

Macroeconomic Theory and Policy

Lecture 12

Economic Growth: Long Run

Levels of GDP per head of population, 1820-2000 (\$ at 1985 US prices)

Country	1950	1989	Multiplication coefficient for income compared to year 1820	Population in 1989 ('000)	Multiplication times of population from 1820
Australia	5931	13584	11	16807	509
Austria	2852	12585	12	7618	2
Belgium	4228	12876	13	9938	3
Canada	6113	17576		26248	41
Denmark	5224	13514	14	5132	5
Finland	3480	13934	22	4962	4
France	4149	13837	13	56160	2
Germany	3339	13989	15	61990	2
Italy	2819	12955	13	57525	3
Japan	1563	15101	26	123116	4
Netherlands	4706	12737	10	14850	6
Norway	4541	16500	19	4228	4
Sweden	5331	14912	16	8439	3
Switzerland	6556	15406		6723	4
UK	5651	13468	8	57202	3
USA	8611	18317	17	248777	26
Average	4693	14456	14	709718	5

Source Angus Maddison (1991) Dynamic Forces in Capitalist Development, Oxford University Press.

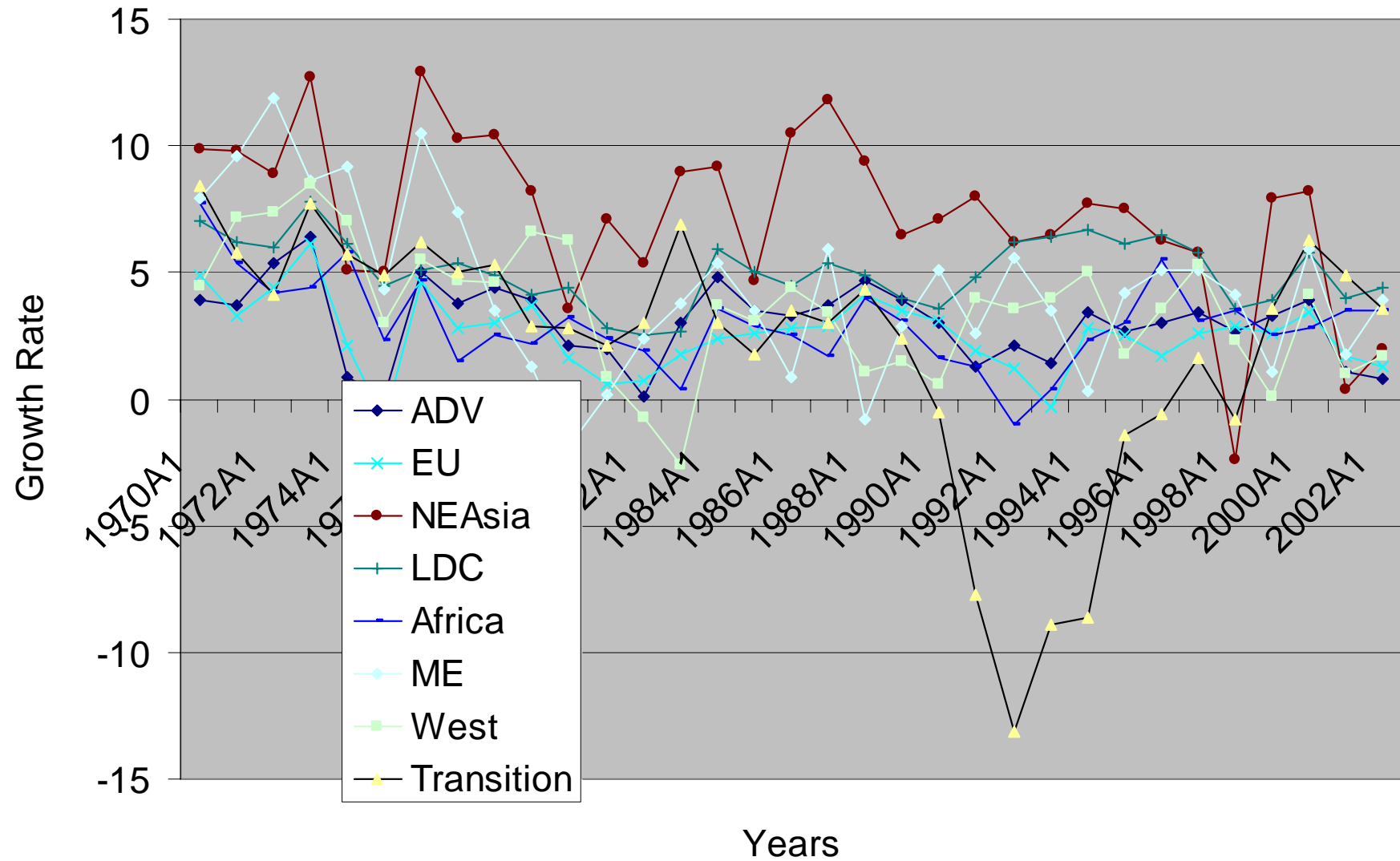
Countries	Growth Experience from 1965-1999					Size of aggregate and per capita GDP in 1999				
	GDP growth (annual %)		Population growth (annual %)		Population in '000	Size of GNP		Size of per capita GNP		Income based ranking out of 206 countries
	Total	Per Head	Total	Labour force		GNP billion of us \$ at 1999	GNP at PPP	Per capita GNP	Per capita GNP at PPP	
China	8.1	6.4	1.7	2.1	1250	980	4112	780	3291	128
Hong Kong	7.3	5.4			7	162	144	23520	20939	26
Japan	4.1	3.4	0.7	1	127	4078	3043	32230	24041	14
Singapore	8.3	6.3	1.9	3.1	3	95	87	29610	27024	7
Pakistan	5.6	2.7	2.8	2.9	135	64	237	470	1757	159
Nepal	3.7	1.2	2.4	2	23	5	29	220	1219	177
Uganda	5.5	2.5	2.9	2.6	21	6.8	24	320	1136	179
Tanzania			3	2.9	33	8	16	240	478	205
Ghana	2	-0.7	2.6	2.6	19	7	34	390	1793	157
UK	2.2	2	0.3	0.5	59	1338	1234	22640	20883	27
USA	3	2	1.1	1.7	273	8351	8350	30600	30600	4
World	3.3	1.8	1.7	2.0	5,975	29232	38405	4890	6490	
Low income	4.1	1.8	2.3	2.2	2,417	988	4315	410	1790	\$755
Middle inc.	4.2	2.4	1.7	2.1	2,667	5323	13022	2000	4880	
High income	3.2	2.4	0.8	1.2	891	22921	21763	25730	24430	\$9266 or >

Source: Derived from the World Development Report, 2001, pp. 274-275.

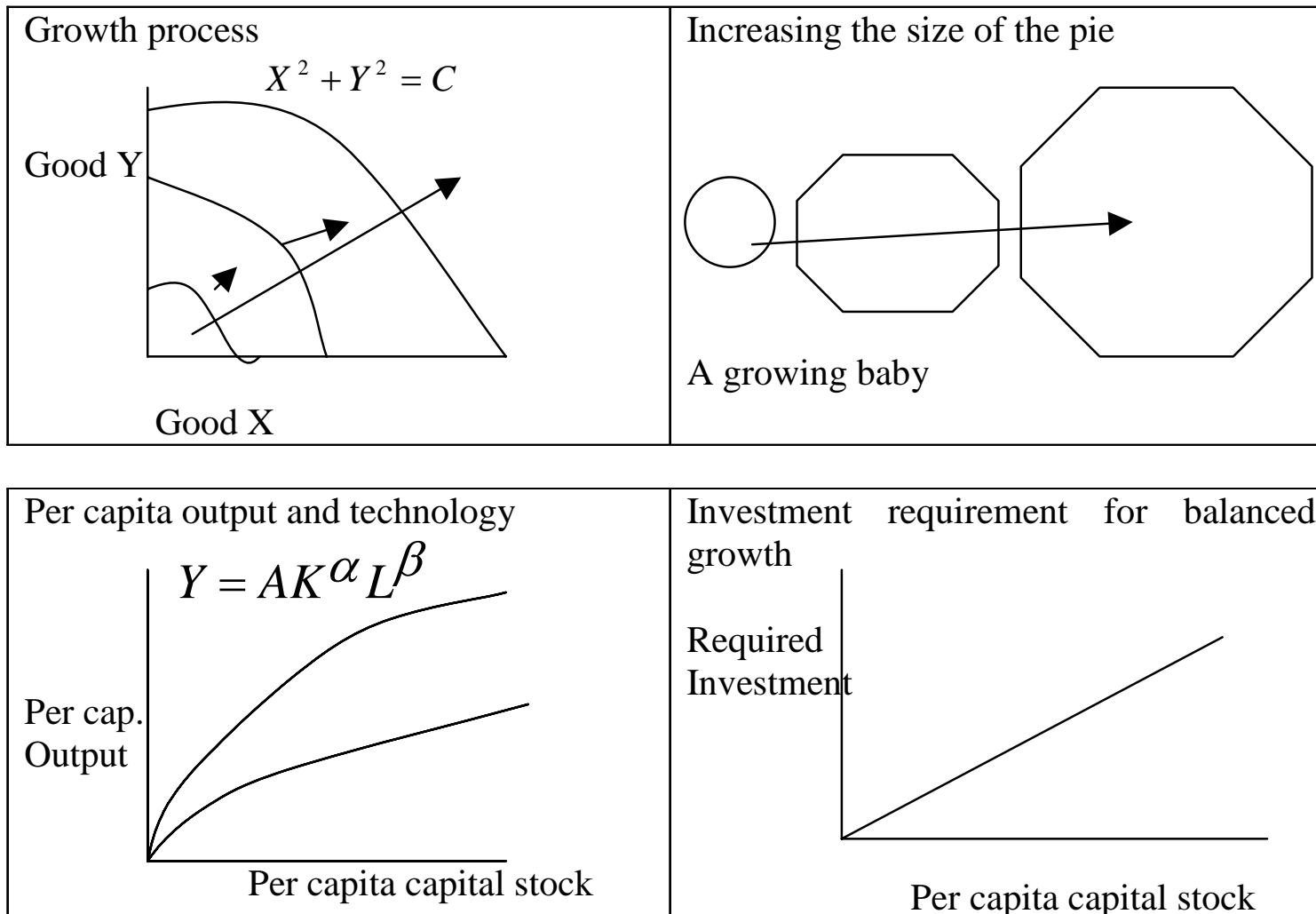
Economic Growth Rates Over Time Across Countries (IMF World Economic Outlook)

	adv	Majadv	Othadv	EurUnion	Euroarea	NIC	DEVC	Africa	Dasia	Mideast	WESH
1970	3.9	3.4	6	4.9	5.5	9.9	7	7.7	8.6	7.9	4.5
1971	3.7	3.4	5	3.3	3.6	9.8	6.2	5.4	4.1	9.6	7.2
1972	5.4	5.3	5.9	4.4	4.7	8.9	6	4.2	2.8	11.9	7.4
1973	6.4	6.3	6.8	6.1	6	12.7	7.8	4.4	8.3	8.6	8.5
1974	0.9	0.3	3.4	2.1	2.9	5.1	6.1	5.8	3.8	9.2	7
1975	0.1	0	0.6	-0.9	-0.9	5	4.5	2.3	6.9	4.3	3
1976	5	5	4.8	4.6	5	12.9	5.1	4.7	2.1	10.5	5.5
1977	3.8	3.9	3.3	2.8	3.1	10.3	5.4	1.5	6.8	7.4	4.7
1978	4.4	4.6	3.2	3	3.1	10.4	4.9	2.5	6.9	3.5	4.6
1979	3.9	3.9	3.8	3.7	3.9	8.2	4.1	2.2	3.8	1.3	6.6
1980	2.1	2	2.8	1.6	2.1	3.6	4.4	3.2	5.8	-1.9	6.3
1981	2	1.8	2.8	0.6	0.9	7.1	2.8	2.4	5.7	0.2	0.9
1982	0.1	-0.2	1.4	0.7	0.5	5.4	2.5	1.9	5.6	2.4	-0.7
1983	3	3.1	2.7	1.8	1.4	9	2.7	0.4	7.1	3.8	-2.6
1984	4.8	5	4.3	2.4	2.3	9.2	5.9	3.6	8.4	5.4	3.7
1985	3.5	3.5	3.5	2.6	2.3	4.7	5	2.9	7.4	3.5	3.1
1986	3.3	3.1	4.2	2.8	2.5	10.5	4.5	2.5	6.5	0.9	4.4
1987	3.7	3.4	5.2	2.9	2.6	11.8	5.4	1.7	7.7	5.9	3.4
1988	4.7	4.6	5	4.1	4	9.4	4.9	4	9.1	-0.8	1.1
1989	3.9	3.7	4.5	3.5	3.9	6.5	4	3.1	6.1	2.9	1.5
1990	3	2.8	4.1	3.1	3.7	7.1	3.6	1.6	5.4	5.1	0.6
1991	1.3	0.9	3	1.9	2.6	8	4.8	1.3	6.5	2.6	4
1992	2.1	2	2.6	1.2	1.5	6.2	6.2	-1	9.3	5.6	3.6
1993	1.4	1.3	1.9	-0.3	-0.8	6.5	6.4	0.4	9.4	3.5	4
1994	3.4	3.1	4.6	2.8	2.3	7.7	6.7	2.3	9.7	0.3	5
1995	2.7	2.3	4.3	2.5	2.3	7.5	6.1	3	9	4.2	1.8
1996	3	2.8	3.8	1.7	1.5	6.3	6.5	5.5	8.3	5.1	3.6
1997	3.4	3.2	4.3	2.6	2.4	5.8	5.8	3.1	6.5	5.1	5.3
1998	2.7	2.8	2.2	2.9	2.9	-2.4	3.6	3.5	4	4.1	2.3
1999	3.3	3	4.9	2.6	2.7	7.9	3.9	2.5	6.2	1.1	0.1
2000	3.9	3.5	5.2	3.4	3.5	8.2	5.8	2.8	6.8	5.9	4.1
2001	1.1	1	1.5	1.7	1.6	0.4	4	3.5	5.6	1.8	1
2002	0.8	0.6	1.9	1.3	1.2	2	4.4	3.5	5.6	3.9	1.7

Economic Growth Around the Globe



Meaning of Economic Growth



Economic Growth

Output: $Y = AK^\alpha L^\beta$

Where Y = output

A = technology

L = Labour input

K = Capital input

α share of capital in output $\rightarrow 0 < \alpha < 1$

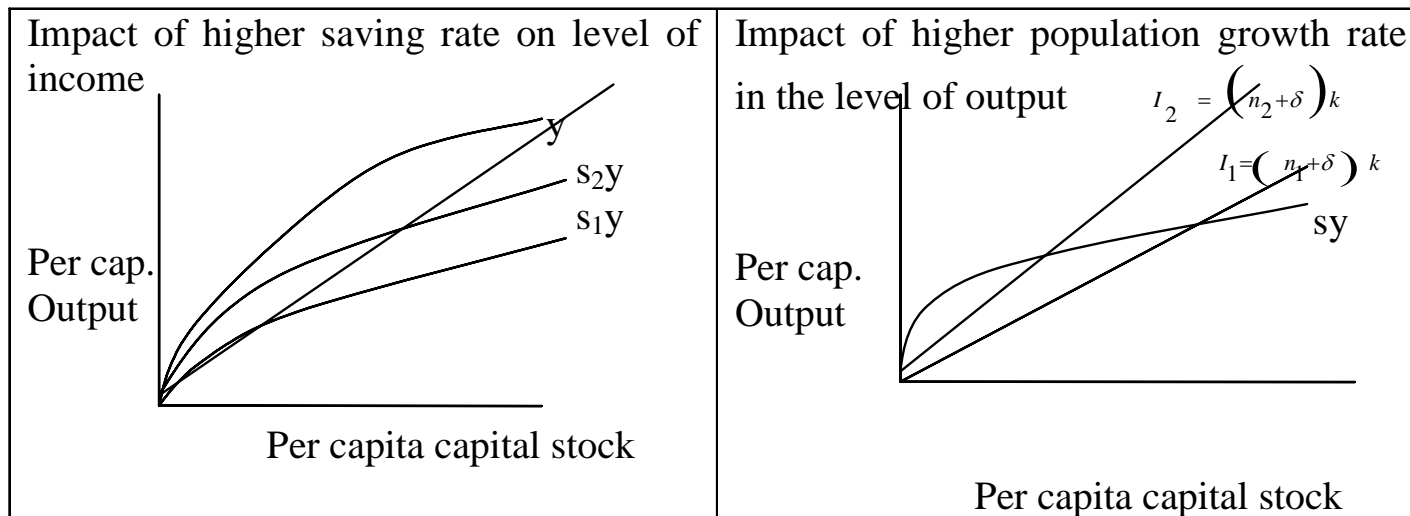
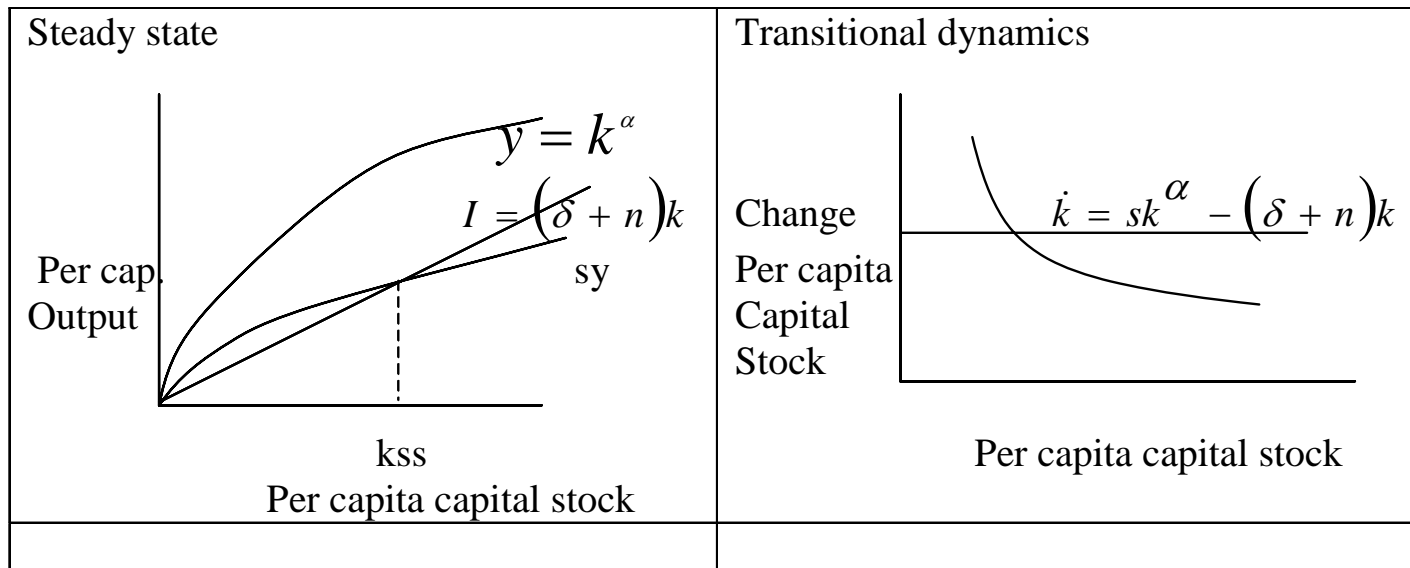
β share of labour in output $\rightarrow 0 < \beta < 1$

Total output goes either to owner of capital or to labour

$$\alpha + \beta = 1$$

Output grows either by increase in technology or in capital stock or in labour input.

Simple graphical illustration of the growth process.



Growth facts and sources of economic growth

1. There is enormous variation in per capita income across countries. The poorest countries have per capita income less than 5 percent of per capita incomes in the richest countries.
2. Rates of economic growth vary substantially across countries.
3. Growth rates are not necessarily constant over time.
4. A country's relative position in the world distribution of per capita incomes is not immutable. Countries can move from being "poor" to being "rich" and vice-versa.
5. Growth in output and growth in the volume of international trade are closely related.
6. Both skilled and unskilled workers tend to migrate from poor to rich countries or regions.

Stylised facts of economic growth

In the US and majority of western countries

1. labour input grows more slowly than capital and output, therefore capital labour ratio (K/L) and per capital output (Y/L) have increased secularly.
2. Capital output ration (K/L) has remained fairly constant, has no discernible trend. These have more or less converged across industrialised economies.
3. rate of return on capital (profit) and the real interest rate (r) have no trend whereas real wages follow a secular rising trend. Secular rise in (Y/L) and (K/L) are translated into higher real wages.
4. share of income devoted to the capital (rK/Y) and labour (wL/Y) shows no trend and remain fairly constant.
5. Capital stock per person tends to grow along with output per person.

Key elements of Solow's Neo-classical Growth Model

- Producers maximise profit by paying factors remuneration equal to their marginal products.
- Consumers maximise utility from consumption given a fixed savings rate.
- Capital and labour are substitutable according to price. Productivity of capital is very high with a low level of capital stock and productivity of labour is higher with lower level labour inputs (Inada conditions). This implies productivity of capital should be higher in labour abundant developing economies than in developed economies. Is that really so?

Public policy can influence level of output by increasing savings rate but growth rate can increase only from technological change

Savings and Investment in the Solow Model

- Savings: fixed fraction of income equals gross investment

$$S = sY = sAK^\alpha L^\beta$$

- Gross investment = net investment plus depreciation.

- Net investment $I_n = sY - \delta K = sAK^\alpha L^\beta - \delta K$

- Change in capital stock $dK = I_n = sY - \delta K = sAK^\alpha L^\beta - \delta K$

- capital accumulation equation $K_t = K_{t-1}(1-\delta) + I_t$

Per capita output $\frac{Y}{L} = \frac{AK^\alpha L^\beta}{L} = A\left(\frac{K}{L}\right)^\alpha$ and per capita

capital $k = \frac{K}{L}$.

Steady State Capital Stock in the Solow Model

- Growth of per capita capital stock:

$$\frac{dk}{k} = \frac{dK}{K} - \frac{dL}{L} = \frac{1}{K} [sY - \delta K] - n$$

$$\frac{dk}{k} = \left[s \frac{Y}{K} - \delta \right] - n; \quad \frac{dk}{k} = \left[s \frac{y}{k} - \delta \right] - n; \quad \frac{dk}{k} = \left[sk^{\alpha-1} - \delta \right] - n$$

- Fundamental equation of economic growth:

$$\frac{dk}{k} = \left[sk^{\alpha-1} - \delta \right] - n$$

In steady state $\frac{dk}{k} = 0$ which implies $sk^{\alpha-1} = \delta + n$ and

$$k^{SS} = \left(\frac{s}{\delta + n} \right)^{\frac{1}{1-\alpha}}$$

Calculation of Steady State A Numerical Example

$$\text{Output in the steady state } y^{ss} = (k^{ss})^\alpha = \left(\frac{s}{\delta + n}\right)^{\frac{\alpha}{1-\alpha}}$$

Steady State Output if $s = 0.32$ and $\delta = 0.08$, and $n = 0$

$$\alpha = \frac{1}{3}.$$

$$y^{ss} = \left(\frac{s}{(\delta + n)}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{0.32}{(0.08 + 0)}\right)^{\frac{\frac{1}{3}}{1-\frac{1}{3}}} = \left(\frac{0.32}{(0.08)}\right)^{\frac{1}{2}} = 2$$

Consumption in the steady state: $C^{ss} = Y^{ss} - sY^{ss} = 2 - 0.32(2) = 1.36$.

when saving rate is low as $s = 0.16$, then steady state output will be lower.

$$y^{ss} = \left(\frac{s}{(\delta + n)}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{0.16}{(0.08 + 0)}\right)^{\frac{\frac{1}{3}}{1-\frac{1}{3}}} = \left(\frac{0.16}{(0.08)}\right)^{\frac{1}{2}} = 1.41$$

1. Countries with higher saving rate have higher steady state level of output and countries with lower saving rate have lower level of output in the steady state.
2. Countries with higher level of technology have higher level of output countries with lower level of technology have lower level of output in steady state.
3. Countries with higher rate of population growth rate have lower level output in the steady state.
4. Countries with higher capital share α have higher output in the steady state.
5. Countries which differ in the initial capital stock eventually reach to the same output level in the steady state.
6. Growth of per capita income is zero in the steady state

Golden Rule of Saving

What is the saving rate that maximises consumption?

$$c = y - i \quad (1)$$

$$c = y - (\delta + n)k \quad (2)$$

$$\text{Max } c = y - (\delta + n)k \quad (3)$$

$$\frac{\partial c}{\partial k} = \alpha k^{\alpha-1} - (\delta + n) = 0 \quad (4)$$

$$\alpha k^{\alpha-1} = (\delta + n) \quad (5)$$

$$\text{MPK} = (\delta + n) \quad (6)$$

Purpose of all economic activities is consumption. The capital stock where the slope of production function (marginal product of capital) equals the slope of the investment requirement line (depreciation plus the growth rate of the population) as given in (6).

A Numerical Example for the Golden Rule of Saving

Take 11 different saving rates, $s = 0, 0.1, 0.2, \dots, 1.0$.

And capital stock like : 1, 4, 9, 16, 25, 36, 49, 64, 81

$$Y = 0.5\sqrt{K} \sqrt{L} \rightarrow \text{in per capita terms } y = 0.5k^{0.5}$$

Assume $\delta = 0.05$, $\alpha = 0.5$, $A=0.5$

Then calculate consumption, mpk and mpk -d as following

s-rate	k	y	dk	c	Saving s	mpk	mpk-d
0.1	1.00	0.50	0.05	0.45	0.05	0.2500	0.2000
0.2	4.00	1.00	0.2	0.8	0.2	0.1250	0.0750
0.3	9.00	1.50	0.45	1.05	0.45	0.0833	0.0333
0.4	16.00	2.00	0.8	1.2	0.8	0.0625	0.0125
0.5	25.00	2.50	1.25	1.25	1.25	0.0500	0.0000
0.6	36.00	3.00	1.8	1.2	1.8	0.0417	-0.0083
0.7	49.00	3.50	2.45	1.05	2.45	0.0357	-0.0143
0.8	64.00	4.00	3.2	0.8	3.2	0.0313	-0.0188
0.9	81.00	4.50	4.05	0.45	4.05	0.0278	-0.0222

For this parametric configuration 50 percent saving rate maximises consumption.

Definition of the Steady State

Define per capita variables as $y = \frac{Y}{L}$ $k = \frac{K}{L}$

- $\frac{dy}{y} = \frac{dY}{Y} - \frac{dL}{L} = 0$ $g_y - g_n = 0$ $g_y = g_n$; or $k = \frac{K}{L}$
 $\frac{dk}{k} = \frac{dK}{K} - \frac{dL}{L} = 0$ $g_k - g_n = 0$ $g_k = g_n$
- $g_y = g_k = g_n$ in the steady state. Capital and output grow growth rate of L.

There is no role of saving in the growth rate

Take log and differentiate both sides with respect to time

$$Y = AK^\alpha L^{1-\alpha} ; \therefore \ln Y = \ln A + \alpha \ln K + (1-\alpha) \ln L$$
$$\frac{dY}{Y} = \frac{dA}{A} + \alpha \frac{dK}{K} + (1-\alpha) \frac{dL}{L}$$

This leads us to following form of growth accounting relation

$$\frac{\Delta y}{y} = g_y = a + \alpha g_k + (1-\alpha) g_n \Rightarrow g - n = \frac{a}{(1-\alpha)}$$

Where does technology come from?

A “manna from heaven!”

Lucas (1988) and Romer (1989) have modified this result by including the human capital in the and endogenous technological growth in the model.

The growth rate effect saving in AK growth model

AK Production technology: $y = Ak$

Assume that the marginal productivity of capital is constant $A = 0.5$ and the saving is over and above the requirement to pay for the depreciation $sy > \delta k$. Thus the change in the output in this case is given by

$$\frac{\dot{y}}{y} = a + \frac{\dot{k}}{k} \text{ or } g = a + \frac{s'y}{k}$$

Growth rate depends on savings rate.

Small Difference in Growth Rate Makes a Huge Difference in Level of Income Over Time

Importance of growth rate: A small difference in growth rate makes a large difference in level of income over years. Low income countries with higher growth rate can catch up higher income but slow growing countries. A simple formula to calculate how long time does it takes to double the level of income can be approximated by

$(70/(\text{growth rate}))$. $Y_0(1+g)^t = 2Y_0$ Here Y_0 is initial income, g is growth rate, t is time period to double the income. Cancel Y_0 both sides and then taking log it becomes

$t \ln(1+g) = \ln(2)$. Using a rule of log for small numbers $\ln(1+g) \approx g$. Therefore time taken to double income is given by $t = \frac{\ln(2)}{g} = \frac{0.7}{g}$. This is a generic formula it can be

applied to find the time taken to quadruple by $t = \frac{\ln(4)}{g} = \frac{1.37}{g}$ or tenfold increase in

income $t = \frac{\ln(4)}{g} = \frac{2.30}{g}$

Thus using this approximation if a country is growing at 5 percent, it takes about 14 years to double the level of income.

How Long Does it Take to Grow n-times?

Growth rate	Double	Treble	Quadruple	Five times	Six times	Seven times	Eight times	Nine times	Ten times
1%	69	110	139	161	179	195	208	220	230
2%	35	55	69	80	90	97	104	110	115
3%	23	37	46	54	60	65	69	73	77
4%	17	27	35	40	45	49	52	55	58
5%	14	22	28	32	36	39	42	44	46
6%	12	18	23	27	30	32	35	37	38
7%	10	16	20	23	26	28	30	31	33
8%	9	14	17	20	22	24	26	27	29
9%	8	12	15	18	20	22	23	24	26
10%	7	11	14	16	18	19	21	22	23
11%	6	10	13	15	16	18	19	20	21
12%	6	9	12	13	15	16	17	18	19
13%	5	8	11	12	14	15	16	17	18
14%	5	8	10	11	13	14	15	16	16
15%	5	7	9	11	12	13	14	15	15
16%	4	7	9	10	11	12	13	14	14

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