Research Methods For Economists

Lecture 7
Theoretical Research
Questions about an Economy:
Input-Output and General Equilibrium
Model for Policy Analysis

Intersectoral Linkages

Optimal allocation and price in an Economy

Resource Balance for the Economy
Supply = Demand

$$X_1 = X_{1,1} + X_{1,2} + F_1$$

$$X_2 = X_{2,1} + X_{2,2} + F_2$$

Liontief Technology Coefficients:

$a_{i,j}$ share of input from row sector i to sector j

$$a_{1,1} = \frac{X_{1,1}}{X_1} \qquad a_{1,2} = \frac{X_{1,2}}{X_2} \qquad a_{2,1} = \frac{X_{2,1}}{X_2} \qquad a_{2,2} = \frac{X_{2,2}}{X_2}$$

$$X_1 = a_{1,1} X_1 + a_{1,2} X_2 + F_1$$

$$X_2 = a_{2,1} X_1 + a_{2,2} X_2 + F_2$$

Input-Output Model

$$\left(1 - a_{1,1}\right) X_1 - a_{1,2} X_2 = F_1$$

$$- a_{2,1} X_1 + \left(1 - a_{2,2}\right) X_2 = F_2$$

$$\begin{bmatrix} \left(1 - a_{1,1}\right) & - a_{1,2} \\ - a_{2,1} & \left(1 - a_{2,2}\right) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} \left(1 - a_{1,1}\right) & - a_{1,2} \\ - a_{2,1} & \left(1 - a_{2,2}\right) \end{bmatrix}^{-1} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$$X = \left(I - A\right)^{-1} F$$

Input-Output Model

$$(I-A)^{-1} = \frac{Adj(A)}{|A|} = \frac{\left[C_{i,j}\right]'}{|A|} = \frac{\begin{bmatrix}(1-a_{2,2}) & a_{2,1} \\ a_{1,2} & (1-a_{1,1})\end{bmatrix}}{(1-a_{1,1})(1-a_{2,2})-a_{1,2}a_{2,1}}$$

$$\begin{bmatrix}X_1 \\ X_2\end{bmatrix} = \frac{\begin{bmatrix}(1-a_{2,2}) & a_{2,1} \\ a_{1,2} & (1-a_{1,1})\end{bmatrix}}{(1-a_{1,1})(1-a_{2,2})-a_{1,2}a_{2,1}}\begin{bmatrix}F_1 \\ F_2\end{bmatrix}$$

$$a_{1,1} = \frac{X_{1,1}}{X_1} = \frac{10}{100} = 0.1 \qquad a_{1,2} = \frac{X_{1,2}}{X_2} = \frac{20}{200} = 0.1$$

$$a_{2,1} = \frac{X_{2,1}}{X_2} = \frac{30}{100} = 0.3 \qquad a_{2,2} = \frac{X_{2,1}}{X_2} = \frac{20}{200} = 0.1$$

A General Equilibrium Model

- Three conditions
  - Demand = supply
  - Income = expenditure
  - Firms maximise profit: zero economic profit in competitive markets

- Major Characteristics
  - Markets allocations depend on relative prices.
  - Relative prices are determined by forces of demand and supply.
  - Demand for a commodity depends on preferences and income.
  - Income of a household is determined by her endowment and price of that endowment.
  - Exchange or trade of goods is mutually beneficial.
  - Each consumer optimises in equilibrium.

An Example of a Pure Exchange Economy

- Two goods: apples and oranges.
- Households A and B
- Production or Supply (Endowments)

	A	B
apples	100	0.00
oranges	0.00	200

- Both have Cobb-Douglas preferences.
- Allocation of income between apples and oranges

	A	B
apples	40%	60%
oranges	60%	40%

- Market structure is competitive and everything is consumed.
- Let apple be X1 and orange be X2.

Household A's Problem

$$\underset{X_1^A, X_2^A}{Max} \quad U^A = \left(X_1^A\right)^{\alpha}\left(X_2^A\right)^{(1-\alpha)}$$

$$Subj\ to \quad I^A = P_1\omega_1$$

$$\omega_1^A = 100 \qquad \alpha = 0.4$$

Household B's Problem

$$\underset{X_1^B, X_2^B}{Max} \quad U^B = \left(X_1^B\right)^{\beta}\left(X_2^B\right)^{(1-\beta)}$$

$$Subj\ to \quad I^B = P_2\omega_2$$

$$\omega_2^B = 200 \qquad \beta = 0.6$$

Question

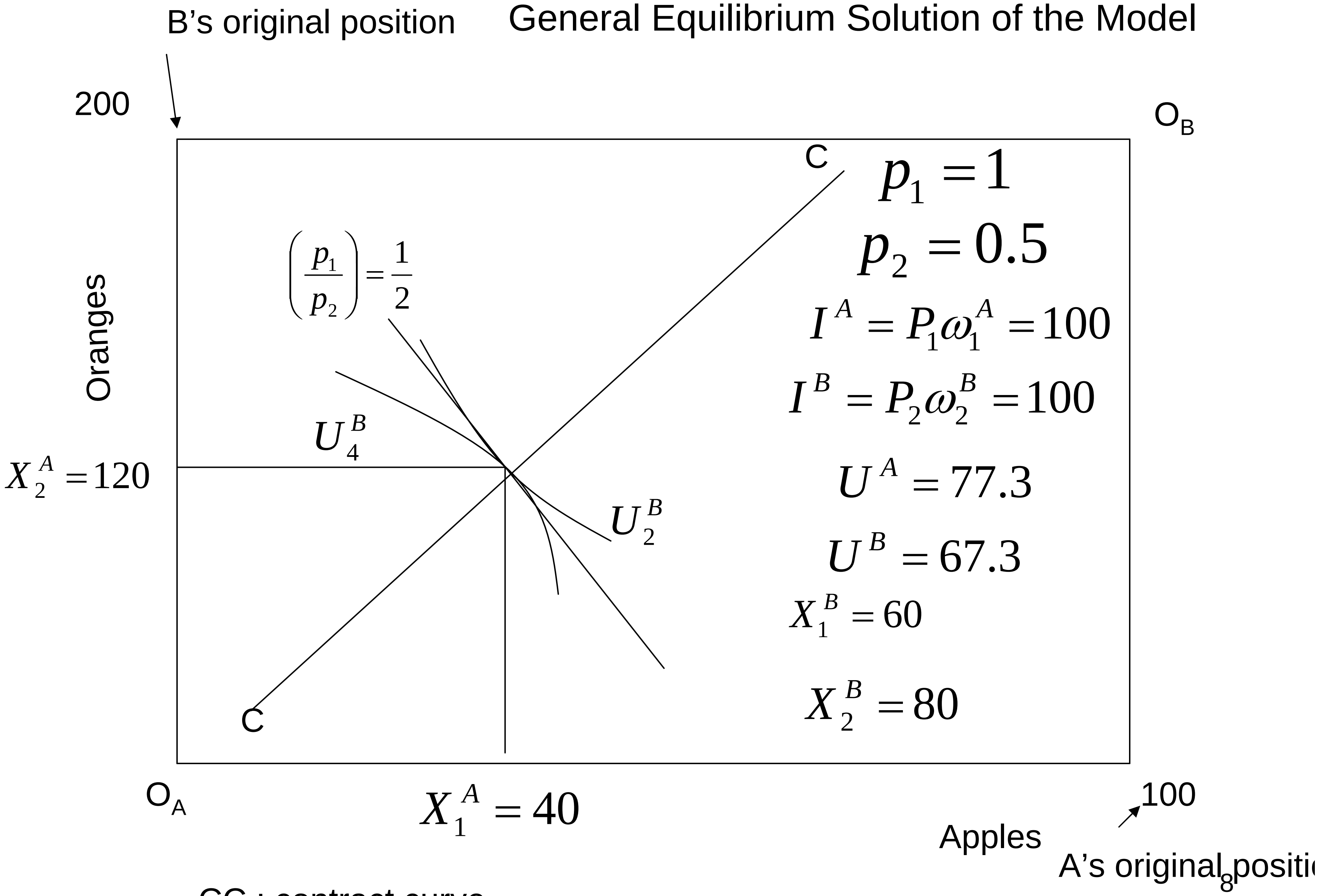
What are the relative prices $\left(\frac{p_1}{p_2}\right)$ that bring optimal allocations in this economy or solves above problems?

What are the demands? $X_1^A \quad X_2^A \quad X_1^B \quad X_2^B$

What are income and welfare? $I^A \quad I^B \quad U^B \quad U^A$

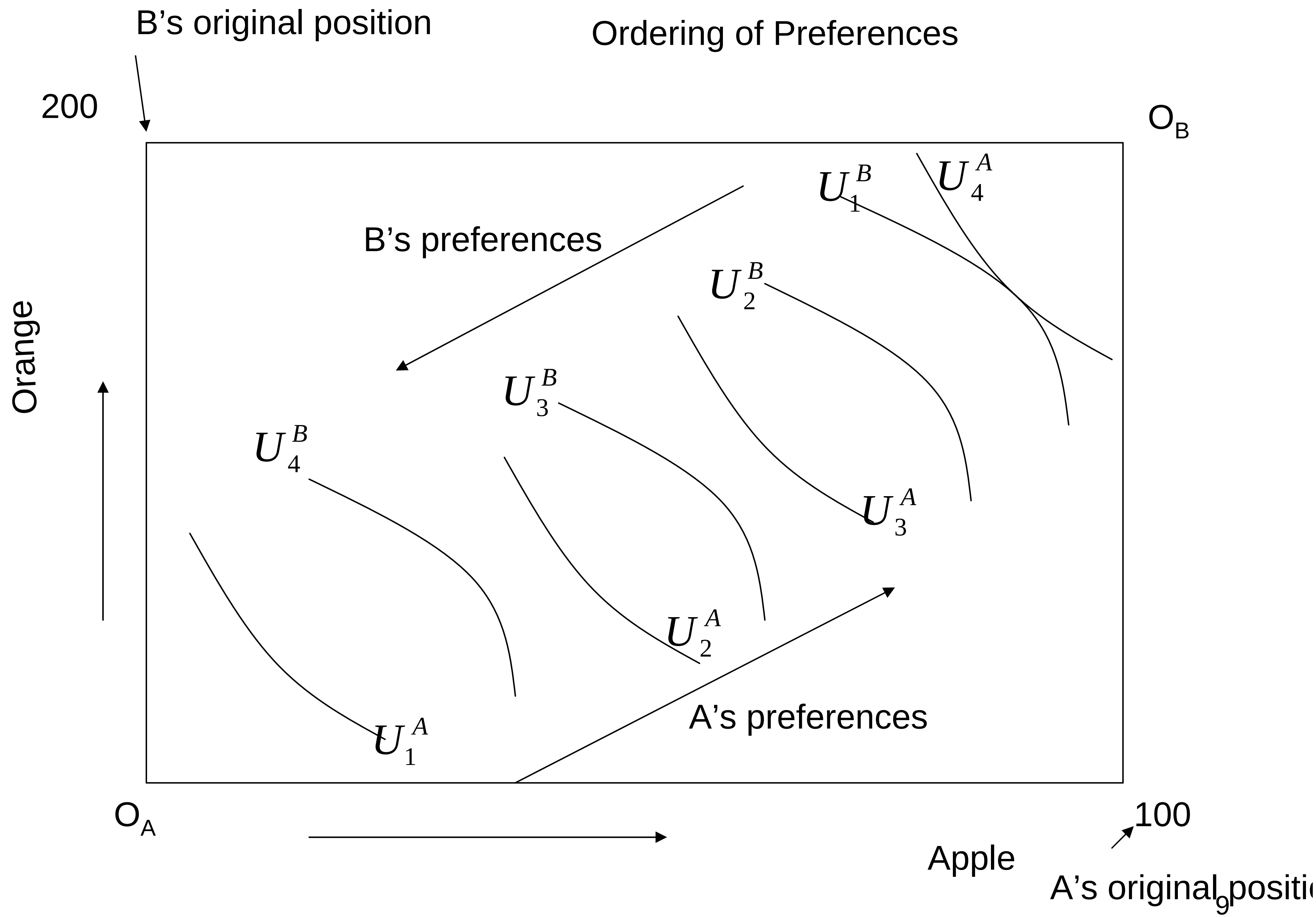
What are the prices? $P_1 \quad P_2$ Numeraire: $P_1 = 1$

General Equilibrium Solution of the Model

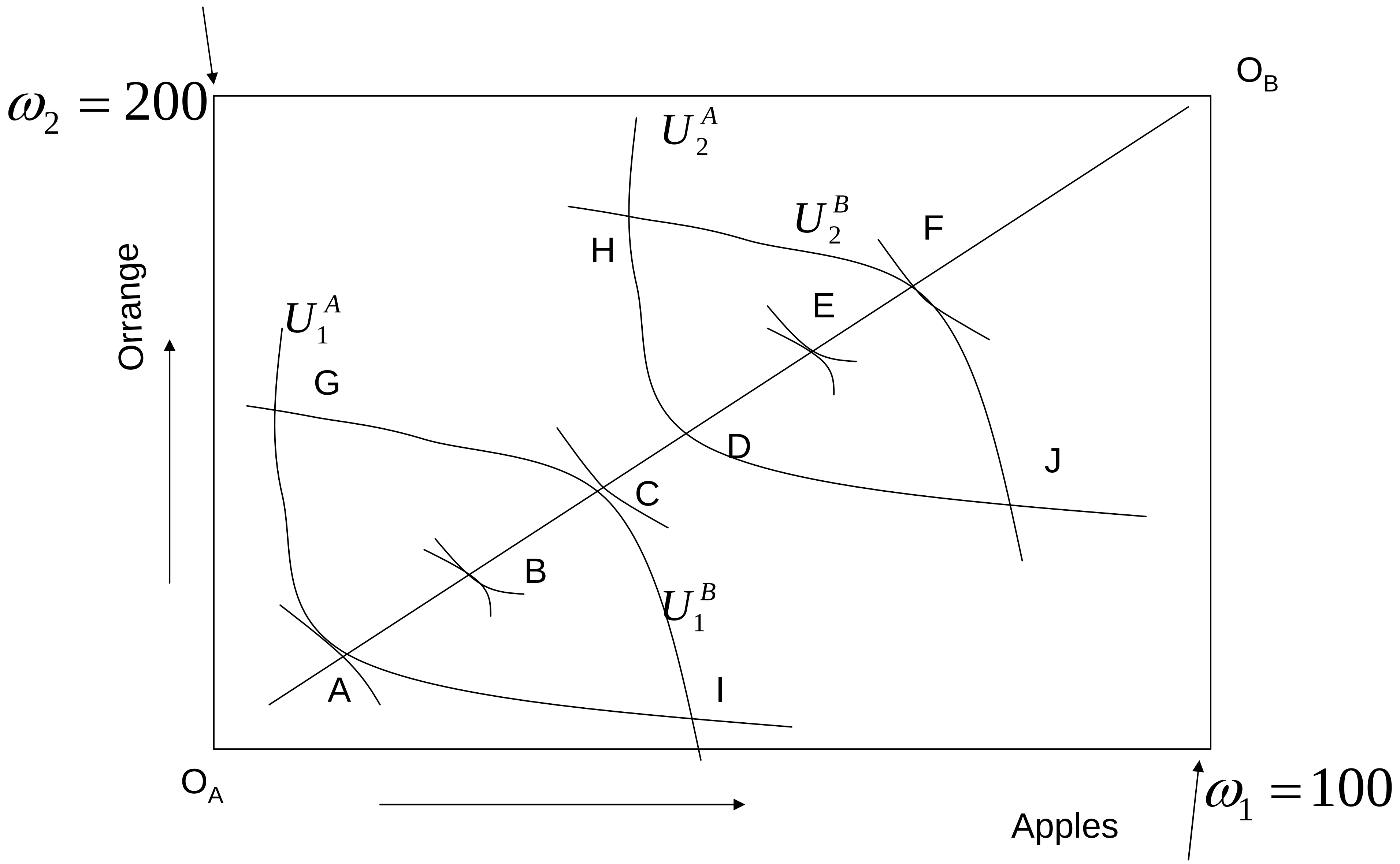


CC : contract curve

Ordering of Preferences

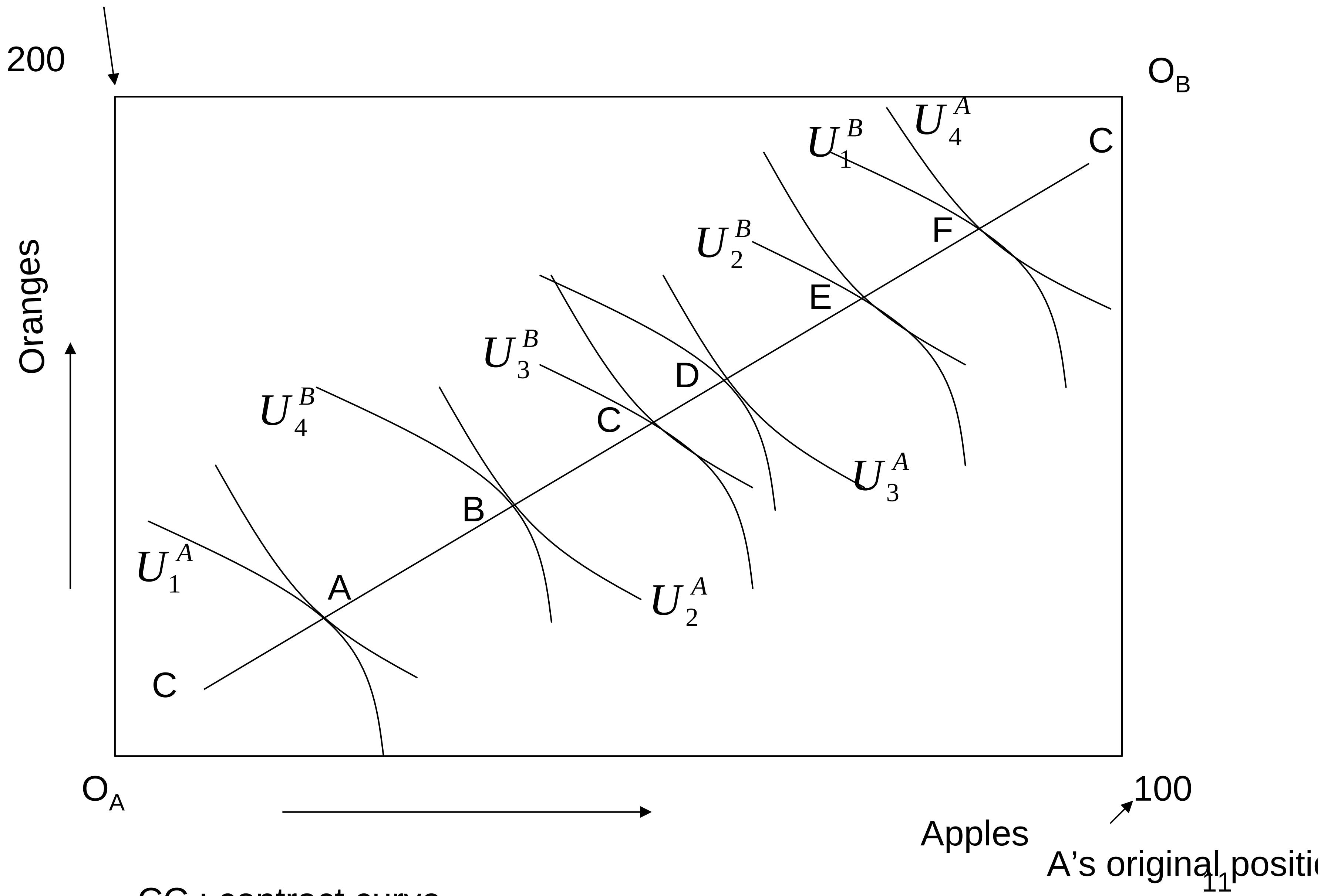


Pareto Optimality Conditions in Two good Two person Economy



Any allocation is possible within this Edgeworth box.

Benefits from Exchange and Contract Curve



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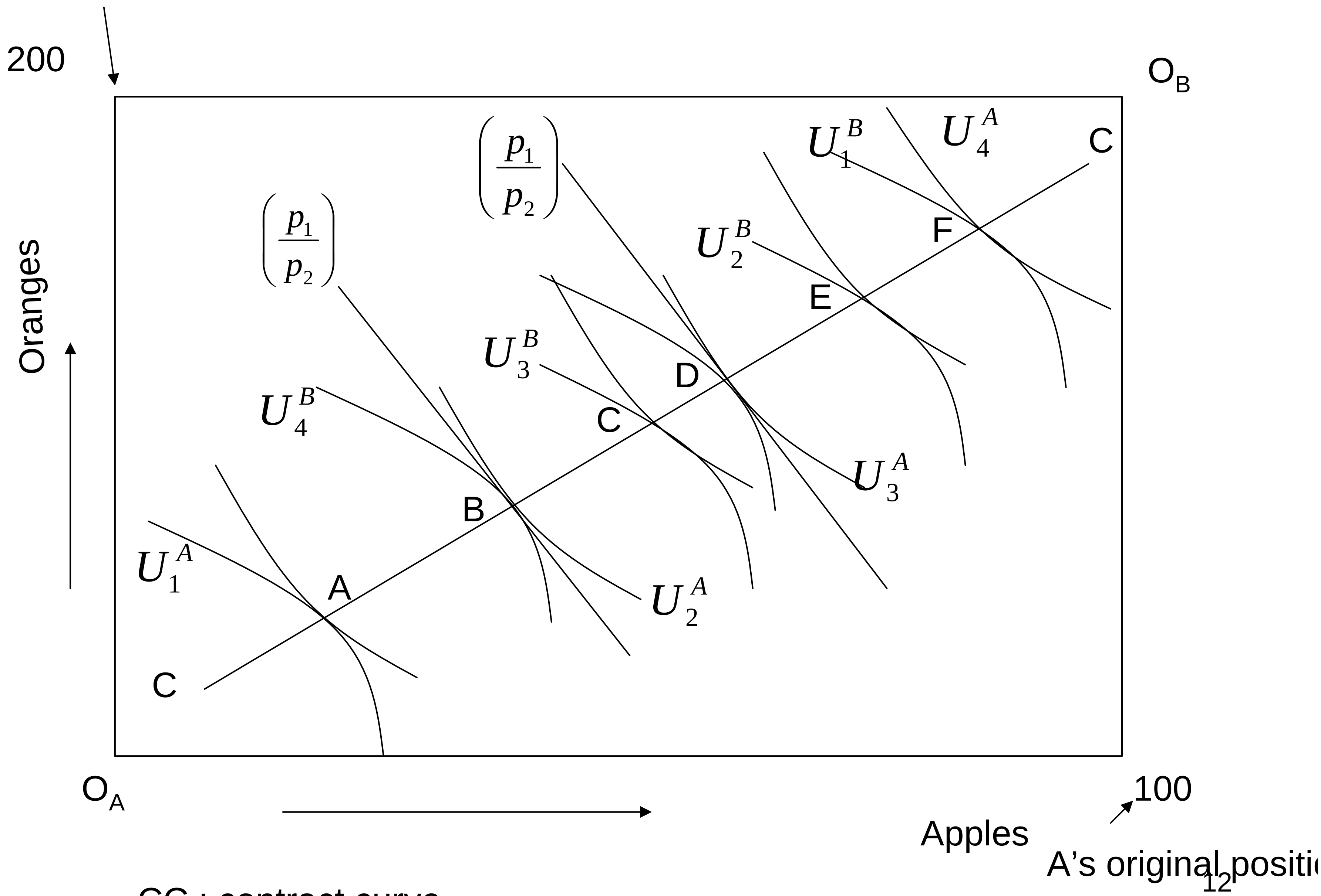
Benefits from Exchange and Contract Curve



CC : contract curve

Demand, Supply and Preferences

$$X_1^A = \frac{\alpha I^A}{P_1}$$

$$X_2^A = \frac{(1-\alpha) I^A}{P_2}$$

$$X_1^B = \frac{\beta I^B}{P_1}$$

$$X_2^B = \frac{(1-\beta) I^B}{P_2}$$

$$\omega_1 = \omega_1^A + \omega_1^B$$

$$\omega_1^A = 100 \qquad \omega_1^B = 0$$

$$\omega_2 = \omega_2^A + \omega_2^B$$

$$\omega_2^B = 200 \qquad \omega_2^A = 0$$

$$\alpha = 0.4 \qquad \beta = 0.6$$

Market Clearing Conditions

$$X_1^A + X_1^B = \omega_1$$

$$I^A = P_1\omega_1$$

$$X_2^A + X_2^B = \omega_2$$

$$I^B = P_2\omega_2$$

$$\frac{\alpha I^A}{P_1} + \frac{\beta I^A}{P_1} = \omega_1$$

$$\frac{(1-\alpha)I^A}{P_2} + \frac{(1-\beta)I^B}{P_2} = \omega_2$$

Prices and Quantities in Equilibrium

$$\frac{\alpha P_1 \omega_1}{P_1} + \frac{\beta P_2 \omega_2}{P_1} = \omega_1 \qquad P_1 = 1 \qquad \alpha_1 \omega_1 + \beta P_2 \omega_2 = \omega_1$$

$$100(0.4) + 200(0.6) P_2 = 100 \qquad 0.4 + 1.2 P_2 = 1$$

$$P_2 = \frac{0.6}{1.2} = 0.5$$

$$X_1^A = \frac{\alpha I^A}{P_1} = \frac{0.4 \times 100}{1} = 40 \qquad X_2^A = \frac{(1-\alpha) I^A}{P_2} = \frac{0.6 \times 100}{0.5} = 120$$

$$X_1^B = \frac{\beta I^B}{P_1} = \frac{0.6 \times 100}{1} = 60 \qquad X_2^B = \frac{(1-\beta) I^B}{P_2} == \frac{0.4 \times 100}{0.5} = 80$$

Sensitivity Analysis -1

$0 < \alpha < 1$ Share of consumption of good 1 by consumer A

alpha	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Beta	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
Income A	100	100	100	100	100	100	100	100	100
Income B	150	133.3333	116.6667	100	83.33333	66.66667	50	33.33333	16.66667
P1	1	1	1	1	1	1	1	1	1
P2	0.75	0.666667	0.583333	0.5	0.416667	0.333333	0.25	0.166667	0.083333
X1A	10	20	30	40	50	60	70	80	90
X1B	90	80	70	60	50	40	30	20	10
X2A	120	120	120	120	120	120	120	120	120
X2B	80	80	80	80	80	80	80	80	80
UA	93.59726	83.85925	79.17047	77.32728	77.45967	79.17047	82.28544	86.75774	92.62674
UB	85.85814	80	73.84053	67.31731	60.34176	52.78032	44.41286	34.82202	22.97397

$0 < \beta < 1$ Share of consumption of good 1 by consumer B

Relative price of good 2 falls as demand for good 1 by consumer A rises.

Sensitivity Analysis -2

alpha	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
Beta	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Income A	100	100	100	100	100	100	100	100	100
Income B	600	300	200	150	120	100	85.714	75	66.667
P1	1	1	1	1	1	1	1	1	1
P2	3	1.5	1	0.75	0.6	0.5	0.4286	0.375	0.3333
X1A	40	40	40	40	40	40	40	40	40
X1B	60	60	60	60	60	60	60	60	60
X2A	20	40	60	80	100	120	140	160	180
X2B	180	160	140	120	100	80	60	40	20
UA	26.39	40	51.017	60.629	69.314	77.327	84.82	91.896	98.625
UB	161.27	131.5	108.58	90.943	77.46	67.317	60	55.326	53.758

Theoretical Observations on Role of Preferences Income and Welfare of Households

- Relative prices of goods, income and amounts of consumption change when preferences change.
- Change in the relative income affects the level of utility and welfare of households
- Household A can make household B worse off by increasing the demand of good 1 that he owns (or supplies to the market).
- Household B can increase his relative income and reduce the relative price of good 1 by increasing the demand for good 2 .
- Relative prices and allocations depend on preferences.

Readings

- Auerbach A.J. and L. J. Kotlikoff (1987), Dynamic Fiscal Policy. Cambridge University Press.
- Bhattarai K (2003) Macroeconomic Impacts of Consumption and Income Taxes: A General Equilibrium Analysis, Research Memorandum 41, University of Hull.
- McKenzie Lionel W. (2002) Classical General Equilibrium Theory, Massachusetts Institute of Technology, Cambridge Massachusetts.
- Debreu Gerard (1959) Theory of Value: An Axiomatic Analysis of Economic Equilibrium, Yale University Press.
- Rutherford T. F. (1995) ,“Extension of GAMS for Complementary Problems Arising in applied Economic Analysis”, Journal of Economic Dynamics and Control, 19, 1299-1324.
- Shoven J.B. and J. Whalley (1992) Applying General Equilibrium, Cambridge University Press.

Texts:

- Pindyck and Rubinfeld (2005) Microeconomics, Chapter 16 and Chapter 4.
- Varian H (2003) Microeconomics, Chapter 30.