Research Methods For Economists

Lecture 4
Choice of Models for Economic Analysis

Structure of the Report or Essay Writing

- Introduction to the Issue: motivation and literature review
- Selection of a Model
  - Choice of the functional form
  - Specification and justification of the Model
  - Mathematical derivations and proof of key equations
  - Assessment of properties of the Key parameters of the Model
- Data collection
- Estimation
- Tests and probing the results with the economic theory.
- Comparison of results with other studies in the literature
- Conclusions
- References

Selection of a Model and Choice of Functional Forms

Regression Model: Time Series and Cross Section
Simultaneous Equation: Some Matrix
- Sales, revenue and price
- Input-Output Model

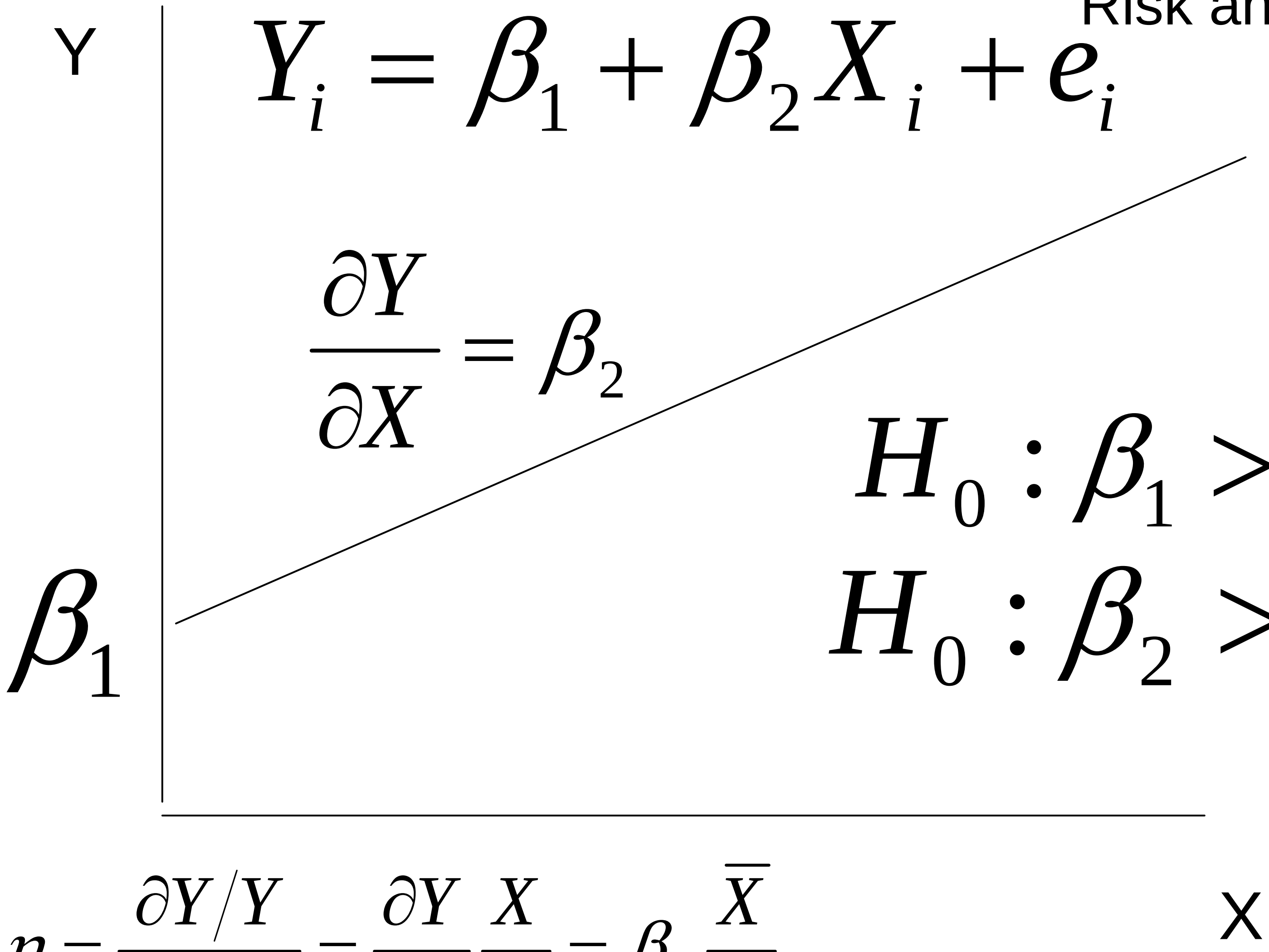
Optimising models
- Linear programming
- Non-linear programming

Game Theory

Positive Linear Relation

Consumption
Supply
Money and price
Risk and return

$$Y_i = \beta_1 + \beta_2 X_i + e_i$$



$$\frac{\partial Y}{\partial X} = \beta_2$$

$$H_0 : \beta_1 > 0$$

$$H_0 : \beta_2 > 0$$

$$\eta = \frac{\partial Y / Y}{\partial X / X} = \frac{\partial Y}{\partial X}\frac{X}{Y} = \beta_2 \frac{\overline{X}}{\overline{Y}}$$

Negative Linear Relation

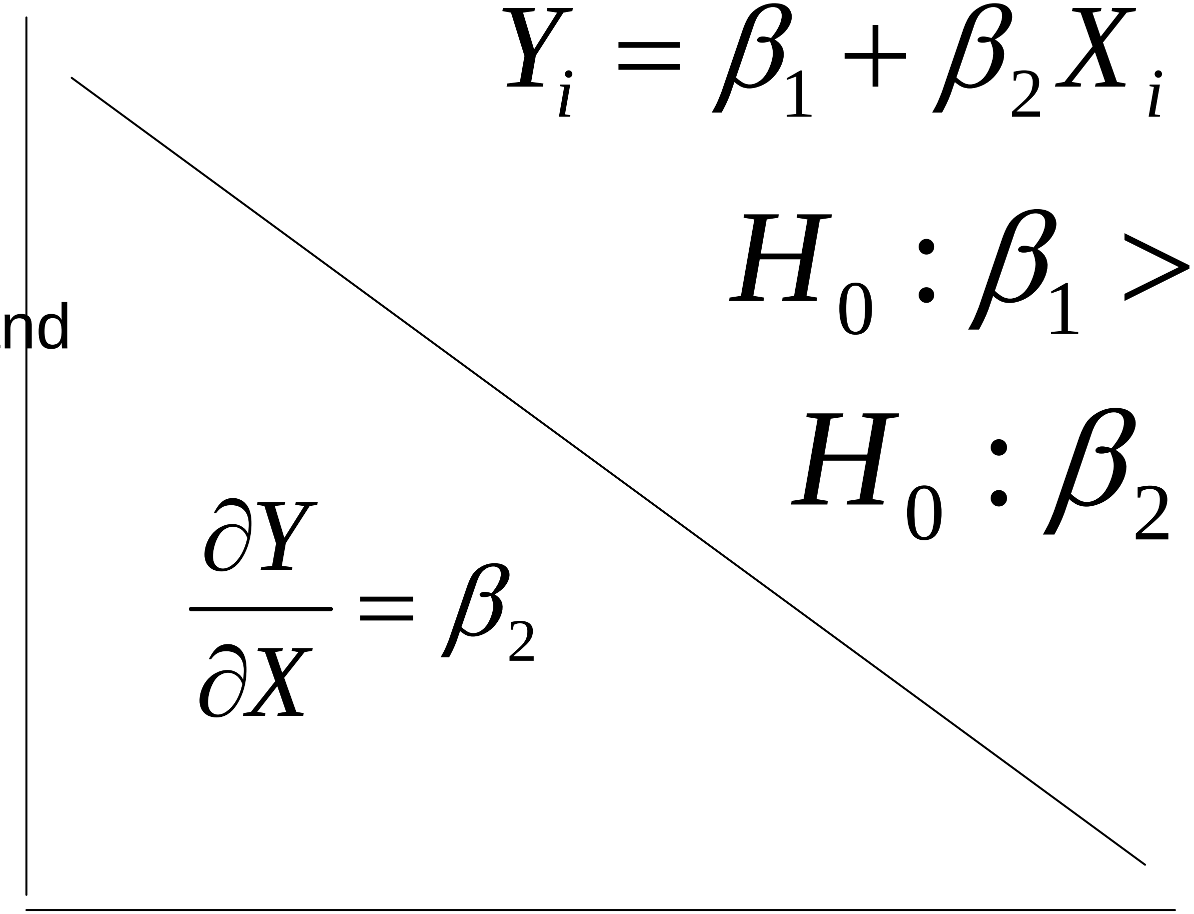
$$Y_i = \beta_1 + \beta_2 X_i + e_i$$

Y

Investment
Demand
Money demand

$$H_0 : \beta_1 > 0$$

$$H_0 : \beta_2 < 0$$



$$\frac{\partial Y}{\partial X} = \beta_2$$

X

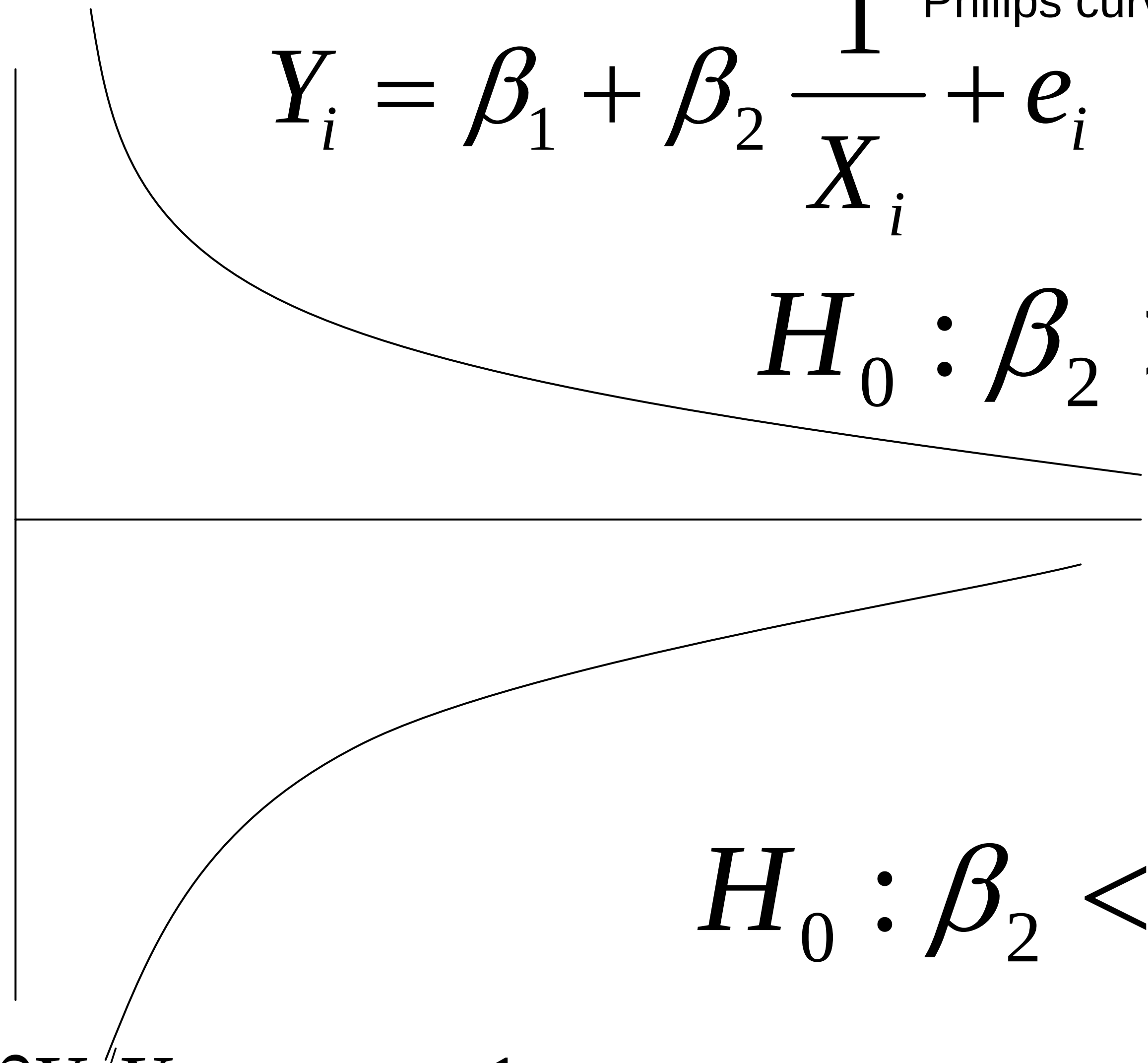
$$\eta = \frac{\partial Y / Y}{\partial X / X} = \frac{\partial Y}{\partial X}\frac{X}{Y} = \beta_2 \frac{\overline{X}}{\overline{Y}}$$

Inverse Reciprocal Relationship

Price and value of money
Philips curve

$$Y_i = \beta_1 + \beta_2 \frac{1}{X_i} + e_i$$

Y



0

X

$$\frac{\partial Y}{\partial X} = -\beta_2 \frac{1}{X^2}$$

$$H_0 : \beta_2 > 0$$

Pollution abatement

$$H_0 : \beta_2 < 0$$

$$\eta = \frac{\partial Y / Y}{\partial X / X} = -\beta_2 \frac{1}{XY}$$

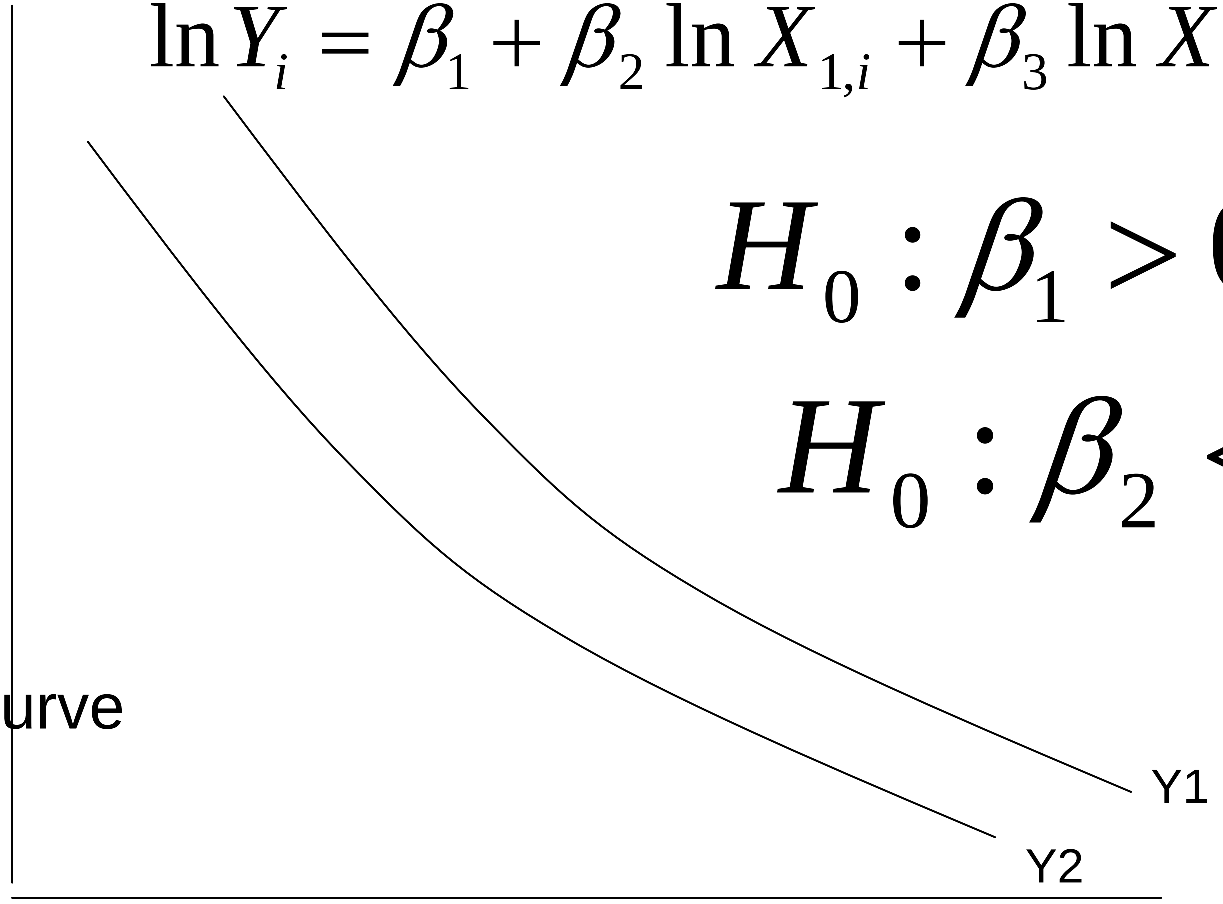
Log -Log Relation $Y_i = \beta_1 X_{1,i}^{\beta_2} X_{2,i}^{\beta_3} e_i$

$$\ln Y_i = \beta_1 + \beta_2 \ln X_{1,i} + \beta_3 \ln X_{2,i} + e_i$$

$$H_0 : \beta_1 > 0$$

$$H_0 : \beta_2 < 0$$

$$\frac{\partial Y}{\partial X_1} = \beta_2 \frac{Y}{X_1}$$



Isoquant
Indifference curve

$$\eta_1 = \frac{\partial Y / Y}{\partial X_1 / X_1} = \beta_2$$

Exponential (Log Linear) Function

$$Y_i = \exp^{\left(\beta_1 + \beta_2 X_{1,i}\right)} e_i$$

$$\ln Y_i = \beta_1 + \beta_2 \ln X_{1,i} + e_i$$

Gas emission and Pollution

Damage from natural disasters

Cost of terrors

Value of education

$$\frac{\partial Y}{\partial X} = \beta_1 Y$$

$$\eta = \frac{\partial Y / Y}{\partial X / X} = \frac{\partial Y}{\partial X}\frac{X}{Y} = \beta_2 Y \frac{X}{Y} = \beta_2 X$$

Semi-Log Linear Function

$$Y_i = \beta_1 + \beta_2 \ln X_{1,i} + e_i$$

$$\frac{\partial Y}{\partial X} = \beta_1 \frac{1}{X}$$

Capital accumulation as a function of investment

Accumulation of Amount in a bank account

$$\eta = \frac{\partial Y / Y}{\partial X / X} = \frac{\partial Y}{\partial X}\frac{X}{Y} = \beta_2 \frac{X}{XY} = \beta_2 \frac{1}{Y}$$

Log Inverse Function

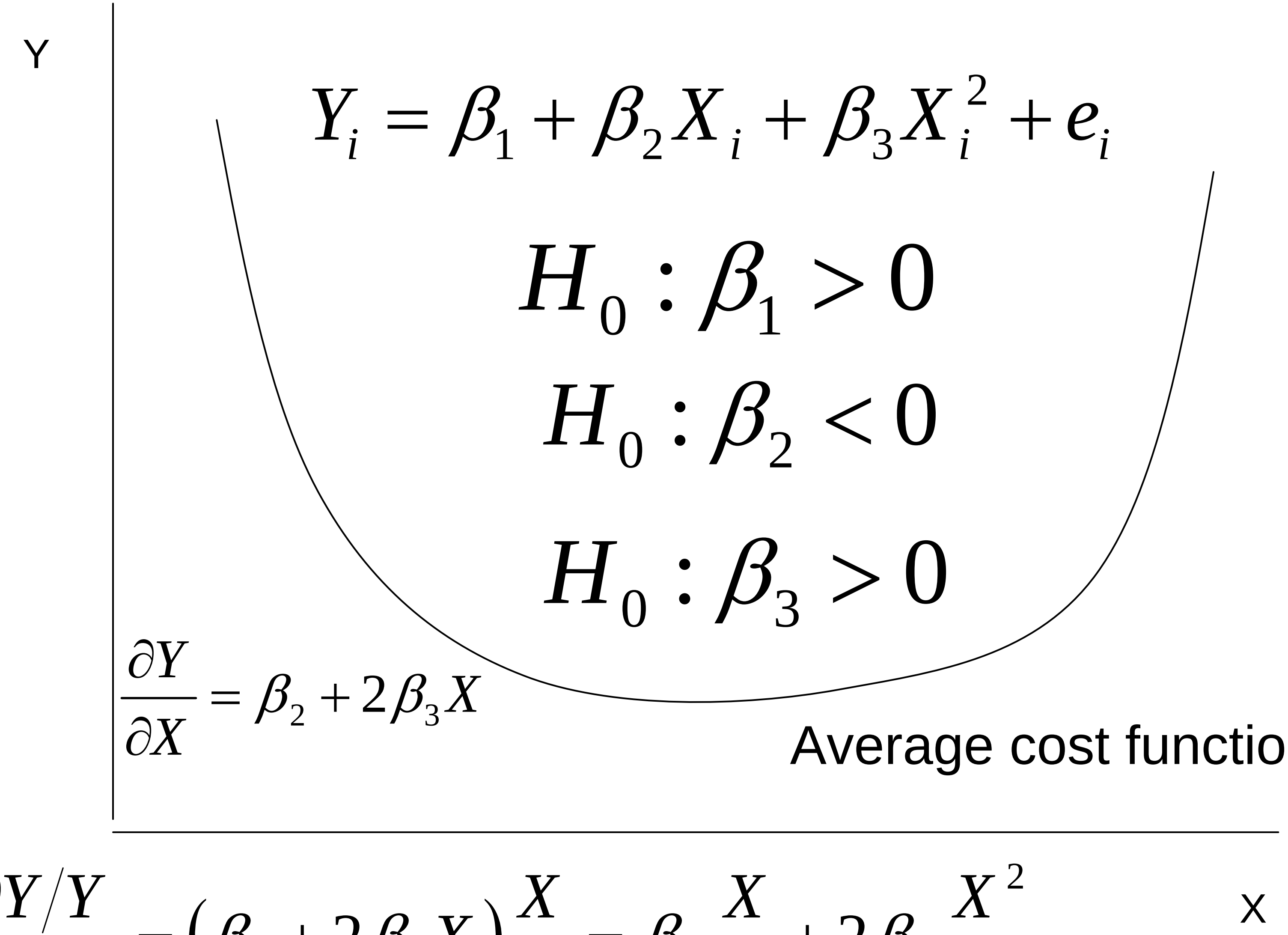
$$\ln Y_i = \beta_1 + \beta_2 \frac{1}{X_i} + e_i$$

$$\frac{\partial Y}{\partial X} = \beta_2 \frac{Y}{X^2}$$

Clean air and disease
Death rates and HIV treatment

$$\eta = \frac{\partial Y / Y}{\partial X / X} = \frac{\partial Y}{\partial X} \frac{X}{Y} = \beta_2 \frac{YX}{X^2 Y} = \beta_2 \frac{1}{X}$$

Polynomial (Quadratic) Relation



$$Y_i = \beta_1 + \beta_2 X_i + \beta_3 X_i^2 + e_i$$

$$H_0 : \beta_1 > 0$$

$$H_0 : \beta_2 < 0$$

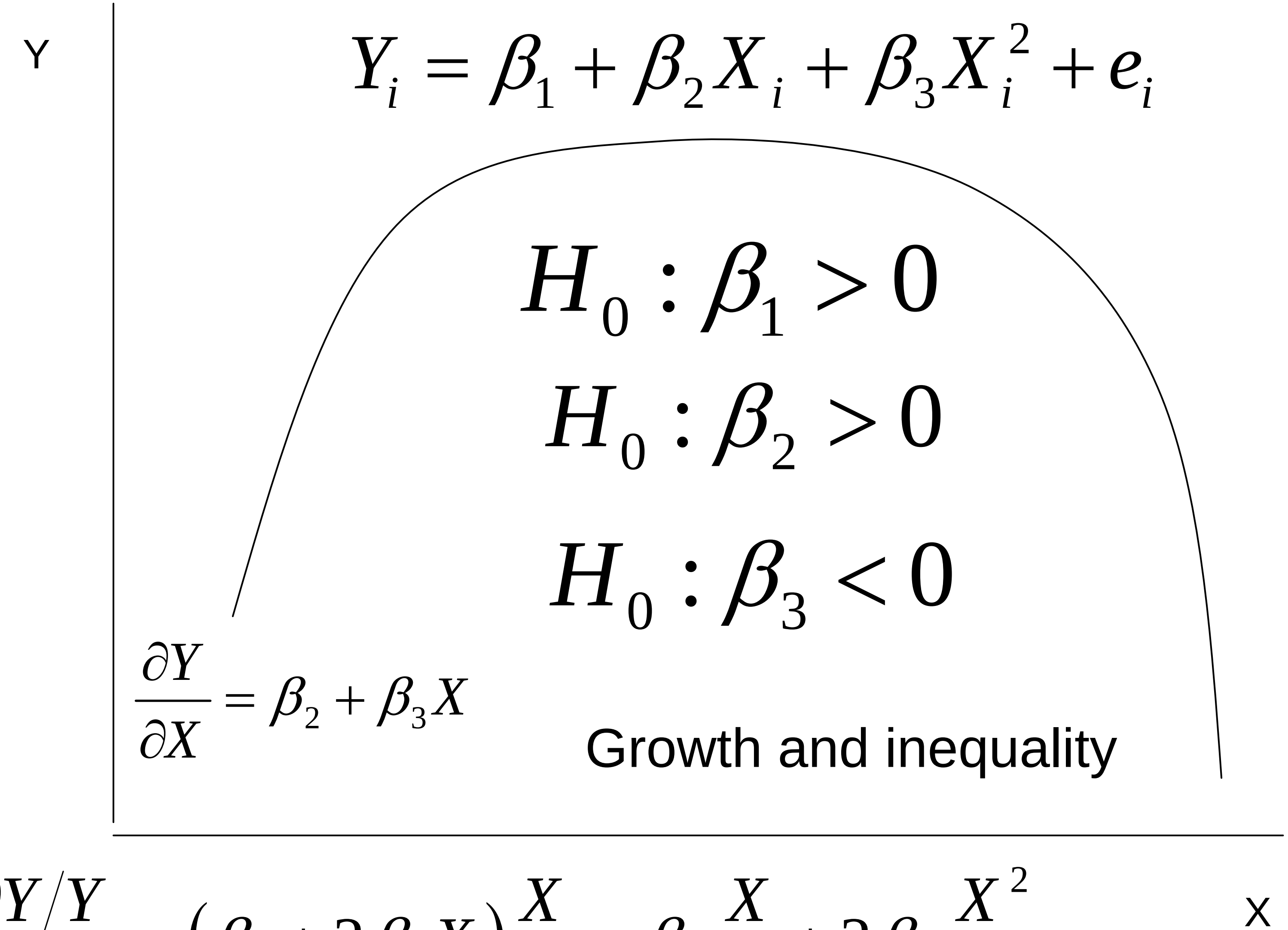
$$H_0 : \beta_3 > 0$$

$$\frac{\partial Y}{\partial X} = \beta_2 + 2\beta_3 X$$

$$\eta_1 = \frac{\partial Y / Y}{\partial X / X} = \left(\beta_2 + 2\beta_3 X\right)\frac{X}{Y} = \beta_2 \frac{X}{Y} + 2\beta_3 \frac{X^2}{Y}$$

Polynomial (Quadratic) Relation

$$Y_i = \beta_1 + \beta_2 X_i + \beta_3 X_i^2 + e_i$$



$$H_0 : \beta_1 > 0$$

$$H_0 : \beta_2 > 0$$

$$H_0 : \beta_3 < 0$$

$$\frac{\partial Y}{\partial X} = \beta_2 + \beta_3 X$$

Growth and inequality

$$\eta_1 = \frac{\partial Y / Y}{\partial X / X} = \left(\beta_2 + 2\beta_3 X\right)\frac{X}{Y} = \beta_2 \frac{X}{Y} + 2\beta_3 \frac{X^2}{Y}$$

Autoregressive Model: AR(1)

$$Y_t = \delta + \theta_1 y_{t-1} + e_t$$

Moving Average Model: MA(1)

$$Y_t = \mu + e_t + \alpha_1 e_{t-1}$$

ARMA(1,1) Model

$$Y_t = \delta + \theta_1 y_{t-1} + e_t + \alpha_1 e_{t-1}$$

Auto Regressive Distributed Lag Model (1,1)

$$Y_t = \beta_1 + \beta_2 Y_{t-1} + \beta_3 X_t + \beta_4 X_{t-1} + \varepsilon_t$$

Distributed polynomial lag model:

$$C_t = \beta_0 + \beta_1 X_t + \beta_2 X_{t-1} + \beta_3 X_{t-2} + \beta_4 X_{t-3} + ... + \beta_k X_{t-k} + .. + u_t$$

Forecasting Models
h =1 period ahead Forecast in AR(1) Model

$$y_t = \delta + \theta_1 y_{t-1} + e_t$$

$$y_{T+1} = \delta + \theta_1 y_T + e_{T+1} \qquad e_{T+1} \sim N(0,1)$$

$$\hat{Y}_{T+1} = E(Y_{T+1}) = \delta + \theta_1 y_T$$

Error of Forecast

$$\hat{e}_{T+1} = Y_{T+1} - \hat{Y}_{T+1} = \delta + \theta_1 y_T + e_{T+1} - \delta - \theta_1 y_T = e_{T+1}$$

$$\operatorname{var}\left(\hat{e}_{T+1}\right) = \sigma_e^2$$

Moving Average Forecasting Model
h =1 period ahead Forecast in MA(1) Model

$$y_t = \mu + e_t + \alpha_1 e_{t-1}$$

$$y_{T+1} = \mu + e_{T+1} + \alpha_1 e_T$$

$$E\left(y_{T+1}\right) = \hat{y}_{T+1} = \mu + \alpha_1 e_T$$

$$\left(y_{T+1} - \hat{y}_{T+1}\right) = \mu + e_{T+1} + \alpha_1 e_T - \mu - \alpha_1 e_T = e_{T+1}$$

$$E\left(y_{T+1} - \hat{y}_{T+1}\right)^2 = \operatorname{var}\left(e_{T+1}\right)^2 = \sigma_e^2$$

ARMA(1,1) Forecasting Model
h =1 period ahead Forecast in ARMA(1,1) Model

$$Y_t = \delta + \theta_1 y_{t-1} + e_t + \alpha_1 e_{t-1}$$

$$y_{T+1} = \delta + \theta_1 y_{t-1} + e_{T+1} + \alpha_1 e_T$$

$$E\left(y_{T+1}\right) = \hat{y}_{T+1} = \delta + \theta_1 y_{t-1} + \alpha_1 e_T$$

$$\hat{e}_{T+1} = \left(y_{T+1} - \hat{y}_{T+1}\right) = \delta + \theta_1 y_{t-1} + e_{T+1} + \alpha_1 e_T - \delta - \theta_1 y_{t-1} - \alpha_1 e_T = e_{T+1}$$

$$\operatorname{var}\left(\hat{e}_{T+1}\right) = E\left(y_{T+1} - \hat{y}_{T+1}\right)^2 = \operatorname{var}\left(e_{T+1}\right) = \sigma_e^2$$

Panel Data Model

$$y_{i,t} = \alpha_i + x_{i,t}\beta_{i,t} + \lambda_t + e_{i,t}$$

$$\begin{bmatrix} y_{i,1} \\ y_{i,2} \\ . \\ . \\ y_{i,T} \end{bmatrix} = \begin{bmatrix} x_{i,1}\beta \\ x_{i,2}\beta \\ . \\ . \\ x_{i,T}\beta \end{bmatrix} + \begin{bmatrix} \alpha_i \\ \alpha_i \\ \\ \\ \alpha_i \end{bmatrix} + \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \\ \\ \lambda_T \end{bmatrix} + \begin{bmatrix} e_{1i} \\ e_{2i} \\ \\ \\ e_{mi} \end{bmatrix}$$

$$y_i = x_i\beta + \lambda_t + \iota_i\alpha_i + e_i$$

α_i Individual effect

λ_t Time specific effects

Dummy Variable Model

$$P_t = \beta_1 + \beta_2 S_t + e_t$$

$$D_t = \begin{cases} 1 \text{if property is in the desirable neighbourhood} \\ 0 \quad \text{if property is not in the desirable area} \end{cases}$$

$$P_t = \beta_1 + \delta D_t + \beta_2 S_t + e_t$$

Expected Price:

$$E(P_t) = \begin{cases} (\beta_1 + \delta) + \beta_2 S_t + e_t \quad if \quad D_t = 1 \\ \beta_1 + \beta_2 S_t + e_t \quad if \quad D_t = 0 \end{cases}$$

Slope Dummy

$$P_t = \beta_1 + \beta_2 S_t + \gamma\left(S_t D_t\right) + e_t$$

$$E\left(P_t\right) = \beta_1 + \beta_2 S_t + \gamma\left(S_t D_t\right) + e_t = \begin{cases} \beta_1 + \left(\beta_2 + \gamma\right) S_t + e_t & if \quad D_t = 1 \\ \beta_1 + \beta_2 S_t + e_t & if \quad D_t = 0 \end{cases}$$

Slope and Intercept Dummy:

$$P_t = \beta_1 + \delta D_t + \beta_2 S_t + \gamma\left(S_t D_t\right) + e_t$$

$$E\left(P_t\right) = \begin{cases} \left(\beta_1 + \delta\right) + \left(\beta_2 + \gamma\right) S_t & if \quad D_t = 1 \\ \beta_1 + \beta_2 S_t & if \quad D_t = 0 \end{cases}$$

Matrix Made Easy

	Firm1	Firm 2
Sales of X_1	50	40
Sales of X_2	70	80
Revenue	200	300

What are the prices of Xx and X2?

$$R_1 = P_1 X_{1,1} + P_2 X_{1,2}$$

$$R_2 = P_1 X_{2,1} + P_2 X_{2,2}$$

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} X_{1,1} & X_{1,2} \\ X_{2,1} & X_{2,2} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

Matrix Example of the Revenue and Price Problem

$$200 = 50P_1 + 40P_2$$

$$300 = 70P_1 + 80P_2$$

$$\begin{bmatrix} 200 \\ 300 \end{bmatrix} = \begin{bmatrix} 50 & 40 \\ 70 & 80 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 50 & 40 \\ 70 & 80 \end{bmatrix}^{-1} \begin{bmatrix} 200 \\ 300 \end{bmatrix}$$

In Excel use 123 help/dta/matrix inverse to find inverse and matrix multiplication.

Solution of the Inverse of the Matrix

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \frac{\begin{bmatrix} 80 & -40 \\ -70 & 50 \end{bmatrix}}{(50\times 80 - 40\times 70)}\begin{bmatrix} 200 \\ 300 \end{bmatrix} = \frac{\begin{bmatrix} 80 & -40 \\ -70 & 50 \end{bmatrix}}{(1200)}\begin{bmatrix} 200 \\ 300 \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 80(200) - 40(300) \\ -70(200) + 50(300) \end{bmatrix}$$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 16000 - 12000 \\ -14000 + 15000 \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 4000 \\ 1000 \end{bmatrix} = \begin{bmatrix} 40/12 \\ 10/12 \end{bmatrix} = \begin{bmatrix} 10/3 \\ 5/6 \end{bmatrix}$$

Cheding the Solutions:

$$200 = 50\times\frac{10}{3} + 40\times\frac{5}{6} = \frac{1000 + 200}{6} = 200$$

$$200 = 70\times\frac{10}{3} + 80\times\frac{5}{6} = \frac{1400 + 400}{6} = 300$$

Solution using the Cramer's Rule

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 50 & 40 \\ 70 & 80 \end{bmatrix}^{-1} \begin{bmatrix} 200 \\ 300 \end{bmatrix}$$

$$P_1 = \frac{\begin{vmatrix} 200 & 40 \\ 300 & 80 \end{vmatrix}}{(50 \times 80 - 40 \times 70)} = \frac{(16000 - 12000)}{1200} = \frac{4000}{1200} = \frac{10}{3}$$

$$P_2 \frac{\begin{vmatrix} 50 & 200 \\ 70 & 300 \end{vmatrix}}{(50 \times 80 - 40 \times 70)} = \frac{(15000 - 14000)}{1200} = \frac{1000}{1200} = \frac{5}{6}$$

Exactly as above.

Solution of the Input-Output Model using Matrix Inverse

$$X_1 = a_{1,1} X_1 + a_{1,2} X_2 + F_1$$

$$X_2 = a_{2,1} X_1 + a_{2,2} X_2 + F_2$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} (1 - a_{1,1}) & -a_{1,2} \\ -a_{2,1} & (1 - a_{2,2}) \end{bmatrix}^{-1} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$$(I - A)^{-1} = \frac{Adj(A)}{|A|} = \frac{[C_{i,j}]'}{|A|} = \frac{\begin{bmatrix} (1 - a_{2,2}) & a_{2,1} \\ a_{1,2} & (1 - a_{1,1}) \end{bmatrix}}{(1 - a_{1,1})(1 - a_{2,2}) - a_{1,2} a_{2,1}}$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \frac{\begin{bmatrix} (1 - a_{2,2}) & a_{2,1} \\ a_{1,2} & (1 - a_{1,1}) \end{bmatrix}}{(1 - a_{1,1})(1 - a_{2,2}) - a_{1,2} a_{2,1}} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

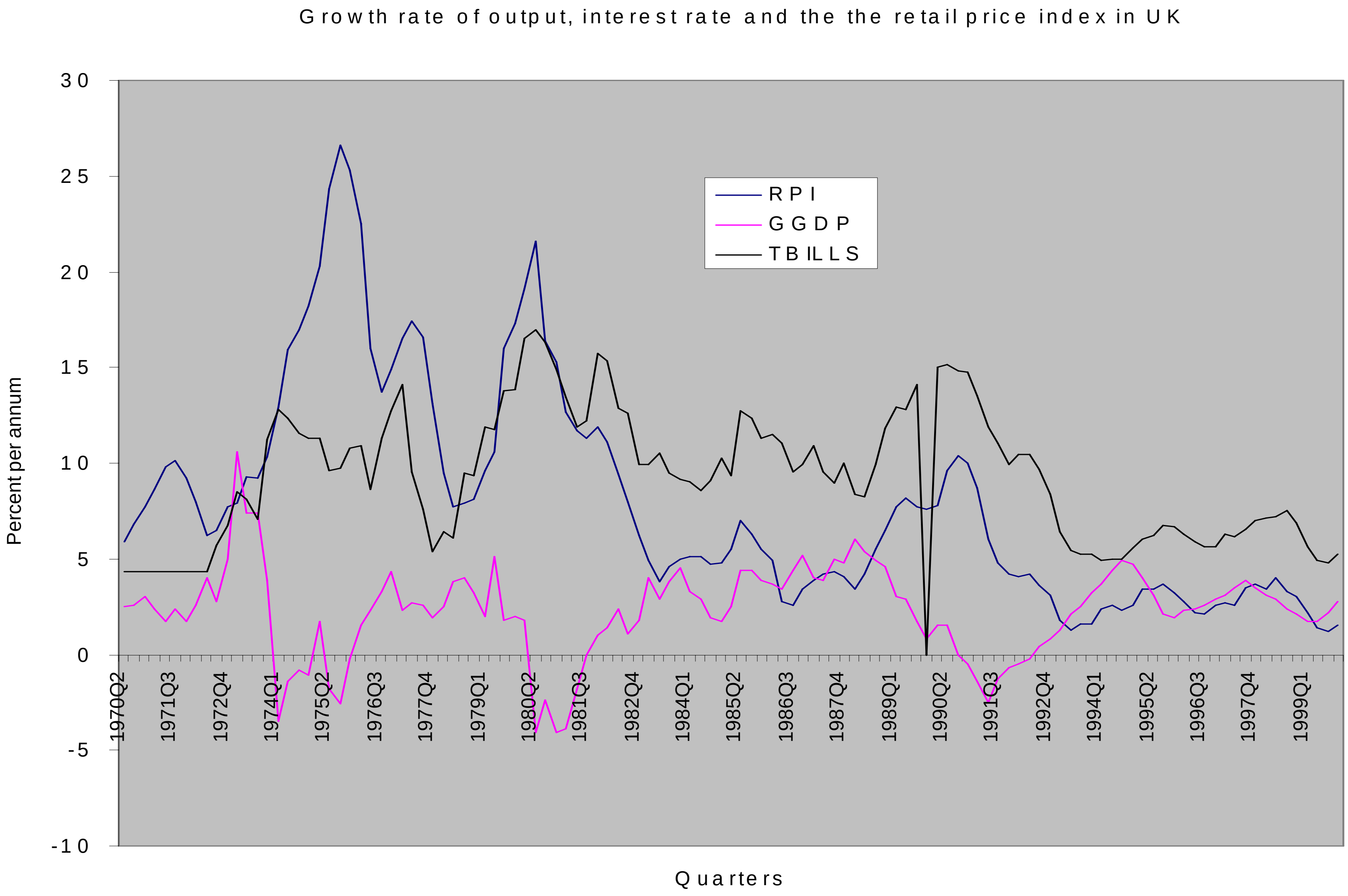
Solution of the Input-Output Model using Cramer’s rule

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} (1 - a_{1,1}) & -a_{1,2} \\ -a_{2,1} & (1 - a_{2,2}) \end{bmatrix}^{-1} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} (1 - a_{1,1}) & -a_{1,2} \\ -a_{2,1} & (1 - a_{2,2}) \end{bmatrix}^{-1} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$$X_2 = \frac{\begin{vmatrix} F_1 & -a_{1,1} \\ F_2 & (1 - a_{2,2}) \end{vmatrix}}{(1 - a_{1,1})(1 - a_{2,2}) - a_{1,2}a_{2,1}}$$

$$X_2 = \frac{\begin{vmatrix} (1 - a_{1,1}) & F_1 \\ -a_{2,1} & F_2 \end{vmatrix}}{(1 - a_{1,1})(1 - a_{2,2}) - a_{1,2}a_{2,1}}$$


Growth rate of output, interest rate and the the retail price index in UK
Percent per annum
30
25
20
15
10
5
0
-5
-10
RPI
GGDP
TBILLS
1970Q2
1971Q3
1972Q4
1974Q1
1975Q2
1976Q3
1977Q4
1979Q1
1980Q2
1981Q3
1982Q4
1984Q1
1985Q2
1986Q3
1987Q4
1989Q1
1990Q2
1991Q3
1992Q4
1994Q1
1995Q2
1996Q3
1997Q4
1999Q1
Quarters

Three Equations of the Interest Determination Rule: Taylor Rule

$$y_t - y_t{}^* = -d\left(i_{t-1} - i_{t-1}^*\right) \qquad d > 0 \qquad (1)$$

where i_t and i_t^* are actual and natural level of output, i_t is the actual rate of interest in period *t*, i is the interest target of the monetary authority.

One period lag is assumed between the interest rate decision and the change in the output.

$$\pi_t = \pi_t^* + c\left(y_{t-1} - y_{t-1}^*\right) \quad c > 0 \qquad (2)$$

where π_t and π_t^* are actual and target inflation rates.

$$i_t = i_t^* + a\left(y_t - y_t^*\right) + b\left(\pi_t - \pi_t^*\right) \quad a > 0; b > 0 \qquad (3)$$

Table 1

Stationarity of variables in the model

ADF tests (T=116, Constant; 5%=-2.89 1%=-3.49)

	Interest rate	Difference of Interest rate	Output gap	Inflation gap
Coefficient	-2.723	-6.463**	-6.160**	-7.428**
Lags	2	2	3	1

$$i_t = 9.446 - 0.183\left(y_t - y_t^*\right) + 0.370\left(\pi_t - \pi_t^*\right)$$

t (32.2) (-1.1) (2.84)

(SE) (0.29) (0.13) (0.181)

Normality test: Chi^2(2) = 11.279 [0.0036]**

Test of Interest Determination Rule for Five Major Economies (Estimates from a 3SLS method)

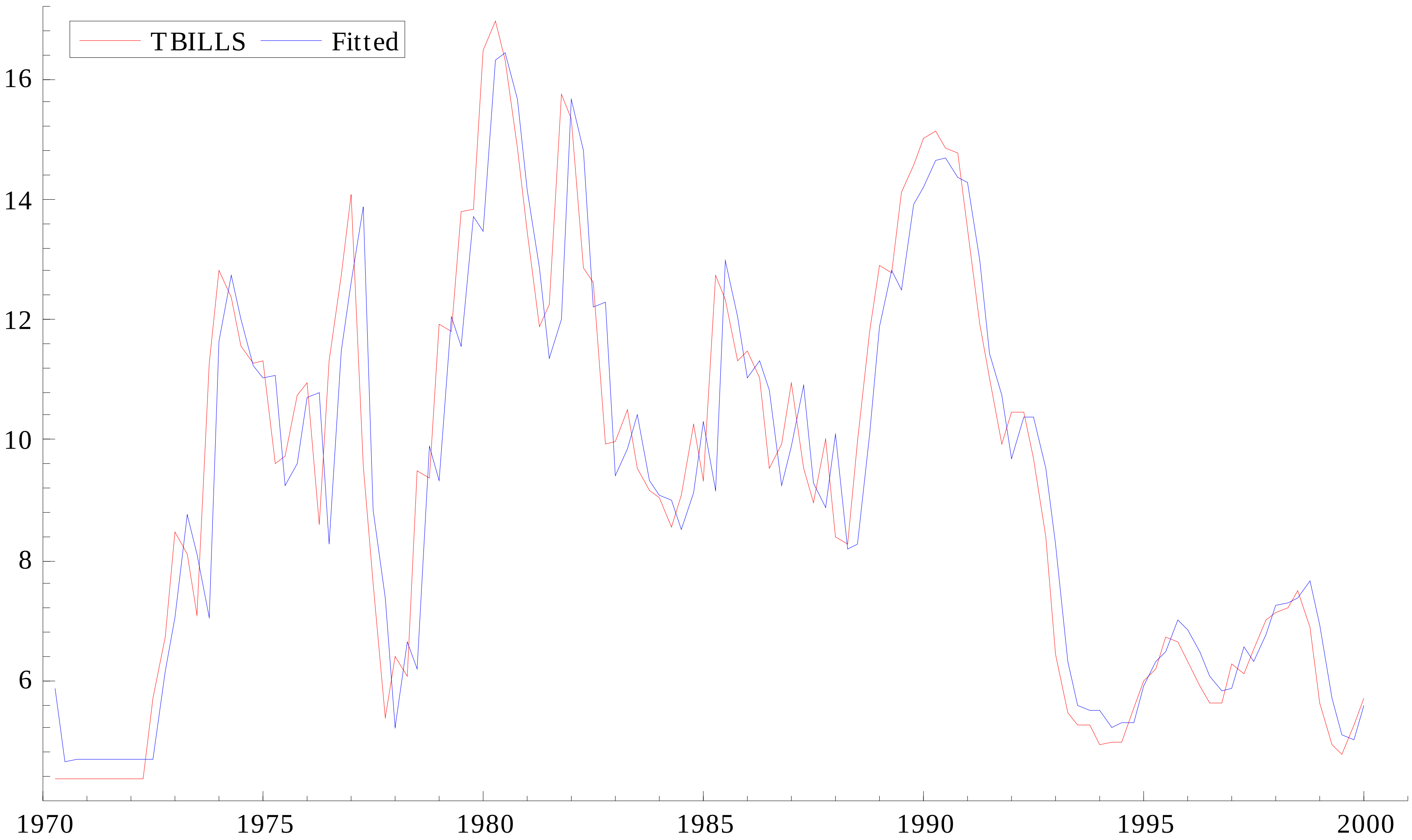
	Output gap	Inflation gap	Constant	R^2
France	-6.641 (-14.778)	0.670 (1.341)	5.900 (1.341)	0.766
Germany	-10.732 (-15.187)	4.335 (4.953)	5.339 (11.898)	0.752
Japan	-6.775 (-6.554)	-1.794 (-7.061)	-1.312 (-3.487)	0.641
UK	-2.941 (-5.885)	1.006 (2.848)	7.416 (10.203)	0.574
USA	-1.794 (-7.061)	0.360 (0.408)	5.337 (18.955)	0.696

Values in the parenthesis represent *t*-statistics.

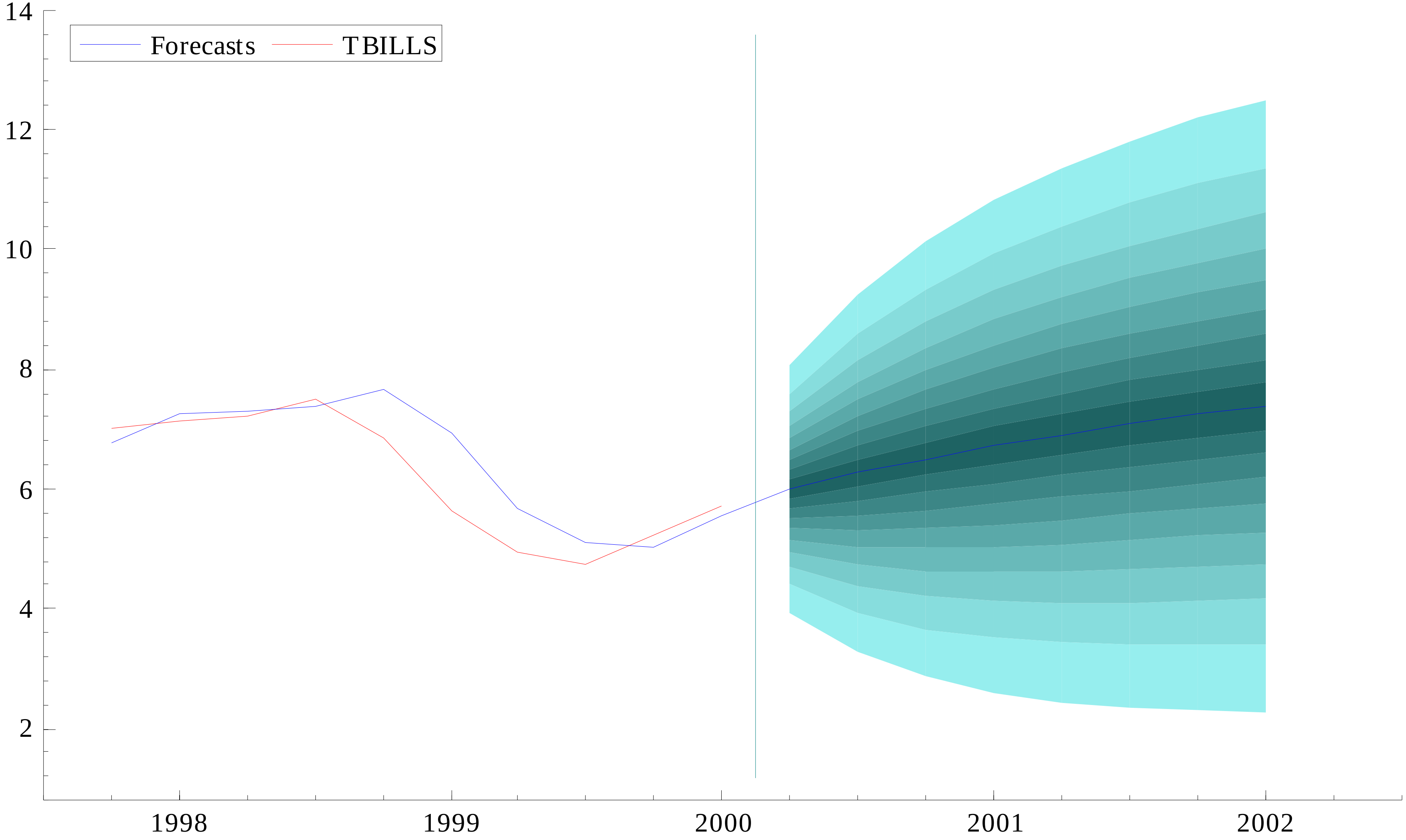
Determinants of Consumption

Explanatory variables	Model 1	Model 2	Model 3
Income	0.70014 (71.826) [0.0000]	0.69521 (78.300) [0.0000]	0.69235 (32.090) [0.0000]
Interest rate		-1228.9 (-3.1305) [0.0035]	-337.44 (-0.89739) [0.3756]
Constant term	-52639 (-9.5303) 0.0000	-40729 (-6.5229) [0.0000]	-42769 (-3.3835) [0.0018]
R-square	0.9931	0.9946	0.9977
Durbin Watson	0.3356	0.5202	1.3540
F-value	5158.946	3214.926	
ABS(E) ON X (GLEJSER) TEST	0.760 (0.38336)	0.130 (0.93720)	1.433 (0.48835)
Sample size	38	38	38

AR(1,1) Model of Interest Rate



ARMA(1,1) Forecast of Interest Rate in the UK



Paned Data Models

Time series for UK available from NAVIDATA at
http://www.statistics.gov.uk,
http://www.statistics.gov.uk/statbase/tsdintro.asp;
www.HM_Treasury.co.uk www.bankofengland.co.uk,
www.mimas.ac.uk, http://www.inlandrevenue.gov.uk,
www.ft.com; http://www.londonstockexchange.com/def;
London metal exchange: http://www.lme.co.uk/ ;
DTI web site http://www.dti.gov.uk

Cross section data (Major National Surveys):
http://www.data-archive.ac.uk/
International data :http://www.esds.ac.uk/international/
www.oecd.org, http://www.imf.org**; http://www.bis.org/index.htm**
Summers and Heston (Penn World Table), **www.iea.org; www.economagic.com,**
http://www.worldbank.org/data/countrydata/countrydata.html
this data also in Start/Applications/Economics
Food and Agricultural Organisation for commodity prices
http://faostat.fao.org/faostat/collections?version=ext&hasbulk=0.

Working papers

http://rfe.wustl.edu/, www.nber.org,

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