Economic Forecasting

Mean and Variance Forecast in AR, MA, ARMA and ARFIMA Models

(Refer to Carter, Griffith and Hill Chap 20 and PcGive III)

h =1 period ahead Forecast in AR(1) Model

$$y_t = \delta + \theta_1 y_{t-1} + e_t$$

$$y_{T+1} = \delta + \theta_1 y_T + e_{T+1} \qquad e_{T+1} \sim N(0,1)$$

$$\hat{Y}_{T+1} = E(Y_{T+1}) = \delta + \theta_1 y_T$$

Error of Forecast

$$\hat{e}_{T+1} = Y_{T+1} - \hat{Y}_{T+1} = \delta + \theta_1 y_T + e_{T+1} - \delta - \theta_1 y_T = e_{T+1}$$

$$\operatorname{var}\left(\hat{e}_{T+1}\right) = \sigma_e^2$$

h =2 periods ahead Forecast in AR(1) Model

$$Y_{T+2} = \delta + \theta_1 y_{T+1} + e_{T+2} \qquad e_{T+2} \sim N(0,1)$$

$$\hat{Y}_{T+2} = E(Y_{T+1}) = \delta + \theta_1 \hat{y}_{T+1}$$

$$\hat{e}_{T+2} = y_{T+2} - \hat{y}_{T+2} = \delta + \theta_1 y_{T+1} + e_{T+2} - \delta - \theta_1 \hat{y}_{T+1} = e_{T+1} + \theta_1 \left(y_{T+1} - \hat{y}_{T+1} \right)$$

$$= e_{T+1} + \theta_1 \left(e_{T+1} \right)$$

$$\operatorname{var}\left(\hat{e}_{T+1} \right) = \sigma_e^2 \left(1 + \theta_1^2 \right)$$

h period ahead Forecast in AR(1) Model

$$y_{T+h} = \delta + \theta_1 y_{T+h-1} + e_{T+h}$$

$$\hat{y}_{T+h} = E(y_{T+h}) = \delta + \theta_1 \hat{y}_{T+h-1}$$

$$\hat{e}_{T+h} = y_{T+h} - \hat{y}_{T+h} = \delta + \theta_1 y_{T+h-1} + e_{T+h} - \delta - \theta_1 \hat{y}_{T+h-1} = e_{T+h} + \theta_1 \left(y_{T+h-1} - \hat{y}_{T+h-1} \right)$$

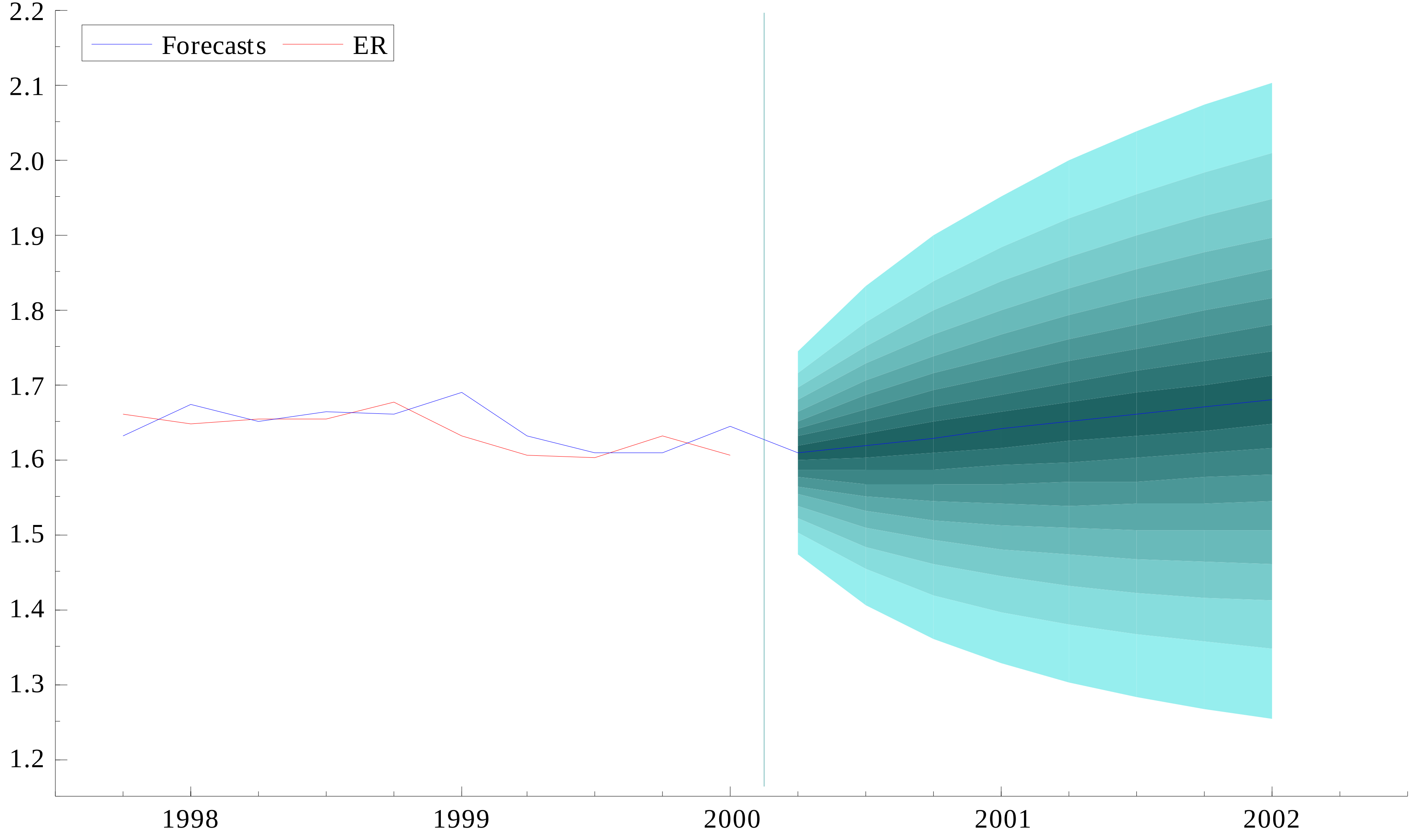
$$\operatorname{var}\left(\hat{e}_{T+h}\right) = \sigma_e^2 \left(1 + \theta_1^2 + \theta_1^4 + \ldots + \theta_1^{2(h-1)}\right)$$

---- Maximum likelihood estimation of ARFIMA(2,d,0) model ----
The estimation sample is: 1970 (3) - 2000 (1)
The dependent variable is: ER (uk_r.xls)

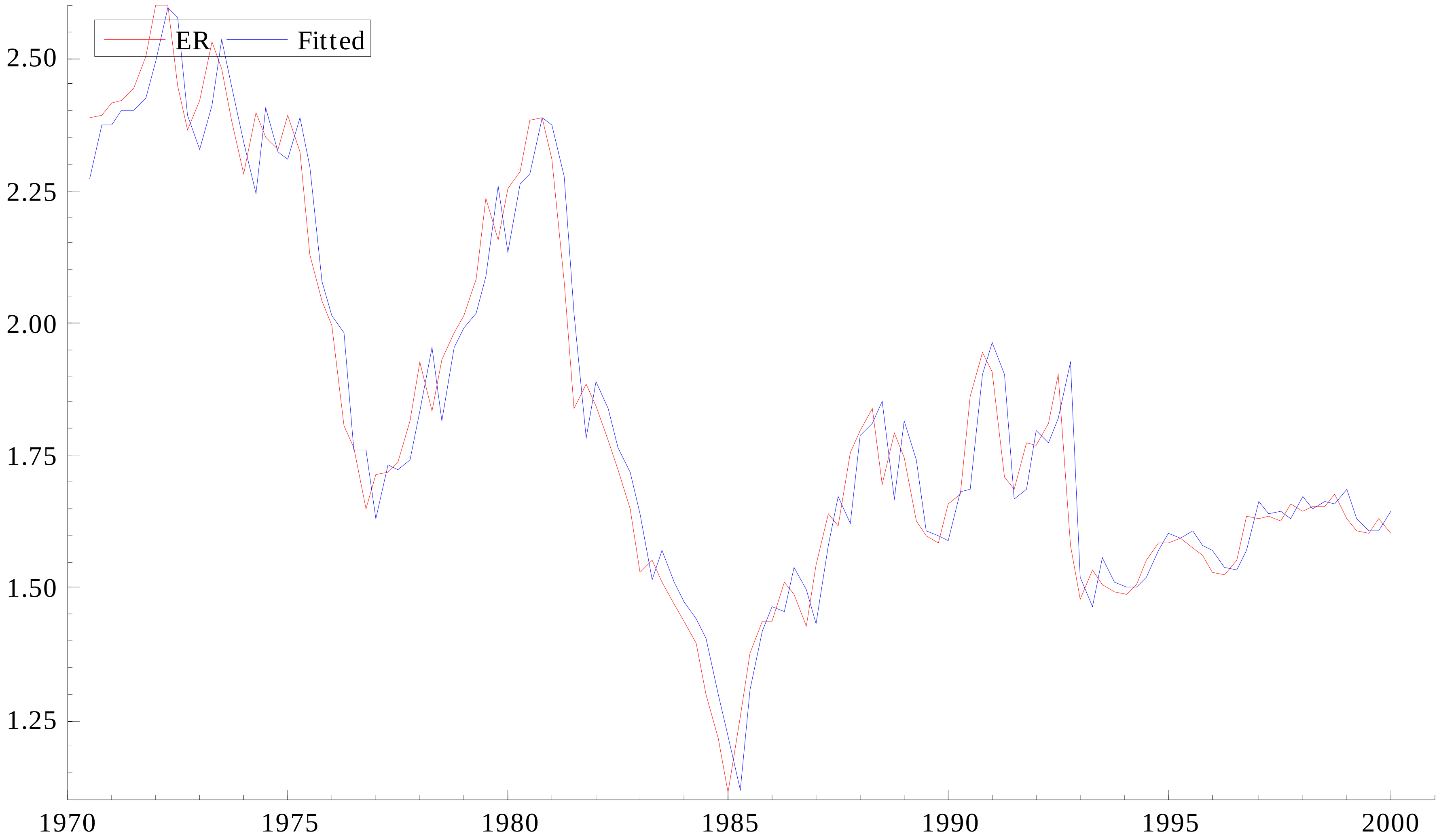
	Coefficient	Std.Error	t-value	t-prob
d parameter	0.0433834	0.2621	0.166	0.869
AR-1	1.14930	0.2840	4.05	0.000
AR-2	-0.191757	0.2352	-0.815	0.417
Constant	1.86839	0.1918	9.74	0.000

log-likelihood	125.731401		
no. of observations	119	no. of parameters	5
AIC.T	-241.462801	AIC	-2.02909917
mean(ER)	1.82648	var(ER)	0.12318
sigma	0.0830594	sigma^2	0.00689886

PcGive AR(2) Forecasts of Exchange Rate



Predicated and actual Values of Forecasts



h =1 period ahead Forecast in MA(1) Model

$$y_t = \mu + e_t + \alpha_1 e_{t-1}$$

$$y_{T+1} = \mu + e_{T+1} + \alpha_1 e_T$$

$$E\left(y_{T+1}\right) = \hat{y}_{T+1} = \mu + \alpha_1 e_T$$

$$\left(y_{T+1} - \hat{y}_{T+1}\right) = \mu + e_{T+1} + \alpha_1 e_T - \mu - \alpha_1 e_T = e_{T+1}$$

$$E\left(y_{T+1} - \hat{y}_{T+1}\right)^2 = \mathrm{var}\left(e_{T+1}\right)^2 = \sigma_e^2$$

h =2 periods ahead Forecast in MA(1) Model

$$y_{T+2} = \mu + e_{T+2} + \alpha_1 e_{T+1}$$

$$E\left(y_{T+2}\right) = \hat{y}_{T+2} = \mu$$

$$\hat{e}_{T+2} = \left(y_{T+2} - \hat{y}_{T+2}\right) = \mu + e_{T+2} + \alpha_1 e_{T+1} - \mu = e_{T+2} + \alpha_1 e_{T+1}$$

$$\operatorname{var}\left(\hat{e}_{T+2}\right) = \operatorname{var}\left(e_{T+2} + \alpha_1 e_{T+1}\right) = \sigma_e^2\left(1 + \alpha_1^2\right)$$

h period ahead forecasts

$$E\left(y_{T+h}\right) = \hat{y}_{T+h} = \mu$$

$$\operatorname{var}\left(\hat{e}_{T+h}\right) = \operatorname{var}\left(e_{T+h} + \alpha_1 e_{T+h-1}\right) = \sigma_e^2\left(1 + \alpha_1^2\right)$$

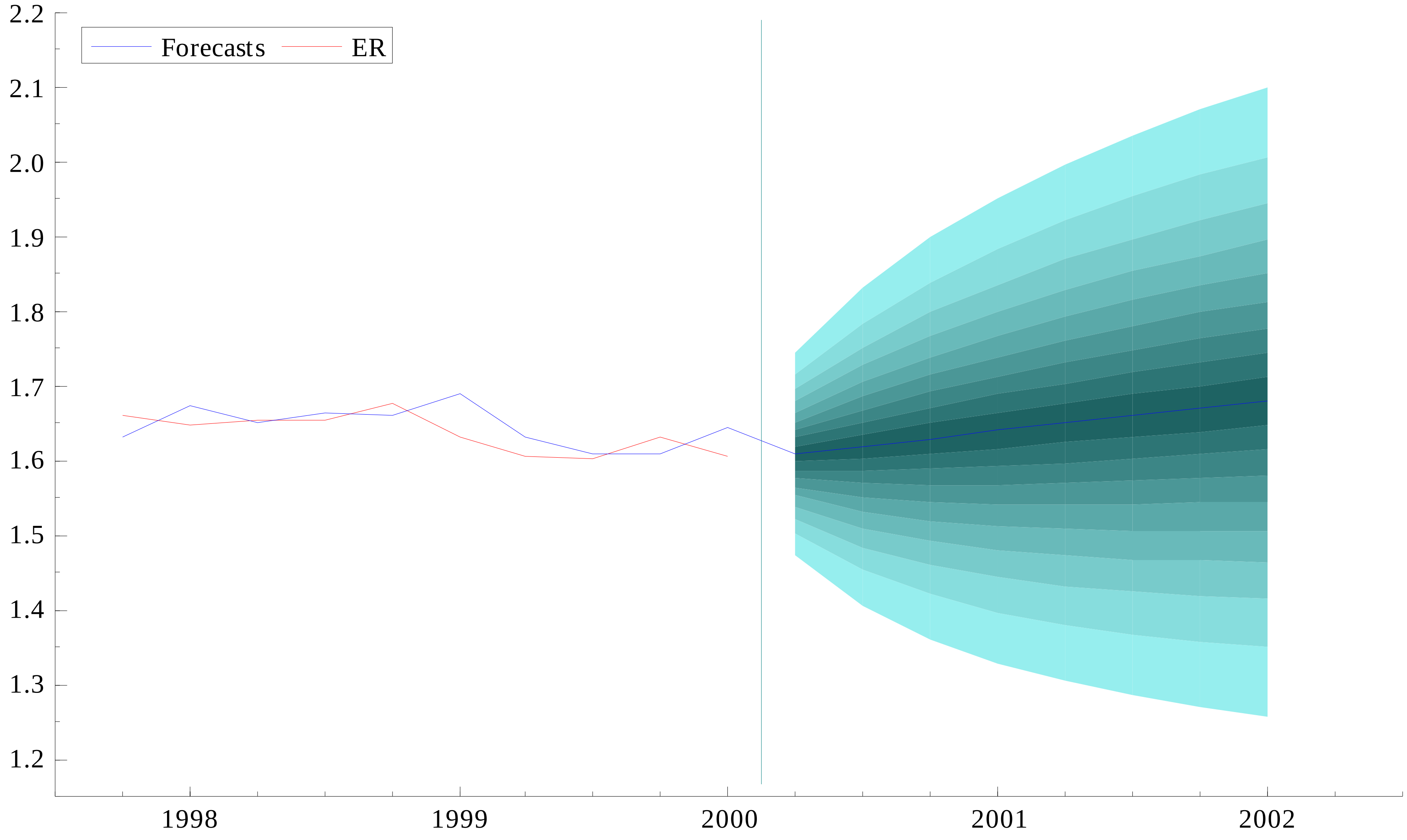
---- Maximum likelihood estimation of ARFIMA(0,d,4) model ----

The estimation sample is: 1970 (3) - 2000 (1)

The dependent variable is: ER (uk_r.xls)

	Coefficient	Std.Error	t-value	t-prob
d parameter	0.464654	0.04509	10.3	0.000
MA-1	0.824885	0.1008	8.18	0.000
MA-2	0.571403	0.1403	4.07	0.000
MA-3	0.388408	0.1284	3.02	0.003
MA-4	0.312203	0.08660	3.60	0.000
Constant	1.88171	0.4767	3.95	0.000

log-likelihood	124.254262		
no. of observations	119	no. of parameters	7
AIC.T	-234.508524	AIC	-1.97065986
mean(ER)	1.82648	var(ER)	0.12318
sigma	0.0834514	sigma^2	0.00696413


2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
Forecasts
ER
1998
1999
2000
2001
2002

h =1 period ahead Forecast in ARMA(1,1) Mode

$$Y_t = \delta + \theta_1 y_{t-1} + e_t + \alpha_1 e_{t-1}$$

$$y_{T+1} = \delta + \theta_1 y_{t-1} + e_{T+1} + \alpha_1 e_T$$

$$E\left(y_{T+1}\right) = \hat{y}_{T+1} = \delta + \theta_1 y_{t-1} + \alpha_1 e_T$$

$$\hat{e}_{T+1} = \left(y_{T+1} - \hat{y}_{T+1}\right) = \delta + \theta_1 y_{t-1} + e_{T+1} + \alpha_1 e_T - \delta - \theta_1 y_{t-1} - \alpha_1 e_T = e_{T+1}$$

$$\operatorname{var}\left(\hat{e}_{T+1}\right) = E\left(y_{T+1} - \hat{y}_{T+1}\right)^2 = \operatorname{var}\left(e_{T+1}\right) = \sigma_e^2$$

h =2 period ahead Forecast in ARMA(1,1) Mode

$$y_{T+2} = \delta + \theta_1 y_{t+1} + e_{T+2} + \alpha_1 e_{T+1}$$

$$E\left(y_{T+2}\right) = \hat{y}_{T+2} = \delta + \theta_1 \hat{y}_{t+1}$$

$$\hat{e}_{T+2} = \left(y_{T+2} - \hat{y}_{T+2}\right) = \delta + \theta_1 y_{t+1} + e_{T+2} + \alpha_1 e_{T+1} - \delta - \theta_1 \hat{y}_{t+1}$$

$$\hat{e}_{T+2} = \theta_1\left(y_{t+1} - \hat{y}_{t+1}\right) + e_{T+2} + \alpha_1 e_{T+1} = \left(\theta_1 + \alpha_1\right) e_{T+1} + e_{T+2}$$

$$\operatorname{var}\left(\hat{e}_{T+1}\right) = \operatorname{var}\left[\left(\theta_1 + \alpha_1\right) e_{T+1} + e_{T+2}\right] = \sigma_e^2\left[\left(\theta_1 + \alpha_1\right)^2 + 1\right]$$

h =3 periods ahead Forecast in ARMA(1,1) Model

$$y_{T+3} = \delta + \theta_1 y_{t+2} + e_{T+3} + \alpha_1 e_{T+2}$$

$$E\left(y_{T+3}\right) = \hat{y}_{T+3} = \delta + \theta_1 \hat{y}_{t+2}$$

$$\hat{e}_{T+3} = \left(y_{T+3} - \hat{y}_{T+3}\right) = \delta + \theta_1 y_{t+2} + e_{T+3} + \alpha_1 e_{T+2} - \delta - \theta_1 \hat{y}_{t+2}$$

$$\hat{e}_{T+3} = e_{T+3} + \alpha_1 e_{T+2} + \theta_1\left(y_{t+2} - \hat{y}_{t+2}\right) = e_{T+3} + \alpha_1 e_{T+2} + (\theta_1 + \alpha_1) e_{T+1} + e_{T+2}$$

$$\hat{e}_{T+3} = e_{T+3} + \alpha_1 e_{T+2} + \theta_1\left(y_{t+2} - \hat{y}_{t+2}\right) = e_{T+3} + \alpha_1 e_{T+2} + (\theta_1 + \alpha_1) e_{T+1} + e_{T+2}$$

$$\operatorname{var}\left(\hat{e}_{T+3}\right) = \operatorname{var}\left(e_{T+3} + \alpha_1 e_{T+2} + (\theta_1 + \alpha_1) e_{T+1} + e_{T+2}\right) = \sigma_e^2\left[1 = \left((1+\alpha_1)^2 + (\theta_1 + \alpha_1)^2\right)\right]$$

---- Maximum likelihood estimation of ARFIMA(2,d,2) model ----

The estimation sample is: 1970 (3) - 2000 (1)

The dependent variable is: ER (uk_r.xls)

	Coefficient	Std.Error	t-value	t-prob
d parameter	0.491109	0.01253	39.2	0.000
AR-2	0.729690	0.09313	7.84	0.000
MA-2	-0.256079	0.1294	-1.98	0.050
Constant	1.89002	1.145	1.65	0.101

log-likelihood	99.8674294		
no. of observations	119	no. of parameters	5
AIC.T	-189.734859	AIC	-1.59441058
mean(ER)	1.82648	var(ER)	0.12318
sigma	0.101928	sigma^2	0.0103893

Forecasts of Exchange Rate from ARMA(2,d,2) Model