of the economic portion of models to allow for substantial richness in statistical specification, the procedure in these models is the opposite. The richness of the economic structure only allows for a crude statistical model that, in the case of calibration to a single year's data, becomes deterministic.

If the widespread use of deterministic calibration in these models is troubling, it is perhaps worthwhile outlining some of the reasons why this calibration approach is so widely used. First, in some of the tax models, several thousand parameters may be involved, and to simultaneously estimate all of the model parameters using time series methods requires either unrealistically large numbers of observations or overly severe identifying restrictions. Partitioning models into submodels (such as a demand and production system) may reduce or overcome this problem, but partitioning does not fully incorporate the equilibrium restrictions that are emphasised in calibration. Also, benchmark data sets are usually constructed in value terms, and their separation into price and quantity observations with consistent units through time as would be required for time series estimation. Finally, the dimensions used in these models make the construction of benchmark equilibrium data sets a nontrivial exercise. Some of the large-scale data sets have required upwards of 18 months work, so that if time series are to be constructed, the required workload may not be sustainable.

Calibration usually involves one year's data. or a single observation represented by an average over a number of years, and it is only in the Cobb-Douglas case that the benchmark data uniquely identify a set of parameter values. In other cases, the required values for the relevant elasticities needed to identify other parameters in the model are usually based on other sources. Typically, heavy reliance is placed on literature surveys of elasticities, and as other modellers have also observed, the literature on elasticity values is sparse (and sometimes contradictory). Also although this procedure might sound straightforward, it can also be difficult because of differences among studies in estimates.

Elasticity values in these models specify the curvature of isoquants and indifference surfaces, with their position given by the benchmark equilibrium data. Because the curvature of CES indifference curves and isoquants cannot be inferred from the benchmark data, extraneous values of substitution elasticities are required. Similarly, for LES demand functions, income elasticities are needed upon which to base the origin coordinates for utility measurement.

In practice, data representing benchmark equilibria for use in calibration are constructed from national accounts and other government data sources. In these data, available information does not satisfy microconsistency conditions (e.g., payments to labour from firms will not equal labour income received by households), and a number of adjustments are needed to ensure that the equilibrium conditions of the models hold. In these, some data are taken as correct and others adjusted to reflect consistency. Tax-related data sets of this type are described in St-Hilaire and Whalley (1983), Piggott and Whalley (1985), and Ballard, Fullerton, Shoven, and Whalley (1985).

Because these benchmark data are usually produced in value terms, in using the data in a general equilibrium model units must be chosen for goods and factors so that separate price and quantity observations are obtained. A commonly used convention, originally adopted by Harberger (1962), is to assume units for both goods and factors such that they have a price of unity in the benchmark equilibrium.

Solving general equilibrium models

The early general equilibrium tax models typically used Scarf's algorithm for their solution (see Scarf (1967), Scarf and Hansen (1973)). More recent models rely on optimization software such as GAMS (the Generalised Algebraic Modelling System). In this latter approach, all equilibrium conditions are written as constraints for an optimization problem with an arbitrary (or convenient) objective function. Any solution to the optimization problem which lies in the contstraint set is, by construction, an equilibrium.

A GAMS model in this format essentially is a set of statements in GAMS syntax (Brook, Kendrick and Meeraus (1992)) which declare sets, data, parameters, variables, equations, and and assigns model relationships in a natural way legible to any model user. A specific GAMS formulation, MPSGE, has also become popular for generating a specific set of large scale standard Arrow-Debreu applied general equilibrium models (Rutherford (1997)). This declares the full model equations in symbolic form. After models are generated, either in GAMS or MPSGE format, a one line command instructs solvers to solve the declared models. It is possible to declare a number of models consisting of different equations in the same file, and include statements to solve these models sequentially with loops over time or counterfactual experiments. The class of these models can reflect linear, non-linear, mixed complementarity, integer or discrete programming problems. Optimization solvers (MINOS5 and CONOPT) are the most popular for solving non-linear and linear programming problems in GAMS, while PATH is a more powerful solver to for mixed complementarity problems (Dirkse and Ferris (1995)). At the end of the GAMS program file, a series of reporting statements are included to handle the report. In recent versions of GAMS a user has more control over the output generated from the model; the results of desired variables can be displayed, put in tabular spreadsheet format, or plotted directly at the end of the model execution.

Evaluating impacts of policy changes

Theoretical literature on welfare economics is usually followed in making comparisons between equilibria in order to arrive at policy evaluation based on these tax models. For welfare impacts, Hicksian compensating (CV) and equivalent variations (EV) are commonly used as summary measures of welfare impact by agent. Economywide welfare measures are often computed by aggregating CVs or EVs over consumer groups. Although this is consistent with practice in the cost-benefit literature, the theoretical shortcomings in using the sum of CVs or EVs as an aggregate welfare criterion are well known.

Models also provide a detailed evaluation of who gains, who loses, and by how much as a result of a policy change. No single summary measure need be chosen if the policy analyst is interested only in the detailed impacts of any policy change. In some tax models, new (policy change) equilibria are computed under the restriction that government revenues remain constant. In these models, this usually implies replacing one set of taxes with another, but with new tax rates endogenously determined to preserve revenues. In other models government revenues change, but where this occurs the welfare impact from changes in the amount of public service needs to be factored into any economywide welfare measure.

In addition to welfare impacts, other impacts of tax changes can be investigated, such as income distribution effects using Lorenz curves or Gini coefficients. Alternative income concepts (e.g., gross of tax or net of tax) can also be used in such calculations. Changes in relative prices can be evaluated, as can changes in the use of factors of production across industries or changes in the product composition of consumer demands.

Uniqueness of equilibrium

One final point to keep in mind is that the applied general equilibrium approach to tax policy may not be particularly instructive if the equilibrium solution in any model is not unique for any particular tax policy. Uniqueness, or lack of it, has been a long-standing interest of general equilibrium theorists (see Kehoe (1980)). There is, however, no theoretical argument that guarantees uniqueness in the tax models currently in use. With some of the models, researchers have conducted *ad hoc* numerical experimentation (approaching equilibria from different directions and at different speeds) but have yet to find a case of nonuniqueness. In the case of the US tax model due to Ballard, Fullerton, Shoven, and Whalley (1985), uniqueness has been numerically demonstrated by Kehoe and Whalley

(1985). The current working hypothesis adopted by most tax modellers seems to be that uniqueness can be presumed in the models discussed here until a clear case of nonuniqueness is found.

3. A UK TAX MODEL

We next discuss a simple version of a UK tax model built under project support. This is a single household model with a variety of tax instruments (sales, income and VAT taxes) similar to that considered earlier by Piggott and Whalley (1985), with a labour-leisure choice component to the model. For this model, we briefly discuss the specification of preferences and technology and then describe the construction of micro-consistent data sets required for model calibration We discuss the methods used for calibration and choice of the elasticity parameters in preferences and production functions. Since the models replicate a base year economy in the presence of taxes, we can solve for a counterfactual equilibrium without taxes in order to evaluate the welfare costs of tax distortions.

The data for this model draws on Input-Output tables for the UK economy (ONS (1995)).[3] We discuss a model for the one household case, with the same sectoral disaggregation as available in the 1990 UK Input-Output table (ONS (1995)). Model parameters are inferred from a fully micro-consistent UK data set for 1990, constructed using these data, and supplemented by literature based estimates of behavioural elasticities. The flexibility of the modelling format allows parameters to be calibrated to later years as and when relevant information becomes available.

Demands

The household maximises a CES utility function defined over commodities and leisure, subject to a budget constraint. This utility function can be written as

$$U_h = \left(\sum_i \alpha_{i,h} C_{i,h}^{\rho_h} + \beta_h L_h^{\rho_h} \right)^{\frac{1}{\rho_h}} \qquad (3.1)$$

[3] We use data for 1990. Data for a later year was not available at construction stage, but 1995 data has since been released.

where U_h is the utility of household *h*, $C_{i,h}$ is the consumption of good *i* by household h, L_h is the leisure taken by household h, $\alpha_{i,h}$ is the share of income of household h spent on consumption of good *i*, β_h is the share of full income spent on leisure, and ρ_h is the elasticity parameter in the utility function, the elasticity of substitution between goods (and leisure) being equal to $\sigma = \dfrac{1}{1-\rho_h}$ (Varian (1992)). In versions of the model in which leisure does not appear β_h=0.

Households receive income from capital and labour endowments, and transfers from the government. They pay taxes on household and capital income. The disposable income of a households is given by

$$I_h = r(1-t_k)\overline{K}_h + (1-t_l^h)w\overline{L}_h + TR_h \tag{3.2}$$

where I_h is the full income of the households, $\overline{K}_h$ is the endowment of capital, $\overline{L}_h$ is the endowment of labour, TR_h are the transfers received by the household h, *r* is the rental rate of capital, w is the wage rate, t_l^h is tax rate on household h's labour income[4], and t_k is tax rate on capital income.

The demand functions for goods and leisure are obtained by maximising (1) with respect to (2), and take the following form

$$C_{i,h} = \left(\frac{\alpha_{i,h} I_h}{\left(P_i(1+t_i^v)\right)^{\sigma_h} \left(\sum_i \alpha_{i,h}\left(P_i\left(1+t_i^v\right)\right)^{1-\sigma_h} + \beta_h\left(w\left(1-t_l^h\right)\right)^{1-\sigma_h} \right)} \right) \tag{3.3}$$

where t_i^v is value added tax rate and t_l^h tax rate on labour income.

[4] The effect of tax distortions on the labour leisure choice can be captured through a subsidy to the consumption of leisure at rate t_l^h.

Consumption of leisure is given by

$$L_h = \left(\frac{\beta_h . I_h}{\left(w\left(1-t_l^h\right)\right)^{\sigma_h} \left(\sum_i \alpha_{i,h} \left(P_i\left(1+t_t^v\right)\right)^{1-\sigma_h} + \beta_h \left(w\left(1-t_t^h\right)\right)^{1-\sigma_h} \right)} \right) \tag{3.4}$$

In versions of the model model where there is no demand for leisure households have demand functions only for goods. In that case equation (3.4) does not exist, and equation (3.3) is modified with the setting β_h=0.

In the one household case, the labour supply of each household LS_h is given by the difference between the household labour endowment, and the demand for leisure, L_h.

$$LS_h = \overline{L}_h - L_h \tag{3.5}$$

In equilibrium, this labour supply by household must be consistent with the total demand for labour derived from the profit maximisation behaviour of firms as set out in the following section.

Production

Producers use labour and capital in each of N sectors to yield value added according to CES functions.

$$Y_i = \Omega_i ((1-\delta_i)(K_i)^{\gamma_i} + \delta_i (LS_i)^{\gamma_i})^{\frac{1}{\gamma_i}} \qquad (i = 1..N) \tag{3.6}$$

where Y_i is the value added of sector i, Ω_i is a shift parameter in the production function, K_i and LS_i are the amounts of capital and labour used in sector *i*, δ_i is the share parameter on labour in the CES function, and γ_i is the CES factor substitution parameter.

The gross output of each sector reflects the use of value added, Y_i and intermediate inputs. We assume fixed coefficients both among intermediate inputs, and between value added and intermediate inputs; M_{ij} is the intermediate use of good *i* in the production of good *j* and X_j is the gross output of sector *j*. At any set of prices, producers in each sector maximise profits subject to their technology constraint

$$\max \Pi_i = P_i X_i - wL_i - rK_i - \sum_j P_j M_{ji} \tag{3.7}$$

where Π_i is the profit of sector *i*. In equilibrium, factor demands by sectors are determined where the value marginal product of factors equal factor prices, and there are no positive profits for producers.

Government Budget

The government collects revenue from taxes on capital and labour income and value-added taxes. All the tax revenues collected are transferred to households in lump sum form; ie.

$$\sum_h TR_h = \sum_i t_k rK_i + \sum_h t_i^v P_i C_{i,h} + \sum_h t_l^h wLS_h \tag{3.8}$$

where t_k is the tax rate on capital income, t_i^v is the *ad valorem* tax rate on final sales, t_l^h is the tax rate on labour income of the household.

These taxes, particularly when they are levied at different rates on different sectors and households, have distortionary impacts on the allocation of resources in the economy. These are captured by the model. Government budget balance is a property of an equilibrium

Model Equilibrium Conditions

In this model a competitive equilibrium is given by prices of consumption goods, P_i; the prices of capital, P^k ; a wage rate for labour, w and levels of gross output, Y_i; capital use, K_i; and sectoral use of labour, L_i; such that, given these,

i) markets for goods and services, labour and capital clear; and

ii) the government budget constraint is satisfied,

More specifically the market clearing condition for the goods market is given by

$$X_i = C_i + \sum_{j=1}^{N} a_{ij} X_j \tag{3.9}$$

where $C_i = \sum_h C_{ih}$ is household final consumption, and $\sum_j a_{ij} X_j$ the intermediate demand.

The capital market clearing condition implies

$$\sum_h \overline{K}_h = \sum_i K_i \tag{3.10}$$

and labour market clearing implies:

$$\sum_h LS_h = \sum_i LS_i \tag{3.11}$$

Government budget balance is given by (3.9). When there are n different markets in the economy, relative prices that clear n-1 markets also clear the nth market as well.

Data Sources and Calibration

The implementation of the UK model begins with input-output data. 1990 input-output information is published in the ONS publication *Input-Output Tables for the United Kingdom*, and are available for up to 123 sectors. The breakdown of consumption spending and the distribution of income across households is limited in this source. We therefore take an eight-sector version of the input-output table (see Table 3) to represent the inter-sectoral linkages in UK production. The input-output table shows the quantities of labour and capital services which producers purchase (see Table 4). It also shows how commodities supplied by the various sectors are bought either by households for final consumption (see Table 5), or by other sectors as intermediate inputs. The zero profit conditions for producers in the benchmark data are met in the input-output data for the various sectors of the economy. This essentially means that the value of output equals payments to labour and capital services and intermediate inputs.

We make two modifications to this input-output data for use in the model. First, in an initial closed economy formulation of the model we do not explicitly consider import and export functions. We modify the data such that exports and imports balance for the economy as a whole, and there are no foreign transfers in the model[5].

[5] Elaboration of this model to more fully include is to follow in the project.

Table 3

Aggregated Input-Output Table of the UK Economy, 1990 (£ millions)

	Agri[1]	Energy[2]	Manuf[3]	Constr[4]	Distr[5]	Transp[6]	BusServ[7]	OtherServ[8]
Agri	2,853	0	9,059	3	536	32	0	88
Energy	550	22,203	5,377	598	2,670	2,241	1,824	694
Manuf	3,774	2,896	60,613	14,961	13,372	4,290	8,763	3,410
Constr	186		791	23,022	760	128	1,808	999
Distr	873	1,220	12,171	2,891	4,438	2,700	2,472	742
Transp	266	1,253	7,251	846	12,956	8,091	10,599	1,303
BusServ	1,010	1,648	21,717	9,008	18,348	7,336	49,008	11,278
OtherServ	375	331	3,518	403	1,118	949	3,314	6,059

Source: ONS (1995), Input-Output Tables of the United Kingdom.

Table 4

UK Value added and indirect business taxes 1990 (£ millions)

	Agri[1]	Energy[2]	Manuf[3]	Constr[4]	Distr[5]	Transp[6]	BusServ[7]	OtherServ[8]
LABOUR	2,966	7,918	69,787	16,703	50,599	22,953	52,776	88,656
CAPITAL	5,228	14,255	29,176	16,923	19,175	16,365	23,047	42,359

Source: ONS, Input-Output Tables of the United Kingdom, 1995.

Table 5

UK Final demands 1990 (£ millions)

	Agri[1]	Energy[2]	Manuf[3]	Constr[4]	Distr[5]	Transp[6]	BusServ[7]	OtherServ[8]
Total FD	5059	18597	109143	57953	105376	23534	40058	140253
VAT	876	2485	9015	655	14943	2852	3224	8518

Source: ONS, Input-Output Tables of the United Kingdom, 1995.

Notes

1. Agriculture, Forestry and Fishing (1-3). Numbers in parentheses in this and other notes refer to input-output classifications.
2. Coal, Oil, Gas, Electricity, Water supply (4-9).
3. All Manufacturing (10-90).
4. Construction (91).
5. Wholesale, Retail Services, Hotel and Entertainment (92-95).
6. Railways, Air, Road, Sea, Postal and Telecommunications (96-102).
7. Business Services, Banking, Insurance, Accountancy (103-114).
8. Other Services, Public Administration, Education, Health (115-123)

The second modification relates to the treatment of final demand. We only consider one category of final demands and do not explicitly incorporate investment and government demands into the model. For simplicity, we also do not separate government consumption and investment.

This model is thus a closed economy single household static model, with the government collecting taxes and redistributing income by means of transfers to households. The government collects revenue from taxes on capital income, value-added taxes on commodities and taxes on household income. The value-added tax influences the composition of final demand. The household income tax and accompanying transfers influence the labour supply decisions of the household sector. Differences in tax rates on capital income across sectors distort the allocation of capital resources among various sectors. In a competitive environment, resources are diverted from heavily taxed sectors to less taxed sectors. Production taxes and subsidies paid to producers both apply, having the effect of reducing the selling price below the cost of production.

Parameters and Elasticities

We use literature based estimates of elasticities of substitution between labour and capital in the production functions in calibration, and elasticities of substitution among various commodities (including leisure) that enter the utility function of households in the model is based on literature values reported in Table 6. Data on input-output transactions, value-added, taxes and final demand as presented above are then used to calibrate parameters on the consumption and production sides of this model. Model parameters are chosen in such a way that its solution replicates base year quantities, when base year prices are given to the model with the calibrated parameters. The resulting calibrated parameters are shown in Table 6.

Table 6

Calibrated share parameters in production and consumption in UK tax model

Sectors	Calibrated Share Parameter on Capital Inputs	Calibrated Share Parameter on Labour Inputs	Calibrated Share Parameters for Household Expenditures by Product
Agri	0.638	0.362	0.006
Energy	0.643	0.357	0.023
Manuf	0.295	0.705	0.137
Constr	0.503	0.497	0.073
Distr	0.275	0.725	0.132
Transp	0.416	0.584	0.030
BusServ	0.304	0.696	0.050
OtherServ	0.323	0.677	0.176

Table 7

Elasticity Parameters used in the UK tax model

Sectors	Elasticity of Substitution in Production Functions	Central tendency values of own price elasticities of household demand functions
Agri	0.50	0.468
Energy	0.75	0.659
Manuf	0.45	0.592
Constr	0.50	0.25
Distr	0.80	0.642
Transp	0.80	0.977
BusServ	0.80	0.461
OtherServ	0.50	0.461

Source: Piggott and Whalley (1985)

This basic model captures three different tax instruments: taxes on capital income, value-added taxes on sales of final goods and taxes on household labour income. Tax rates on capital income are taken from information in the UK input-output tables. Value added tax rates and tax rates on household labour income are calculated from the data on income and sales taxes contained in *Economic Trends* (ONS, 1995). (See Tables 8 and 9).

Table 8

Production Side Tax Rates in the UK tax model

Sectors[1]	Tax Rate on Capital Income (%)	Value-added Tax Rates (%)
Agri	8.6	23
Energy	23.1	23
Manuf	6.0	23
Constr	1.7	23
Distr	46.5	23
Transp	6.2	23
BusServ	25.2	23
OtherServ	1.7	23

1. See Table 3 for a key to sectoral abbreviations

Table 9

Household tax rates in the UK tax model

Household labour income tax rate (%)	Transfers as fraction of household income
22	0.273

To assess the welfare impacts of alternative UK tax policies, the model computes compensating and equivalent variations based on consumers' money metric welfare associated with various tax changes. It also computes the marginal excess burdens involved when raising additional tax revenues from various sources.

Model results (Table 10) indicate that the removal of all taxes in the UK model yields a gain equal to 1.762 percent of the benchmark income. The largest welfare gain results from the removal of household labour income tax distortions which equal to 1.132 percent of benchmark income. Among the three tax measures, taxes on capital income show the lowest welfare cost, equal to 0.33 percent of income in the benchmark equilibrium.

Table 10

Welfare effects of the removal of tax distortions in the UK model (EV as a % of GDP)

	Removal of VAT	Removal of taxes on capital income	Removal of taxes on household labour income	Removal of all three taxes
Hicksian EV for representative household as % of GDP	1.002	0.33	1.132	1.762
	VAT	Capital income	Household labour income	All taxes
MEB £1 of extra revenue when raised by taxes	0.132	-0.028	0.234	0.191

Table 10 also reports the marginal cost of public funds (MEB) for the three categories of taxes in this initial UK model. The marginal excess burden is 23 pence per pound of revenues raised from taxes on household income and 13 pence per pound if extra revenues are raised by means of VAT. However, revenues raised by capital taxes improve the welfare in the economy. This outcome is due to the distortionary nature of capital taxes in the base year, which partly reflects distortions elsewhere in sales and other taxes. In the model data, for instance, capital income from the distribution sector is taxed at 46.5 percent, while capital income from construction and other sectors is subject only to 1.7 per cent tax rate on capital income.

4. EXTENSIONS OF THE BASIC APPROACH

Disaggregation and Elaboration

This basic UK model has already been expanded on in several ways in the project. We have disaggregated the households sector into ten different categories so that we can analyse incidence effects of taxes. We use data reported in the Family Expenditure Survey and income distribution data published in *Economic Trends*. As data tables in the latter sources are presented for ten deciles ranked by household income, we have expanded the UK model set out above to capture ten household groups. In the model, households are grouped by income decile, each endowed with labour and capital. Households receive income by supplying labour and capital, and receive transfers from the government. Rich households have proportionately larger capital endowments, and are liable for higher rates of taxes than poor households. In turn, poor households receive larger transfers from the government than those received by rich households. The income distribution data in the model includes income on wages and salaries, income from investment, and transfers by decile.

We have also extended the model to include an international trade sector, using two different variants. One is a small open economy model of the UK and the other is a world trade model, including the UK. The latter can be used to analyse multilateral trade policy initiatives, such as the impacts of the Uruguay Round decisions in the WTO.

Discreteness of Labour Supply Choices and Tax Distortions of Labour Supply

Bhattarai and Whalley[6] have also use this modelling approach to investigate the significance of discrete choice for calculations of the welfare cost of tax distortions of labour supply by constructing observationally equivalent discrete and continuous choice UK based

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[6] See Bhattarai and Whalley (1997a).

labour supply models[7]. The discrete analogue models they use embody varying forms of agent heterogeneity (over share parameters in preferences, substitution elasticities, endowments), while maintaining equivalence to identical single agent continuous models through similar model generated aggregate uncompensated labour supply elasticities. Issues arise as to how discrete choice is modelled in the with tax case, and they present two alternative formulations. In one, tax revenues are returned to those who pay the tax; no interagent redistribution occurs. In the other, households have proportional claims on revenues generated by the tax, and redistribution occurs. The discrete analogue forms both asymptotically approach the continuous case as the grid over which discrete choice occurs becomes everywhere dense.

Results from these exercises are striking. In simple numerical examples using their first discrete formulation (tax revenues recycled to those who pay the tax), the discrete choice model produces sharply lower welfare costs of taxes if model calibration is made to the same uncompensated labour supply elasticities in both discrete and continuous models. Welfare costs of similar *ad valorem* labour supply tax distortions differ between models by factors of around 5, being sharply lower in the discrete formulation. On the other hand, under their second discrete choice formulation, which maintains fixed shares of households in tax revenues, the discrete choice model produces sharply higher welfare cost estimates. This is because households who switch from high to low labour supply inflict a fiscal externality on those remaining in their original state, since revenues fall. For an individual close to indifferent between high and low discrete labour supply states, when switching to the low labour supply state they experience little or no gain, while all other individuals collectively experience a loss represented by the tax revenue foregone. The factors of proportionality in results between discrete and continuous models change as the form of discreteness changes

[7] See the discussion of the related but different issue of non linear budget constraints and the welfare cost of

(step size across uniform parameter distributions, numbers of individuals, model parameters used for the population distributions), but not markedly.

They also consider cases where there is discrete choice for one household member (the primary worker) and continuity of choice for the other (the secondary worker); and consider household rather than individual optimization. Differences in results between mixed discrete-continuous choice models are smaller, but other insights also emerge. These include the influence of discreteness of choice for one household member over the labour supply behaviour of the other, with implications for the literature on labour supply elasticities for primary and secondary workers. They investigate differences in model results empirically using UK data and literature based parameter estimates instead of numerical examples. Results indicate slightly smaller but still significant factors of proportionality to those which occur in numerical examples. The conclusion is that discreteness in modelling labour supply behaviour matters, not only for labour supply estimation but also for the calculation of welfare costs of labour supply taxation. The formulation chosen also critically affects whether estimates are raised or lowered.

The Redistributive Effects of Transfers

Bhattarai and Whalley (1997b) also use this modelling approach to investigate the redistributive effects of transfers in the UK. The point of departure in their analysis is the observation, seemingly not in the literature, that with voluntary participation in transfer programmes (with tax back or withdrawal provisions), the utility, or real income, value of transfers to participants is typically less than the cash transfers received. This is because individuals (or households) compare utility across two regimes; one with benefits and tax back arrangements, and the other with no benefits and no tax back, choosing the higher utility

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taxes in Preston and Walker (1992).

regime. A money metric utility comparison between the regimes (the real income difference) bears no direct relation to the net of tax back cash transfers actually received. Indeed, in an extreme case where an individual (or household) is indifferent between participating and not participating, if they were recorded as a benefit programme participant, the real income gain to them from participation in the transfer programme would be zero, despite the disbursement of public funds to the recipient. This analysis argues that, in general, and ignoring wage rate effects, the real income received by transfer recipients is smaller than the cash transfers actually made, since to the recipient the reference point for valuing them should be the reservation utility in the no benefit-no tax back regime.

Given tax back rates, individuals (or households) will also not participate in transfer programmes until some threshold level of transfers is reached, raising utility under participation above that in the no participation regime. Tax back conditions in benefit programmes thus impose a form of real income entry fee on participation, which must be deducted when evaluating the redistributive effects of transfer programmes. Conditional on participation in transfer programmes, transfers shift recipients' budget constraints parallel. In the homothetic case, after the entry fee has been paid, marginal increases in transfers (for fixed tax back rates) provide recipients with corresponding marginal increases in real income in the fixed wage case.

They also argue that an additional effect, also seemingly missing from existing literature, needs to enter analyses of the redistribution effects of transfers, namely that when the poor participate in a conditional transfer scheme, the associated withdrawal of low wage labour from the market place drives up the wage of low wage(skill) workers relative to high wage(skill) workers. Thus, with heterogeneous labour income(skill) range, the withdrawal of low wage labour due to high tax back conditional transfers to the poor raises their wage rate. This set aside effect potentially substantially raises the welfare of the poor. Thus, conventional redistribution to the poor through transfers may be overestimated for the reasons given above, but induced redistribution via changes in the wage distribution can provide significant redistribution.

Bhattari and Whalley use conditional choice general equilibrium models to assess the importance of these two effects. These models embody both endogenous participation decisions in transfer schemes by households, and heterogeneous labour in production (by income range or skill type). In such models, programme participation by low income households is endogenously determined in equilibrium, along with the wage distribution since wage rates change as transfer programmes characteristics (benefit levels, tax-back rates) change. The budget set for households are non-convex, presenting special computation problems not tackled, as far as we are aware, in existing general equilibrium tax computation literature. Commodity demands by households, including leisure, are no longer analytic, even for conventional CES or Cobb-Douglas utility functions. This means that household demands have to be evaluated numerically using optimisation techniques within a larger equilibrium structure, including the production side modelling of the economy. The larger model is itself solved through separate application of numerical optimization methods; in essence optimization embedded within a wider optimization, since consumer demands are non-analytic.

The bottom line conclusion they offer is that assessing the redistributive effects of conditional transfers is neither as straightforward as it at first sight appears, nor is the real income value of such transfers seemingly so large as is implicitly assumed in the literature. Regime choice significantly affects such measures, and their general equilibrium effects through programme-induced low wage labour withdrawal need to be taken into account. These effects seem to be virtually ignored in available literature in the area, and are important because they can radically change perceptions as to the redistributive effects of transfers.

5. CONCLUDING REMARKS

This paper discusses applied general equilibrium tax modelling, and initial applications to UK policy issues under an ESRC supported research project on General Equilibrium Modelling of UK Policy Issues. Future activity on the project will reflect the objectives outlined in the original project proposal. Further work also remains to be done on model applications to the analysis of other UK policy issues (e.g. effects on the UK of the Uruguay Round of GATT negotiations, possible effects of a carbon tax on UK economic performance, and analysis of UK social security issues) through the development of a other of issue-driven modelling structures. The original project proposal set out a research agenda through the years 1998-99. Furthering the development of general equilibrium and dynamic modelling techniques and providing a better integration with other disciplines, including econometrics, while contributing to the dissemination of general equilibrium modelling techniques and relevant software tools will remain the focus of the remainder of project.

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