THE BUSINESS SCHOOL

26207

Microeconomics

Lecture Outlines and Problems

(Sept 25-December 13, 2006)

Level: 5
Semester: 1
Credits: 20

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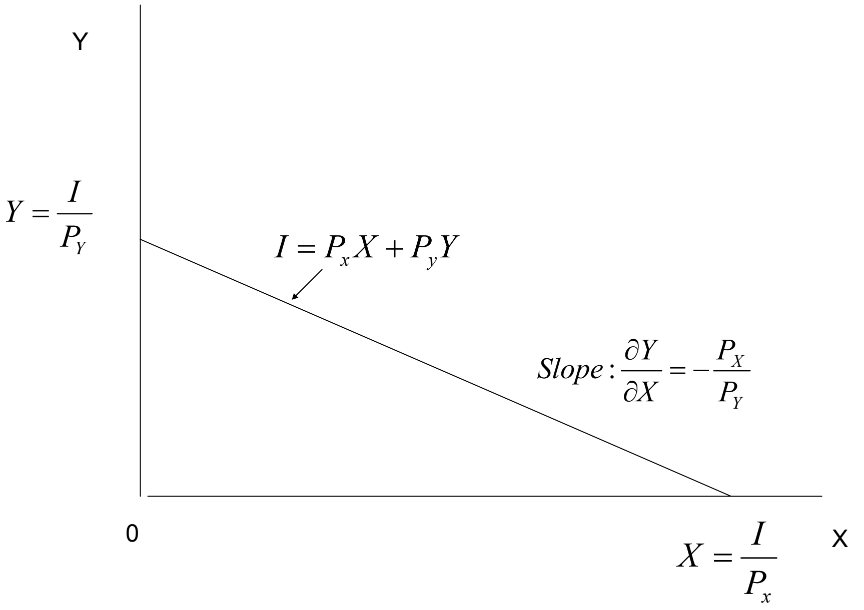
Lecture 1
Budget constraint

A person consumes X and Y. His/her budget constraint is

$$I = P_x X + P_y Y$$

where I is income P_x and P_y, are prices of commodities X and Y.



How many units of X can he/she buy?

$$X = \frac{I}{P_x} - \frac{P_y}{P_x} Y$$

How many units of Y can he/she buy?

$$Y = \frac{I}{P_Y} - \frac{P_X}{P_Y} X$$

Draw a diagram and determine feasible and infeasible sets.

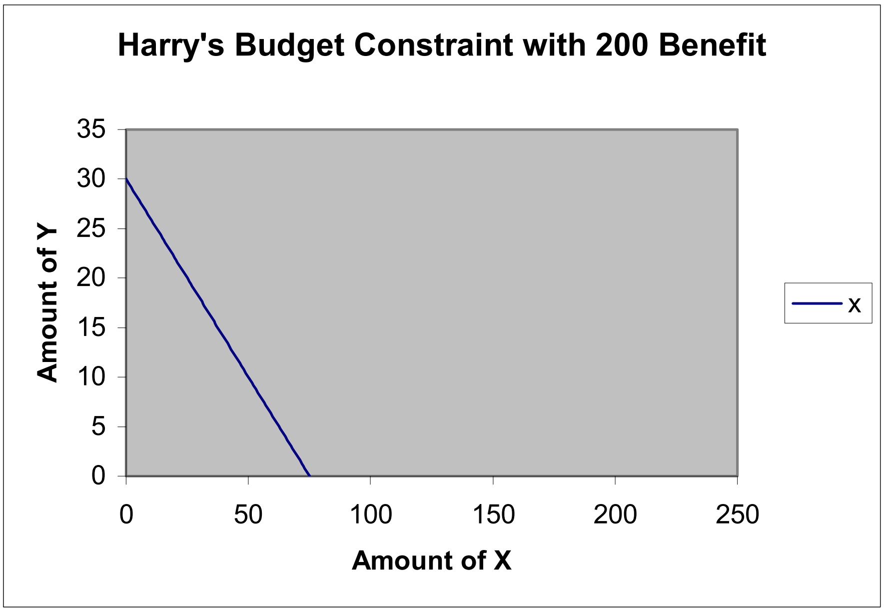
Numerical Example:

$$100 = 4X + 10Y$$

Fill the blanks in the following Table for this budget constraint

	1	2	3	4	5	6	7	8	9	10
X	1	2	4	7	10	14	18	20	22	25
Y										

What is price ratio? Draw the diagram.



Key questions: How does economic power relate to the budget constraint?

How does consumer expenditure survey provide information about the budget constraints of different categories of households? http://www.data-archive.ac.uk/

In real life people buy N number of commodities. If N = 123 how does this budget constraint look like?

$$I = P_1X_1 + P_2X_2 + ... + ... + P_{123}X_{123} = \sum_{i=1}^{N} P_iX_i$$

Readings: VAR 2, MK4,

Lecture 2
Slope and intercepts of a budget line

In budget line $I = P_x X + P_y Y$ what is X intercept? What is Y intercept? What the economic meaning of these two?

X intercept: $X = \frac{I}{P_x}$; Y intercept: $Y = \frac{I}{P_Y}$

Define the slope of the budget line.

$$Y = \frac{I}{P_Y} - \frac{P_X}{P_Y}X \quad \frac{\partial Y}{\partial X} = -\frac{P_X}{P_Y}$$

What does it mean when the slope is greater than one? Is X cheaper than Y?

What does it mean when the slope is less than one? Is Y cheaper than X?

What is the slope in $200 = 5X + 10Y$?

$$Y = \frac{200}{P_Y} - \frac{P_X}{P_Y}X \quad Y = \frac{200}{10} - \frac{5}{10}X \quad \frac{\partial Y}{\partial X} = -\frac{P_X}{P_Y} = \frac{5}{10} = 0.5$$

One unit of X buys half of Y.

How does slope of budget line change when X is cheaper? Say if the price of X reduces to 2.

$\frac{\partial Y}{\partial X} = -\frac{P_X}{P_Y} = \frac{2}{10} = 0.2$ One X buys only 1/5th of Y.

How does slope of budget line change when the price of X is 2 and Y becomes twice as expensive than it was before, i.e. when P_y is 20?

$\frac{\partial Y}{\partial X} = -\frac{P_X}{P_Y} = \frac{2}{20} = 0.1$ One X buys only 1/10th of Y.

Think of a car and agricultural crops.

Draw the original and new budget lines. Indicate which points have become feasible and which have become infeasible?

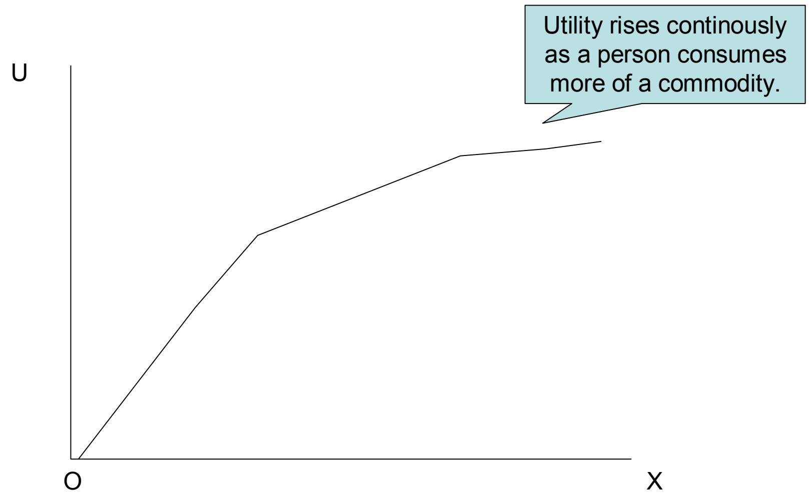
Key questions: Consider expenses in housing and transport. Is real income of people living in small villages higher than those of similar income living in urban areas?

Readings: VAR2, MK4

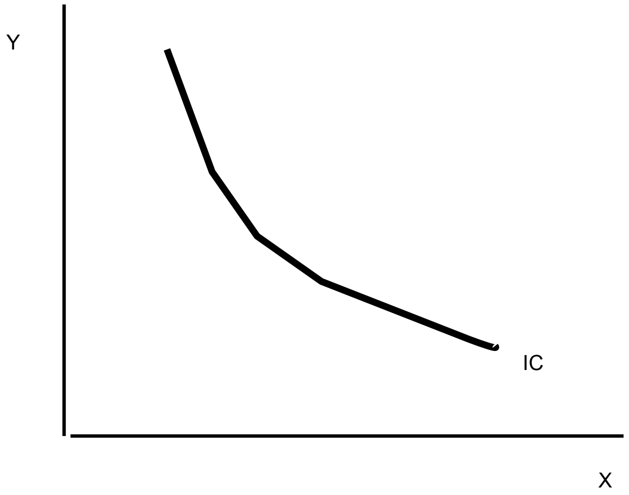
Lecture 3
Preferences

What is the meaning of preferences and utility? People are rational. Their economic choices are consistent, complete and transitive. Non-satiation: people like more and more; never satisfied.

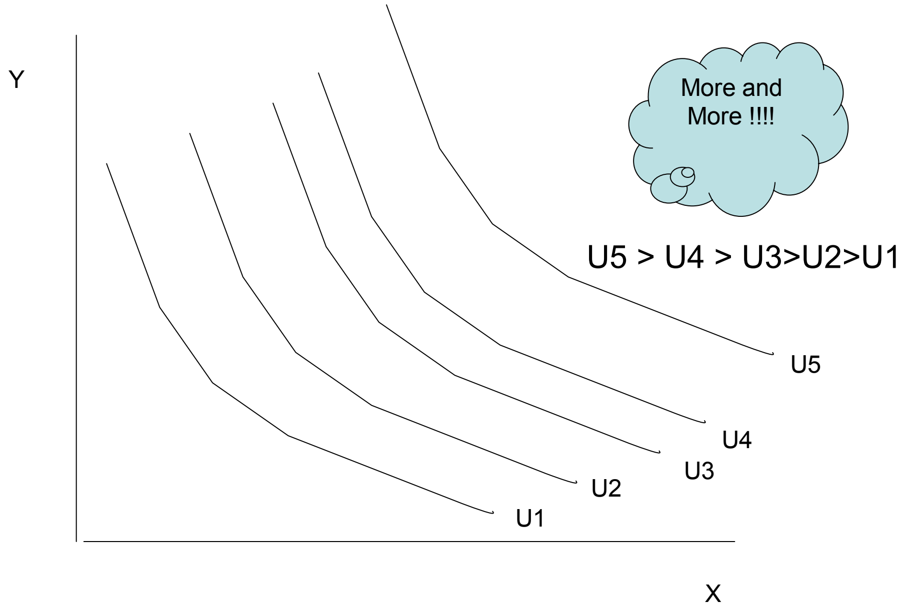
What is the meaning of utility? How does it relate to consumer psychology and characteristic of a commodity?



Draw (X,Y) diagram and show points that are more desirable and less desirable? Indifference curve. What does it show? Why is the consumer indifferent along this curve?

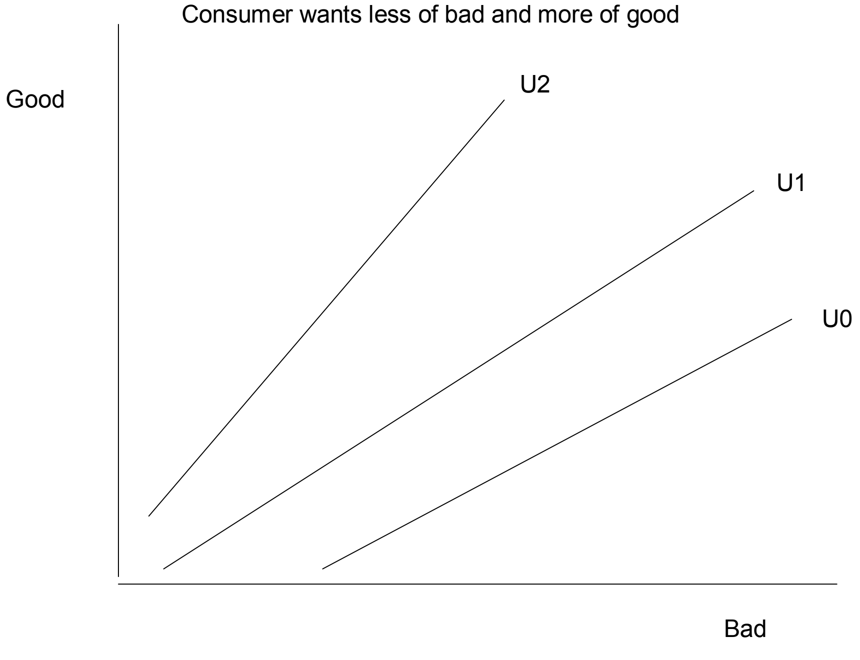


Draw families of indifference curves. Where do you like to be? Is that possible?

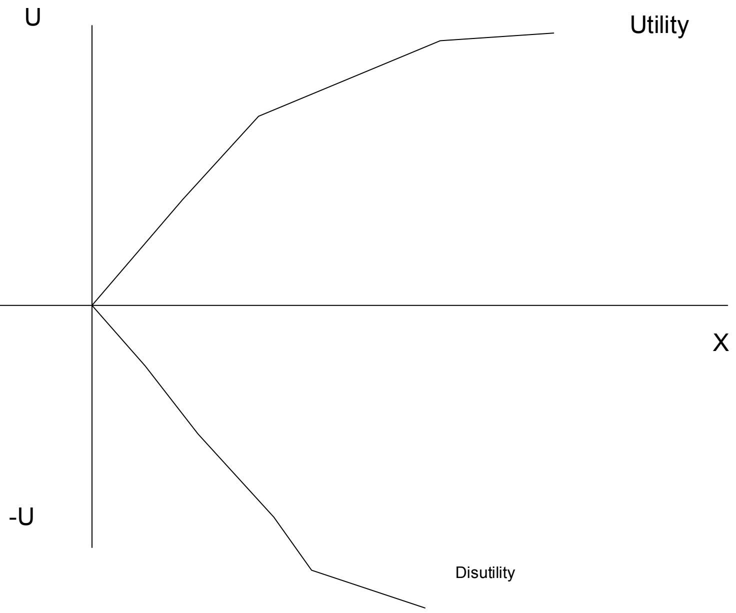


U5 > U4 > U3>U2>U1

Consumer wants less of bad and more of good



How can total utility be represented from good that has both good and bad characteristics? Energy and pollution; sweet and obesity



Driving and pollution; production and pollution.

Can one compare utilities of two individuals? How representative is a representative consumer?
Key questions: Can we rank individuals with their level of income and utilities? What is the meaning of money metric utility?
A consumer consume n number of goods X_1, X_2 …… X_{123}

Utility $U(X_1, X_2, ..., X_{123})$

Optimality condition $\frac{MU_{X1}}{P_{X1}} = \frac{MU_{X2}}{P_{X2}} = ... = \frac{MU_{X123}}{P_{X123}}$

Marginal utility from consumption

	MUX	MUY
1	25	18
2	20	14
3	12	9
4	3	4
5	-6	-2
6	-20	-6

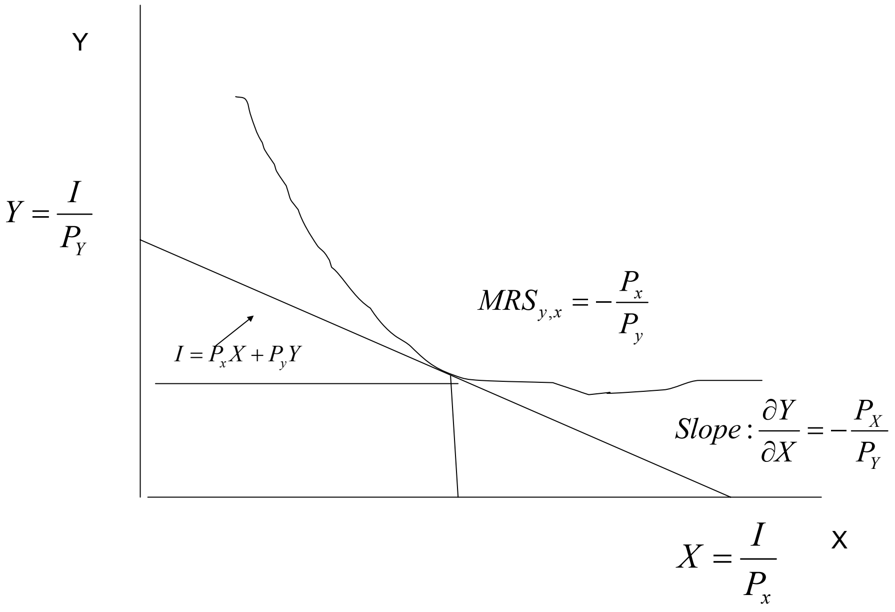
Questions

a) If the price of X is 5 and price of Y is 3 how many X and how my Y should the consumer buy if her budget is 11? What will be the total utility?
b) How much of X and Y should this consumer buy in those prices she has 24 to spend?

Readings: VAR3, MK4

Lecture 4
Consumer Optimisation: demand

Put the budget constraint and the indifference curves together.



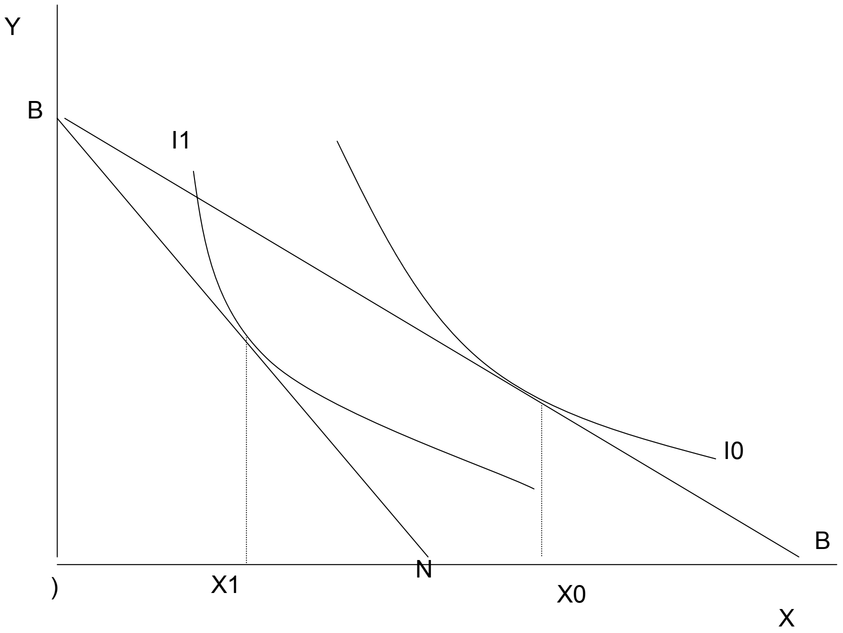
What is meaning of equilibrium and optimal choice?

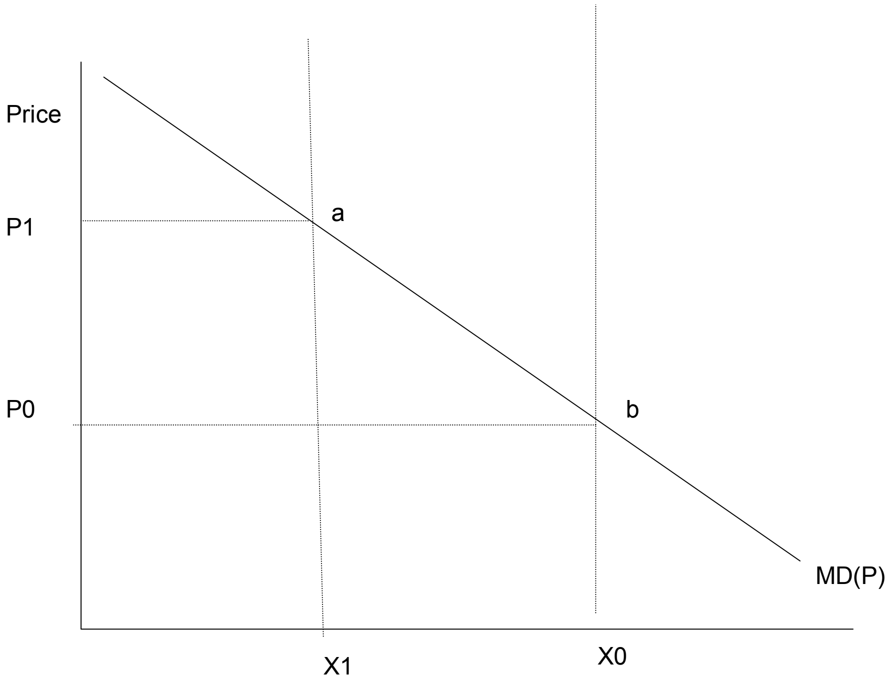
How does this optimal change when X is cheaper than before?

How does the optimal change when X is even cheaper?

Derive a demand curve that shows the amount demanded at each price. What are the properties of this demand curve?

Good Y becomes more expensive: Consumer buys X1 instead of X0





Derivation of a demand curves

Main Properties:

1. A demand curve is downward sloping, shows people buy more when goods are cheaper.
2. It represents optimal choices given those prices as seen from the constrained optimisation process as shown in above diagram.
3. It can shift up or down wards with other exogenous factors such as income or preferences.

How does a price subsidy to Y changes the shape of the budget line and optimal choice and the demand function?

Four numerical problems on deriving the individual demand functions from constrained optimisation.

1. Demand for X and Y, two normal goods

$Max \quad U = X^{\alpha} Y^{\beta}$

subject to $\quad I = P_x X + P_y Y$

$$X > 0; Y > 0; U > 0 \quad P_x \geq 0; \;\; P_y \geq 0; \;\; I \geq 0; \;\; \alpha + \beta = 1 \quad \alpha > 1 \;\; \beta > 1$$

Lagrange function for constrained maximisation

$$L(X, Y, \lambda) = X^{\alpha} Y^{\beta} + \lambda \left[I - P_x X - P_y Y \right]$$

Three first order conditions for maximization

(1) $\dfrac{\partial L(X, Y, \lambda)}{\partial X} = \alpha X^{\alpha - 1} Y^{\beta} - \lambda P_x = 0$

(2) $\dfrac{\partial L(X, Y, \lambda)}{\partial Y} = \beta X^{\alpha} Y^{\beta - 1} - \lambda P_Y = 0$

(3) $\dfrac{\partial L(X, Y, \lambda)}{\partial \lambda} = I - P_x X - P_y Y = 0$

Dividing (1) by (2) and rearranging the equations

(4) $\dfrac{\alpha X^{\alpha-1} Y^{\beta}}{\beta X^{\alpha} Y^{\beta-1}} = \dfrac{P_x}{P_Y}$ or $\dfrac{\alpha Y}{\beta X} = \dfrac{P_x}{P_Y}$ or $P_x X = \dfrac{\alpha P_Y Y}{\beta}$

Substituting this result in (3)

$$I - P_x X - P_y Y = 0 \quad I = P_x X + P_y Y \quad I = \frac{\alpha P_Y Y}{\beta} + P_y Y \quad I = \left(\frac{\alpha + \beta}{\beta} \right) P_Y Y$$

$I = \left(\dfrac{1}{\beta} \right) P_Y Y \qquad Y = \dfrac{\beta \cdot I}{P_Y}$ This is the demand for Y.

For demand for X substitute Y in $P_x X = \dfrac{\alpha P_Y Y}{\beta} \quad X = \dfrac{\alpha P_Y}{P_x \beta} \dfrac{\beta \cdot I}{P_Y} \quad X = \dfrac{\alpha \cdot I}{P_x}$

Demands derived from Cobb-Douglas utility functions

$$Y = \frac{\beta \cdot I}{P_Y} \;, \quad X = \frac{\alpha \cdot I}{P_x}$$

Level of Utility implied by optimal choice $U = X^{\alpha} Y^{\beta} = \left(\frac{\alpha \cdot I}{P_x}\right)^{\alpha} \left(\frac{\beta \cdot I}{P_Y}\right)^{\beta}$

If some one has $\alpha = 0.6 \;\; \beta = 0.4 \;\; I = 500$ and $P_x = 5; \;\; P_y = 10$ then

$$X = \frac{\alpha \cdot I}{P_x} = \frac{0.6 \times 500}{5} = 60; \;\; Y = \frac{\beta \cdot I}{P_Y} = \frac{0.4 \times 500}{10} = 20;$$

$$U = X^{\alpha} Y^{\beta} = \left(\frac{\alpha \cdot I}{P_x}\right)^{\alpha} \left(\frac{\beta \cdot I}{P_Y}\right)^{\beta} = (60)^{0.6} (20)^{0.4} = 38.66$$

$$I = P_x X + P_y Y = 5 \times 60 + 10 \times 20 = 500$$

2. Optimal choice of consumption and leisure

$$Max \quad U = C^{\alpha} l^{\beta}$$

subject to $P \cdot C + w \cdot l = w \cdot L$

$$C > 0; l > 0; U > 0 \quad P \geq 0; \;\; w \geq 0; \;\; L \geq 0; \;\; \alpha + \beta = 1 \quad \alpha > 1 \;\; \beta > 1$$

Further assume that $\alpha = 0.7 \;\; \beta = 0.3 \quad L = 68$ and $P = 20 \;\; P_y = 10 \quad w = 10$ then

$$C = \frac{\alpha \cdot I}{P} = \frac{0.7 \times 10 \times 68}{20} = 0.7 \times 34 = 23.8$$

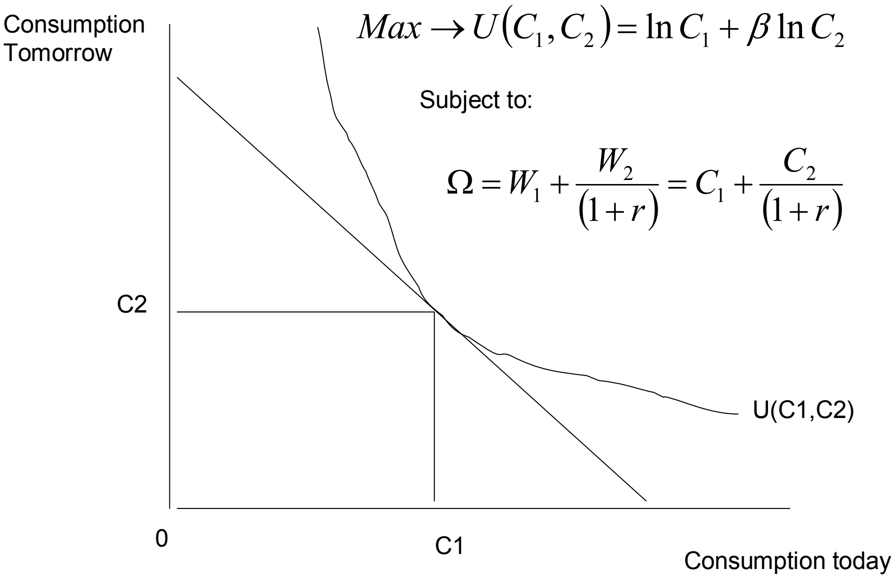
$$l = \frac{\beta \cdot wL}{w} = \frac{0.3 \times 10 \times 68}{10} = 20.4$$

$$U = C^{\alpha} l^{\beta} = \left(\frac{\alpha \cdot wL}{P}\right)^{\alpha} \left(\frac{\beta \cdot wL}{w}\right)^{\beta} = (23.8)^{0.7} (20.4)^{0.3} = 22.72$$

$$P \cdot C + w \cdot l = w \cdot L \quad 20 \times 23.8 + 10 \times 20.4 = 10 \cdot 68; \;\; 476 + 204 = 680$$

3. Choice on consumption today and saving for tomorrow

A consumer lives for two periods and has income of 400 and 800 in the first and second periods respectively. He/she values consumption of both periods equally. What would be the optimal value of consumption in the first and the second periods, (i) at zero rate of the real interest? or (ii) at 10 percent rate of real interest?



$$Max \rightarrow U\left(C_1, C_2\right) = \ln C_1 + \beta \ln C_2$$

Subject to:

$$\Omega = W_1 + \frac{W_2}{\left(1+r\right)} = C_1 + \frac{C_2}{\left(1+r\right)}$$

When r =0 and β =1

$$C_1 = \frac{1}{2}\Omega = \frac{1}{2}\left((400+800) = 1200\right) = 600$$

$$C_2 = \frac{1}{2}\Omega = \frac{1}{2}\left(1200\right) = 600$$

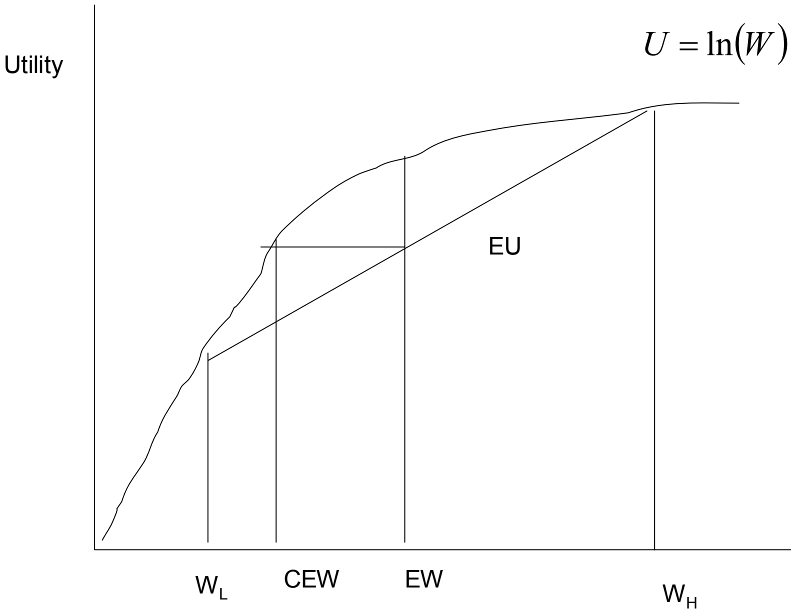
When r = 0.1 and β =1 $C_1 = \frac{1}{2}\Omega = \frac{1}{2}\left((400+\frac{800}{10.1}) = 1127.3\right) = 563.6$

$$C_2 = \frac{1}{2}\Omega = \frac{1}{2}\left((400+\frac{800}{10.1}) = 1127.3\right) = 563.6$$

4. Choice under uncertainty and risk and insurance

Utility from wealth for a person living in Fairfield village is given by $U = \ln(W)$, where U is the utility and W is the level of wealth. This person has a prospect of good income of 4000 with probability 0.4 and of bad prospect of low income of 1000 with probability of 0.6. How much would this person pay to insure against income uncertainty?

A) 459.38 B) 100.03 C) 50.02 D) 30.02



First find the expected wealth $EW = \pi_L W_L + (1-\pi_H)W_H$, then find utility from expected wealth $UEW = \ln(EU)$. Then find the certainty equivalent wealth by solving $UEU = \ln(CEW)$. Finally the insurance is the difference between the expected wealth and certainty equivalent wealth. $EW = 0.4(4000) + 0.6(1000) = 2200$ $EU = 0.4\ln(4000) + 0.6\ln(1000) = 7.462$; $UEW = \ln(EU) = \ln(2200) = 7.692$; Given $\ln(CEW) = 7.462$ find $CEW = e^{7.462} = 1740.62$; insurance = 2200-1740.62 = 459.38

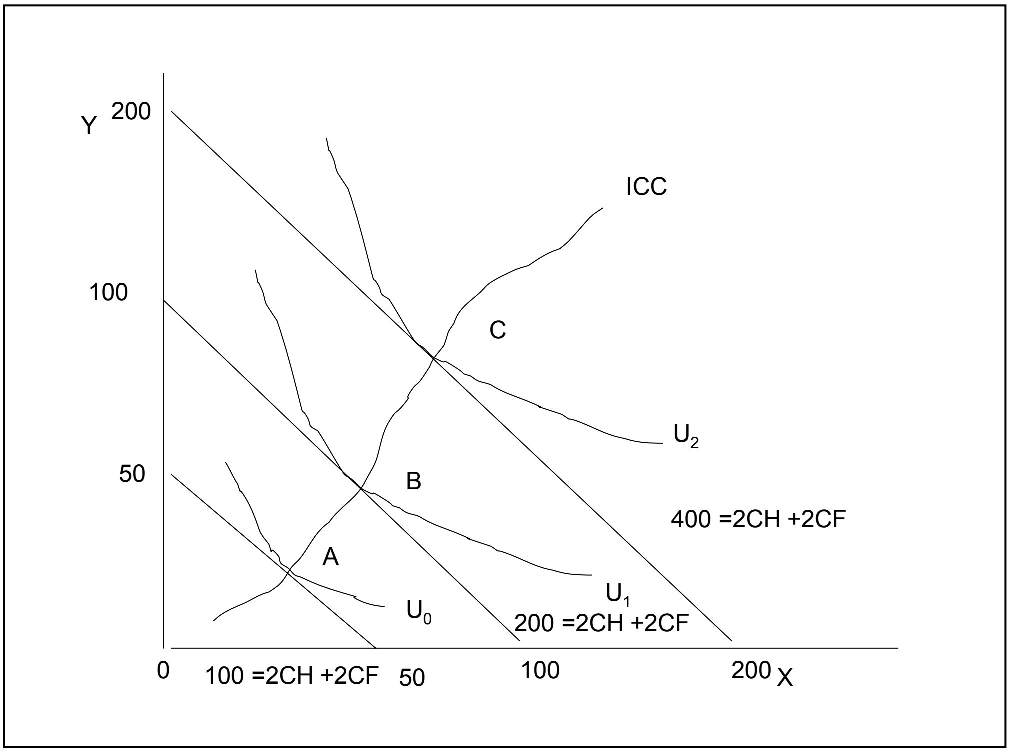
Key questions: Why is consumer the king of a market economy?

Readings: VAR 6, MK5

Lecture 5
Income effect: normal, superior and inferior goods; substitutes and complements

How does optimal consumption of X and Y relate to income?

Kathy consumes chocolate and coffee. Her preference is given by $U = XY$. She had 100 to spend on these two goods last year. This has increased to 200 this year and will rise to 400 next year. Price of chocolate is 2 and price of coffee also is 2. Demonstrate Kathy's income consumption line using diagrams and optimal solutions.



Since Kathy prefers both goods equally and prices of goods are the same her income and consumption patterns are very easy to calculate. She spends 50 percent of income on X and remaining 50 percent on Y. With 100 income she buys 25 of X and 25 of Y, with 200 income she buys 50 of X and 50 of Y and with 400 income she buys 100 of X and 100 of Y.

	100	200	400
X	25	50	100
Y	25	50	100

Her income consumption line can be expressed by $X = \frac{P_x}{P_Y} Y$. This is derived from the optimality condition that requires marginal utility ratios be equal to price ratios.

Income expansion path with homogenous and homethetic preferences? Engel curve.

In above example, if Kathy's income doubles to 800 but the prices of both chocolate and coffee also double to 4, what would be her utility? Will it be the same as when income was 400 and prices were 2?

What makes a commodity a normal good?

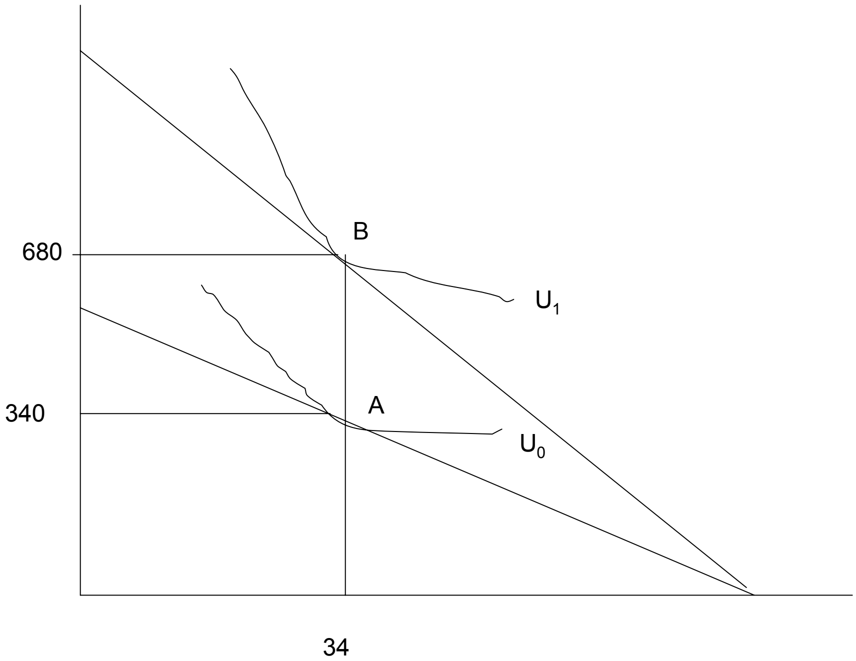
Green has written and published a book on game theory. It is up to date and very comprehensive in theories and examples relating to the game theory.

a) Is this book a normal good for students of economics?
b) Inferior good for a Ted Belly, the carpenter?
c) A snobbish good for Sara, a student in mathematical physics?

Do people have choice between consumption and leisure? Is leisure a normal good?

Able has 68 hours of productive time each week, which he can either spend to work or spend in leisure time in which he can either watch TV, play pool or sleep and relax, $L + l = 68$. He has equal preference for leisure and consumption, $U = C \times l$, where U is utility, C is composite consumption and *l* is leisure. He can get market wage of 10 for his work and price of composite consumption is 1. How many hours are optimal hours for him to work? How many hours are optimal leisure hours for him? What will be his utility? If the wage rate rises to 20, with this change his choice of consumption and leisure hours? Why or why not?

$U = C \times l$



Demonstration:

$$L(C, l, \lambda) = C \times l + \lambda\left[w\overline{L} - PC - wl\right]$$

Three First order conditions for maximization

(1) $\dfrac{\partial L(C, l, \lambda)}{\partial C} = l - \lambda = 0$

(2) $\dfrac{\partial L(C,l,\lambda)}{\partial l} = C - 10\lambda = 0$

(3) $\dfrac{\partial L(C,l,\lambda)}{\partial \lambda} = 680 - C - 10l = 0$

Marginal rate of substitution between leisure and consumption

$\dfrac{l}{C} = \dfrac{1}{10}$ $C = 10l$, and by substituting this into the budget constraint

$680 - C - 10l = 680 - 10l - 10l = 0$ $l = \dfrac{680}{20} = 34$ and $C = 10l = 10 \times 34 = 340$

When the wage rate is 20, $L(C,l,\lambda) = C \times l + \lambda\left[w\overline{L} - PC - wl\right]$

(1) $\dfrac{\partial L(C,l,\lambda)}{\partial C} = l - \lambda = 0$

(2) $\dfrac{\partial L(C,l,\lambda)}{\partial l} = C - 20\lambda = 0$

(3) $\dfrac{\partial L(C,l,\lambda)}{\partial \lambda} = 1360 - C - 20l = 0$

$C = 20l$ and $l = \dfrac{1360}{40} = 34$ but $C = 20l = 20 \times 34 = 680$

When wage rate doubles it allows consumption to double.

How would above results change if he valued consumption more than leisure, $U = C^{0.7} \times l^{0.3}$?

What do snobbish people do?

Key questions: Do rich people take more leisure or do they work more? What about low income people?

How does a particular commodity such a car can be a normal good in one country but a superior good in another country?
Is going to holidays a normal good?

Substitutes and complements

Readings: VAR 6, MK6

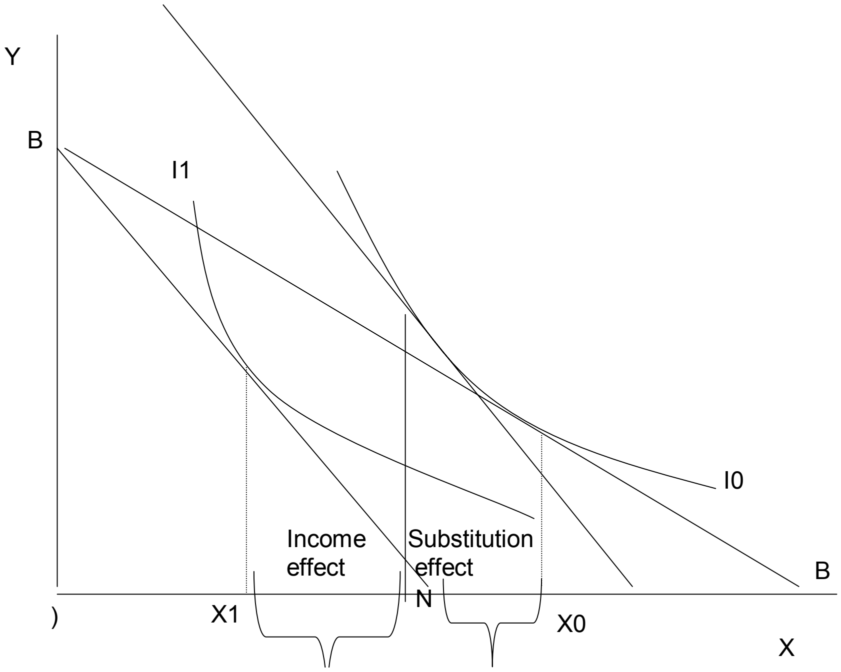
Lecture 6
Substitution and income effects: Hicks and Slutsky

What makes one thing be substituted by another? Are there substitutes for oil, gas or electricity?

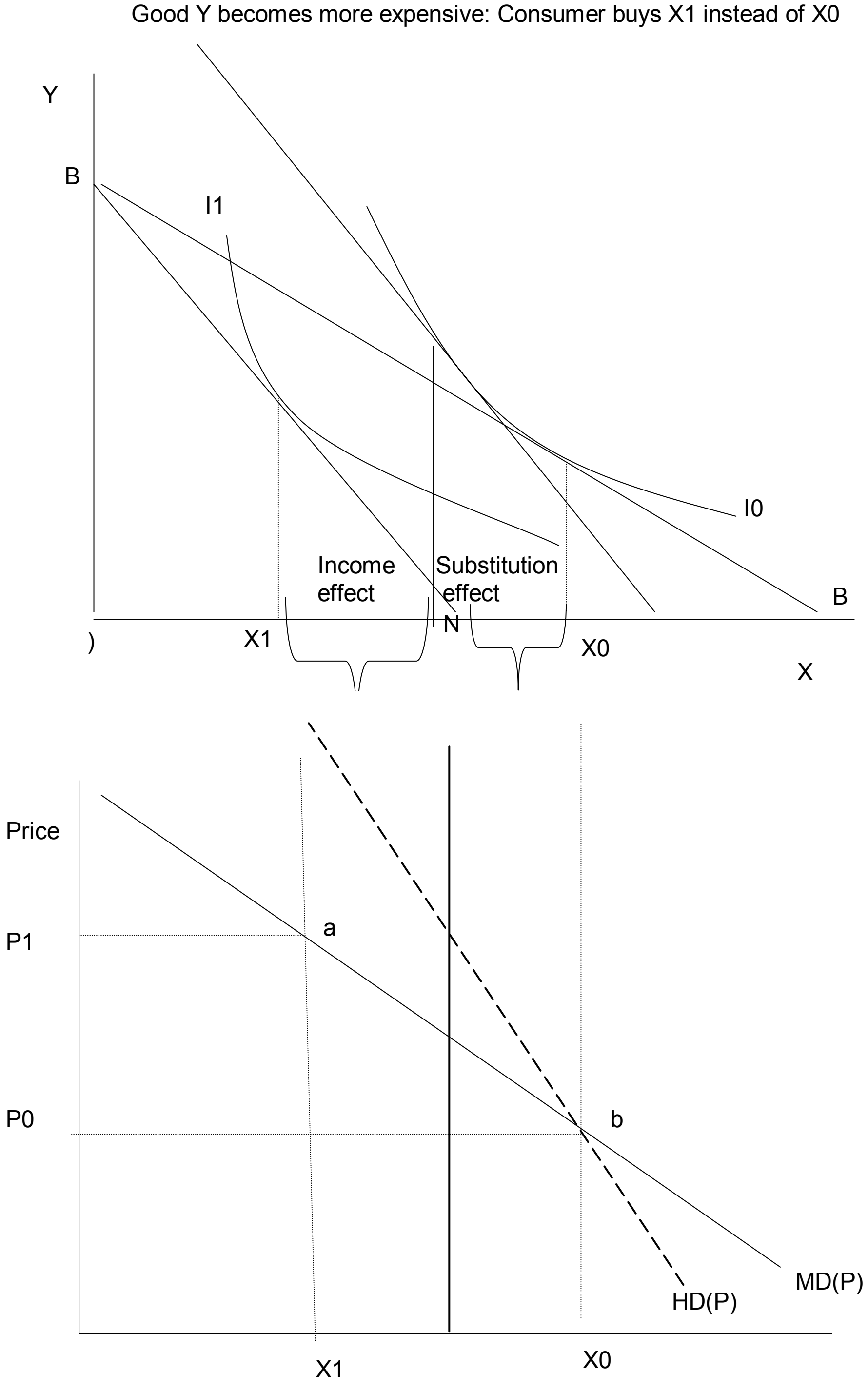
Provide examples good and bad substitutions:
Food:
Drink:
Transportation:
Housing:
Books:
Workers
Players
Singers
Officials

Decomposition of income and price effects: Hicksian equivalent and compensating variation.

Good Y becomes more expensive: Consumer buys X1 instead of X0



Marshallian demand curve and Hicksian compensated demand curve.



Derivation of the Marshallian and Hicksian demand curves

A numerical example

Robinson gets utility from consumption of ice-cream and salads in every lunch. He spends 40 percent of his lunch budget in ice-cream and remaining 60 percent is salads. His utility function can be written as $U_0 = X_1^{0.4} X_2^{0.6}$ where X_1 is amount of icecream and X_2 is amount of salads. Price of ice-cream is 3. The demand function of icecream is

$$X_1(P_1, m,) = \frac{0.4 \times m}{P_1} = \frac{0.4 \times 150}{3} = 20$$

Now the price of ice-cream falls to 2. The demand for ice-cream at the new price with income constant at 150 is

$$X_1(P_1^{'}, m,) = \frac{0.4 \times m}{P_1} = \frac{0.4 \times 150}{2} = 30$$

The change in demand $X_1(P_1, m,) - X_1(P_1^{'}, m,) = 30 - 20 = 10$

How much of this change is due to pure change in price. For, this first find out how much additional money is required to buy the original bundle of icecream. This can be done by multiplying the change in price by the original quantities.

$$\Delta m = (P_1^{'} - P_1)X_1 = (2-3) \times 20 = -20$$

The amount of income to be adjusted for price change is 20, that means (150-20 =130, would be enough to buy the original bundle. Since the price has changed consumer can buy more.

$$X_1(P_1^{'}, m',) = \frac{0.4 \times m'}{P'_1} = \frac{0.4 \times 130}{2} = 26$$

The pure substitution effect is the difference $X_1(P_1^{'}, m',) - X_1(P_1^{'}, m) = 26 - 20 = 6$.
The income effect is $X_1(P_1^{'}, m) - X_1(P_1^{'}, m') = 30 - 26 = 4$

This implies the fall in the price of ice-cream has made Robinson's real income increase of 4.
Key questions: Can everything that can be bought in the market be substituted?

Compensating and equivalent variations:

Base utility $u_0 = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$, with budget $m = p_1 x_1 + p_2 x_2$ if $(p_1, p_2) = (1,1)$ demand functions $x_1 = \frac{m}{2p_1}$, $x_2 = \frac{m}{2p_2}$, given m =100 $(x_1, x_2) = (50,50)$. Now price of good one rises to 2, $(p_1, p_2) = (2,1)$, income remains the same m =100, $x_1 = \frac{m}{2p_1} = \frac{100}{2 \times 2} = 25$, $x_2 = \frac{m}{2p_2} = \frac{100}{2 \times 1} = 50$. How much income need to be compensated to this consumer to maintain at the old level of utility,

$u_0 = 50^{\frac{1}{2}} 50^{\frac{1}{2}} = \left(\frac{m'}{4}\right)^{\frac{1}{2}} \left(\frac{m'}{2}\right)^{\frac{1}{2}} = 50 \Rightarrow$ m '=141, Therefore compensating variation is 141-100=41. Compensating variation is positive for a price rise.

Equivalent variation: How much money should be taken away from the consumer in the original prices to make him/her achieve the utility level after the price change.

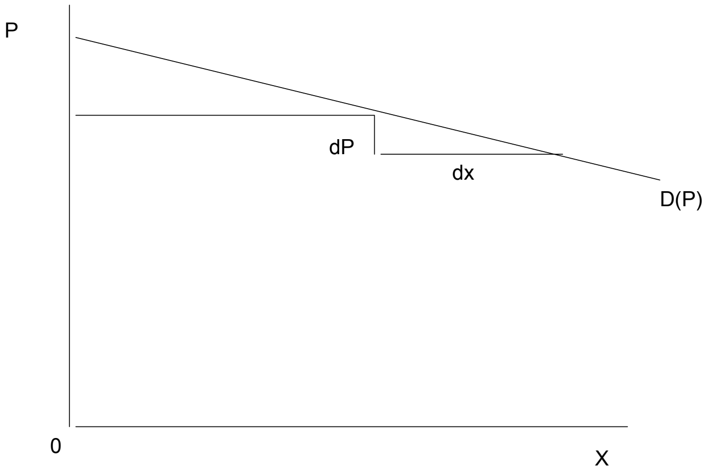
$$u_n = 25^{\frac{1}{2}} 50^{\frac{1}{2}} = \left(\frac{m'}{2}\right)^{\frac{1}{2}} \left(\frac{m'}{2}\right)^{\frac{1}{2}} = 5 \times 5 \times \sqrt{2} \quad m' = 2 \times 5 \times 5 \times \sqrt{2} = 70.7$$

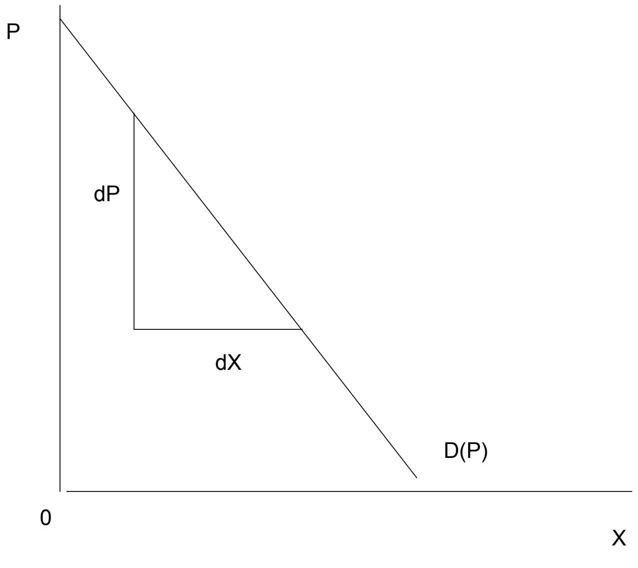
EV = 100-70.7 = 29.3.
Readings: VAR8, MK5

Lecture 7
Elasticity: own price, cross price, income

Draw diagrams for elastic and inelastic demands

Own price





Elasticity: $e_p = (dx/x)/(dp_x/p_x)$

Elasticity: $e_p = (dx/x)/(dp_x/p_x)$

p	10	9	8	7	6	5	4	3	2
x	1	3	8	27	64	125	216	343	512
elasticity		-20.00	-15.00	-19.00	-9.59	-5.72	-3.64	-2.35	-1.48
dp		-1	-1	-1	-1	-1	-1	-1	-1
dx		2	5	19	37	61	91	127	169

0.50 1.33 1.80 2.00 2.08 2.12 2.13 2.13 2.13

Cross price elasticity $e_x = (dx/x)/(dp_y/p_y)$

Py	1	2	3	4	5	6	7	8	9
X	100	150	250	400	600	850	1150	1500	1900
Cross e		0.50	1.33	1.80	2.00	2.08	2.12	2.13	2.13
dp		1	1	1	1	1	1	1	1
dx		50	100	150	200	250	300	350	400

Income elasticity $e_I = (dx/x)/(dI/I)$

Income	100	200	300	400	500	600	700	800	900
Demand	100	600	1600	3100	5100	7600	10600	14100	18100
Income e		5.00	3.33	2.81	2.58	2.45	2.37	2.31	2.27
dI		100	100	100	100	100	100	100	100
Dx		500	1000	1500	2000	2500	3000	3500	4000

What are the determinants of elasticity of demand?
Substitutability; Need; Advertisement; Time

Expenditure function: minimum income required to achieve a certain level of utility.

budget that minimises expenditure $E = p_1 x_1 + p_2 x_2$ subject to $\bar{u} = x_1^{\alpha} x_2^{\beta}$

$$L(x_1, x_2, \lambda) = p_1 x_1 + p_2 x_2 + \lambda\left[\bar{u} - x_1^{\alpha} x_2^{\beta}\right]$$

Three first order conditions for maximization

(1) $$\frac{\partial L(x_1, x_2, \lambda)}{\partial x_1} = p_1 - \lambda \alpha x_1^{\alpha - 1} x_2^{\beta} = 0$$

(2) $$\frac{\partial L(x_1, x_2, \lambda)}{\partial x_2} = p_2 - \lambda \beta x_1^{\alpha} x_2^{\beta - 1} = 0$$

(3) $$\frac{\partial L(x_1, x_2, \lambda)}{\partial \lambda} = \bar{u} - x_1^{\alpha} x_2^{\beta} = 0$$

Dividing (1) by (2) and rearranging the equations

(4) $$\frac{p_1}{p_2} = \frac{\alpha x_1^{\alpha-1} x_2^{\beta}}{\beta x_1^{\alpha} x_2^{\beta-1}} = \frac{\alpha}{\beta}\frac{x_2}{x_1} \quad \text{or} \quad x_2 = \frac{\beta}{\alpha}\frac{p_1}{p_2} x_1 \text{ or}$$

Substituting this result in (3) $\bar{u} = x_1^{\alpha}\left(\frac{\beta}{\alpha}\frac{p_1}{p_2} x_1\right)^{\beta}$ $\quad x_1 = \bar{u}\left(\frac{\alpha}{\beta}\frac{p_2}{p_1}\right)^{\beta}$

$$x_2 = \frac{\beta}{\alpha}\frac{p_1}{p_2} x_1 \quad x_2 = \bar{u}\,\frac{\beta}{\alpha}\frac{p_1}{p_2}\left(\frac{\alpha}{\beta}\frac{p_2}{p_1}\right)^{\beta} = \bar{u}\left(\frac{\beta}{\alpha}\right)^{\alpha}\left(\frac{p_1}{p_2}\right)^{\alpha}$$

Put these values in the expenditure function.

$$E = p_1 x_1 + p_2 x_2 \quad E = p_1 \bar{u}\left(\frac{\alpha}{\beta}\frac{p_2}{p_1}\right)^{\beta} + p_2 \bar{u}\left(\frac{\beta}{\alpha}\right)^{\alpha}\left(\frac{p_1}{p_2}\right)^{\alpha}$$

$$E = \bar{u} p_1^{\alpha} p_2^{\beta}\left(\frac{\alpha}{\beta}\right)^{\beta} + \bar{u} p_1^{\alpha} p_2^{\beta}\left(\frac{\beta}{\alpha}\right)^{\alpha} = \bar{u} p_1^{\alpha} p_2^{\beta}\left[\left(\frac{\alpha}{\beta}\right)^{\beta} + \left(\frac{\beta}{\alpha}\right)^{\alpha}\right]$$

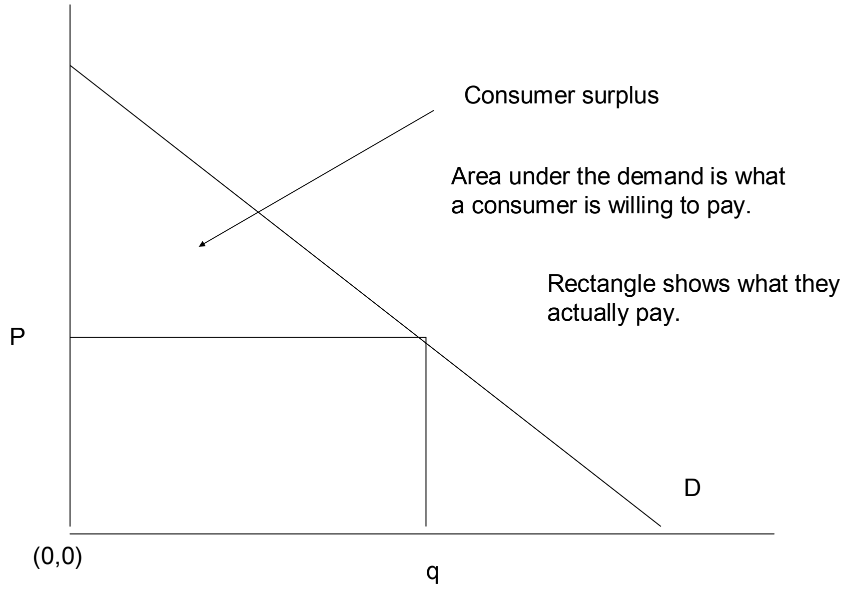
Thus the minimum level of expenditure is function of the level of utility, prices of x_1 and x_2 and the preference parameters α and β.

Key questions: Maximisation of utility is the same as minimisation of expenditure for a target level of utility. Prove this statement using duality theorem in diagrams and derivations.

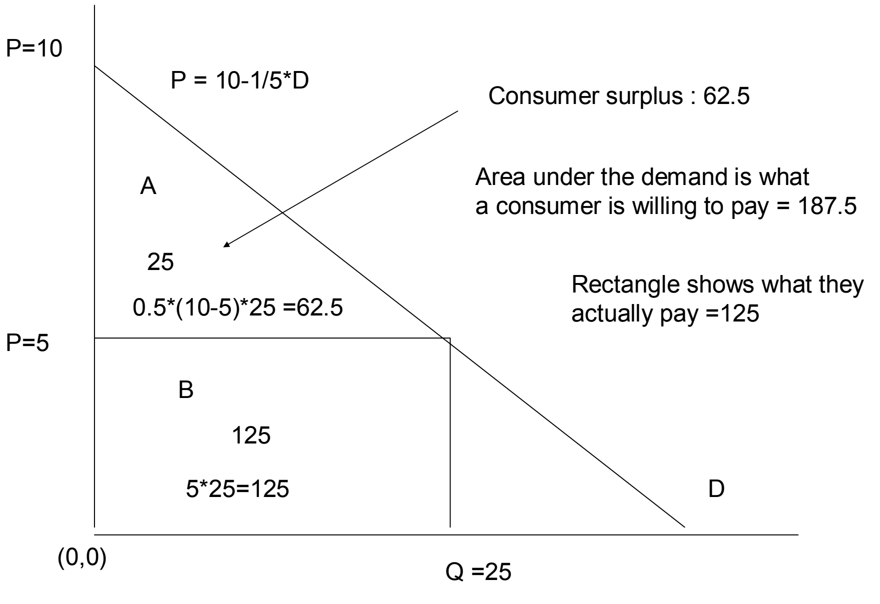
Readings: VAR 15, MK7

Lecture 8
Consumer surplus and deadweight loss

Draw a demand curve and show the difference between how much consumers want to pay and what they actually pay.



Calculate the amount of consumer surplus for a product whose demand equals $D = 50 - 5P$ when the market price is 5?



What is the consumer surplus if $D = 50 - 4P$, $D = 50 - 3P$ $D = 50 - 2P$ $D = 50 - P$?

D = a -bP

B	5	4	3	2	1
A	50	50	50	50	50
P	5	5	5	5	5
Demand	25	30	35	40	45
Willing to pay	187.5	262.5	379.2	600.0	1237.5
Actual payment	125	150	175	200	225
Consumer surplus	62.5	112.5	204.2	400.0	1012.5

When technical innovation reduces prices of computers how does it improve the consumer surplus?

B	5	5	5	5	5
A	50	50	50	50	50
P	5	4	3	2	1
Demand	25	30	35	40	45
Willingness to pay	187.5	210.0	227.5	240.0	247.5
Actual payment	125	120	105	80	45
Consumer surplus	62.5	90.0	122.5	160.0	202.5

Why taxes make economic system inefficient? (Deadweight loss of taxes)

Take regular demand and supply functions D =a-bP and S = -c+dP, and find the equilibrium price and quantity demanded and supplied. $P = \frac{a+c}{b+d}$ $q = \frac{ad+bc}{b+d}$.

Now impose a tax on the commodity. This creates a wedge between the price received by suppliers and price paid by consumers. This distorts the market equilibrium and allocation.

Slightly reformulate above equations reflecting these features.

$$D = a - bP^D$$

$$S = -c + dP^S$$

Price that consumers pay is higher $P^D = P^S + t$ where t represent tax.

Demand equals supply in equilibrium: $a - bP^D = -c + dP^S$
$a - b\left(P^S + t\right) = -c + dP^S$ or $a - bP^S - bt = -c + dP^S$ or $a + c - bt = bP^S + dP^S$

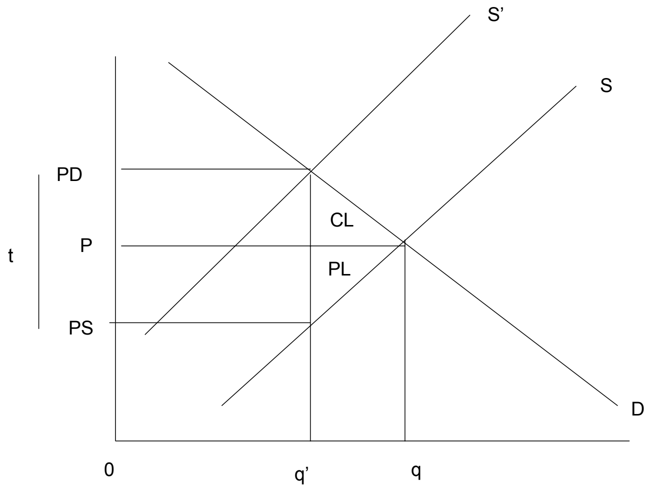
Price received by suppliers: $P^S = \frac{a+c-bt}{b+d}$

Price paid by consumers: $P^D = P^S + t = \frac{a+c-bt}{b+d} + t = \frac{a+c+dt}{b+d}$

The equilibrium quantity in the distorted market is :

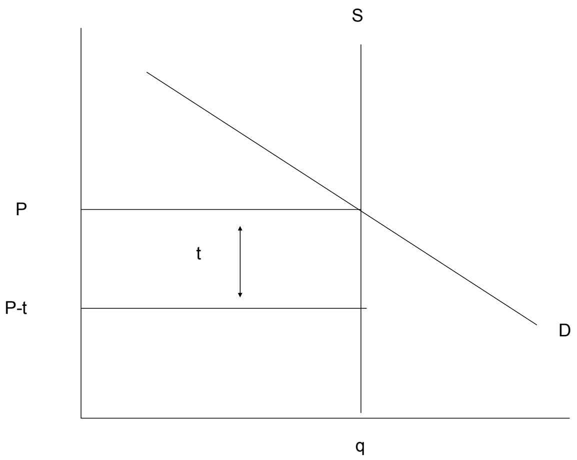
$$D = a - bP^D = a - b\left(\frac{a+c+dt}{b+d}\right) = \frac{ad-bc+bdt}{b+d}$$

Obviously price paid by consumers is higher than the price received by the suppliers, the exact price depends very much on demand and supply side parameters, a, b, c,. d and t.



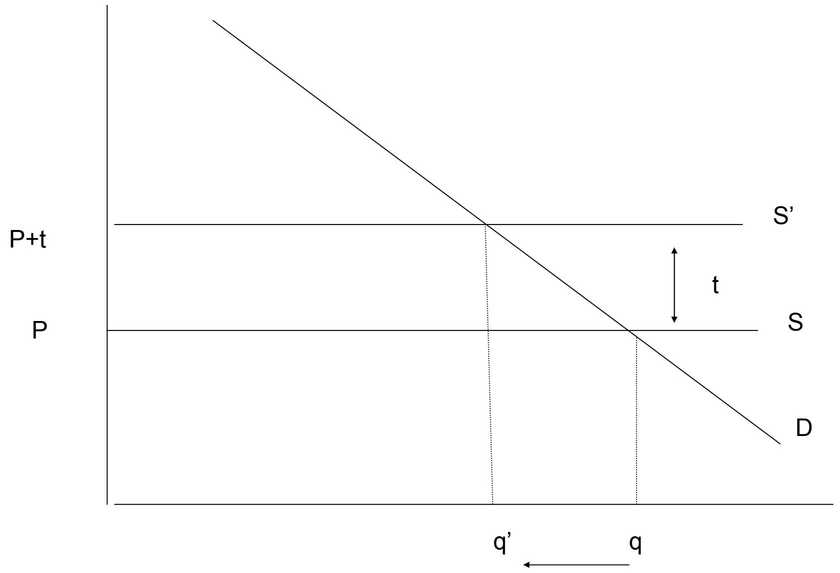
The loss to consumer because of taxes is CL and to producers is PL.. Total deadweight loss is sum of these two. This represents distortions or inefficiency due to taxes.

Who bears this excess burden? This depends on the elasticity of demand and supply. When the supply is perfectly inelastic, the producers bear the burden of whole taxes and when the supply is perfectly elastic all burden is passed onto consumers.



All burden of tax fall on producers her as supply is completely inelastic; raw fish market.

When supply is perfectly elastic all burden of tax falls on to the consumers.



Key questions: Is income tax less distortionary than a tax on sales? Explain how the elasticities of demand are important in analysing the burden of taxes.

Readings: VAR 16

Lecture 9
Production function: input and output

Linear production function: $Y = aL$

Non-Linear production function (Isoquant: locus of capital and labour that gives the same amount of output):

$$Y = K^{\alpha} L^{\beta} \text{ with } \alpha + \beta = 1$$

Marginal product of capital $\frac{\partial Y}{\partial K} = \alpha K^{\alpha-1} L^{\beta} \quad MPK = \alpha \left(\frac{L}{K} \right)^{\beta}$

Marginal product of labour: $\frac{\partial Y}{\partial L} = \beta K^{\alpha} L^{\beta-1} \quad MPK = \beta \left(\frac{K}{L} \right)^{\alpha}$

This is a constant return to scale production function:

$$(\lambda K)^{\alpha} (\lambda L)^{\beta} = \lambda^{\alpha+\beta} K^{\alpha} L^{\beta} = \lambda Y$$

Total output is divided as remunerations to capital and labour when it is paid according to the marginal productivity principle.

$$Y = rK + wL$$

In equilibrium interest rate equals the marginal product of capital and wage rate equals the marginal product of labour

$$r = MPK = \alpha K^{\alpha-1} L^{\beta}$$

$$w = MPL = \beta K^{\alpha} L^{\beta-1}$$

Rate of technical substitution $\frac{MPL}{MPK} = \frac{\partial Y}{\partial L} \Big/ \frac{\partial Y}{\partial K} = \frac{\beta K^{\alpha} L^{\beta-1}}{\alpha K^{\alpha-1} L^{\beta}} = \frac{w}{r}$

$$Y = rK + wL = MPK \times K + MPL \times L = \alpha K^{\alpha-1} L^{\beta} \times K + \beta K^{\alpha} L^{\beta-1} \times L = K^{\alpha-1} L^{\beta} (\alpha + \beta) = K^{\alpha-1} L^{\beta}$$

Elasticity of substitution

$\sigma = \frac{\Delta (K/L)/(K/L)}{\Delta (w/r)/w/r}$ in $Y = K^{\alpha} L^{\beta}$ $\frac{w}{r} = \frac{\beta K^{\alpha} L^{\beta-1}}{\alpha K^{\alpha-1} L^{\beta}} = \frac{K}{L}$ $\sigma = \frac{\Delta (K/L)/(K/L)}{\Delta (K/L)/(K/L)} = 1$

A quadratic production function
Short run and long run
Total product

Marginal product

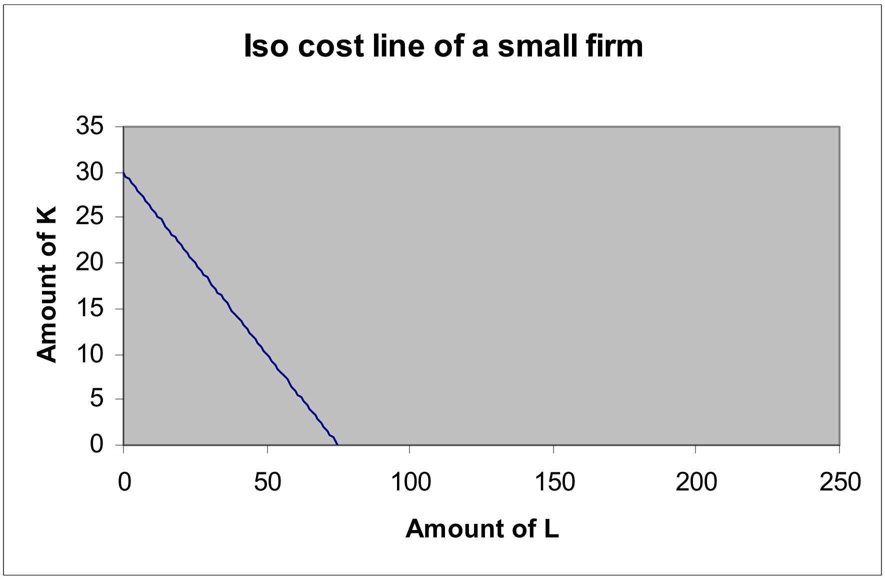
Key questions: Is an economic system fair if the factors are paid according to their marginal productivities?
Readings: VAR 18, 19

Lecture 10

Iso-cost and iso quant

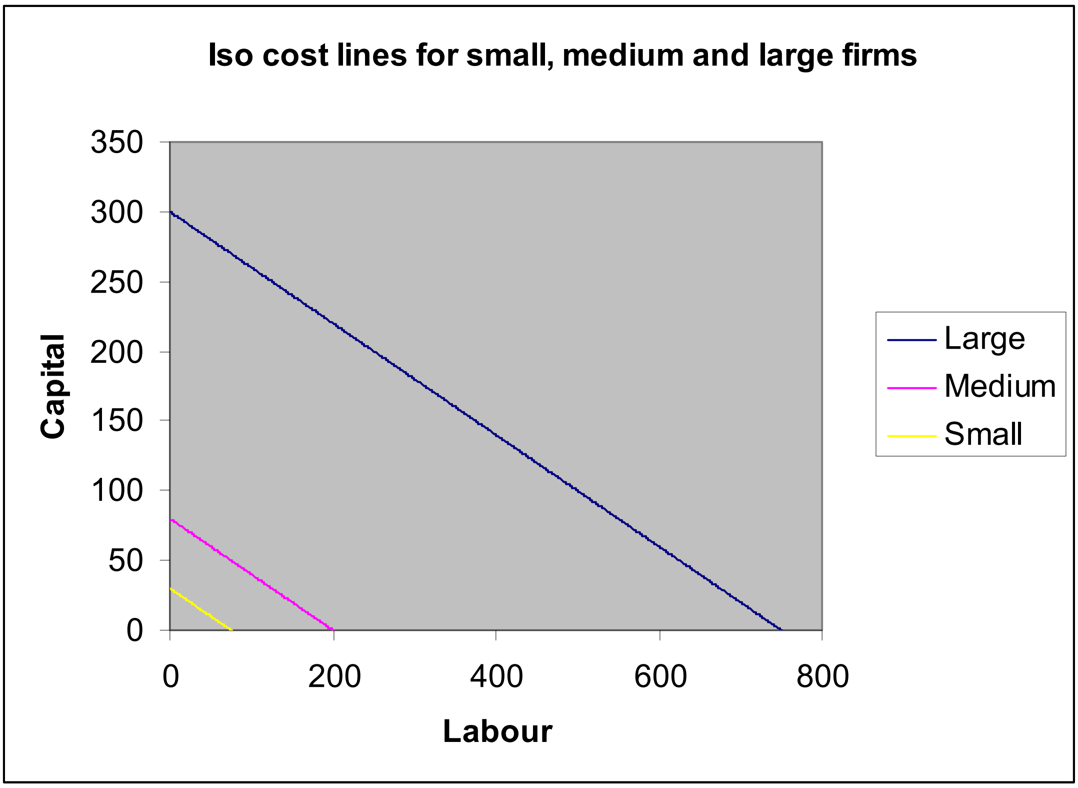
Iso- cost line and relative prices of capital and labour

$\overline{C} = wL + rK \qquad 300 = 4L + 10K$



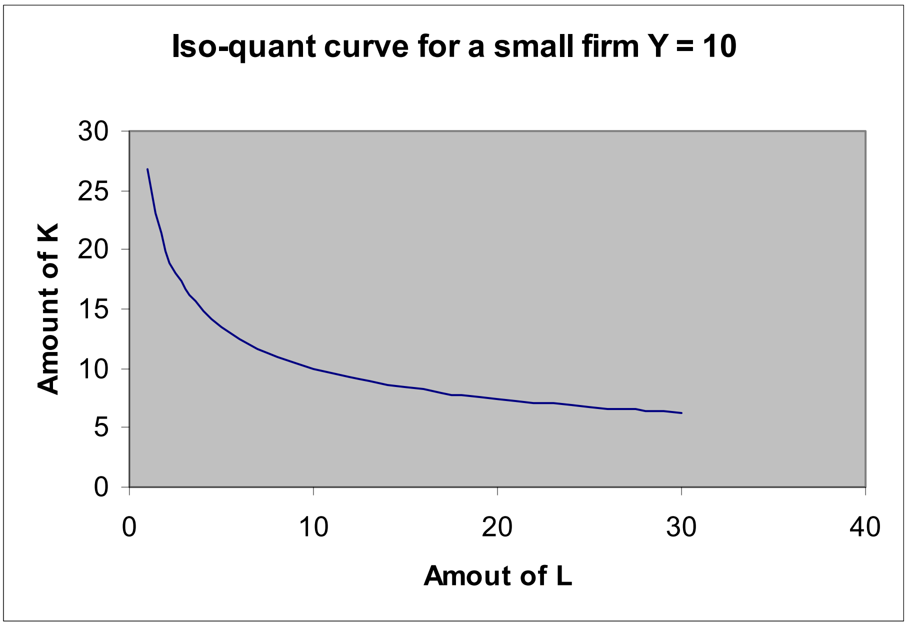
Iso cost lines for small, medium and large firms

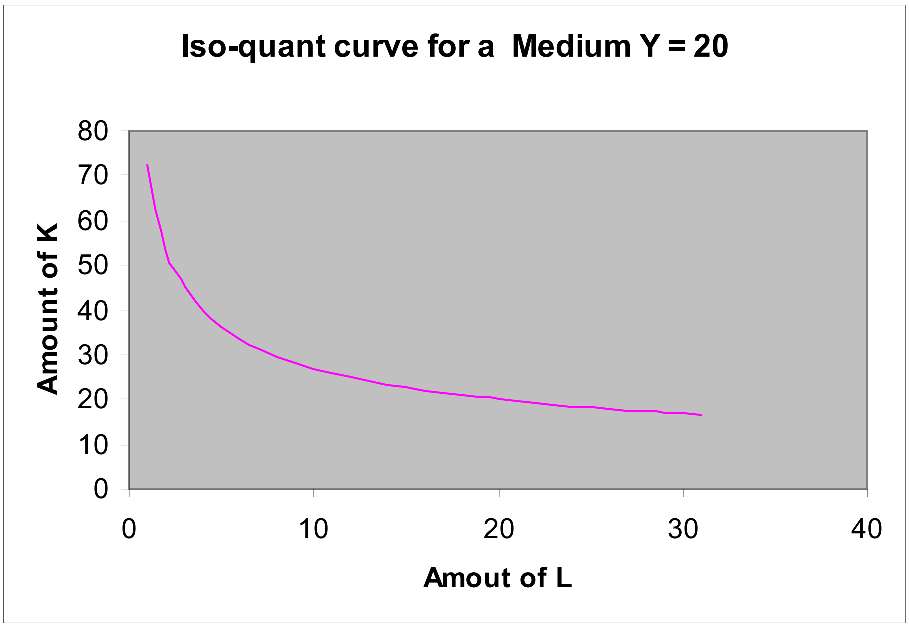
Small: $300 = 4L + 10K$; Medium $800 = 4L + 10K$; Large: $3000 = 4L + 10K$

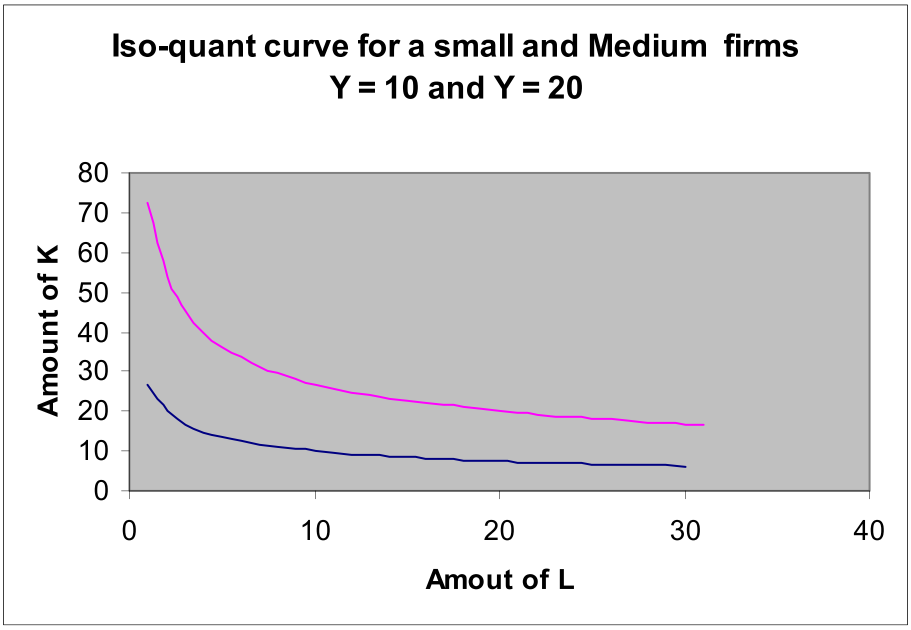


Iso-quant for small and medium scale firms

Small $K^{\alpha}L^{\beta}=10$ medium $K^{\alpha}L^{\beta}=20$ $K^{0.7}L^{0.3}=20$







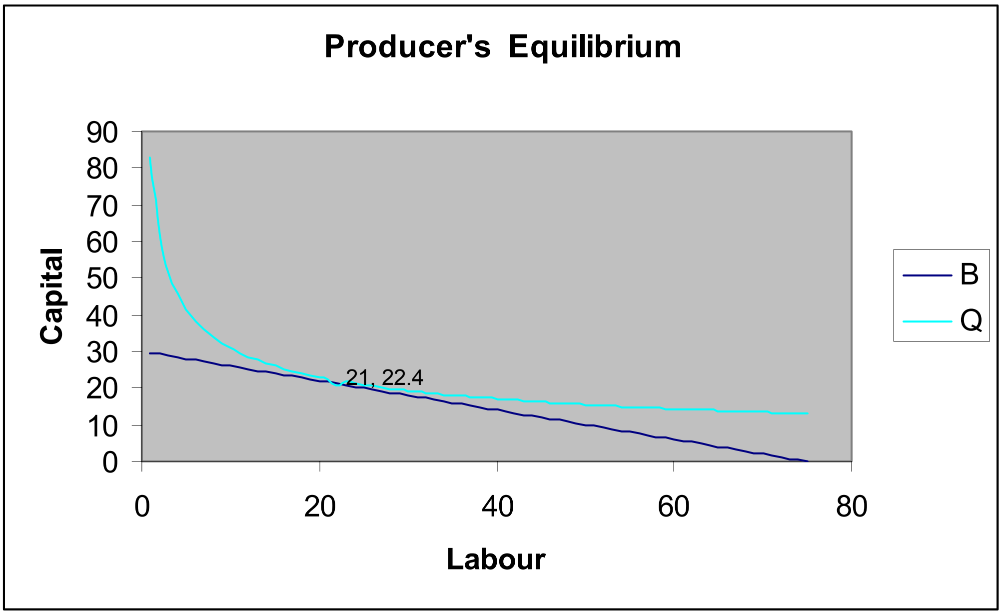
Marginal productivity of labour and demand for labour input

Marginal product of labour: $\frac{\partial Y}{\partial L} = \beta K^{\alpha} L^{\beta - 1}$ $MPK = \beta \left(\frac{K}{L} \right)^{\alpha}$

Marginal productivity of capital and demand for capital input

Marginal product of capital $\frac{\partial Y}{\partial K} = \alpha K^{\alpha - 1} L^{\beta}$ $MPK = \alpha \left(\frac{L}{K} \right)^{\beta}$

Producers' equilibrium



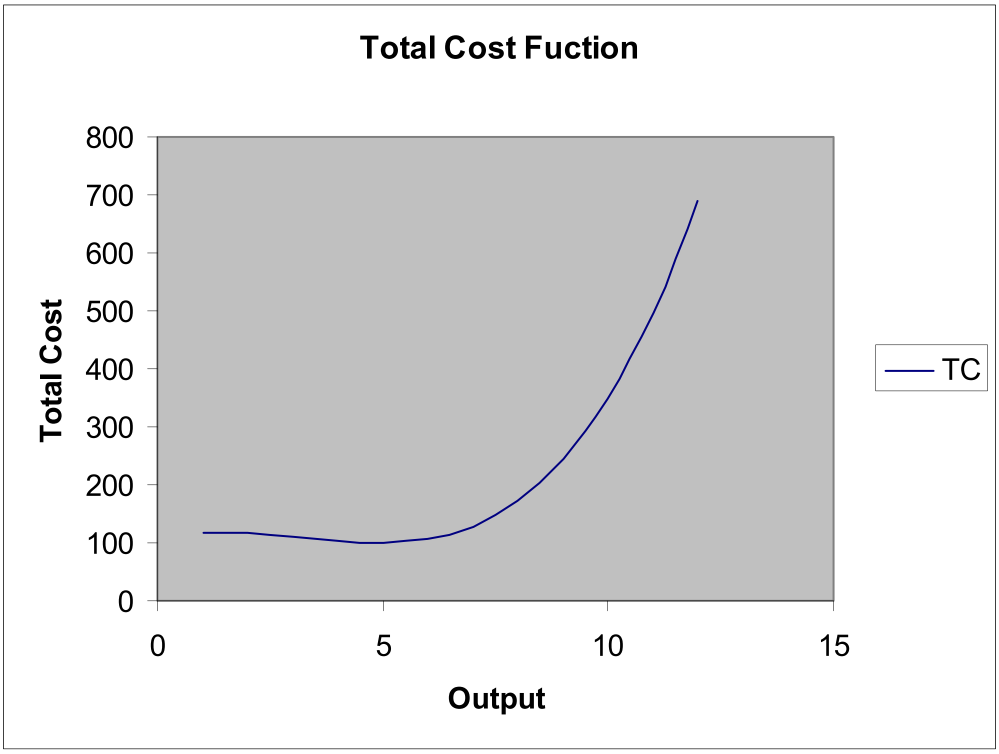
Rate of technical substitution $\frac{MPL}{MPK} = \frac{\partial Y}{\partial L} \Big/ \frac{\partial Y}{\partial K} = \frac{\beta K^{\alpha} L^{\beta - 1}}{\alpha K^{\alpha - 1} L^{\beta}} = \frac{w}{r}$

Key questions: How does technical innovation affect the marginal productivity of capital and labour?

Readings:

Lecture 11
Cost functions

Total cost a) $C = aQ^3 - bQ^2 + cQ + d$ b) $C = Q^3 - 10Q^2 + 25Q + 100$



Average cost: $AC = \dfrac{C}{Q} = Q^2 - 10Q + 25 + 100/Q$

Average variable cost: $AVC = Q^2 - 10Q + 25$ Minimum point of the AVC:

$\dfrac{AVC}{\partial Q} = 2Q - 10 = 0$ Q = 5

Average fixed cost $AFC = 100/Q$

Marginal cost

$MC = \dfrac{\partial C}{\partial Q} = 3Q^2 - 20Q + 25$ MC when AC is minimum.

$MC = 3 \times 5^2 - 20 \times 5 + 25 = 75 - 100 + 25 = 0$

Short run supply curve: P = MC

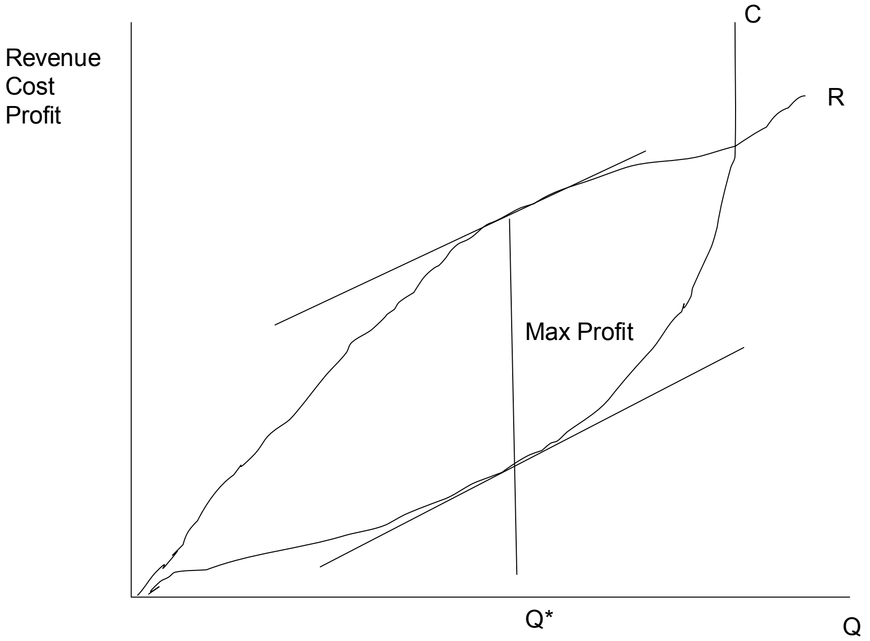
Long run cost: There is no fixed cost in the long run.

Key questions: How is the long run cost of curve of is an envelope of short run cost curves?

Readings: VAR 21

Lecture 12
Profit maximisation, cost minimisation

Relation between revenue cost functions and does the profit function relate to these two functions.



A numerical example

A quadratic revenue function

$$R = 5900Q - 10Q^2$$

What is the marginal revenue implied by this revenue function?

A Cubic cost function.

$$C = 2Q^3 - 4Q^2 + 140Q + 845$$

What are the marginal cost and average cost functions derived from this function?

What is the profit function implied by above revenue and cost functions?

$$\Pi = 5900Q - 10Q^2 - 2Q^3 + 4Q^2 - 140Q - 845$$

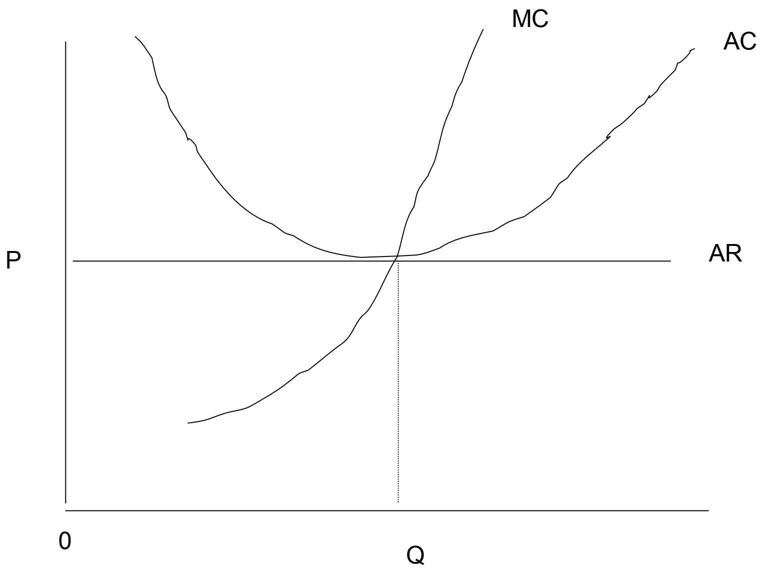
What is the level of profit maximising output ?

Key questions: Does a firm have any other ohjective that profit maximisation?

Readings: VAR 19-21

Lecture 13
Price and output under perfect competition

Pricing and output of a firm under perfect competition



Cost function $C = Q^3 - 5Q^2 + 60Q$

Average cost function $AC = \frac{C}{Q} = Q^2 - 5Q + 60$

Marginal cost function $MC = \frac{\partial C}{\partial Q} = 3Q^2 - 10Q + 60$

Equilibrium

Market price = min (AC) = MR = AR

$$\frac{\partial(AC)}{\partial Q} = 2Q - 5 = 0 \;\Rightarrow\; Q = \frac{5}{2} = 2.5$$

AC when Q =2.5, $AC = Q^2 - 5Q + 60 = (2.5)^2 - 5\times(2.5) + 60 = 53.75$

MC = AC at equilibrium: $MC = 3Q^2 - 10Q + 60 = 3(2.5)^2 - 10\times 2.5 + 60 = 53.75$

Price = 53.75 output = 2.5.

Profit: zero in perfect competition otherwise firms will enter the market.

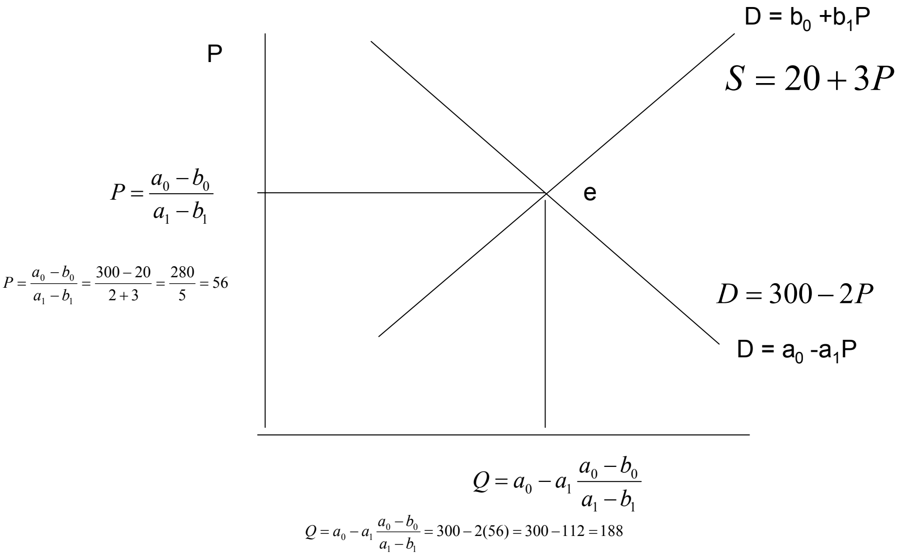
Supply function of a firm is equivalent to its MC $P = 3Q^2 - 10Q + 60$

Market supply: sum of supply of all individual firms.

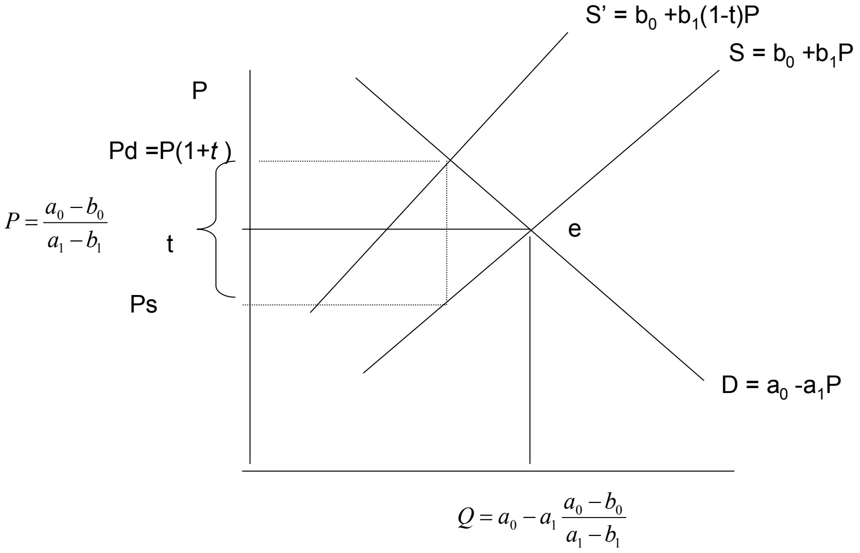
Number of firms: indefinite in perfect competition, because of free entry.

Do above exercise when $C = Q^3 - 21Q^2 + 500Q$

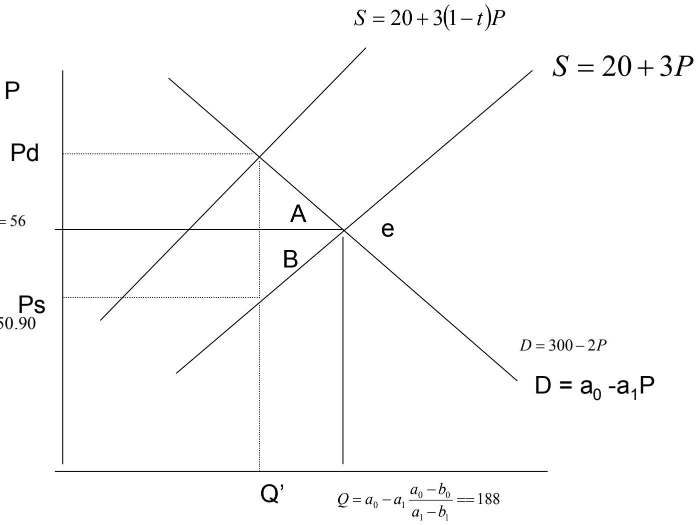
Equilibrium in Price and Quantity in Demand and Supply Model
A Numerical Example



Impact of Taxes in Price and Quantity in the Demand and Supply Model



Deadweight Loss of Tax in the Demand and Supply Model



$$Pt = \frac{a_0 - b_0}{a_1 - b_1(1-t)} = \frac{300-20}{2+3(1-0.2))} = \frac{280}{4.4} = 63.64$$

$$P = \frac{a_0 - b_0}{a_1 - b_1} = \frac{300-20}{2+3} = \frac{280}{5} = 56$$

$$Ps = Pt(1-t) = 63.63 \times 0.8 = 50.90$$

$$Q = a_0 - a_1 \frac{a_0 - b_0}{a_1 - b_1} == 188$$

$$Qt = a_0 - a_1 \frac{a_0 - b_0}{a_1 - b_1(1-t)} = 300 - 2(63.64) = 300 - 127.3 = 172.2$$

Revenue of the supplier before tax:

$$PQ = 56 \times 188 = 10528$$

Revenue of the supplier after tax:

$$Ps \times Qt = 50.90 \times 172.2 = 8764.98$$

Revenue for the government:

$$R = t \times Pt \times Qt = 0.2 \times 63.64 \times 172.2 = 2191.76$$

Consumers' loss:

$$\frac{1}{2} \times (Pt - P) \times \Delta Q = 0.5 \times (63.63 - 56) \times (172.2 - 188) = -60.35$$

Producers' loss:

$$\frac{1}{2} \times (Ps - P) \times \Delta Q = 0.5 \times (50.90 - 56) \times (172.2 - 188) = 40.29$$

Deadweight loss of taxes: Consumers loss + Producer's loss = 60.3+40.3 = 100.6

Key questions: Is the competitive market the most efficient form of resource allocation? What are alternatives?

Readings: VAR 22

Lecture 14

Sample Mid Term Exam

Answer any two questions. Marks for each subsection is as indicated in the margins.

1. Susan is a representative consumer from country Norfolk. She has 1000 to spend on household goods and holidays. Each household goods on average costs 25 and each item of holidays also costs 25. She has equal preference for both households goods (G) and holidays (H), as shown by $U = G \times H$.

a) Represent Susan's budget constraint in a diagram with G in X-axis and H in Y-axis. How many holiday items can she buy if she spends all money in them or how many household goods can she buy if she spends all money in them? Represent her budget by an equation. [12]

b) Show Susan's indifference curve representing her preferences for goods and holidays in (G, H) space. [12]

c) Illustrate optimal choices of G and H by Susan that maximise her utility subject to her budget constraint in the above diagram. [12]

d) Using first order conditions for constrained optimisation derive the demand curve for G and H and determine the optimum quantities. Check that it is consistent with the Susan's budget constraint. [16]

e) If the average price of goods rises to 50 that of holidays unchanged at 25, what will be the new equilibrium? Derive a demand curve deriving from it in a diagram. [12]

f) What is the substitution effect of increase in the prices of goods? [12]

g) What is the income effect of increase in price of goods? [12]

h) What is the cross price elasticity of demand for holidays? [12]

2. Nick is a young man with some plans to start a small production plant in the East Yorkshire. He has overall resource of 10,000. He will hire workers and rent the capital for production. Rental rate of capital is 20 and the wage rate is 10.
    a) Show Iso-cost line for Nick's plant in a diagram and represent it by an equation. [10]

    b) This plant uses 60 percent of its resources in labour and remaining 40 percent in capital. Show production technology available to Nick in an iso-quant equation and represent it in (L,K) space. [10]

    c) Show optimal choice of labour and capital inputs by Nick using the above iso-cost line and the iso-quant curve [10]

    d) Find out the optimal amounts of labour and capital that Nick will employ in production and associated level of output. [10]

    e) If the wage rate rises to 15 but the rental rate of capital remains the same what will be optimal amount of labour input and capital input and the level of output? [10]

    f) What will be the optimal amount of labour and capital input if the rental rate falls to 30 and wage rate rises to 20 and the associated level of output? [10]

    g) Show how the proceeds of the plant are divided between the labour and capital according to their marginal productivities. [10]

    h) Show that this plant has a constant return to scale. If Nick doubles the amount of labour and capital input, that will also double the output. [10]

    i) Prove that the elasticity of substitution between labour and capital is one in this type of production technology. [10]

3. Market demand and supply for a product is given by $D = 50 - 5P$ and $S = -2 + 4P$ respectively.

a) Represent these equations in one diagram and indicate the point ot equilibrium, excess demand and excess supply. Briefly mention the mechanism that brings this market system towards equilibrium point. [12]

b) Find out the equilibrium price and quantity demanded and supplied solving this market system. [11]

c) What are the amount of consumer and producer surplus in this equilibrium? [11]

d) Government imposes sales tax of 1 in production of this good. How does it affect the demand and supply curves. Illustrated using in another diagram. [11]

e) How can this tax be incorporated in the above model by using post tax prices paid by buyers and received by supplies? [11]

f) Solve the tax distorted system and find out the amount of quantities bought and sold, the prices paid by buyers, received by suppliers, government revenue. [11]

g) What are the deadweight losses to consumers, producers and the economy as a whole from the imposition of this tax? [11]

h) If the technology reduces the cost of production if this commodity and supply curve changes to $S = -2 + 2P$ what will be equilibrium quantities before and after taxes and the deadweight loss of taxes. [11]

i) How does a massive advertisement campaign affect the demand curve in this market. Does it make it more elastic or less elastic? [11]

4. The revenue and cost functions for a particular firm are given as following:

Revenue: $R = 5900Q - 10Q^2$

Cost: $C = 2Q^3 - 4Q^2 + 140Q + 845$

a) Sketch the revenue and cost functions in one diagram and indicate a point where the profit is maximised. [12]
b) What is its marginal revenue? [12]
c) What is its marginal cost? [12]
d) What is it's the profit function? How much should this firm produce to maximise profit? What will be the amount of profit? [12]
e) Is this consistent to the MR=MC criteria? [12]
f) How much should it produce to maximise revenue? [12]
g) How much to minimise average variable cost? [12]
h) If this firm is to operate under the perfect competitive market what price should it set and how much should it produce? Repeat above exercise when $R = 1000Q - 4Q^2$ and $C = 2Q^3 - 7Q^2 + 64Q + 256$. [16]

Lecture 15
Monopoly

Demand function of a monopolist:

$$Q = 50 - \frac{1}{10}P$$

Reverse demand function: $P = 500 - 10Q$

$$R = P \times Q = (500 - 10Q)Q = 500Q - 10Q^2$$

Cost function

$$C = 8Q$$

Profit

$$\Pi = P \times Q - C = 500Q - 10Q^2 - 8Q$$

Marginal Revenue function

$$MR = \frac{\partial R}{\partial Q} = 500 - 20Q$$

Marginal cost function

$$MC = \frac{\partial C}{\partial Q} = 8$$

Output and price

$MR = MC$ ➔ $500 - 20Q = 8$

Output $Q = \frac{492}{8} = 24.6$

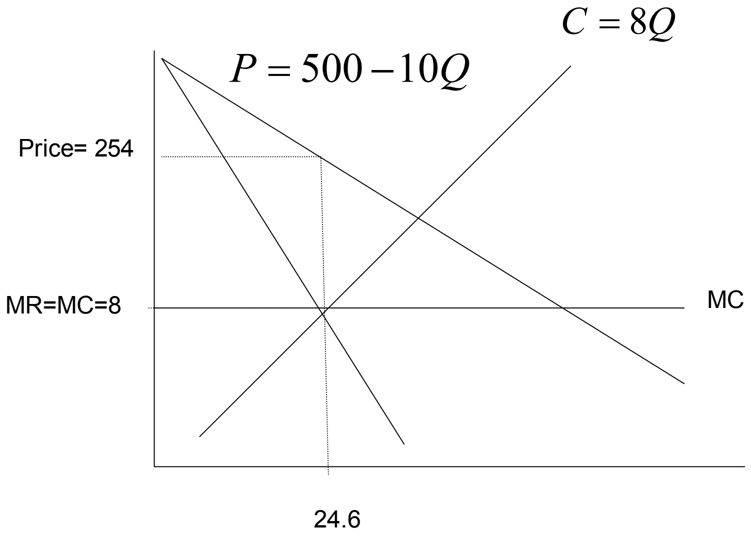
Price: $P = 500 - 10Q = 500 - 246 = 254$

Revenue: $R = P \times Q = 254 \times 24.6 = 6223$

Cost: $C = 8Q = 8 \times 24.6 = 196.8$

Profit: $\Pi = R - C = 6223 - 196.8 = 6026.2$

Price and Output Decision of a Monopolist



Charges more, supplied less: inefficient from a social point

Why is a firm under monopoly inefficient?

Price discrimination

A monopolist facing two different demand functions at home and abroad discriminates between two market to maximise profit

Demand at home: $Q_1 = 21 - 0.1 \times P_1$; Inverse demand: $P_1 = 210 - 10 \times Q_1$
Demand abroad: $Q_2 = 50 - 0.4 \times P_2$; Inverse demand $P_2 = 125 - 2.5 \times Q_2$
Common cost function: $C = 2000 + 10Q$
Profit maximising condition: $MR1 = MR2 = MC$
Total revenue from home market: $R_1 = P_1 \times Q_1 = (210 - 10Q_1)Q_1 = 210Q_1 - 10Q_1^2$;
Marginal revenue from home market: $MR1 = 210 - 20Q_1$
Total revenue from foreign market: $R_2 = P_2 \times Q_2 = (125 - 2.5Q_2)Q_2 = 125Q_2 - 2.5Q_2^2$
Marginal revenue from foreign market: $MR2 = 125 - 5Q_2$
Marginal cost of production $MC = 10$
$MR1 = MC \rightarrow 210 - 20Q_1 = 10 \rightarrow Q_1 = 10 \rightarrow P_1 = 210 - 10 \times Q_1 \rightarrow P_1 = 110$
$MR2 = MC \rightarrow M125 - 5Q_2 = 10 \rightarrow Q_1 = 23 \rightarrow P_2 = 125 - 2.5 \times Q_2 \rightarrow P_1 = 67.5$
$R_1 = P_1 \times Q_1 = 110 \times 10 = 1100$; $R_2 = P_2 \times Q_2 = 67.5 \times 23 = 1552.5$;
Total cost production: $C = 2000 + 10(Q_1 + Q_2) = 2000 + 10(10 + 23) = 2330$
Profit: 1100+1552.5-2330 =322.5

Exercise: Contrast this to a monopolist that does not discriminate. Add up two demand function fro a market demand.
Natural monopolies: Yorkshire water. City council and local public services.
Key questions: Do public utilities such as water and sewage, electricity distribution, TV-telephone lines have a case of monopolistic industrial set up?
Readings: VAR 24-25.

Lecture 16
Oligopoly: duopoly

Market demand curve for a product is

$$Q = 10 - \frac{1}{5}P$$

Implied reverse demand function

$$P = 50 - 5Q = 50 - 5(q_1 + q_2)$$

Two firms exists in the market to supply this product.. Their cost functions are

$$C_1 = 4q_1 \quad C_2 = 5q_2$$

Implied profit functions

$$\Pi_1 = Pq_1 - C_1 = [50 - 5(q_1 + q_2)]q_1 - 4q_1 = 50q_1 - 5q_1^2 - 5q_1q_2 - 4q_1$$

$$\Pi_2 = Pq_2 - C_2 = [50 - 5(q_1 + q_2)]q_2 - 5q_2 = 50q_2 - 5q_2^2 - 5q_1q_2 - 5q_2$$

In Cournot model each firm takes other's action as a given parameter and tries to maximize its own output.

Reaction function of firm 1

$$\frac{\partial \Pi_1}{\partial q_1} = 50 - 10q_1 - 5q_2 - 4 = 0$$

$10q_1 + 5q_2 = 46$; $10q_1 = 46 - 5q_2$ or $q_1 = 4.6 - \frac{1}{2}q_2$ (R1)

Reaction function of firm 2

$$\frac{\partial \Pi_2}{\partial q_2} = 50 - 10q_2 - 5q_1 - 5 = 0$$

$10q_2 + 5q_5 = 45$; $10q_2 = 45 - 5q_1$ or $q_2 = 4.5 - \frac{1}{2}q_1$ (R2)

Solving two reaction functions

$10q_1 + 5q_2 = 46$ (R1)

$5q_1 + 10q_2 = 45$ (R2)

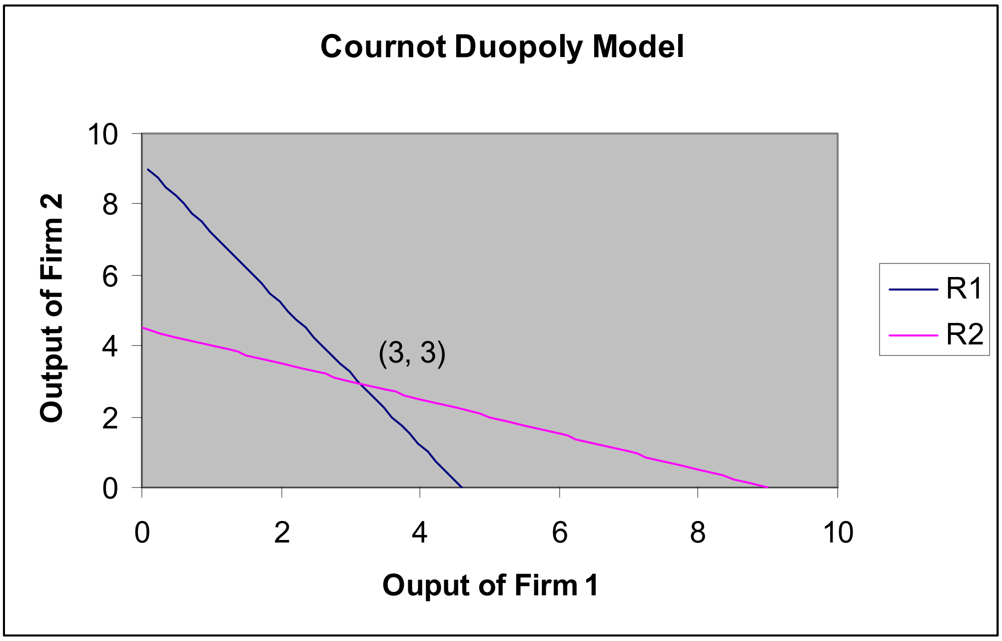
One can solve by 2*R2-R1

$10q_1 + 20q_2 = 90$

-($10q_1 + 5q_2 = 46$) or $15q_2 = 44$; $q_2 = \frac{44}{15} = 2.93$

Substituting this value in $q_1 = 4.6 - \frac{1}{2} q_2$

$$q_1 = 4.6 - \frac{1}{2}\left(\frac{44}{15}\right) = 4.6 - \frac{22}{15} = 4.6 - 1.47 = 3.13$$



See: Cournot_Duopoly.xls

Market price:

$$P = 50 - 5Q = 50 - 5(3.13 + 2.93) = 19.7$$

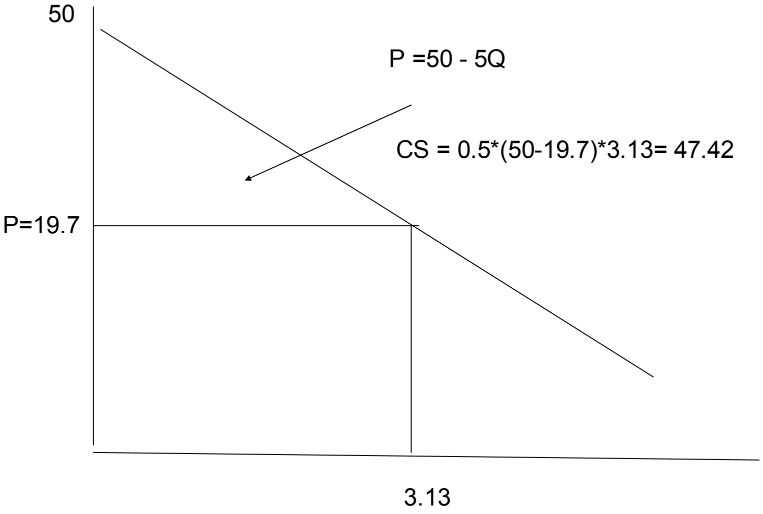
Profit of firm1

$$\Pi_1 = Pq_1 - C_1 = 19.7 \times 3.13 - 4 \times 3.13 = 61.66 - 12.52 = 49.14$$

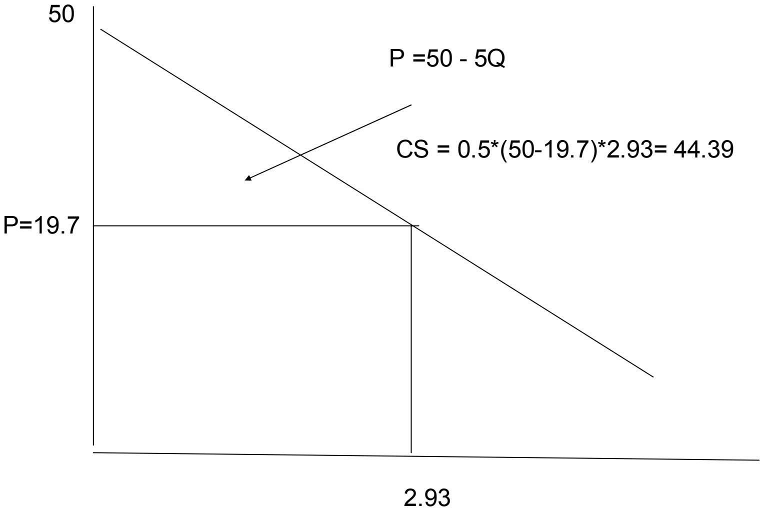
Profit of firm 2

$$\Pi_2 = Pq_2 - C_2 = 19.7 \times 2.93 - 5 \times 2.93 = 57.72 - 14.65 = 43.07$$

Consumer Surplus from Firm 1



Consumer Surplus from Firm 2



Total consumer surplus = CS1+CS2 = 47.42 +44.39

Stakelberg Leadership Model.

Firm 1 acts as a leader: It needs to take account of follower's reaction function while determining its level of output. Taking the same model

$$Q = 10 - \frac{1}{5}P$$

Implied reverse demand function

$$P = 50 - 5Q = 50 - 5(q_1 + q_2)$$

Assume that two firms exist in the market to supply this product.. Firm 1 is the leader and firm 2 the follower with the following linear cost functions.

$$C_1 = 4q_1 \quad C_2 = 5q_2$$

Implied profit functions

$$\Pi_1 = Pq_1 - C_1 = [50 - 5(q_1 + q_2)]q_1 - 4q_1 = 50q_1 - 5q_1^2 - 5q_1q_2 - 4q_1$$

$$\Pi_2 = Pq_2 - C_2 = [50 - 5(q_1 + q_2)]q_2 - 5q_2 = 50q_2 - 5q_2^2 - 5q_1q_2 - 5q_2$$

As found before the reaction function of firm 2

$$\frac{\partial \Pi_2}{\partial q_2} = 50 - 10q_2 - 5q_1 - 5 = 0$$

$10q_2 + 5q_5 = 45$; $10q_2 = 45 - 5q_1$ or $q_2 = 4.5 - \frac{1}{2}q_1$ (R2)

Put this into the profit function of firm 1

$$\Pi_1 = Pq_1 - C_1 = [50 - 5(q_1 + q_2)]q_1 - 4q_1 = 50q_1 - 5q_1^2 - 5q_1q_2 - 4q_1$$

$$\Pi_1 = 50q_1 - 5q_1^2 - 5q_1\left(4.5 - \frac{1}{2}q_1\right) - 4q_1 = 50q_1 - 5q_1^2 - 22.5q_1 + 2.5q_1^2 - 4q_1$$

$$\Pi_1 = 27.5q_1 - 2.5q_1^2 - 4q_1$$

$$\frac{\partial \Pi_1}{\partial q_1} = 27.5 - 5q_1 - 4 = 0 \quad q_1 = \frac{23.5}{5} = 4.7$$

Given this output of the leader, the follower firms decides its own output

$$q_2 = 4.5 - \frac{1}{2}q_1 = 4.5 - \frac{1}{2}(4.7) = 2.15$$

Market price $P = 50 - 5Q = 50 - 5(4.7 + 2.15) = 15.75$

Profit of the leader $\Pi_1 = Pq_1 - C_1 = 15.75 \times 4.7 - 4 \times 4.7 = 74.03 - 18.8 = 55.23$

Profit of the follower $\Pi_2 = Pq_2 - C_2 = 15.75 \times 2.15 - 5 \times 2.15 = 33.86 - 10.75 = 23.11$

Follower gets less profit than the leader.

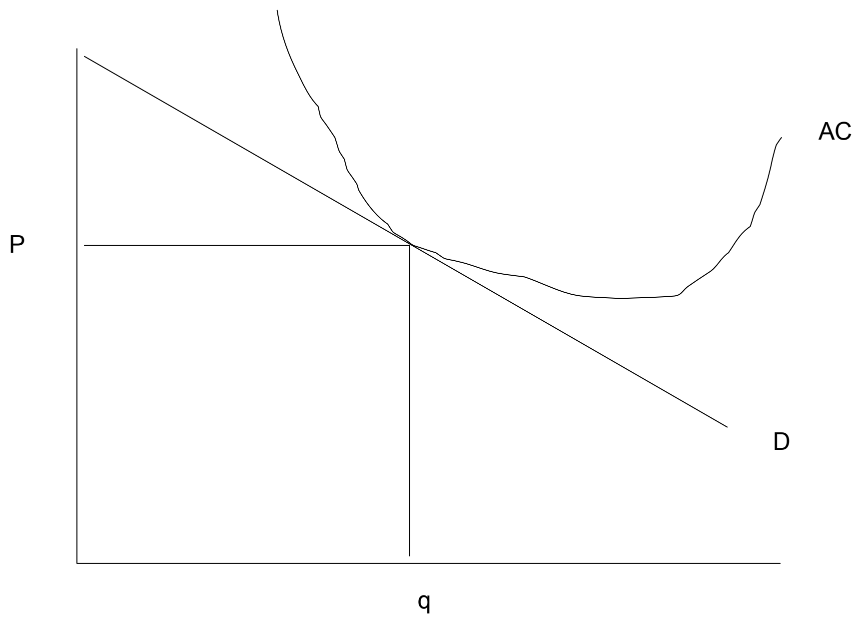
Key questions: How can game theory with complete and incomplete information be useful in analysing behaviour of firms in an oligopolistic industry?
Readings: VAR 27

Lecture 17
Monopolistic competition - (Chamberlin)

Product differentiation and close substitutes: ipod, CD, DVD, diskettes
Soft drinks: Coke, Pepsi, Fanta, Tango, Sprite, 7 Up, Dr. Pepper,
Cars: BMW, Voxhaul, Poeguet, Chrisler, Ford, GM, Toyota, Nissan, Hyundai, Fiat.
Cosmetics:
Shoes:
Watches:
Camera:
PC Computers:
Fast food:
Yoghurt:
Aspirins:
Pens:
Books in microeconomics or macroeconomics:

Firm and industry: A firm has its own downward sloping demand and so has some monopolistic power in pricing but faces competition from firms producing close substitutes. If it charges higher prices looses consumers to other producers. Free entry implies zero profit for the incumbent firms. Firms do not produce at the most efficient point, therefore less efficient than firms in perfectly competitive markets.

Price and output decisions: Long run equilibrium

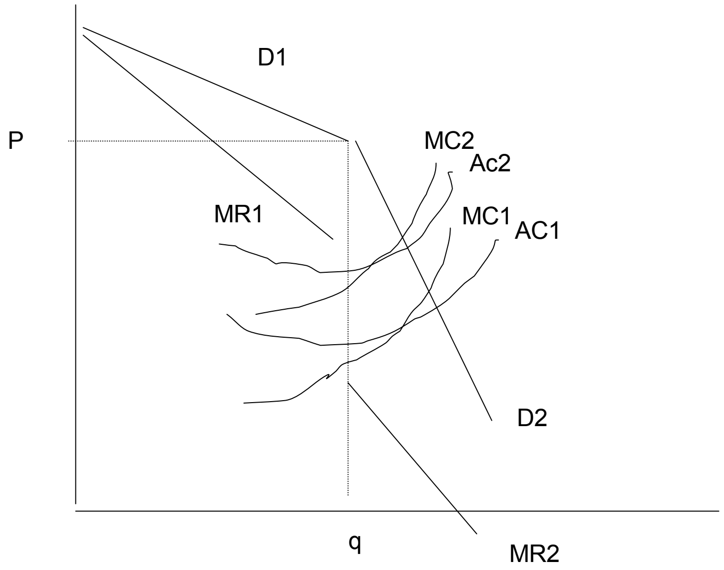


It is possible the firms make positive economic profit in the short run.

Kink in demand - Sweezy model of price and quantity rigidity

If a firm reduces its own price other firms will reduce it, when it raises its own price none of the others will raise their prices. A firm reduces its own price when another firm reduce it but does not raise its own price when another raise their prices.

A firm is reluctant to change it price as it does not want to stir and disturb the other firms in the market by sending wrong signals. Both price and quantity are fixed.



Prices and quantities are fixed, firms do not follow MR = MC principle.

Example from industries

There are two firms in a market, I and II. The marked demand and cost functions faced by each is as following (See Agarwal(1978)):

$$P_1 = 105 - 2q_1 - q_2 ;\ \ C_1 = 5q_1^2$$

$$P_1 = 35 - q_1 - q_2 ;\ C_1 = q_2^2$$

The base line Counot duopoly equilibrium

$$\Pi_1 = Pq_1 - C_1 = \left[105 - 2q_1 - q_2\right]q_1 - 5q_1^2 = 105q_1 - 7q_1^2 - q_1 q_2$$

$$\Pi_1 = 105q_1 - 7q_1^2 - q_1 q_2$$

$$\frac{\partial \Pi_1}{\partial q_1} = 105 - 14q_1 - q_2 = 0 \quad 14q_1 + q_2 = 105$$

$$\Pi_2 = Pq_2 - C_2 = \left[35 - q_1 q_2 - q_2^2\right]q_2 - q_2^2 = 35q_2 - q_1 q_2 - 2q_2^2$$

$\Pi_2 = 35q_2 - q_1q_2 - 2q_2^2$

$\frac{\partial \Pi_2}{\partial q_2} = 35 - q_1 - 4q_2 = 0$; $q_1 + 4q_2 = 35$

Solving two reaction functions

$14q_1 + q_2 = 105$ (R1)

$q_1 + 4q_2 = 35$ (R2)

4*(R1) -R2

$56q_1 + 4q_2 = 420$

$q_1 + 4q_2 = 35$

$q_1 = 7 \quad q_2 = 7$

$P_1 = 105 - 2q_1 - q_2 = 105 - 2(7) - 7 = 84$; $C_1 = 5q_1^2 = 5(7^2) = 245$

$P_2 = 35 - q_1 - q_2 = 35 - 7 - 7 = 21$; $C_2 = q_2^2 = 7^2 = 49$

$\Pi_1 = 105q_1 - 7q_1^2 - q_1q_2 = 105 \times 7 - 7 \times 7^2 - 7 \times 7 = 343$

$\Pi_2 = 35q_2 - q_1q_2 - 2q_2^2 = 35 \times 7 - 7 \times 7 - 2 \times 7^2 = 98$

Now consider that firm 2 raises its price but the firm two does not react.

$P_1 = 84 + 2 = 86$ but $P_2 = 21$

First get the reaction function of firm II that does not change its price, i.e. $P_2 = 21$

$P_2 = 35 - q_1 - q_2$ or $21 = 35 - q_1 - q_2 \quad q_2 = 14 - q_1$

Use this reaction function of II into the price function of I to get output of firm I.

$P_1 = 105 - 2q_1 - q_2 = 105 - 2(7) - 7 = 84$

$86 = 105 - 2q_1 - (14 - q_1) \quad 86 = 91 - q_1 \quad q_1 = 5$

Using II's reaction function

$q_2 = 14 - q_1 \quad q_2 = 9$

$\Pi_1 = 105q_1 - 7q_1^2 - q_1q_2 = 105 \times 5 - 9 \times 5^2 - 5 \times 9 = 305$; $C_1 = 5q_1^2 = 5(5^2) = 125$

$\Pi_2 = 35q_2 - q_1q_2 - 2q_2^2 = 35 \times 9 - 5 \times 9 - 2 \times 9^2 = 108$; $C_1 = q_2^2 = 9^2 = 81$

If the duopolist I reduces price by 2 the firm II will also follow the suit.

$P_1 = 84 - 2 = 82$ but $P_2 = 21$

Given that demand functions are: $P_1 = 105 - 2q_1 - q_2$; $P_2 = 35 - q_1 - q_2$

Firm I reduces its price by 2 i.e. $P_1 = 82$
Get firm II's reaction function from $P_2 = 35 - q_1 - q_2$ or $21 = 35 - q_1 - q_2$ $q_2 = 14 - q_1$
Use this reaction function of II into the price function of I to get output of firm I.

$82 = 105 - 2q_1 - (14 - q_1)$ $82 = 91 - q_1$ $q_1 = 9$
Using II want to maintain the old level of output by reducing its price
$P_2 = 35 - q_1 - q_2$ $P_2 = 35 - 9 - 7$ $P_2 = 19$

$\Pi_1 = 105q_1 - 7q_1^2 - q_1q_2 = 105 \times 9 - 7 \times 9^2 - 7 \times 9 = 333$; $C_1 = 5q_1^2 = 5(9^2) = 405$

$\Pi_2 = 35q_2 - q_1q_2 - 2q_2^2 = 35 \times 7 - 7 \times 9 - 2 \times 7^2 = 84$; $C_1 = q_2^2 = 7^2 = 49$

Summary of the Monopolistic Competition Model

	P1	P2	Q1	Q2	R1	R2	C1	C2	PR1	PR2
Base line Cournot Model	84	21	7	7	588	147	245	49	343	98
When I raises P1 by 2	86	21	5	9	430	189	125	81	305	108
When I reduces P1 by 2	82	19	9	7	738	133	405	49	333	84

When I raises price II does not raise its price and gets more profit by supplying more but charging the same price. When I reduces price II also reduces price and produces same as before but gets less profit.

Key questions: Why do airlines compete in cutting prices?
Readings: Glenn and Patrick (2006, Chap. 12)

Lecture 18
Strategic Choices: Game

Elements of a game: rational players, strategy and payoff.
How they play: static and dynamic; two players and multiple players.

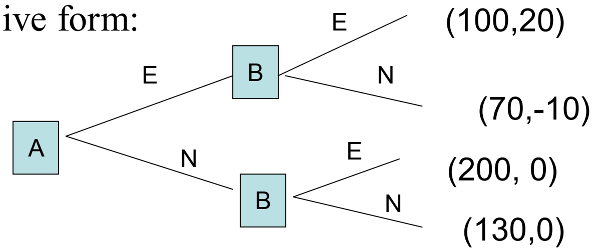
Normal and Extensive Form Representation of a Game

Normal form:

$$\begin{array}{ccccc} & & & A & \\ & & & \text{Enter} & \text{Don't Enter} \\ & \text{Enter} & \left[\begin{array}{c} (100,20) \\ (70,-10) \end{array}\right. & & \\ B & & & & \\ & \text{Don't Enter} & & & \end{array}$$

$$\begin{array}{ccc} & \multicolumn{2}{c}{A} \\ & \text{Enter} & \text{Don't Enter} \\ B \; \text{Enter} & (100,20) & (200,0) \\ B \; \text{Don't Enter} & (70,-10) & (130,0) \end{array}$$

Extensive form:



Gain of one player equals the loss of another in a zero sum game as in the following table.

		Player 2	
		Buy	Sell
Player 1	Buy	(10 , -10)	(-20 , 20)
	Sell	(20 , -20)	(-10 , 10)

Consider a market where there are only two firms. They share the market and have a cartel. They have two strategies. Either to cooperate and honour the agreement on the level of output or to cheat and supply more than the agreed amount. Each gets £25,000 if both cooperate. If one is able to deceit another and raise the amount of output he will be able to get 40,000 but the another party will get just 10000. If the opponent finds that he has been cheated he can retaliate by further cheating. Both players end up playing cheat strategy. Alternatively they can play a tit for tat strategy. Cooperate if another cooperates and cheat if other cheats. If they play like this it is optimal for both to cooperate and not to cheat. The repeated solution to the game is to cooperate. One cooperates if the another does so

	Cooperate	Cheat
Cooperate	(25,000, 25,000)	(1000, 40,000)
Cheat	(40,000, 1000)	(1000 , 1000)

Cooperative strategy is Pareto superior to non-cooperative strategy.

Stackelberg Game

Consider a simple model of firm, like in Lecture 17 assume the market demand is given by

$Q = 10 - P$

Implied reverse demand function

$P = 10 - Q = 10 - (q_1 + q_2)$

Two firms exists in the market to supply this product.. Their cost functions are

$C_1 = 0 \quad C_2 = 0$

Implied profit functions

$$\Pi_2 = Pq_1 - C_1 = [10 - (q_1 + q_2)]q_1 - 0 = 10q_1 - q_2^2 - q_1 q_2$$

$$\Pi_2 = Pq_2 - C_2 = [10 - (q_1 + q_2)]q_2 - 0 = 10q_2 - 1q_2^2 - q_1 q_2$$

Reaction function of firm 1

$$\frac{\partial \Pi_1}{\partial q_1} = 10 - 2q_1 - q_2 = 0$$

$2q_1 + q_2 = 10$

$q_1 = 5 - \frac{1}{2} q_2$ (R1)

Reaction function of firm 2

$$\frac{\partial \Pi_2}{\partial q_2} = 10 - 2q_2 - q_1 = 0$$

$q_1 + 2q_2 = 10 \quad 2q_2 = 10 - q_1$ or $q_2 = 5 - \frac{1}{2} q_1$ (R2)

Solving two reaction functions

$2q_1 + q_2 = 10$ (R1)

$q_1 + 2q_2 = 10$ (R2)

One can solve by 2*R2-R1

$2q_1 + q_2 = 10$

-($2q_1 + 4q_2 = 20$) or $3q_2 = 10$; $q_2 = \frac{10}{3} = 3.3333$

Substituting this value in $q_1 = 10 - 2q_2 \quad q_1 = 10 - 2(3.333) = 10 - 6.6666 = 3.3333$

Market price:

$P = 10 - Q = 10 - (3.3333 + 3..333) = 3.333$

Profit of firm1

$$\Pi_1 = Pq_1 - C_1 = 3.333 \times 3.33 - 0 = 11.11$$

Profit of firm 2

$$\Pi_2 = Pq_2 - C_2 = 3.333 \times 3.333 - 0 = 11.11$$

Thus when both firms act as a follower in the strategic game each get profit of 11.11 .

If firm 1 acts as a leader then it need to take account reaction of the follower

$$\Pi_2 = Pq_1 - C_1 = [10 - (q_1 + q_2)]q_1 - 0 = 10q_1 - q_2^2 - q_1 q_2$$

$$q_2 = 5 - \frac{1}{2} q_1$$

$$\Pi_1 = Pq_1 - C_1 = 10q_1 - q_1^2 - q_1 q_2 = 10q_1 - q_1^2 - q_1\left(5 - \frac{1}{2} q_1\right)$$

$$\Pi_1 = 10q_1 - q_1^2 - 5q_1 + \frac{1}{2} q_1^2 = 5q_1 - \frac{1}{2} q_1^2$$

$$\frac{\partial \Pi_1}{\partial q_1} = 5 - q_1 = 0 \quad q_1 = 5$$

The followers looks at the leader's output to determine its output

$$q_2 = 5 - \frac{1}{2} q_1 = 5 - \frac{1}{2}(5) = 5 - 2.5 = 2.5$$

$$P = 10 - Q = 10 - (5 + 2.5) = 2.5$$

Profit of leading firm

$$\Pi_1 = Pq_1 - C_1 = 2.5 \times 5 - 0 = 12.5$$

Profit of follower firm 2

$$\Pi_2 = Pq_2 - C_2 = 2.5 \times 2.5 - 0 = 6.25$$

If the firm2 acts as a leader, by symmetry firm 2 will get 12.25 and firm 1 gets 6.25.

$$\Pi_2 = Pq_2 - C_{12} = 10q_2 - q_2^2 - q_1 q_2 = 10q_2 - q_2^2 - q_2\left(5 - \frac{1}{2} q_2\right)$$

$$\Pi_2 = 10q_2 - q_2^2 - 5q_2 + \frac{1}{2} q_2^2 = 5q_2 - \frac{1}{2} q_2^2$$

$$\frac{\partial \Pi_2}{\partial q_2} = 5 - q_2 = 0 \quad q_2 = 5$$

The followers looks at the leader's output to determine its output

$$q_1 = 5 - \frac{1}{2} q_2 = 5 - \frac{1}{2}(5) = 5 - 2.5 = 2.5$$

$$P = 10 - Q = 10 - (5 + 2.5) = 2.5$$

Profit of follower firm

$$\Pi_1 = Pq_1 - C_1 = 2.5 \times 2.5 - 0 = 6.25$$

Profit of the leading firm 2

$$\Pi_2 = Pq_2 - C_2 = 2.5 \times 5 - 0 = 12.5$$

If both firms act as a leader each will produce 5 then from the price equation

$$P = 10 - Q = 10 - (q_1 + q_2) = 10 - (5 + 5) = 0$$

$$\Pi_1 = Pq_1 - C_1 = 0 \times 5 - 0 = 0$$

$$\Pi_2 = Pq_2 - C_2 = 0 \times 5 - 0 = 0$$

This is in fact the equilibrium in the competitive market because in competitive market $P = MC = 0 = 10 - Q$ $Q = 10$ $\Pi = PQ = 0 \times 10 = 0$

If these firms agree to form a cartel, then

$$\Pi = PQ - C = (10 - Q)Q = 10Q - Q^2 \quad \Pi = 10Q - Q^2$$

$$\frac{\partial \Pi}{\partial Q} = 10 - 2Q = 0 \quad 2Q = 10 \quad Q = 5 \quad P = 10 - Q = 10 - 5 = 5$$

$$\Pi = 10Q - Q^2 = 10 \times 5 - 5^2 = 25 .$$

Firm 1

		Leader	Follower
Firm 2	Leader	(0 , 0)	(12.5 , 6.25)
	Follower	(6.25 , 12.25)	(11.11 , 11.11)

Thus profit under perfect competition is zero, under collusion is 25, under Counot conjecture is 11.11, under Stackelberg leadership is 12.25 for leader and 6.25 for the follower and under leadership competition is 0 for both.

Solutions of a game: Dominant strategy, Nash equilibrium, Mixed strategy

Dominant strategy,

Does subsidy to the Airbus by EU countries deter Boeing from Producing a New Aircraft?

$$\begin{array}{ll} & \text{Airbus} \\ & \begin{array}{cc} \text{S1} & \text{S2} \end{array} \\ \text{Boeing} \begin{array}{r} \text{S 1} \\ \text{S2} \end{array} & \begin{bmatrix} (-10,-10) & (100,0) \\ (0,100) & (0,0) \end{bmatrix} \end{array}$$

GAME 1

$$\begin{array}{ll} & \text{Airbus} \\ & \begin{array}{cc} \text{S1} & \text{S2} \end{array} \\ \text{Boeing} \begin{array}{r} \text{S 1} \\ \text{S2} \end{array} & \begin{bmatrix} (-10,10) & (100,0) \\ (0,120) & (0,0) \end{bmatrix} \end{array}$$

GAME 2

Nash equilibrium,

Game with A Nash Equilibrium

Prisoners dilemma

$$\begin{array}{ll} & \text{A} \\ & \begin{array}{cc} \text{Confess} & \text{Do not confess} \end{array} \\ \text{B} \begin{array}{r} \text{Confess} \\ \text{Do not confess} \end{array} & \begin{bmatrix} (-5,-5) & (-1,-10) \\ (-10,-1) & (-2,-2) \end{bmatrix} \end{array}$$

$$\begin{bmatrix} (-\underline{5},-\underline{5}) & (-\underline{1},-10) \\ (-10,-\underline{1}) & (-2,-2) \end{bmatrix}$$

Nash Solution: (-5,-5)

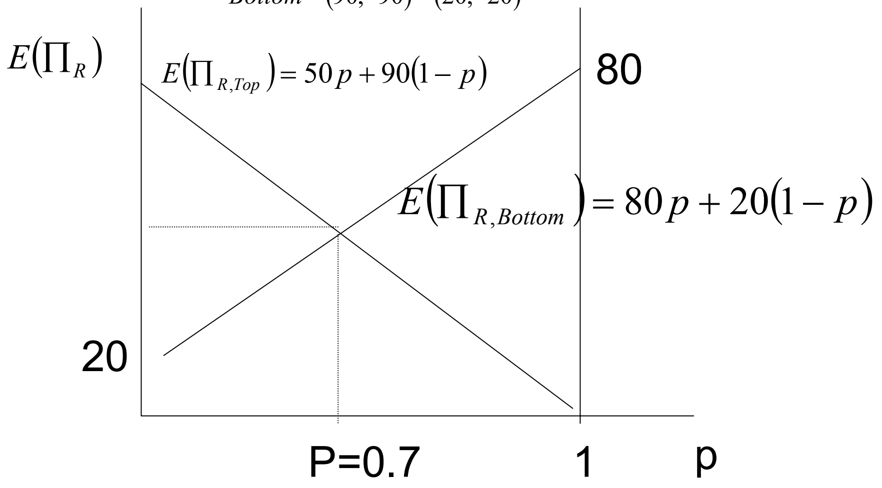
Cooperation was better: (-2,-2)

Cooperation is better but each think that other player will cheat and therefore they Don’t cooperate, therefore stay longer in jail.

Mixed strategy

Finding the Mixed Strategy in a Competitive Game

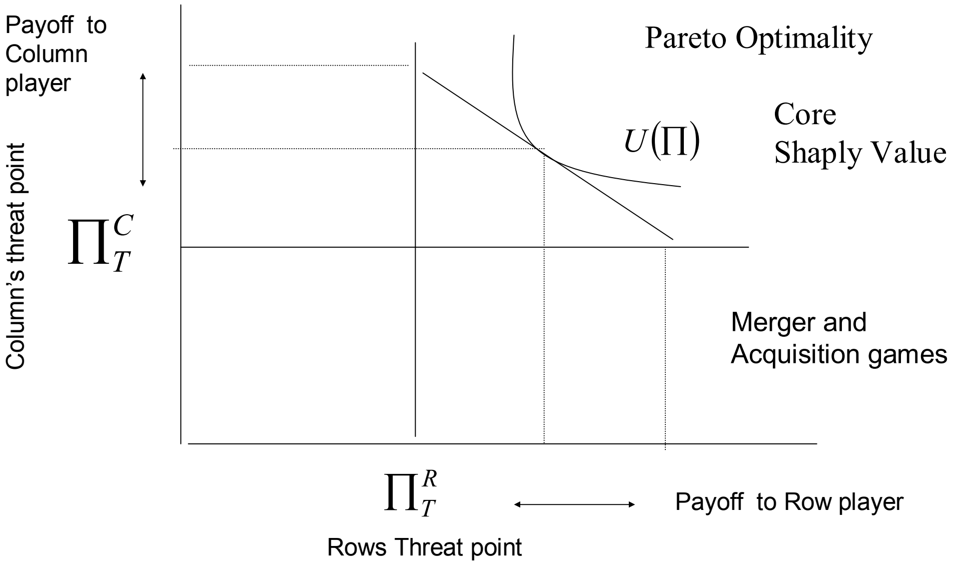
$$\begin{array}{lcc} & left & right \\ top & (50,-50) & (80,-80) \\ Bottom & (90,-90) & (20,-20) \end{array}$$



$$50p + 90(1-p) = 80p + 20(1-p) \qquad 100p = 70$$

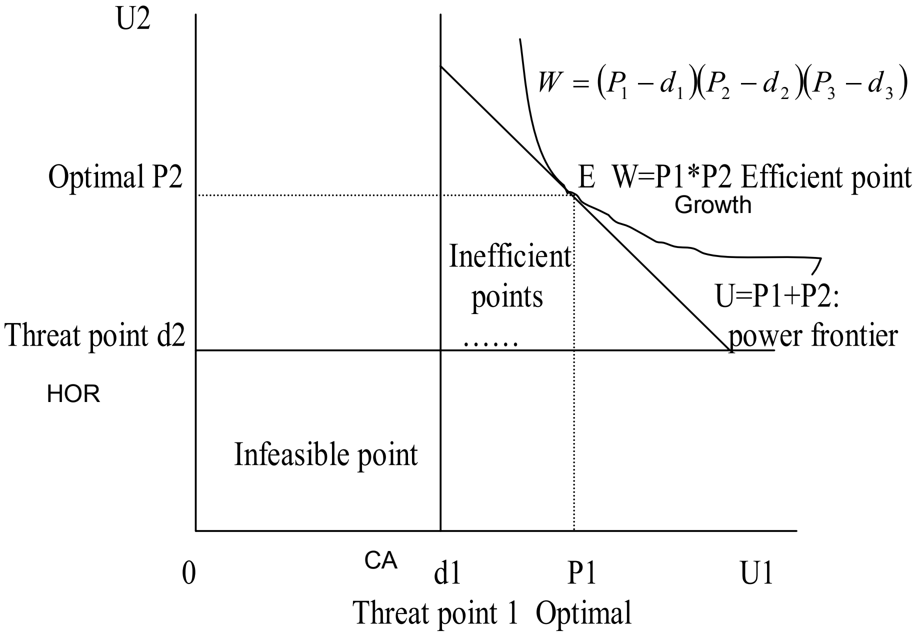
Pareto Optimality in bargaining, solution

Gains from Co-operative Solutions and Room for a Bargain

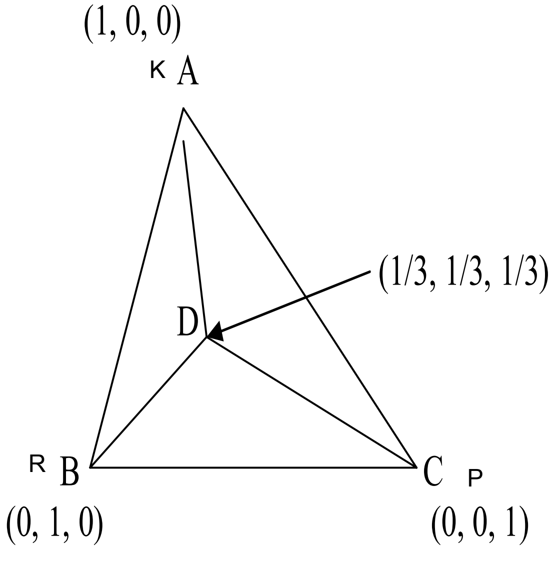


With many players there can be several coalitions.

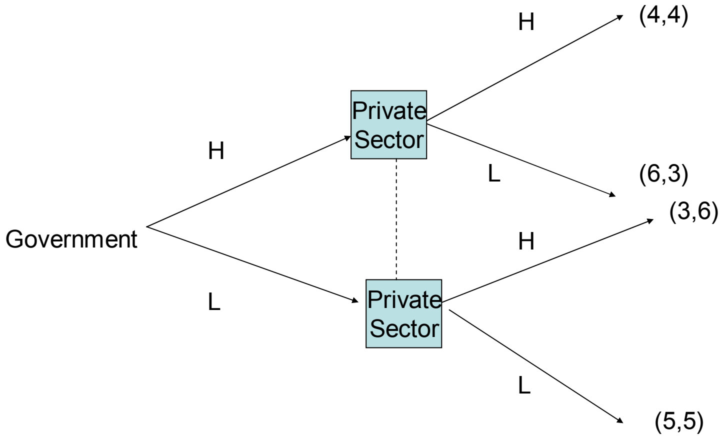
Efficient and Inefficient Bargaining Solutions



Bargaining Process and Equilibrium



GAMES with Incomplete Information



Battle of sexes

		Girls	
		Play	Sing
Boys	Play	(5,3)	(0,0)
	Sing	(0,0)	(5,3)

Players' objective is to maximise the expected payoff. Actions chosen by players (boys) depend on what they believe that others (girls) will choose. It is unpredictable what others will choose each player can only form a probability as following

		Girls	
		Play (c)	Sing (1- c)
Boys	Play (r)	(5,3)	(0,0)
	Sing (1-r)	(0,0)	(5,3)

Where boys play top with probability r and bottom with probability (1-r). Girls play left column with probability c and right column with probability (1-c).

Calculate the expected pay-off of boys

$$E\Pi^{B} = 5rc + 5(1-c)(1-r) = 5rc + 5(1-c-r+rc) = 5rc + 5 - 5c - 5r + 5rc = 10cr - 5c - 5r + 5$$

$$E\Pi^{G} = 3rc + 3(1-c)(1-r) = 3rc + 3(1-c-r+rc) = 3rc + 3 - 3c - 3r + 3rc = 6cr - 3c - 3r + 3$$

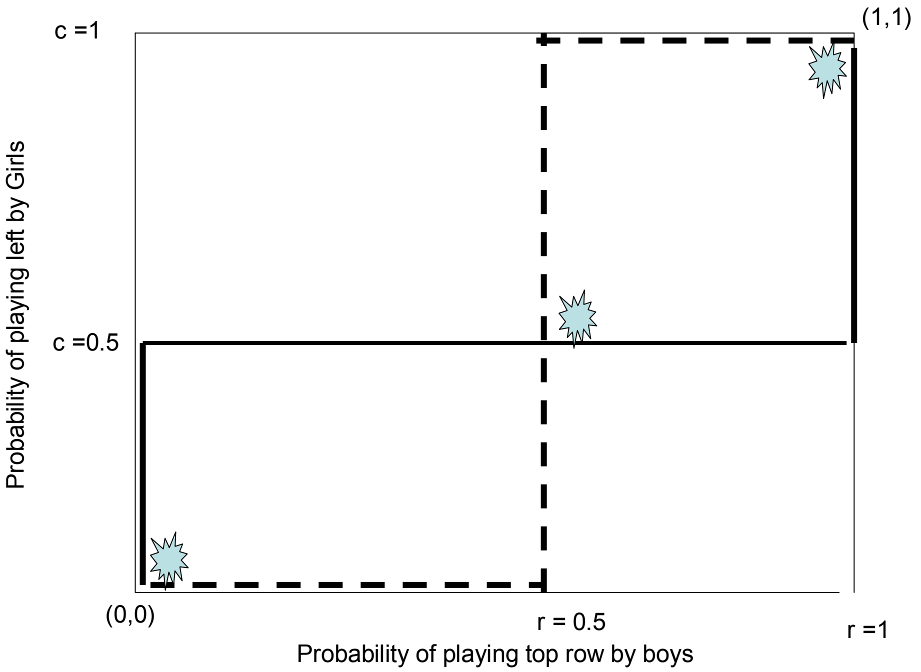
Given this objective function what is likely probability that boys will play top?
Consider increasing probability by a bit and check how it affects the expected payoff.

$\Delta E\Pi^{B} = 10c\Delta r - 5\Delta r = (10c - 5)\Delta r \quad \Delta E\Pi^{B} > 0$ only if $(10c \ > \ 5)$ or $\left(c \ > \frac{1}{2}\right)$

$\left(\frac{1}{2} < c < 1\right)$. Similarly for girls consider incremental probability of playing the left.

$\Delta E\Pi^{G} = 6\Delta cr - 3\Delta c = (6r - 3)\Delta c \quad \Delta E\Pi^{G} > 0$ only if $(6r \ > \ 3)$ or $\left(r \ > \frac{1}{2}\right)$

Summarise these results in the best response curve



The ✷ symbol indicated the probability in equilibrium, one in the middle indicated the mixed strategy and other two solutions indicate the pure strategy.

Asymmetric information and collapsing of markets (See Varian 36, P&R 17)

Asymmetric information: Market Failure in Plum and Lemon cars (Akerlof)
Lower quality products (car) drive the high quality products out of the market. If many low quality products are sold in the market it is difficult to sell high quality products.

Seller knows the exact quality of car but buyer does not. It is safe for him to assume that all cars are of low quality. Demand for high quality cars fall and that for low quality cars rise. In fact only low quality cars remain in the market. Plums disappear and only lemon remain in the market.
Signalling: Sellers can use warranty and guarantee system to signal quality of their cars. It is cheaper for the sellers of high quality cars to provide extra warranty as these are expected to last long but such warranty is costly for low quality cars as they frequently break down.

Adverse selection (hidden information): low quality items crowd out high quality items; good people are driven out by bad one. Consider theft insurance; health insurance; risky vs. gentle borrowers in a financial market. Healthy people are less likely to buy health insurance, people from safe area are less likely to buy theft insurance and honest borrowers less likely to borrow at higher interest rates. However companies offering these services do not have a priori information on this.

Moral hazard (hidden action): People who have theft insurance are likely to have easy to break locks in their bicycle (car) and most likely to claim insurances. Remedy: deductible amount; to ensure that some customers take care in security.

Missing markets: problem of low and high quality umbrella

	High quality	Low quality	Cost
Consumer's offer	14	8	11.5

Let q be proportion of high quality then equilibrium applies

$$p = 14q + 8(1-q) = 11.5$$

$$6q = 3.5 \quad q = 3.5/6 = 7/12$$

Education as a signal of quality of workers:

Type 1 is less productive than type 2 worker but an employer cannot distinguish off-hand. The marginal productivity of type 1 is less than that of type 2, $a_1 < a_2$. Production is a linear function of these two types of labour: $y = a_1L_1 + a_2L_2$. In a perfect market wage rates are according to the marginal productivity of labour $w_1 = a_1$ and $w_2 = a_2$; but when the market is not perfect the wage rate depends on the average productivity. For any $0 \le b \le 1$, the average wage rate is $w = bw_1 + (1-b)w_2$. Market is inefficient, it drives out more productive workers.
If workers can signal their quality by the level of educational attainment, then market may work well.
Low quality workers may find obtaining certain education costlier than high quality workers, $c_1e^* \ge c_2e^*$, $c_1 \ge c_2$, Let there be an educational level

$$\frac{a_2 - a_1}{c_1} < e^* < \frac{a_2 - a_1}{c_2}$$

It is costly for low quality worker to get the specified education, $(a_2 - a_1) < c_1 e^*$

It is beneficial for high quality worker to get education $(a_2 - a_1) > c_2 e^*$; so the low quality worker gets no education, but the higher quality worker gets education. Employers pay according to the level of education. Therefore education works as a signalling device and makes the market efficient. Education separates the equilibrium.

Incentive System (Michael Spence)

If a worker puts x amount of effort, the land produces $y = f(x)$. Then the land owner pays worker $s(y)$. The land owner wants to maximise

$\pi = f(x) - s(y) = f(x) - s(f(x))$

Worker has cost of putting effort $c(x)$ and has a reservation utility $\bar{u}$. The participation constraint is given by $s(f(x)) - c(x) \geq \bar{u}$. Including this constraint, the maximisation problem of the landlord becomes

$Max \quad f(x) - s(f(x))$ subject to $s(f(x)) - c(x) \geq \bar{u}$

$\underset{x}{Max} \quad f(x) - c(x) - \bar{u}$ Since $\bar{u}$ is a constant, this turns to $f'(x^*) = c'(x^*)$ where the marginal product of putting x^* amount of effort equals the marginal cost of it. As Varian summarises, there are a number of ways for designing an incentive compatible contract

(a) renting the land where the workers pays a fixed rent R to the owner and takes the residual amount of output, at equilibrium $f(x^*) - c(x^*) - R = \bar{u}$

(b) Take it or leave it contract where the owner gives some amount such as B^*, $B^* - c(x^*) = \bar{u}$,

(c) hourly contract $s(f(x)) = wx + K$

(d) sharecropping, in which both worker and owner divide the output in a certain way.

In (a)-(c) burden of risks due to fluctuations in the output falls on the worker but it is shared by both owner and worker in (d). Which of these incentives work best depends on the situation.

St. Petersburg Paradox:

How much should one pay to play a game that promises to pay 2^n if the head turns up in the nth trial? Bernouli's Game.

Expected payoff: $E(\pi) = \pi_1 \cdot 2 + \pi_2 \cdot 2^2 + \pi_3 \cdot 2^3 + .. + \pi_n \cdot 2^n = 1 + 1 + 1 + ... + 1 + = \infty$

$\pi_1 > \pi_2 > \pi_3 > .... > \pi_n \quad \pi_1 = 1/2$; $\pi_2 = 1/2^2$; $\pi_3 = 1/2^3$; .... $\pi_n = 1/2^n$

The above calculations ignore the fact that utility increases at decreasing rate as:

$$E(u) = \frac{1}{2}\ln(2) + \frac{1}{2^2}\ln(2^2) + \frac{1}{2^3}\ln(2^3) + .. + \frac{1}{2^n}\ln(2^n) < \infty$$

Solutions $E(u) = \frac{1}{2}\ln(2) + \frac{1}{2^2}\ln(2^2) + \frac{1}{2^3}\ln(2^3) + .. + \frac{1}{2^n}\ln(2^n) < \infty$

$$E(u) = \sum_i^\infty \frac{1}{2^i} i \ln 2 = \ln 2 \sum_i^\infty \frac{i}{2^i} = \ln 2 \cdot 2 = 1.39$$

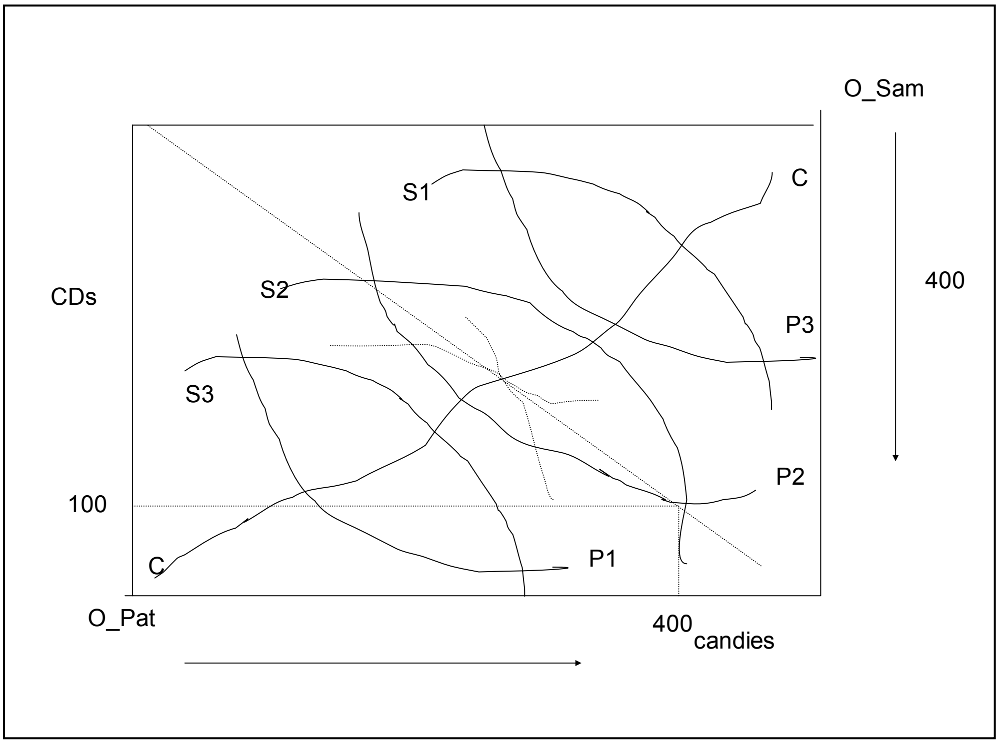
They should not pay more than 1.39

Key questions: More information makes market efficient. Discuss how the internet is making this happen.

Readings: VAR 28, MK 18

Lecture 19
General Equilibrium

Consider an economy of Pat and Sam. Pat has 400 of candies and 100 CDs and Sam has 100 candies and 400 CDs. Represent total amounts and individual endowments of candies and CDs using an Edgeworh box diagram (EDB). Draw reasonable preference functions (indifference curves) of Pat and Sam facing against each other (from the north east and south west quadrants) in that diagram.



What is the level of welfare that they are able to attain before any exchange?

Show how each can gain by exchanging goods that they have more with they have less using the preference line.

Show much both can gains from exchange and how that depends on their bargaining power.

Draw an Edgeworth box diagram (EBD) where capital and labour inputs are used in producing goods X and Y.

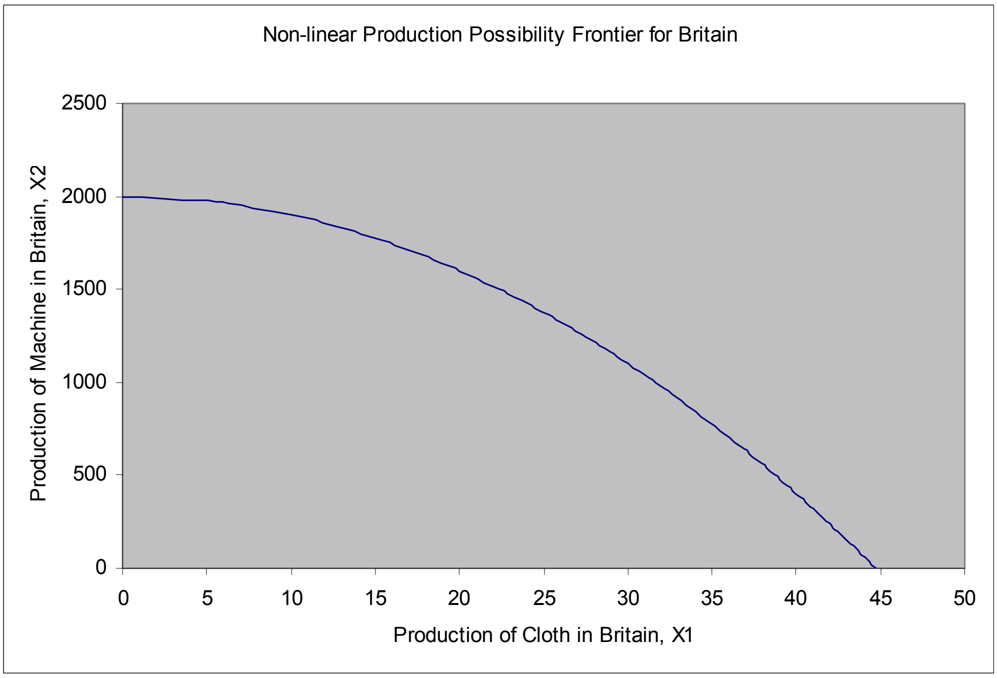
Derive a production possibility frontier using EBD of production

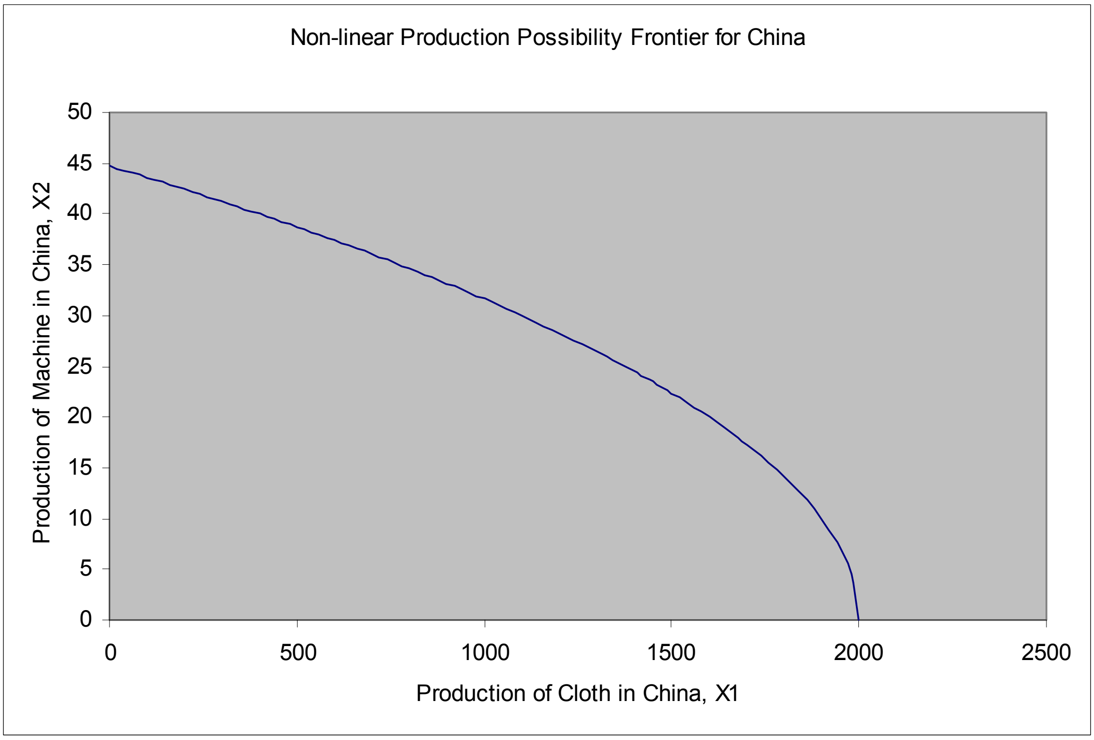
Key questions: How does a competitive price system guarantee efficiency? How does it allocate scarce resources to their best use both in consumption and production sides in a decentralised economy?

Why did central command system failed in efficiency?

Consider two countries, say, Britain and China that can produce two goods, clothes and machine. Britain is more efficient in making machines and China in making clothes. One production possibility frontier that reflects these realities is $X_1 + X_2^2 = 2000$ for China and $X_1^2 + X_2 = 2000$ for Britain where X_1 is cloths and X_2 machines.

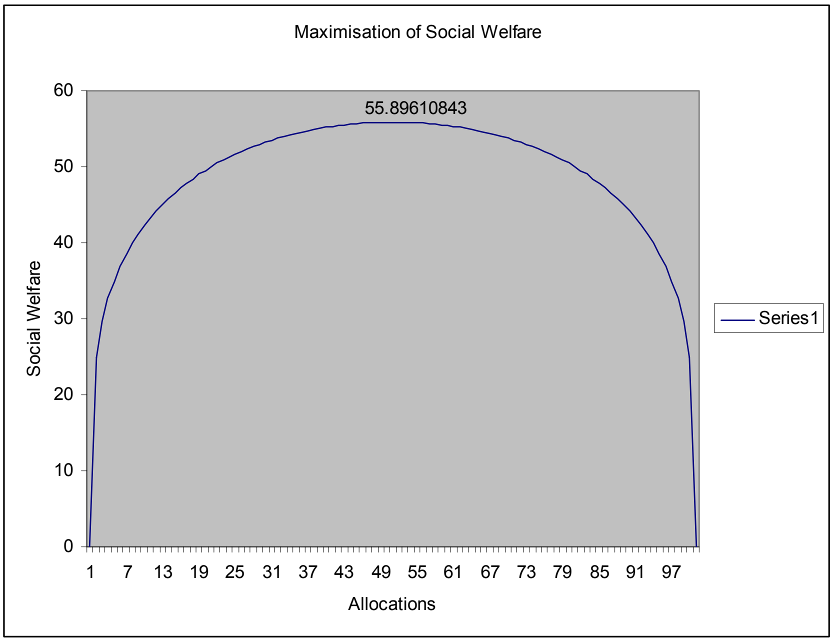
If China only produces cloth it can produce 2000 cloths, if it produces only machines it can produce only 44.7 machines. In contrast, Britain can produce 2000 machines if she puts all resources in machines and does not produce any cloth. It is economically efficient to specialise and trade, as both can have more of both goods.





Demand side of Britain is represented by a Cobb-Douglas utility functions, $U^B = X_1^B X_2^B$, its budget constraint $I^B = P_1 X_1^B + P_2 X_2^B$ and income $I^B = P_1 Y_1^B + P_2 Y_2^B$, where Y_1^B is production of cloths by Britain and Y_2^B is production of machines by Britain. Similarly the demand side of China is represented by $U^{Ch} = X_1^{Ch} X_2^{Ch}$, $I^{Ch} = P_1 X_1^{Ch} + P_2 X_2^{Ch}$, $I^{Ch} = P_1 Y_1^{Ch} + P_2 Y_2^{Ch}$. What are the equilibrium prices and demand of clothes and machines and what is the income of China and Britain if they specialise only in one good? What is be equilibrium in autarchy? What would be equilibrium price and demand if the preferences are represented by $U^B = \left(X_1^B\right)^{0.3}\left(X_2^B\right)^{0.7}$ $U^{Ch} = \left(X_1^{Ch}\right)^{0.3}\left(X_2^{Ch}\right)^{0.7}$ Show these results in an Edgeworth box diagram.

Consider an economy inhibited by two individuals with aggregate income of 50,000, which can be divided between the two in any way, $y_1 + y_2 = Y = 50{,}000$. Each person receives utility from income and is given by $u_1 = \sqrt{y_1}$ $u_2 = \frac{1}{2}\sqrt{y_2}$. Person 1 is a bit happier than person two when he/she gets one unit of income. Government likes to maximise the social welfare function that takes account of utilities of both these individuals $W = u_1^{\frac{1}{2}} u_2^{\frac{1}{2}}$. Consider 100 different allocations Y (use excel to calculate), in y_1 and y_2 in such a way that each allocation differs from another one by 500 plus or minus in acceding or descending order. Which of these allocations maximises the social welfare? Show that there is an unique allocation that maximises the social welfare function.



1. Two people, husband and wife, live in a house. Their total annual income is 50,000 pounds and all of it is consumed in the same year. Utility function of husband and wife are given by $U_H = \sqrt{Y_H}$ and $U_W = \frac{1}{2}\sqrt{Y_W}$ respectively where Y is income, subscripts H denotes husband and W for wife.

(a) What are the values of utilities of husband and wife if the income is divided equally between them?
(b) How should the income be divided so that each of them can get the same level of utility?
(c) What should be the distribution of income to maximise the total utility of the household given above utility functions?
(d) What should the distribution of household income be if wife needs the minimum utility level of 120?
(e) How should the income be distributed between them to maximise household welfare if the household welfare function is given by $W = U_H^{\frac{1}{2}} U_W^{\frac{1}{2}}$?

Answer (a)

Here $Y = Y_H + Y_W = 50{,}000$

If distributed equally each husband and wife get 15000.

$$U_H = \sqrt{Y_H} = \sqrt{25000} = 158.11 \quad U_W = \frac{1}{2}\sqrt{Y_W} = \frac{1}{2}\sqrt{25000} = 79.06$$

$$U^T = U_H + U_W = \sqrt{Y_H} + \frac{1}{2}\sqrt{Y_W} = \sqrt{25000} + \frac{1}{2}\sqrt{25000} = 158.11 + 79.06 = 233.17$$

Answer (b) For both of them to get same level of utility:

$$U_H = U_W \;\Rightarrow\; \sqrt{Y_H} = \frac{1}{2}\sqrt{Y_W} \text{ or } \sqrt{Y_H} = \frac{1}{2}\sqrt{Y - Y_H} \text{ or } Y_H = \frac{1}{4}(Y - Y_H)$$

or $4Y_H = (Y - Y_H)$ or $5Y_H = Y$ or $5Y_H = 50{,}000$ $\therefore Y_H = 10000$ and $\therefore Y_W = 40000$

$$U^T = U_H + U_W = \sqrt{Y_H} + \frac{1}{2}\sqrt{Y_W} = \sqrt{10000} + \frac{1}{2}\sqrt{40000} = 100 + 100 = 200$$

Answer (c) What should be the distribution of income to maximise the total utility of the household given above utility functions?

Total utility of the household

$$U^T = U_H + U_W = \sqrt{Y_H} + \frac{1}{2}\sqrt{Y_W} \qquad (1)$$

Household budget constraint is $50{,}000 = Y_H + Y_W$ (2)

Lagrangian for the household utility is

$$L(Y_H, Y_W, \lambda) = \sqrt{Y_H} + \frac{1}{2}\sqrt{Y_W} + \lambda[50{,}000 - Y_H - Y_W] \qquad (3)$$

Three first order conditions

$$\frac{\partial L(Y_H, Y_W, \lambda)}{\partial Y_H} = \frac{1}{2}(Y_H)^{-\frac{1}{2}} - \lambda = 0 \qquad (4)$$

$$\frac{\partial L(Y_H, Y_W, \lambda)}{\partial Y_W} = \frac{1}{4}(Y_W)^{-\frac{1}{2}} - \lambda = 0 \qquad (5)$$

$$\frac{\partial L(Y_H, Y_W, \lambda)}{\partial \lambda} = 50{,}000 - Y_H - Y_W = 0 \qquad (6)$$

Use equations (4) to (6) to solve for the optimal value of Y_H, Y_W and λ. By dividing (4) by (5) we get

$$\frac{\dfrac{\partial L(Y_H, Y_W, \lambda)}{\partial Y_H}}{\dfrac{\partial L(Y_H, Y_W, \lambda)}{\partial Y_W}} = \frac{\dfrac{1}{2}(Y_H)^{-\frac{1}{2}}}{\dfrac{1}{4}(Y_W)^{-\frac{1}{2}}} = 1 \quad \Rightarrow 2(Y_W)^{\frac{1}{2}} = (Y_H)^{\frac{1}{2}} \text{ or } 4Y_W = Y_H$$

Now using this in (6) $50{,}000 = Y_H + Y_W$

$50{,}000 = 4Y_W + Y_W$ or $50{,}000 = 5Y_W \quad \therefore Y_W = 10000 \quad \therefore Y_H = 40000$

$$U^T = U_H + U_W = \sqrt{Y_H} + \frac{1}{2}\sqrt{Y_W} = \sqrt{40000} + \frac{1}{2}\sqrt{10000} = 200 + 50 = 250$$

Answer (d) What should the distribution of household income be if wife needs the minimum utility level of 120?

$$U_W = \frac{1}{2}\sqrt{Y_W} = 120 \Rightarrow \sqrt{Y_W} = 240 \Rightarrow Y_W = (240)^2 = 57600$$

But the household income is only 50000. Household has to borrow 7600 and husband gets nothing. Doubling wife's living standard given the current income implies greater amount of the debt for the household.

Answer (e) How should the income be distributed between them to maximise household welfare if the household welfare function is given by $W = U_H^{\frac{1}{2}} U_W^{\frac{1}{2}}$?

Here solution procedure is the same as in (b) above. Just the utility function is different

$$L(Y_H, Y_W, \lambda) = U_H^{\frac{1}{2}} U_W^{\frac{1}{2}} + \lambda[50{,}000 - Y_H - Y_W]$$

$$L(Y_H, Y_W, \lambda) = \left(\sqrt{Y_H}\right)^{\frac{1}{2}} \left(\frac{1}{2}\sqrt{Y_W}\right)^{\frac{1}{2}} + \lambda[50{,}000 - Y_H - Y_W]$$

$$\frac{\partial L(Y_H, Y_W, \lambda)}{\partial Y_H} = \left(\frac{1}{2}\right)^{\frac{1}{2}} \frac{1}{4}(Y_H)^{\frac{1}{4}-1}(Y_W)^{\frac{1}{4}} - \lambda = 0 \qquad (4')$$

$$\frac{\partial L(Y_H, Y_W, \lambda)}{\partial Y_W} = \left(\frac{1}{2}\right)^{\frac{1}{2}} \frac{1}{4}(Y_H)^{\frac{1}{4}}(Y_W)^{\frac{1}{4}-1} - \lambda = 0 \qquad (5')$$

$$\frac{\partial L(Y_H, Y_W, \lambda)}{\partial \lambda} = 50{,}000 - Y_H - Y_W = 0 \qquad (6')$$

Use equations (4') to (6') to solve for the optimal value of Y_H, Y_W and λ. By dividing (4') by (5') we get

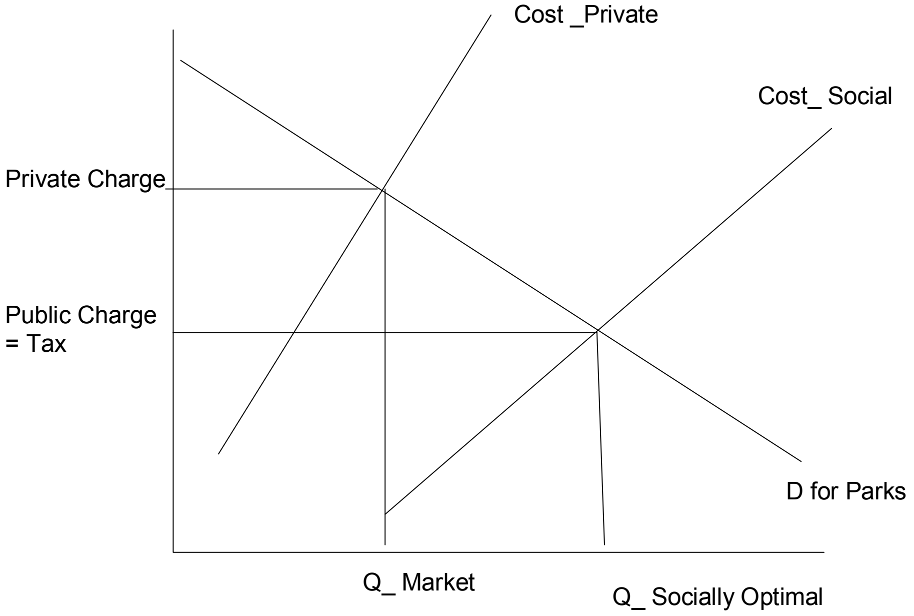
$$\frac{\dfrac{\partial L(Y_H, Y_W, \lambda)}{\partial Y_H}}{\dfrac{\partial L(Y_H, Y_W, \lambda)}{\partial Y_W}} = \frac{\left(\dfrac{1}{2}\right)^{\frac{1}{2}} \dfrac{1}{4}(Y_H)^{\frac{1}{4}-1}(Y_W)^{\frac{1}{4}}}{\left(\dfrac{1}{2}\right)^{\frac{1}{2}} \dfrac{1}{4}(Y_H)^{\frac{1}{4}}(Y_W)^{\frac{1}{4}-1}} = 1 \quad \Rightarrow \frac{Y_W}{Y_H} = 1 \text{ or } Y_W = Y_H = 25000$$

Both husband and wife should get the same income if the household utility function is defined like this.

Readings: VAR 31

Lecture 20
Externality

What would happen to public parks if city councils do not maintain? Personal and social benefits of a beautiful garden?



Why does not produce efficient amount of education and health?

Why will market produce excessive amount of water, air or noise pollution?

Why many cities in England are introducing congestion charges?

Numerical example of positive and negative externalities:

A classic example of positive externality: bees pollinate apple trees and they get materials for honey from apples. For instance cost of producing apples is $C_a = a^2$ and cost of producing honey $C_h = h^2 - a$, my marginal cost pricing rule

$\Pi_a = P_a a - C_a = P_a a - a^2$ $\frac{\partial \Pi_a}{\partial a} = P_a - 2a = 0$ $P_a = 2a$ and

hence supply of apples $a = \frac{P_a}{2}$; Similarly $\Pi_h = P_h a - C_h = P_h h - h^2 + a$

$\frac{\partial \Pi_h}{\partial a} = P_h - 2h = 0$ $P_h = 2h$ Supply of honey by the private market $h = \frac{P_h}{2}$.

Private market does not consider positive externality.

$\Pi = P_a a - a^2 + P_h h - h^2 + a$

$$\frac{\partial \Pi}{\partial a} = P_a - 2a + 1 = 0 \quad a = \frac{P_a}{2} + \frac{1}{2}$$

$$\frac{\partial \Pi}{\partial h} = P_h - 2h = 0 \quad h = \frac{P_h}{2}$$

It is optimal to produce more apples taking account of its positive externality.

Negative externality production of electricity and pollution and food production

Electricity production using coal generates electricity as well as pollution. This pollution raises production cost in the food industry.

Cost of electricity production when the environment is not taken into account

$C_e = e^2 - (x-3)^2$ and its profit function is: $\Pi_e = P_e e - C_e = P_e e - e^2 + (x-3)^2$ the cost of food production $C_f = f^2 + 2x$ and its profit $\Pi_f = P_f f - C_f = P_{fe} f - f^2 + 2x$. Pollution adds extra cost in food production.

Private market solution

$\frac{\partial \Pi_e}{\partial e} = P_e - 2e = 0 \quad P_e = 2e$ and $e = \frac{P_e}{2}$

hence supply of apples $e = \frac{P_e}{2}$; $\frac{\partial \Pi_f}{\partial f} = P_f - 2f = 0 \quad P_f = 2f$ Supply of honey by the private market $f = \frac{P_f}{2}$. Here pollution is produced more than optimal.

$$\frac{\partial \Pi_e}{\partial x} = 2(x-3) = 0 \quad x = 3$$

Socially optimal solution : $\Pi = P_e e - e^2 + (x-3)^2 + P_f f - f^2 + 2x$

$$\frac{\partial \Pi}{\partial e} = P_e - 2e = 0 \quad P_e = 2e \quad e = \frac{P_e}{2}$$

$$\frac{\partial \Pi_f}{\partial f} = P_f - 2f = 0 \quad P_f = 2f \quad h = \frac{P_h}{2}$$

$$\frac{\partial \Pi}{\partial x} = 2(x-3) + 2 = 0 \quad x = 2$$

Social solution generates less pollution than the market solution.

Marginal social benefit and personal social cost

Economics of Information

Provision of public goods: two consumers, and public and private goods, but not clear how they should pay for it; valuation of each person is different.

Proposition: Pareto optimality requires that sum of the marginal rate of substitution between private and public goods by two individuals should equal the marginal cost of provision of public goods (see Varian 35).

$$Max \quad u_1 = u_1(x_1, G)$$

subject to 1) $\bar{u}_2 = u_1(x_2, G)$

2) $x_1 + x_2 + c(G) = w_1 + w_2$

Lagrangian of the problem:

$$L = u_1(x_1, G) - \lambda(u_1(x_2, G) - \bar{u}_2) - \mu(x_1 + x_2 + c(G) - w_1 - w_2)$$

Three first order conditions:

1. $\frac{\partial L}{\partial x_1} = \frac{\partial u_1(x_1, G)}{\partial x_1} - \mu = 0$ or $\mu = \frac{\partial u_1(x_1, G)}{\partial x_1}$

2. $\frac{\partial L}{\partial x_2} = -\lambda \frac{\partial u_2(x_2, G)}{\partial x_2} - \mu = 0$ or $-\frac{\partial u_2(x_2, G)}{\partial x_2} = \frac{\mu}{\lambda}$

3. $\frac{\partial L}{\partial G} = \frac{\partial u_1(x_1, G)}{\partial G} - \lambda \frac{\partial u_2(x_2, G)}{\partial G} - \mu \frac{\partial c(G)}{\partial G} = 0$ or

$$\frac{1}{\mu}\frac{\partial u_1(x_1, G)}{\partial G} - \frac{\lambda}{\mu}\frac{\partial u_2(x_2, G)}{\partial G} = \frac{\partial c(G)}{\partial G}$$

$$\frac{\frac{\partial u_1(x_1, G)}{\partial G}}{\frac{\partial u_1(x_1, G)}{\partial x_1}} + \frac{\frac{\partial u_2(x_2, G)}{\partial G}}{\frac{\partial u_2(x_1, G)}{\partial x_2}} = \frac{\partial c(G)}{\partial G}$$ or $MRS_1 + MRS_2 = MC(G)$ Q.E.D.

Key questions: Pollution controls are less important in developing countries such as China and India. How does it affect the global environment?

Readings: VAR 33

Lecture 21 and 22
Review on lectures 1-20

Key questions:

Policy applications
Readings:

http://en.wikipedia.org/wiki/Main_Page

Background in Mathematics Required for Microeconomics

Four Rules of Differentiation

I. Power Rule $Y = K^{\alpha}$ $\quad \frac{\partial Y}{\partial K} = \alpha K^{\alpha - 1}$

II. Product Rule $R = PY$ $\quad dR = Y \times dP + P \times dY$

III. Quotient Rule $y = \frac{Y}{L}$ $\quad dy = \frac{L \times dY - dL \times Y}{L^2} = \frac{dY}{L} - \frac{dL}{L}\frac{Y}{L}$

IV. Chain Rule $Y = L^{0.5}$ $\quad L = 50\left(\frac{w}{p}\right)^2$ $\quad Y = \left[50\left(\frac{w}{p}\right)^2\right]^{0.5}$

$$\frac{\partial Y}{\partial (w/p)} = \frac{\partial Y}{\partial L}\frac{\partial L}{\partial (w/p)} = 0.5L^{-0.5} 50 \times 2(w/p) = \left(\frac{50}{L}\right)(w/p)$$

Five Rules of log

I. Power Rule $Y = K^{\alpha}$ $\quad \ln(Y) = \alpha \ln(K)$

II. Product Rule $R = PY$ $\quad \ln(R) = \ln(Y) + \ln(P)$

III. Quotient Rule $y = \frac{Y}{L}$ $\quad \ln(y) = \ln(Y) - \ln(L)$

IV. Log of Exponentials $Y_t = Y_0 e^{gt}$ $\quad \ln(Y_t) = \ln(Y_0) + gt$

V. Differentiation of log with respect to time

$$\frac{d(\ln Y_t)}{dt} = \frac{dY_t}{Y_t}$$

Six rules of approximation for small numbers

I. Log $\quad \ln(1 + r) = r$

II. Product $(1 + \pi)(1 + g) \cong (1 + \pi + g)$ $\quad (1 + g)^n \cong 1 + ng$

III. Division $\quad (1 + r) = \frac{1 + i}{1 + \pi^e} \approx (1 + i - \pi^e)$

IV. Growth of a product $\quad Y = LW$ $\quad g_Y = g_L + g_W$

V. Growth of a Ratio $\quad y = \frac{Y}{L}$ $\quad g_y = g_Y - g_L$

VI. Sum of a geometric Series

$$S = 1 + X + X^2 + ... + X^n = \frac{1 - X^{n+1}}{1 - X}$$

Problem 1
Budget constraints

1.1. Harry is a poor person and has 200 to spend in food (X) and recreation (Y). Sally is rich and has 1000 to spend on X and Y. Market prices of X and Y, denoted by P_x and P_y, were 4 and 10 respectively.

   a. Formulate budget constraints for Harry and Sally.
   b. How much can Harry buy X if all is spent in X? How much of Y can he buy if he dose not spent anything in X? Using Excel show 10 combinations of X and Y that Harry can buy with his income.
   c. Do the same exercise for Sally.
   d. Compare the feasibility set for Harry and Sally and explain differences in their living standards implied by such configurations of their income.

1.2. Harry gets benefit worth 200 to complement his income. How does this change his budget constraint? Sally pays 200 in taxes. How does this affect her budget constraint?

1.3. Technological innovations make X cheaper and prices of X now reduce to 2 but the prices of Y remain unchanged. How does this affect budget constraints of Harry and Sally?

1.4. Prices of X remain at 4 but the due to rising fuel prices Y becomes more expensive. It costs 20 for one unit of Y. How does it affect the budget constraints of Harry and Sally?

1.5. Compare new budget constraints for Harry and Sally when X is cheaper and Y more expensive to the original budget constraints. Show feasible and infeasible points in these budget sets.

Basics of Differentiation and its application in micro

Find the first derivative of following functions:

i) $y = a$. ii) $y = a+bx$ iii) $y = 5+5x-2x^2$ iv) $y = xz$ v) $y = x/z$

Find minimum point of the average cost function $AC = 3q \quad -4q+6$

What is the profit maximising output in $\Pi = R - TC = 100Q - \frac{1}{10}Q^2 - 6Q$.

Problem 2
Marginal utility, indifference curves and demand

2.1 Graph total and marginal utilities for following utility functions. Provide examples.

a. Linear utility function $U = 2X$ and $U = 0.5X$

b. Non linear (log linear) utility function $U = \ln(x)$

c. Exponential utility function $U = X^{0.5}$

c. Reciprocal utility function $U = \dfrac{100}{X}$

2.2. Graph indifference curves for following functions

a. $U = X^{0.5}Y^{0.5}$ when the utility level is 10.

b. $U = X^{0.5}Y^{0.5}$ when the utility level is 15.

c. What is the marginal rate of substitution between X and Y? Why should this differ at different points of an indifference curve?

2.3. Using a graphical method show the equilibrium point for a consumer whose utility function is $U = X^{0.5}Y^{0.5}$ and who faces a budget constraint $I = P_x X + P_y Y$.

2.4. Derive demand functions for products X and Y with the utility function $U = X^{\alpha}Y^{(1-\alpha)}$ and the budget constraint $I = P_x X + P_y Y$. Explain how the demands for X and Y depend on their prices and income of the consumer. Show this demand in a diagram.

2.5 Demonstrate how the market demand curve can be derived using individual demand curves, with a numerical example for any two individuals.

Problem 3
Income and substitution effects and elasticity

3.1 A person consumes X and Y two normal commodities.

a. Demonstrate equilibrium for this consumer in X, Y space.
b. Commodity X becomes cheaper and consumer buys more of X. Show the change in equilibrium in a diagram.
c. Derive demand curve for X using above results.
d. Reduction in price of X has increased the real income of the consumer. Decompose the effect of price change in income and substitution effects.
e. Indicate Hicksian equivalent and compensating variations.
f. Compare the compensated demand curve with the uncompensated demand curve.
g. What would happen to demand for X if it was an inferior or a superior good?

3.2 Define price elasticity, income elasticity and cross elasticity of demand for a product. Provide numerical examples for each.

3.3 Robinson gets utility from consumption of ice-cream and salads in every lunch. He spends 40 percent of his lunch budget in ice-cream and remaining 60 percent in salads. His utility function can be written as $U_0 = X_1^{0.4} X_2^{0.6}$ where X_1 is amount of icecream and X_2 is amount of salads. Price of ice-cream is 3. What is the demand function of icecream? If the price of ice-cream falls to 2 what will be the demand for ice-cream? Decompose total change in demand between substitution and income effects.

Problem 4
Production and cost function

4.1 What is the difference between the short run and long run in production?

4.2 Find total, marginal and average product for the following production functions. Y is output, *L* labour and *K* capital, *a* and *b* are parameters of the model.

a. Linear production function: $Y = a + bL$. Show TPL, MPL and APL in a diagram. Is this a reasonable production function?

b. Cobb-Douglas production function $Y = L^{\alpha} K^{\beta}$.

4.3. Write an iso-cost line (resource constraint) faced by a firm that uses capital and labour in production. Show a short run cost function for a firm with such technology using a diagram.

4.4. What is an iso-quant? Show how the same level of output can be produced by labour intensive, capital intensive or equal amounts of labour and capital.

4.5. Show optimum choices of inputs by a producer using Iso-cost and iso-quant lines. How does a firm react to changes in the interest rate or reduction in wage rate due to influx of immigrants?

4.6. What is the meaning of the rate of technical substitution? Why should it equal to the ratios of input prices.

4.7. What is the meaning of the elasticity of substitution in production? Prove that the elasticity of substation is one for a Cobb-Douglas production function.

4.8. A firm is producing for two markets, locating plants in A and B regions. It can borrow to invest from banks at a standard interest rate r. At the moment plant A is more capital intensive than the plant B. Use the marginal productivity theory of capital to show how it should allocate capital stock between these two plants.

4.7 Consider a total cost function of the following form: $C = Y^3 - 10Y^2 + 25Y + 100$. Find the associated marginal and average cost functions. Plot average and marginal cost functions in one diagram. The total cost function in another diagram.

4.8 Compare long run average costs for increasing, decreasing and constant returns to scale technology.

Problem 5

Market prices: Price and output decision under perfect competition

5.1 Use diagrams to show optimal level of output for a firm under the perfectly competitive market.

5.2 Derive the long run cost function for firms with production function $Q = 10L^{0.5}K^{0.5}$ and cost function $C = P_k K + P_L L$ where the price of capital P_k is 40 and price of labour P_L is 10. What is the marginal cost for this form? How much should the price of the product be and how much should this firm produce in a competitive market?

5.3 Consider revenue and cost functions for a firm operating under the perfectly competitive market:

Revenue: $R = PQ$

Cost function: $TC = \frac{1}{3}Q^3 - 5Q^2 + 40Q + 10$

What is the price and the level of output optimal for a profit maximising firm under this market condition?

5.4 Market demand and supply for a product is given by $D = 50 - 5P$ and $S = -2 + 4P$ respectively.

a) Represent these equations in one diagram and indicate the point ot equilibrium, excess demand and excess supply. Briefly mention the mechanism that brings this market system towards equilibrium point.

b) Find out the equilibrium price and quantity demanded and supplied solving this market system.

c) What are the amounts of consumer surplus and producer surplus in this equilibrium?

d) Government imposes sales tax of 1 in production of this good. How does it affect the demand and supply curves? Illustrated using in another diagram.

e) How can this tax be incorporated in the above model by using post tax prices paid by buyers and received by supplies?

f) Solve the tax distorted system and find out the amount of quantities bought and sold, the prices paid by buyers, received by suppliers, the government revenue.

g) What are the deadweight losses to consumers, producers and the economy as a whole from the imposition of this tax?

h) If the technology reduces the cost of production if this commodity and supply curve changes to $S = -2 + 2P$ what will be equilibrium quantities before and after taxes and the deadweight loss of taxes?

i) How does a massive advertisement campaign affect the demand curve in this market? Does it make it more elastic or less elastic?

Problem 6
Price and output under monopoly

6.1 Illustrate pricing and output decisions of a monopolist using a diagram. Explain aims and objectives of a monopolist and why it is socially inefficient in allocating resources for production?

6.2 Demand for a monopolist is given by $P = 52 - 1.5Q$ and it has a cubic cost function $C = \frac{1}{3}Q^3 - 5Q^2 + 34Q + 169$. How much price does this monopolist charge to maximise its profit? How much does it sell? What is its profit?

6.3 Illustrate the difference between a monopolist and a firm under perfect competition if the market demand is $Q = 10{,}000 - 500P$ and the short run total cost function is $SRTC = 160 + 0.025Q^2$

6.4 POP is a monopoly supplier of passenger parking services for ferries from Hull to Rotterdam. For a certain day its demand function is given by $Q = 50 - \frac{1}{10}P$ and its cost function is $C = 8Q$.

a) How should POP determine its prices and quantity supplied to the market?

b) What is POP's profit and consumer surplus from its services?

c) After a study POP finds that it can operate in both sides of the North Sea. Its demand curve at Hull is given by $Q_1 = 21 - 0.1 \times P_1$ and that in Rotterdam by $Q_2 = 50 - 0.4 \times P_2$ If it operates to both sides of the market it faces a common cost function $C = 2000 + 10Q$. How much price should POP charge for per unit of parking services in Hull? How much profit does it earn from Hull?

d) How much price should POP charge for per unit of parking services in Rotterdam? How much profit does it earn from Rotterdam? What is the aggregate profit?

e) What is the consumer surplus in Hull and in Rotterdam? How does it compare to the consumer surplus when it did not discriminate the market in this manner?

Problem 7
Price and output under duopoly

7.1 Consider a market with two firms facing a market given by $P = 50 - (q_1 + q_2)$ and each faces a cost function $C_i = 6q_i$.

a. Illustrate this market using a diagram
b. Derive the reaction functions for firm 1 and 2 under Cournot strategy.
c. Determine the level of output chosen by each firm, market price and level of profit for each.

7.2 What would be the equilibrium solution if both firms acted as a cartel to maximise joint profits?

7.3 Now assume the firms 1 acts as a Stackelberg leader and firm 2 follows the decisions made by firm 1. What are the levels of output and profit of each firm and the market price of the commodity?

7.4. Market demand curve for mobile phones is given by $Q = 10 - \frac{1}{5}P$. Two firms exists in the market to supply this product. Their cost functions are $C_1 = 4q_1$ and $C_2 = 5q_2$.

a) Write the profit function for each firm in this market. [12]
b) Each operates under Cournot model, takes other's action as a given parameter while making its production decision. Determine the reaction functions for both firms and represent it in a diagram. [12]
c) How much will each produce? What will be the market price? How much profit will each make from selling mobiles? [12]
d) What will be the consumer surplus from each of them? Show these consumer surpluses in a diagram. [12]
e) Firm 1 acts as a leader and takes account of the reaction function of its follower. How is it's profit function modified. What will be output level of firm 1? [12]
f) What will be the output of the follower firm? What will be the market price and profit of firm 1 and firm 2? [12]
g) What would the output and profit levels of each firm and the market price would have been if the firm 2 acted as a leader and firm 2 acted as a follower? Present these results of the Leader-Follower game in a pay-off matrix of the following form. [16]

	Leader	Follower
Leader		
Follower		

h) What would have been the level of output, market price and profit if they had formed a collusive cartel? [12]

Problem 8
Basics of Game theory

8.1 Draw a pay-off matrix in the following format for a market only with two producers under Cournot and Stackelberg model presented above, if their market demand is $P = 100 - (q_1 + q_2)$ and the cost function is $C_i = 6q_i$.

	Leader	Follower
Leader		
Follower		

Explain strategic choices of for each player.

8.1 Formulate a two by two game which can be solved using a dominant strategy for one or both players.

		Firm 2	
		Enter	Stay out
Firm 1	Enter		
	Stay out		

8.2 Formulate a game which has can be solved using a Nash equilibrium method. Explain why any other solution is not a stable solution.

		Firm 2	
		Deal	No Dealt
Firm 1	Deal		
	No Deal		

8.3. Consider wage bargaining between unions and firms. Using diagrams illustrate how unions or firms can settle a wage dispute using their credible threat points, utility possibility frontier and the Nash product. Why is cooperative solution Pareto superior to a non-cooperative solution?

8.4 Consider a game called battle of sexes.

		Girls	
		Play	Sing
Boys	Play	(5,3)	(0,0)
	Sing	(0,0)	(5,3)

Show that this game has multiple equilibria in pure and mixed strategies. Illustrate results using the best response curves of both players (see Varian Chapter 29).

8.5 Use the St. Petersburg Paradox and Bornouli game to illustrate how much money can people bet on to play games like the “Who wants to be a millionaire?”.

Problem 9
General equilibrium under pure exchange

10.1 Use an Edgeworth box diagram to illustrate how two individuals A and B can benefit from exchanging goods X and Y in a market economy.

10.1 Derive production possibility frontier for an economy using Edgeworth diagram where the labour and capital inputs are used producing goods X and Y.
10.2 What drives the prices of these goods. Can you specify a model?

10.3. Consider two countries, say, Britain and China that can produce two goods, clothes and machines. Britain is more efficient in making machines and China in making clothes. Production possibility frontiers that reflects these realities are $X_1 + X_2^2 = 2000$ for China and $X_1^2 + X_2 = 2000$ for Britain where X_1 is clothes and X_2 machines.

a) How much of clothes and machines can be produced in autarchy? How much of clothes and machines can be produced if China completely specialises in clothes and Britain specialises only in producing machines?

b) Represent these solutions in an Edgeworth box diagram with cloths on X axis and Machines in Y axis and Britain's moving towards the Northeast direction and Chin moving towards Southwest for optimisation in production and consumption.

c) Show the potential gains from specialisation and trade and what sort of bargaining in the division of gains is implied by this model.

d) Demand side of Britain is represented by a Cobb-Douglas utility functions, $U^B = X_1^B X_2^B$, its budget constraint $I^B = P_1 X_1^B + P_2 X_2^B$ and income $I^B = P_1 Y_1^B + P_2 Y_2^B$, where Y_1^B is production of cloths by Britain and Y_2^B is production of machines by Britain. Similarly the demand side of China is represented by $U^{Ch} = X_1^{Ch} X_2^{Ch}$, $I^{Ch} = P_1 X_1^{Ch} + P_2 X_2^{Ch}$, $I^{Ch} = P_1 Y_1^{Ch} + P_2 Y_2^{Ch}$. What are the equilibrium prices and demand of clothes and machines and what is the income of China and Britain if they specialise only in one good?

e) What is the equilibrium in autarchy implied by these preferences?

f) What would be equilibrium price and demand if the preferences are represented by $U^B = \left(X_1^B\right)^{0.3}\left(X_2^B\right)^{0.7}$ $U^{Ch} = \left(X_1^{Ch}\right)^{0.3}\left(X_2^{Ch}\right)^{0.7}$

Show these results in an Edgeworth box diagram.

Problem 10
Externality

10.1 Give five examples of negative and positive externalities in consumption and in production.

10.2 Illustrate a market for pollution? Explain why the market fails to provide correct amount of clean air?

10.3 Explain positive externality of internet or education? Justify cases for subsidising research on the ground of positive externality.

10.4 Consider the positive externality from the education sector to industries. Universities and labs generate useful knowledge that can be applied to increase productivity in industries. For instance cost of producing education is $C_E = E^2$ and profit of producing education services is $\Pi_E = P_E E - C_E = P_E E - E^2$ and the cost of producing goods is $C_G = G^2 - E$ and its profit function taking account of the positive externality is $\Pi_G = P_G G - C_G = P_G G - G^2 + E$.

a) What is the optimal amount of education in the private market solution?
b) What will be socially optimal amount of education? Illustrate these solutions using a diagram.

10.5 Electricity production using coal generates electricity as well as pollution which raises cost of production in the food industry. Cost of electricity production without any concerns for environment is given by $C_e = e^2 - (x-3)^2$, where *x* is the amount of pollution caused by producing *e* amount of energy and C_e denotes the private cost of generating electricity. The profit function for the electricity plant thus is given by $\Pi_e = P_e e - C_e = P_e e - e^2 + (x-3)^2$

The cost of food production is given by $C_f = f^2 + 2x$ where the pollution puts extra cost in producing food. The profit for food plant is $\Pi_f = P_f f - C_f = P_{fe} f - f^2 + 2x$

a) What it the optimal amount of electricity and food supply and amount of pollution generated by the electricity plant in the private market solution?

b) How much pollution is reduced if the electricity is produced to minimise the extra social cost generated by the pollution?

Sample Final Exam: Microeconomics (26207)

Two hours

Answer any four questions. Marks are as indicated at the margins.

1. Sharon has 68 hours of productive time each week, which he can either spend to work or spend on leisure, $L + l = 68$. She earns money if she works but can either watch TV, play pool or sleep and relax in here leisure time. She has equal preference for leisure and consumption, $U = C \times l$, where *U* is utility, *C* is composite consumption and *l* is leisure. She can get market wage of 10 for his work and price of composite consumption is 1.

   a) Represent Sharon's budget and constraint in (*l*, C) space. [14]
   b) Show how she chooses optimal hours of leisure, work and consumption in that one diagram. [14]
   c) Solve her constrained optimisation problem and find out the optimal hours of leisure and amount of amount of her optimal consumption. [14]
   d) How many hours are optimal hours for him to work? How many hours are optimal leisure hours for him? What will be his utility? [14]
   e) How will her optimal leisure hours change if the wage rate rises to 20? What would be her optimal amount of consumption? Draw her implied labour supply curve. [14]
   f) Decompose the change in leisure in income and substitution effects. [14]
   g) Kelly values consumption more than leisure and her preferences are given by $U = C^{0.6} \times l^{0.4}$, otherwise her work-hours, wage rate and price of consumption good are same as that of Sharon. How do Kelly's hours of leisure, consumption, and work hours compare to that of Sharon. [16]

2. POP is a monopoly supplier of passenger parking services for ferries from Hull to Rotterdam. For a certain day its demand function is given by $Q = 50 - \frac{1}{10}P$ and its cost function is $C = 8Q$.

   f) How should POP determine its prices and quantity supplied to the market? [20]
   g) What is POP's profit and consumer surplus from its services? [20]
   h) After study POP finds that it can operate in both sides of the terminal. Its demand curve at Hull is given by $Q_1 = 21 - 0.1 \times P_1$ and in Rotterdam by $Q_2 = 50 - 0.4 \times P_2$ It it operates to both sides of the market it faces a common cost function $C = 2000 + 10Q$. How much price should POP charge for per unit of parking services in Hull? How much profit does it earn from Hull? [20]
   i) How much price should POP charge for per unit of parking services in Rotterdam? How much profit does it earn from Rotterdam? What is the aggregate profit? [20]
   j) What is the consumer surplus in Hull and in Rotterdam? How does it compare to the consumer surplus when it did not discriminate the market in this manner? [20]

3. Market demand curve for mobile phones is given by $Q = 10 - \frac{1}{5}P$. Two firms exists in the market to supply this product. Their cost functions are $C_1 = 4q_1$ and $C_2 = 5q_2$.

i) Write the profit function for each firm in this market. [12]

j) Each operates under Cournot model, takes other's action as a given parameter while making its production decision. Determine the reaction functions for both firms and represent it in a diagram. [12]

k) How much will each produce? What will be the market price? How much profit will each make from selling mobiles? [12]

l) What will be the consumer surplus from each of them? Show these consumer surpluses in a diagram. [12]

m) Firm 1 acts as a leader and takes account of the reaction function of its follower. How is it's profit function modified. What will be output level of firm 1? [12]

n) What will be the output of the follower firm? What will be the market price and profit of firm 1 and firm 2? [12]

o) What would the output and profit levels of each firm and the market price would have been if the firm 2 acted as a leader and firm 2 acted as a follower? Present these results of the Leader-Follower game in a pay-off matrix of the following form. [16]

	Leader	Follower
Leader		
Follower		

p) What would have been the level of output, market price and profit if they had formed a collusive cartel? [12]

4. Passenger car market is operating under a monopolistic competition. A particular firm under this market will cut down it own prices if any another firm reduces its price but will not raise its price if another firm raises its price. For simplicity assume that there two firms in the market and their inverse demand and cost functions are as following:

Demand and cost function of firm 1: $P_1 = 105 - 2q_1 - q_2$; $C_1 = 5q_1^2$

Demand and cost function of firm 2: $P_1 = 35 - q_1 - q_2$; $C_1 = q_2^2$

a) Find the Cournot duopoly equilibrium as a base line for comparison. What are output, price and profit of each firm under this market conditions. [20]

b) Now firm two raises prices of its own cars by 2 but the firm two will not change its own price. What will be the prices, output and profit of each firm? [16]

c) If the firm one reduces its price below the base line price. How much firm 2 reduce its price to maintain its market share? What are the profits, level of output and prices for each firm? [16]

d) Put every result of the calculation in one table and explain the underlying factors behind these results. [16]

e) A firm under the monopolistic competition does not follow MR = MC rule. Explain this concept using the Kink-Demand curve hypothesis. [16]

f) In what sense this is a monopoly? In what sense a competitive market? Illustrate discussion with some real world examples. [16]

5. Consider two countries, say, Britain and China that can produce two goods, clothes and machine. Britain is more efficient in making machines and China in making clothes. Production possibility frontiers that reflects these realities are $X_1 + X_2^2 = 2000$ for China and $X_1^2 + X_2 = 2000$ for Britain where X_1 is clothes and X_2 machines.

g) How much of clothes and machines can be produced in autarchy? How much of clothes and machines can be produced if China completely specialises in clothes and Britain specialises only in producing machines? [16]

h) Represent these solutions in an Edgeworth box diagram with cloths on X axis and Machines in Y axis and Britain's moving towards the Northeast direction and Chin moving towards Southwest for optimisation in production and consumption. [16]

i) Show the potential gains from specialisation and trade and what sort of bargaining in the division of gains is implied by this model. [16]

j) Demand side of Britain is represented by a Cobb-Douglas utility functions, $U^B = X_1^B X_2^B$, its budget constraint $I^B = P_1 X_1^B + P_2 X_2^B$ and income $I^B = P_1 Y_1^B + P_2 Y_2^B$, where Y_1^B is production of cloths by Britain and Y_2^B is production of machines by Britain. Similarly the demand side of China is represented by $U^{Ch} = X_1^{Ch} X_2^{Ch}$, $I^{Ch} = P_1 X_1^{Ch} + P_2 X_2^{Ch}$, $I^{Ch} = P_1 Y_1^{Ch} + P_2 Y_2^{Ch}$. What are the equilibrium prices and demand of clothes and machines and what is the income of China and Britain if they specialise only in one good? [16]

k) What is be equilibrium in autarchy implied by these preferences? [20]

l) What would be equilibrium price and demand if the preferences are represented by $U^B = \left(X_1^B\right)^{0.3}\left(X_2^B\right)^{0.7}$ $U^{Ch} = \left(X_1^{Ch}\right)^{0.3}\left(X_2^{Ch}\right)^{0.7}$ Show these results in an Edgeworth box diagram. [16]

6. Write short note in any four of the following [25 marks for each]
   a) Equivalent and compensating variation measures of price change
   b) Normal, inferior and superior goods
   c) Aggregate deadweight loss of taxes in market model of demand and supply
   d) Axioms of utility theory and cardinal and ordinal measures of utility Assumptions and limitations of perfect competition
   e) Every game has a solution in mixed strategy
   f) Role of bargaining in cooperative games
   g) Normal and extensive form of a game
   h) Utility from wealth for a person living in Fairfield village is given by $U = \ln(W)$, where U is the utility and W is the level of wealth. This person has a prospect of good income of 4000 with probability 0.4 and of bad prospect of low income of 1000 with probability of 0.6. How much would this person pay to insure against income uncertainty?
   i) A consumer lives for two periods and has income of 400 and 800 in the first and second periods respectively. He/she values consumption of both periods equally. What would be present value of consumption in the first and the second periods
      i. at zero rate of the real interest (ii) at 10 percent rate of real interest?

II. Micro-founded general equilibrium model with a representative household and a representative firm (Taken from Bhattarai (2003) Research Memorandum no.41, Business School, Hull).

A simple general equilibrium model represents an economy in which a representative household maximises utility by consuming goods and services supplied by producers and paying for them by income that it receives in return of supply of labour and capital inputs to the producers. Firms optimise profit combining inputs with the existing technology in production and rewarding inputs according to its marginal productivity. Tax policies of government influence both production and consumption sides of the economy by affecting prices of inputs and outputs. By distorting the marginal conditions of optimisation, these taxes influence choices of goods and services by households and use of inputs by producers. The incidence and impact of taxes on consumption may differ from taxes on labour income depending on the key parameters for share or elasticities of substitution in consumption or in the production sides of the economy.

A general equilibrium implies a set of prices that eliminate excess supply or excess demand and where these prices and wage rates are consistent with the preferences and endowments of households and technology of firms. The perfect match between demand and supply for both goods and services and inputs of production follow from the properties of utility and production functions as given by explicit analytical solutions in the next section.

Household's Problem

Consider an economy with a representative household and a representative firm. The household tries to maximise utility by consuming goods and services and enjoying leisure subject to its budget constraint. The producer wants to maximise profit by selling goods produced using the labour supplied by the household. The household maximisation problem can be stated in terms of preferences, budget and time constraints as the following:

$$Max \quad U = c^{\phi} l^{1-\phi}$$

Subject to:

i. $l + h^s = 1$ time constraint

ii. $wh^s + \pi = pc$ budget constraint

iii. $c \geq 0; l \geq 0; h^s \geq 0$ non-negativity constraint (1)

where c is consumption, l is leisure and h^s is labour supply, p is the price of the commodity, w is the wage rate; π is the profit from owning the firm, ϕ is weight of consumption and $(1-\phi)$ weight of leisure in utility.

For simplicity, normalise labour endowment to 1, define leisure as $l = 1 - h^s$ and substitute it into the utility function to get, $U = c^{\phi}\left(1-h^s\right)^{1-\phi}$.

The Lagrangian function for the constrained optimisation is given by

$$L(c,l,\lambda) = c^{\phi}\left(1-h^s\right)^{1-\phi} + \lambda\left[wh^s + \pi - pc\right] \tag{2}$$

It has three choice variables (c,l,λ), λ is shadow price of income. The first order conditions (FOC) for utility maximisation are:

$$\frac{\partial L(c,l,\lambda)}{\partial c} = \phi c^{\phi-1}\left(1-h^s\right)^{1-\phi} - \lambda p = 0 \tag{3}$$

$$\frac{\partial L(c,l,\lambda)}{\partial h^s} = (1-\phi)c^{\phi}\left(1-h^s\right)^{-\phi}(-1) + \lambda w = 0 \tag{4}$$

$$\frac{\partial L(c,l,\lambda)}{\partial \lambda} = wh^s + \pi - pc = 0 \tag{5}$$

Equation (3) and (4) give the marginal utility from consumption and leisure respectively. These three equations can be used to solve for three choice variables. Demand for consumption goods can be derived by dividing FOC (3) by FOC (4) and solving for c.

$$\frac{\frac{\partial L(c,l,\lambda)}{\partial h^s}}{\frac{\partial L(c,l,\lambda)}{\partial c}} = \frac{(1-\phi)c^{\phi}\left(1-h^s\right)^{-\phi}(-1)}{\phi c^{\phi-1}\left(1-h^s\right)^{1-\phi}} = \frac{w}{p} \tag{6}$$

This optimising condition implies that the marginal rate of substitution between leisure and consumption should equal the real wage rate in equilibrium. It is clear that the consumption demand depends on real wage rate and the work efforts.

$$c = \left(\frac{\phi}{1-\phi}\right)\left(1-h^s\right)\frac{w}{p}. \tag{7}$$

In the absence of taxes, the market clearing condition in this single good economy implies that the income of the household equals spending on consumption: $wh^s + \pi = pc$. Using (7) this budget constraint can be rewritten as:

$$h^s\frac{w}{p} + \frac{\pi}{p} = c = \left(\frac{\phi}{1-\phi}\right)\left(1-h^s\right)\frac{w}{p} \tag{8}$$

Now (8) can be solved for the household's labour supply

$$h^s\frac{w}{p} + \frac{\pi}{p} = \left(\frac{\phi}{1-\phi}\right)\left(1-h^s\right)\frac{w}{p} => h^s\left(1+\frac{\phi}{1-\phi}\right)\frac{w}{p} = \left(\frac{\phi}{1-\phi}\right)\frac{w}{p} - \frac{\pi}{p}$$

$$h^s\left(\frac{1}{1-\phi}\right)\frac{w}{p} = \left(\frac{\phi}{1-\phi}\right)\frac{w}{p} - \frac{\pi}{p}\frac{1-\phi}{1-\phi}$$

$$h^s\frac{w}{p} = \phi\frac{w}{p} - \frac{\pi}{p}(1-\phi) \tag{9}$$

From the optimising behaviour of a representative household the supply of labour, demand for leisure and consumption become functions of the real wage rate and profit.

$$h^s = \frac{\phi\frac{w}{p} - \frac{\pi}{p}(1-\phi)}{\frac{w}{p}}; \text{ leisure } l = 1-h^s = 1 - \frac{\phi\frac{w}{p} - \frac{\pi}{p}(1-\phi)}{\frac{w}{p}} \tag{10}$$

$$c = \left(\frac{\phi}{1-\phi}\right)\left(1-h^s\right)\frac{w}{p} = c = \left(\frac{\phi}{1-\phi}\right)\left(1 - \frac{\phi\frac{w}{p} - \frac{\pi}{p}(1-\phi)}{\frac{w}{p}}\right)\frac{w}{p} \tag{11}$$

The real wage rate that is consistent with optimal demand for labour and profit from the firm's side of optimisation is found in the next section.

Firm's Problem

The representative firm receives the net of tax price of goods by selling them to the households and pays the gross of tax wage rate for labour. Its maximisation problem, given the technology, input and output prices can be stated as:

$$Max \quad \pi = py - wh^d$$

subject to :

i. $y \leq \left(h^d\right)^\alpha$ technology constraint

ii. $y \geq 0; h^d \geq 0$ non negativity constraint (12)

where y is the output supplied by the firm and h^d is its demand for labour.

All above conditions can be collapsed into one objective function for the firm.

$$\text{Max } \pi = py - wh^d = p\left(h^d\right)^\alpha - wh^d \tag{13}$$

From the first order condition, firm's demand for labour input can be derived using the profit maximisation condition, which implies

$$\frac{\partial \pi}{\partial h^d} = p\alpha\left(h^d\right)^{\alpha-1} - w = 0 => h^d = \left(\frac{1}{\alpha}\frac{w}{p}\right)^{\frac{1}{\alpha-1}} \tag{14}$$

This optimal demand for labour by the firm in terms of the real wage rate can be substituted in the objective function of the firm (12) to express profit in terms of the real wage rate.

$$\frac{\pi}{p} = y - w\frac{h^d}{p} = \left(h^d\right)^\alpha - \frac{w}{p}h^d = \left(\frac{1}{\alpha}\frac{w}{p}\right)^{\frac{\alpha}{\alpha-1}} - \frac{w}{p}\left(\frac{1}{\alpha}\frac{w}{p}\right)^{\frac{1}{\alpha-1}} = \left(\frac{w}{p}\right)^{\frac{\alpha}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right] \tag{15}$$

Definition of a Competitive Equilibrium

In this model a competitive equilibrium is obtained by a real wage rate that guarantees the demand for labour by firms to be exactly equal to the supply of labour by households, the time endowments of households to be exactly equal to the sum of supply of labour and demand for leisure, and the production (supply) of output to be exactly equal to the demand for consumption by the household. Given the convexity of preferences and technology such equilibrium exists, is unique and stable.

The real wage rate can be determined by using the labour market clearing condition where the demand for labour by the firm equals the supply of labour by the household, $h^d = h^s$. That implies in equilibrium the demand for labour, $h^d = \left(\frac{1}{\alpha}\frac{w}{p}\right)^{\frac{1}{\alpha-1}}$, should equal the supply of labour, $h^s = \frac{\phi\frac{w}{p} - \frac{\pi}{p}(1-\phi)}{\frac{w}{p}}$. Then using the expression for the real profit from (15) this equality should fulfil the following identity.

$$h^d = \left(\frac{1}{\alpha}\frac{w}{p}\right)^{\frac{1}{\alpha-1}} = h^s = \frac{\phi\frac{w}{p} - \frac{\pi}{p}(1-\phi)}{\frac{w}{p}} = \frac{\phi\frac{w}{p} - \left(\frac{w}{p}\right)^{\frac{\alpha}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right](1-\phi)}{\frac{w}{p}} \tag{16}$$

Two sides of the labour market in (16) can be solved for equilibrium real wage rate as a function of preference and technology parameters of the model.

$$\frac{w}{p} = \frac{\phi^{\alpha-1}}{\left[(1-\phi)\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} + \phi\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]^{\alpha-1}} \tag{17}$$

The equilibrium quantities of *l* and h^d are determined by this equilibrium wage rate:

$$\hat{h}^d = \hat{h}^s = \left(\frac{1}{\alpha}\frac{\phi^{\alpha-1}}{\left[(1-\phi)\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} + \phi\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]^{\alpha-1}}\right)^{\frac{1}{\alpha-1}} \tag{18}$$

the demand for leisure: $\hat{l} = 1 - \hat{h}^s = 1 - \left(\frac{1}{\alpha} \frac{\phi^{\alpha-1}}{\left[(1-\phi)\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} + \phi\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}} \right]^{\alpha-1}} \right)^{\frac{1}{\alpha-1}}$ (19)

Given the real wage rate and labour supply and leisure, the amount of consumption (*c*) and the level of output supplied (*y*) in the no tax case can be obtained by substituting real wage rate from (17) into the demand for consumption function in (7) and labour demand function in (14)

$$\hat{c} = \left(\frac{\phi}{1-\phi}\right)\left(1 - \left(\frac{1}{\alpha} \frac{\phi^{\alpha-1}}{\left[(1-\phi)\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} + \phi\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}} \right]^{\alpha-1}} \right)^{\frac{1}{\alpha-1}} \right) \frac{\phi^{\alpha-1}}{\left[(1-\phi)\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} + \phi\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}} \right]^{\alpha-1}} \qquad (20)$$

Equilibrium output is then given by substituting the equilibrium labour input in the production function in (12)

$$\hat{y} = \left(\frac{1}{\alpha} \frac{\phi^{\alpha-1}}{\left[(1-\phi)\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} + \phi\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}} \right]^{\alpha-1}} \right)^{\frac{\alpha}{\alpha-1}} \qquad (21)$$

The representative household maximises utility using these optimal values of consumption and leisure in the utility function, $\hat{U} = \hat{c}^{\phi}\hat{l}^{1-\phi}$. This example explicitly shows how the price system allocates resources in an economy.

Two sector model of an economy

Input-output model do not include income and substitution effects. A general equilibrium model includes both income, substitution and cross price effects in the allocation resources in the entire economy. This example and GAMS program illustrate how a general equilibrium can be applied to analyse resource allocation in these two sector economies.

Input output table for a two sector economy

	Primary	**Secondary**	**C**	**I**	**G**	**X**	**M**	**Total**
Primary	x_{11}	x_{12}	C_1	I_1	G_1	X_1	M_1	Y_1
Secondary	x_{21}	x_{22}	C_2	I_2	G_2	X_2	M_2	Y_2
Labour	L_1	L_2						wL
Capital	K_1	K_2						rK
Taxes	T_1	T_2						T
Total	Y_1	Y_2	C	I	G	X	M	

$X_{i,j}$ are intermediate inputs from firm i to sector j, i.e. X12 is the intermediate input supplied by firm 1 to firm 2; L_i is the labour demanded by firm i from the household; K_i is the capital demanded by firm i from the household; T_i are taxes paid by labour employed in sector i. Similarly C_i, I_i, G_i and X_i are consumption, investment, government consumption and export demand for product supplied by a firm i respectively. M_i is import to firm-i type good, Y_i is the gross production of sector i. L and K are endowment of labour and capital of the household.

A hypothetical data set for this two sector economy is given in the following **input-output** table.

	Primary	**Secondary**	**C**	**I**	**G**	**X**	**M**	**Total**
Primary	15	25	50	10	10	5	-15	100
Secondary	20	25	70	20	5	15	-5	150
Labour	45	55						100
Capital	15	35						50
Taxes	5	10						15
Total	100	150	120	30	15	20	-20	

First two rows give component of demand; and row total = total demand

First two columns give cost of production by sector; and column total = total supply

Sectoral value added is given by the total of labour and capital income per sector.

Check total demand = total supply in equilibrium

Income balance

Trade balance

Revenue balance

- Consumers preferences: $U(C_1,C_2)=C_1^{\alpha}C_2^{\beta}$ with shares spend on these commodities add up to 1, $\alpha+\beta=1$.
- Profit of firms: $\Pi_i = P_iY_i - wL_i - r(1-t_k)K_i$
- Technology: $Y_i = A_iK_i^{\alpha}L_i^{\beta}$ or with human capital: $Y_i = A_iK_i^{\alpha}L_i^{\beta}H_i^{\gamma}$

- A competitive economy is a price system {p1, p2, w, r} such that given these prices
  - Households maximise their utility subject to their budget constraint
  - Demand and supply of both goods are equal, there is no excess demand

- Demand for labour and supply of labour is equal
- Demand and supply for capital is equal
- Government budget is balanced.

Tax rate in the primary sector = (5/45) = 0.1111
Tax rate in the secondary sector = (10/55) = 0.1818
Tax rate in the capital income from primary sector = (5/15) = 0.33333
Tax rate in the capital income from secondary sector = (10/35) = 0.2857

A rough estimation of sectoral capital stock in the steady state: $rK_i = I_i^K$; $K_i = \frac{I_i^K}{r}$

Pre-requisite reading
Varian Hall R. Intermediate Microeconomics - A Modern Approach, 6th edition, 2003, ISBN 0-393-92671-0

Core text(s)
Pindyck R.S. and D.L. Rubinfeld (2005) Microeconomics, 6th Edition, Pearson; ISBN 0-13-191207-0.
John Hey (2003) Intermediate Microeconomics-McGraw Hill, 0-07-710364-5.

Equivalent reading
Perloff Jeffrey M. (2007) Microeconomics, 4th Edition, Pearson ISBN: 0321-41057-2.
Mathis Stephen A and J. Koscianski (2002) Microeconomic Theory: An integrated Approach, Pearson Education, ISBN: 0-13-011418-9.

Further reading
Nicholson W. (2002) Microeconomics principles and extensions, Norton.
Kreps David M (1990) A Course in Microeconomic Theory, Princeton, ISBN:0-691--04264-0.
Mas-Colell A, M.D.Whinston and J.R.Green (1995) Microeconomic Theory, Oxford University Press.
Tirole Jean (1995) Theory of Industrial Organisation, MIT Press, ISBN: 9-780262-200714.

Journal list
Economics Letters, Applied Economics, Applied Economics Letters, Economic System Research, Journal of Development Studies, Macroeconomics Journal, National Institute's Bulletin of Economic Research, Oxford Economic Bulletin, Quarterly Bulletin of Bank of England and Yorkshire Bulletin. Construct your list of references using electronic databases such as the Econlit and the JOSTR available through the www.hull.ac.uk/lib/.

Websites
http://www.competition-commission.org.uk/ http://www.fairtrade.org.uk/
Industry data base www.Northcote.co.uk. http://www.competition-commission.org.uk/