THE ROLE OF LABOUR DEMAND ELASTICITIES IN TAX INCIDENCE ANALYSIS WITH HETEROGENEOUS LABOUR

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Abstract

Whether labour bears full burden of household level income and consumption taxes ultimately depends on the degree of substitutability among different types of labour in production. We find more variation in incidence patterns across households with less than perfectly substitutable heterogeneous labour than with perfectly substitutable homogeneous labour in production. This finding is based on results obtained from homogeneous and heterogeneous labour general equilibrium tax models calibrated to decile level income and consumption distribution data of UK households for the year 1994. We use labour supply elasticities implied by the substitution elasticity in households' utility functions and derive labour demand elasticities from the substitution elasticity in the production function.

Key words: Elasticities, labour demand, labour supply, welfare, incidence, redistribution
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I. Introduction

This paper builds on the observation that existing empirically based incidence analyses drawing either on shifting assumptions[2], or on general equilibrium tax models treat labour as bearing the burden of its own income and payroll taxes[3]. The implicit assumption is that labour is a homogenous input which is perfectly mobile across industries and yields leisure which is consumed by households. A key set of parameters in incidence analyses conducted with these models have been presumed to be the labour supply elasticities which are the subject of some attention in both calibration and sensitivity analysis. Labour demand elasticities do not enter these analyses.

We argue that with homogeneous labour in production, the implicit labour demand function facing each household is highly elastic since, if small relative to the aggregate, each is a price taker in labour markets. The implication is that labour will bear most of the burden of its own labour taxes, independently of labour supply elasticities used. Conventional sensitivity analysis on labour supply elasticities will show little variation in tax incidence profiles. On the other hand, varying labour demand elasticities will allow for the shifting of tax burdens to other groups or households.

We investigate differences in model analyses of tax incidence using comparable nested models in which labour is either homogenous or heterogeneous in production, so that labour demand elasticities also enter. Each model is calibrated to a ten decile household data set containing data on consumption, taxes, and leisure for the UK for 1994. Labour supply elasticity calibration is based on estimates from Killingsworth (1983), with the labour

[2] See Pechman and Okner (1974) , and Gillespie (1965) as examples of this approach.

[3] See Shoven and Whalley (1972) for a simple 2 sector 2 household Harberger tax model; Piggott and Whalley (1985) for a 100 household model of the UK; Ballard, Fullerton, Shoven and Whalley (1985) for a 15 household model of the US; and Auerbach and Kotlikoff (1987) for a 55 overlapping generations dynamic structure applied to US data.

demand elasticity used in calibration in the heterogeneous labour model drawing on estimates from Hamermesh (1993). Significant differences in incidence profiles are found across the two models. The heterogeneous model shows significant variations in incidence profiles as labour demand elasticities change, while the homogeneous good model shows little sensitivity to labour supply elasticities. The implication drawn is the need to more carefully specify labour demand elasticities in tax incidence analyses.

II. Tax Incidence Models with Heterogeneous and Homogeneous Labour Supply

It seems clear that if labour supplied by household groups is heterogeneous with imperfect substitutability in production across skill levels, then both labour demand and labour supply elasticities are needed in tax incidence analyses. If labour is treated as homogeneous across households, then if each household is small and a taker of wage rates, they implicitly face a perfectly elastic labour demand function. In this case, varying the labour supply elasticity will not change the conclusion that labour bears the burden of their own income taxes, even if tax rates differ across households.

If the labour demand elasticity is less than infinite, as labour supply functions shift due to household specific taxes, some of the burden of the tax is shifted elsewhere. The implication is that if heterogeneous labour models are used for tax incidence analysis and model parameters calibrated to both labour demand and labour supply elasticities, tax incidence results can differ[4] (and potentially sharply so) from those generated by the homogenous labour model conventionally used.

We choose the household as the basic unit of tax incidence analysis not only because consumption and labour-leisure choice decisions are made at the household level, but also it is adapted in the data on income and consumption used in the empirical analysis of incidence profiles in the paper.

<u>A heterogeneous labour household tax model</u>

We consider an economy with households differentiated according to their skill levels, which, for our empirical application, we take to be collinear with income ranges. Each household is endowed with a fixed amount of time, which it can divide between leisure and work. A production function specifies how the various labour types combine to yield a single consumption good. Each of them buys the consumption good using income earned by selling its labour on the market along with transfers received from government, effectively buying back its leisure at its net of tax wage. Households maximize utility by choosing bundles of consumption goods and leisure subject to their budget constraint. Hours of work (labour supply), consumption and leisure are thus obtained by solving each household's optimization problem.

Taxes distort the consumption-leisure choice of households. Tax rates are household specific, and government budget balance holds with transfers the sole expenditure item. In empirical implementation, we use a single tax rate on labour income for each household to represent the composite of income and payroll taxes, and a composite indirect tax rate for each household which reflects sales (VAT) and excise taxes. These latter rates differ by household due to differing consumption patterns in the data.

More specifically, we assume CES preferences for each household as

$$U^h = \left[\alpha^h C^{h\frac{\sigma^h - 1}{\sigma^h}} + (1-\alpha^h) L^{h\frac{\sigma^h - 1}{\sigma^h}} \right]^{\frac{\sigma^h}{\sigma^h - 1}} \qquad (1)$$

where U^h is utility, α^h is the share of income spent on the consumption good, (1-α^h) is the share parameter on leisure, C^h and L^h are consumption and leisure respectively of

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[4]One reason for the relative absence of heterogeneous labour models in empirically based general equilibrium work is the seeming difficulty of solution in the non-nested production function case. We solve it using the new PATH algorithm developed and employed with GAMS by Michael Ferris (see Appendix 2 for details).

household *h*, and σ^h is the elasticity of substitution between consumption and leisure of household *h*.

The income for household *h* equals the time endowment valued at the net of tax wage plus transfers received from government, i.e.

$$I^h = (1 - t_I^h) w^h \overline{L}^h + RH^h \qquad (2)$$

where I^h is the full income of household *h*, w^h is the gross of tax wage rate for household *h*, and t_I^h is the household specific income tax rate. $\overline{L}^h$ is the time endowment of household *h* to be divided between labour supply and leisure, and RH^h are transfers received by household *h*.

Maximization of utility (1), subject to (2), yields demand functions for consumption and leisure for each household as,

$$C^h = \left[\frac{\alpha^h}{P(1+t_C^h)}\right]^{\sigma^h} \left[\frac{I^h}{\alpha^h (P(1+t_C^h))^{1-\sigma^h} + (1-\alpha^h)(w^h(1-t_I^h))^{1-\sigma^h}}\right] \qquad (3)$$

$$L^h = \left[\frac{(1-\alpha^h)}{w^h(1-t_I^h)}\right]^{\sigma^h} \left[\frac{I^h}{\alpha^h (P(1+t_C^h))^{1-\sigma^h} + (1-\alpha^h)(w^h(1-t_I^h))^{1-\sigma^h}}\right] \qquad (4)$$

where w^h is the gross of tax wage rate for labor of type *h* (supplied by household *h*), *P* is the price of the consumption good, and t_C^h is the consumption (or indirect) tax rate faced by household *h*. The budget balance condition for households implies that on the expenditure side

$$I^h = P(1+t_c^h)C^h + w^h(1-t_l^h)L^h \qquad (5)$$

Each household supplies labour to the market which reflects the difference between its labour endowment and its demand for leisure,

$$LS^h = \overline{L}^h - L^h \qquad (6)$$

where LS^h is labour supplied by household *h*. The economy wide labour supply is the sum of labour supplied across the individual households. In equilibrium, equation (6) is also the labour market clearing condition for labour of type *h*.

In the model, we assume that the labour each household supplies is differentiated by skill level from the labour supplied by all other households, and we represent this through a CES production technology for the single output *Y* in which all labour types enter, i.e.

$$Y = \lambda \left(\sum_h \delta^h LS^{h\frac{\sigma_p - 1}{\sigma_p}} \right)^{\frac{\sigma_p}{\sigma_p - 1}} \tag{7}$$

where LS^h is the input (labour supply) of type *h*, δ^h is the share parameter in production on each category of labour, λ is a units term and σ_p is the elasticity of substitution among labour types in production.

Producers pay the gross of tax wage rate when hiring labour from each household, and households receive the net of income tax wage. For simplicity, we assume that only one consumption good is produced in this economy, and producers maximize profit, Π, given by

$$\Pi = PY - \sum_h w^h LS^h \tag{8}$$

where the *P* is price of the consumption good, and LS^h is the labour input of type *h*.

Profit maximization results in the labour demand function for each labour type *h* as,

$$LD^h = \frac{1}{\lambda} \left(\delta^h + \sum_{hh} \delta^{hh} \left(\frac{\prod_{hh \neq h} w^{hh}}{w^h} \right)^{\sigma_{p-1}} \right)^{\frac{\sigma_p}{\sigma_p - 1}} \tag{9}$$

where LD^h is labour demand of type *h*, and in equilibrium also equals labour supply LS^h.

The government in this economy raises revenues, *R*, by taxing income and consumption, i.e,

$$R = \sum_h t_I^h w^h LS^h + \sum_h t_c^h PC^h \tag{10}$$

We assume a single period in which all output is consumed (there is no saving). In equilibrium, *P* and the household wage rates, w^h, are endogeneously determined such that there is market clearing in the consumption good

$$Y = \sum_h C^h \tag{11}$$

and there is market clearing for each labour type,

$$LS^h = LD^h . \tag{12}$$

As transfers to households are the only government expenditure item, government budget balance also requires that in equilibrium $R = \sum_h RH^h$. Thus, for each household *h*, in equilibrium labour supplied of each type equals its use in production, and a profile of skill specific equilibrium wage rates will be determined. For convenience, in this model, we can choose the numeraire of this system to be that the price of the consumption good is equal to 1.

A homogeneous labour tax model

In contrast to the heterogeneous labour model, when labour is homogeneous in production we need only specify a model with labour as the single input into production. We assume a constant marginal product of labour production function, which is linear in (total) labour [5].

$$Y = \lambda \sum_{h} LS^{h} \qquad (13)$$

A single wage applies to all households' labour supply because of the homogeneity of labour inputs, and the relationship between the wage rate and the price of the consumption good is given by

$$P = \frac{w}{\lambda}. \qquad (14)$$

Households still differ in their preferences as in (1), still face household specific income and consumption tax rates, but unlike in the heterogeneous labour model face a common wage rate. Equilibrium in this case is given by market clearing for the single labour type in the model; with, in this case, one single wage rate endogenously determined.

This homogeneous labour model is thus a special case of the model presented above, and nests into the more general heterogeneous labour model.

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[5] This is a special case of the production function (7) in the model above, for the case where σ_p becomes infinite.

III. Implementation of Homogeneous and Heterogeneous Labour Tax Incidence Models

We perform tax incidence analyses using two models above by calibrating[6] each to a base year data set, specifying labour demand and labour supply elasticity values, and performing counterfactual equilibrium analyses. The base case data set we use reflects the UK economy for the UK tax year 1994/95[7]. We use data on incomes, taxes and benefits by household decile compiled by the UK Treasury and reported in the UK government statistical publication Economic Trends (1996). This source reports data for non retired households grouped by income[8] decile, benefits received both in cash and in kind, and direct and indirect taxes paid by each household decile.

Base case data

For modelling purposes, we require a base case data set which is fully model admissible. This means that all variables which appear in each model should be identified in the data set, and all of the model equilibrium conditions need to be satisfied. Among these are conditions that the value of consumption across households should equal the value of production (a zero profit condition in production implies that income received from supplying labour equals the value of production). All households should also be represented by data which satisfies household budget balance, and government expenditures equal government receipts (government budget balance).

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[6] Calibration, here, denotes the exact calibration of each model to a model admissible data set which is constructed from unadjusted data from a variety of statistical sources. This is the sense of calibration discussed by Mansur and Whalley (1984) and differs from the calibration procedures used by real business cycles researchers (see Kydland and Prescott (1982)). In this latter work, no readjustments are made to data, and model parameter values chosen by reference to literature sources with a view to seeing how close model solution can be made to actual data. See also Watson (1993).

[7] April 5th 1994 to April 4th 1995. This is the year used in recording tax and household income data by the UK tax authorities.

[8] The income concept used in the published data is "household equivalized disposable income".

The basic data we use, while having most of the information we need, is deficient for our purposes in number of respects. Household leisure consumption is not identified. Government budget balance is violated in the data, since all taxes paid by households are identified but only those government expenditures leading directly to direct household benefits (cash transfers, education, and health care appear, but defence, for instance, is missing). In the basic data, government expenditures in aggregate are thus substantially less than tax revenues. Also, individual household budget constraints do not automatically hold.

A series of adjustments and modifications are therefore necessary to the basic data set before it can be used for model calibration. These are set out in more detail in Appendix 1, but can be summarised as follows. We scale the in kind portion of government benefits for each decile such that government expenditures equal taxes collected. Transfers received by each decile in the model are thus the sum of cash and in kind benefits provided by government. We use wage rate data by household and UK time use survey data to construct data on the value of leisure time by household for each decile, valuing time at the net of tax wage. We then make further adjustments to ensure full consistency of the data set to the model, including modifications such that household budget constraints hold in the data.

The resulting model admissible data set across ten UK households is displayed in Table 1. In this data, gross income is concentrated in the higher deciles, with transfers concentrated in the lower deciles. The household profile of leisure consumption reflects the interaction of hours (falling as we move to the higher income ranges) and wage rates at which leisure is valued (rising by income range). The two tax rate profiles are for average (not marginal) tax rates. For income taxes they rise by income range, but not by as much as might be thought from an examination of tax rate schedules. This is because of income tax allowances, caps on social (national) insurance contributions, untaxed housing capital income, and UK tax shelters (pensions, savings in tax sheltered vehicles), all of which have a major influence on the average tax rate profile. The indirect tax rates fall by income range due to the influence of excise taxes, particularly on petrol, but also on drink, both of which are a considerably larger fraction of expenditures for the poor than the rich.

Table1
Model admissible household data set by deciles of income for non-retired households, UK 1994/95[9]

Households[10]	(a) Gross of tax labor income[a]	(b) Transfers[a]	(c) Income and other direct taxes paid[a,11]	(d) Indirect taxes paid[a,12]	(e) Consumption gross of indirect taxes [a,13]	(f) Leisure[a,14]	(g) Income tax rate[15]	(h) Indirect tax rate[16]
Decile 1 (Poor)	3079	14638	930	2139	16787	14491	0.05	0.13
Decile 2	5918	12880	1194	2183	17604	17085	0.06	0.12
Decile 3	11021	11040	1880	2759	20181	14076	0.09	0.14
Decile 4	16111	9867	2815	3213	23163	11934	0.11	0.14
Decile 5	21184	8559	3831	3572	25912	10895	0.13	0.14
Decile 6	26161	8205	4745	3957	29621	10455	0.14	0.13
Decile 7	30140	6858	5531	4266	31467	12293	0.15	0.14
Decile 8	34614	5645	6576	4362	33683	13895	0.16	0.13
Decile 9	41918	5166	8175	4537	38909	15449	0.17	0.12
Decile 10 (Rich)	71147	4440	15082	5551	60505	6871	0.20	0.09

Notes: All figures in this table noted with superscript **a** are millions of £, for the tax year 1994/95.

Using information on elasticities, we calibrate both the homogeneous and heterogeneous labour models to this benchmark equilibrium data set. To do this, we first take the benchmark data set from Table 1 in value terms, and decompose it into separate price and quantity observations. Following Harberger (1962), and Shoven and Whalley (1992) we choose units both for labour by type, and for consumption, as those amounts

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9 See Appendix 1 for more detail.
10 These households are grouped by "original" household income as in Economic Trends (1995). Original income is pre tax / pre transfer income.
11 This includes all social insurance contributions.
12 This includes VAT and all excises (especially on petrol, tobacco, drink).
13 This is gross of indirect taxes.
14 This is from UK time use survey data; leisure time is valued at the net of tax wage.
15 This includes income tax and social insurance contributions.
16 This includes the VAT plus specific excise taxes.

which sell for £1 in the base case equilibrium. Using this convention all prices and wages are one in the base case, and all quantities are as given by the base case observations in Table 1. The calibrated versions of each model replicate this base case data as a model solution.

Elasticities

The elasticity parameters needed for the models are the ten household specific substitution elasticities in consumption (CES preferences) which are used in both models, and the substitution elasticity in production (in the CES production function) in the heterogeneous labour model. Direct estimates of these elasticities are not available in the literature. Elasticities in consumption over goods and leisure must be inferred from literature estimates of labour supply elasticities. Elasticities of substitution in production between labour types must be inferred from literature estimates of labour demand elasticities for various types of labour.

From the production function (7), we can use the first order conditions for profit maximization and the derived labour demand function (9). The elasticity of labour demand in production is given by $e_{LD}^{h} = \frac{\partial LD^{h}}{\partial w^{h}} \frac{w^{h}}{LD^{h}}$ and differentiating (9) w.r.t. w^{h} gives

$$e_{LD}^{h} = \frac{-\sigma_{p} \sum_{hh \neq h} \delta^{hh} \left(\prod_{h \neq h} w^{hh} \right)^{\sigma_{p}-1}}{w^{h^{\sigma_{p}-1}} \left[\delta^{h} + \sum_{hh} \delta^{hh} \left(\frac{\prod_{h \neq h} w^{hh}}{w^{h}} \right)^{\sigma_{p}-1} \right]} \qquad (15)$$

In the base case all gross of tax wage rates are unity, and if, in addition, household labour shares are small, e_{LD}^{h} effectively collapses to $-\sigma^{p}$. We choose the elasticity of substitution between labour types in production which, from (15), we can calibrate numerically to values of labour demand elasticities found in the literature (Hamermesh

(1993)), and use other sensitivity cases discussed below to reflect ranges around a central case value. As there are ten household demand elasticities in the model around the base case equilibrium, and only one free parameter, σ^p, in calibration we choose σ^p such that elasticities across households (which do not vary that much) are within a desired range.

Labour supply elasticities, in contrast, are found using the leisure demand function (4). Point estimates of labour supply elasticities for each household in the neighbourhood of the benchmark equilibrium can be generated by noting that

$$\frac{\partial LS^h}{\partial w^h}\frac{w^h}{LS^h} = \frac{\partial LS^h}{\partial L^h}\frac{\partial L^h}{\partial w^h}\frac{w^h}{L^h}\frac{L^h}{LS^h} = (-1)\eta_{LE}\frac{L^h}{LS^h} \tag{16}$$

We use the leisure demand function (4) to derive the leisure demand elasticity, η_{LE} which is given by

$$\eta^h_{LE} = -\left(\sigma^h + \frac{(\sigma^h - 1)(1-\alpha^h){w^h}^{1-\sigma^h}}{\alpha^h P^{1-\sigma^h} + (1-\alpha^h){w^h}^{1-\sigma^h}}\right) \tag{17}$$

The point estimate of the labour supply elasticity for each household, given σ^h, is :

$$e^h_{LS} = \left(\sigma^h + \frac{(\sigma_p - 1)(1-\alpha^h){w^h}^{1-\sigma^h}}{\alpha^h P^{1-\sigma^h} + (1-\alpha^h){w^h}^{1-\sigma^h}}\right)\frac{L^h}{LS^h} \tag{18}$$

In the neighbourhood of the base case, where the price of goods and all wage rates are unity, this collapses to

$$e^h_{LS} = \left(\sigma^h + (\sigma_p - 1)(1-\alpha^h){w^h}^{1-\sigma^h}\right)\frac{L^h}{LS^h} \tag{19}$$

and if $(1-\alpha^h)$ is small[17] $e^h_{LS} \approx \sigma^h \frac{L^h}{LS^h}$.

We choose values for labour supply elasticities from literature estimates and using (16) these imply leisure demand elasticities. Using those as point estimates around the benchmark equilibrium, and using (18), elasticities of substitution in preferences are determined for each household decile. The equation (18) yields an implicit function for σ^h, which we solve numerically.

Table 2
Model production and consumption side elasticities, and literature justification

A. Range of labour supply elasticities based on those reported in Killingsworth (1983)

Range of values	Labour supply elasticity assumed for each household	Range of elasticities of substitution in consumption implied for household deciles
High	1.0	0.52-10.5
Mid (central case)	0.3	0.38-3.50
Low	0.15	0.32-1.57

B. Range of labour demand elasticities based on those reported in Hamermesh (1993)

Range	Range of labour demand elasticities by decile	Elasticity of substitution used in production
High	-1.81 to -2.10	1.93
Mid Range (central case)	-1.05 to -1.24	1.32
Low	-0.58 to -0.67	0.71

Table 2 sets out the elasticity ranges we use for in the two models and the implied substitution elasticities in consumption and production. These are approximately consistent with ranges of parameter estimates reported by Hamermesh (1993) for labour demand, and Killingsworth (1983) for labour supply.

In the model, there are 10 separate labour demand elasticities for each labour type. These elasticities vary, and hence we calibrate the model to point estimates of these elasticities in the neighbourhood of the benchmark equilibrium. In addition, there is only one free model parameter (the elasticity of substitution among labour types in production) so that exact calibration for each household type labour demand elasticity is not feasible. We thus

[17] Typically, however, consumption share parameters on leisure are not small.

vary the single production side elasticity until the household labour demand elasticities, which are similar across households, are within the desired range.

IV. Results

We have used the heterogeneous and homogeneous labour models described above and calibrated to UK data for the 1995/96 tax year to investigate the behaviour of tax incidence results across models. We do this for different tax experiments, different labour supply elasticity (consumption/leisure substitution) configurations, freezing the labour demand elasticity specification; and for different labour demand elasticity specifications, freezing the labour supply elasticity specification.

The essence of tax policy analysis lies in comparing welfare changes between benchmark and counterfactual equilibria. How much a typical household has gained or lost because of changes in policy in money metric terms, or how much money is required to bring him/her back to their original welfare can be measured at either original or new prices. The Hicksian equivalent variation (*EV*) is a money metric measure of the welfare change between benchmark and counterfactual scenarios using benchmark (old) prices. It is the difference in money metric utility at old prices corresponding to benchmark and counterfactual model solutions; i.e.

$$EV = E(U^N, P^0) - E(U^0, P^0) \qquad (20)$$

where superscripts *N* and *O* represent new and old values for the variable on which they appear, U is the utility, and *E* is the expenditure function which depends on prices and utility level.

If utility functions are of the linear homogeneous type, then original and new equilibria can be thought of in terms of a radial expansion of the utility surface. In this case the change in money metric welfare between benchmark and counterfactual solutions of the model is proportional to the change in utility or the percentage change along the radial projection between the two consumption points.

$$EV = \frac{U^N - U^0}{U^0} I^0 \qquad (21)$$

where *N* and *O* represent new and old values of the variables as before, and *I* represents the income.

In Table 3 we report tax incidence calculations for a case where we replace the pre existing pattern of labour income tax rates by household by a yield preserving uniform rate income tax across households. We consider cases where we freeze labour supply elasticities first at 0.3, and then at 1.0, and vary the ranges we use for labour demand elasticities in the heterogeneous labour model. We compare model results across the homogeneous labour model and the various specifications of the heterogeneous model for these labour supply elasticities.

Table 3
Comparing homogeneous and heterogeneous labour models

Specification

- Experiment: replacing existing labour income taxes by yield preserving uniform rate
- Elasticity specification: Labour supply elasticity 0.3 and 1.0 in two cases, labour demand elasticities range from -0.58 to -2.1.

Results:

Welfare gains/losses by decile in terms of Hicksian EV as a fraction of base income (with low labour supply elasticity (0.3))

Decile	Homogeneous Labour model	Heterogeneous Labour model Labour demand elasticities ranges as specified in Table 2 Low (-0.58 to -0.67)	Middle (-1.05 to -1.24)	High (-1.8 to-2.1)
1 poor	-0.0581	-0.0134	-0.0501	-0.0591
2	-0.0485	-0.0173	-0.0434	-0.0493
3	-0.0424	-0.0334	-0.0401	-0.0430

4	-0.0299	-0.0306	-0.0296	-0.0303
5	-0.0143	-0.0211	-0.0153	-0.0146
6	-0.0064	-0.0159	-0.0078	-0.0066
7	0.0044	-0.0059	0.0028	0.0042
8	0.0173	0.0060	0.0155	0.0171
9	0.0272	0.0154	0.0253	0.0270
10 rich	0.0670	0.0562	0.0660	0.0666

Results:

Welfare gains/losses by decile in terms of Hicksian EV as a fraction of base income (with low labour supply elasticity (1.0))

Decile	Homogeneous Labour model	Heterogeneous Labour model Labour demand elasticities ranges as specified in Table 2		
		Low (-0.58 to -0.67)	Middle (-1.05 to -1.24)	High (-1.8 to-2.1)
1 poor	-0.0577	-0.0376	-0.0503	-0.0585
2	-0.0481	-0.0709	-0.0442	-0.0486
3	-0.042	-0.2191	-0.0420	-0.0421
4	-0.0295	-0.2944	-0.0312	-0.0294
5	-0.014	-0.3300	-0.0150	-0.0140
6	-0.0061	-0.1114	-0.0060	-0.0061
7	0.0047	0.3908	0.0066	0.0045
8	0.0176	0.0739	0.0221	0.0172
9	0.0276	0.1476	0.0349	0.0270
10 rich	0.0676	-0.0025	0.0737	0.0670

Results in Table 3 show the redistribution across households in these cases. Richer households gain because their taxes fall, and poorer households lose since the replacement tax is at a uniform rate and their taxes rise. However, there is considerable variation in the redistribution profile across the two elasticity cases for the heterogeneous labour model. Considerably more redistribution occurs in the high elasticity case since wage rate changes in response to the tax replacement are small; low income households bear most of the burden of their higher taxes, and high income households gain by most of their tax saving.

As one moves across labour demand elasticity specifications the redistributive effects from the tax changes become larger, with wage changes less pronounced. Labour demand elasticities have a significant influence on the perceived tax incidence effects from the tax replacement. From various sensitivity analyses, we find that as labour demand elasticities increase, the income tax incidence profile in the heterogeneous labour model approaches that of the homogeneous labour model for any given value of the labour supply elasticity.

In Table 4 we explore how the model comparisons change as we vary labour supply elasticities. We consider the same tax replacement, i.e. replacing the existing profile of income tax rates by a uniform rate income tax across households, but only consider the mid range specification of labour demand elasticities, changing labour supply elasticities in both homogeneous and heterogeneous labour models.

Results in Table 4 show that the incidence profile changes relatively little between heterogeneous and homogeneous labour models as labour supply elasticities increase. The low income households lose about 5-6 percent of the base year income in both heterogeneous and homogeneous labour models irrespective of different values of labour supply elasticities. For middle range values of labour demand elasticities (irrespective to any specific value of labour supply elasticity), income tax incidence profiles of replacing base case labour income taxes by yield preserving labour income tax rates become comparable across two models.

Table 4

Impacts of Varying Labour Supply Elasticities on Incidence Profile Comparisons

Specification

- Experiment: replacing existing labour income taxes by yield preserving uniform rate
- Elasticity specification: Labour supply elasticity 0.3, mid range demand elasticities in the heterogeneous labour model.

Results:

Welfare gains/losses by households, Hicksian EV as a fraction of base income

Decile	Labour supply elasticity (0.15)		Labour supply elasticity (0.3)		Labour supply elasticity (1.0)	
	Homogen	Heterogen	Homogen	Heterogen	Homogen	Heterogen

	eous labour Model	eous labour model	eous labour Model	eous labour model	eous labour Model	eous labour model
1 poor	-0.0582	-0.0500	-0.0581	-0.0501	-0.0577	-0.0585
2	-0.0486	-0.0431	-0.0485	-0.0434	-0.0481	-0.0486
3	-0.0426	-0.0395	-0.0424	-0.0401	-0.0420	-0.0421
4	-0.0300	-0.0290	-0.0299	-0.0296	-0.0295	-0.0294
5	-0.0144	-0.0149	-0.0143	-0.0153	-0.0140	-0.0140
6	-0.0065	-0.0075	-0.0064	-0.0078	-0.0061	-0.0061
7	0.0043	0.0028	0.0044	0.0028	0.0047	0.0045
8	0.0172	0.0150	0.0173	0.0155	0.0176	0.0172
9	0.0270	0.0244	0.0272	0.0253	0.0276	0.0270
10 rich	0.0668	0.0650	0.0670	0.0660	0.0676	0.0670

As final sets of results in Table 5 we present incidence results for three different yield preserving tax changes; the first one only involves income taxes, the second one involves income and sales taxes, and the last one involves only sales taxes. We use a 0.3 labour supply elasticity and mid range labour demand elasticities.

Table 5

Incidence Comparison for Different Tax Changes for the Heterogeneous Labour and Homogeneous Labour Models

Specification

- Labour supply elasticity set at 0.3
- Labour demand elasticities set at mid range values in the heterogeneous labour model.

Results:

Welfare gains/losses by households, Hicksian EV as a fraction of base income

	Only income tax		Income and sales tax		Only sales tax	
Decile	Homogeneous labour Model	Heterogeneous labour model	Homogeneous labour Model	Heterogeneous labour model	Homogeneous labour Model	Heterogeneous labour model
1 poor	-0.0581	-0.0501	-0.0551	-0.0489	0.0029	0.0026
2	-0.0485	-0.0434	-0.0478	-0.0434	0.0006	0.0006
3	-0.0424	-0.0401	-0.0324	-0.0307	0.0101	0.0096
4	-0.0299	-0.0296	-0.0167	-0.0166	0.0132	0.0129
5	-0.0143	-0.0153	-0.0008	-0.0014	0.0134	0.0134
6	-0.0064	-0.0078	0.0039	0.0032	0.0100	0.0102

7	0.0044	0.0028	0.0164	0.0155	0.0116	0.0118
8	0.0173	0.0155	0.0235	0.0225	0.0058	0.0061
9	0.0272	0.0253	0.0217	0.0208	-0.0057	-0.0054
10 rich	0.0670	0.0660	0.0314	0.0307	-0.0337	-0.0338

In both the income tax case and the combined income and sales tax case we find low income households lose and high income households gain when base case taxes are replaced by yield preserving tax rates, but the pattern of gains is different in the upper tail of the distribution across heterogeneous and homogeneous models.

V. Conclusions

In this paper we analyze how labour demand elasticities, long neglected in empirically based tax incidence analysis, affect incidence conclusions. Using a data set for the UK for tax year 1994/95 covering 10 household deciles we use two models to evaluate the incidence effect of various tax changes, specially the replacement of the existing pattern of income tax rates by a uniform rate yield preserving alternative. We consider two models, one with labour heterogeneous in production, i.e. 10 different labour types (one for each decile) in production; and the other with labour homogeneous across households i.e. only one type of labour in production. The substitution elasticity among labour types in production determines labour demand elasticities. Our results suggest that labour demand elasticities do indeed matter for tax incidence conclusions.

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Appendix 1
Data Sources

This appendix presents details on various data sources and adjustments that underlie Table 1. The main data sources are Table 3A (Appendix 1) of Economic Trends, 1995/96, p. 36, New Earnings Survey 1995 and Time Use Survey reported in Dex et al. (1995).

The gross income in column (a) of Table 3A comprises original income and direct taxes (see Table 1 in text). Original income includes wages and salaries, imputed income from benefits in kind, self-employment income, occupational pensions, annuities and other income. Direct taxes include employees' national insurance (NI) contributions. The household average direct tax rate to be income and other taxes divided by gross of tax income plus transfers.

A1
Components of Gross of tax labour income in Table 1 (see Text)
(£ million 1995/1996 UK Tax year)

	H1	H2	H3	H4	H5	H6	H7	H8	H9	H10	Total
Original income2	2149	4724	9141	13296	17353	21416	24609	28038	33743	56065	210534
Direct taxes5	930	1194	1880	2815	3831	4745	5531	6576	8175	15082	50759
Total	3079	5918	11021	16111	21184	26161	30140	34614	41918	71147	261293

The UK Economic Trends data distinguishes five different concepts of income: original, income, gross income, disposable income, post tax income and final income. Original income plus cash benefits equal gross income, disposable income is gross income minus direct taxes. Post tax income is disposable income minus indirect taxes
Final income equals post tax income plus in kind benefits.

Table A2
Gross consumption by Households in Table 1 (see Text)
(£ million 1995/1996 UK Tax year)

Decile	H1	H2	H3	H4	H5	H6	H7	H8	H9	H10	Total
Gross income	3079	5918	11021	16111	21184	26161	30140	34614	41918	71147	261293
Transfers	14638	12880	11040	9867	8559	8205	6858	5645	5166	4440	86479
Direct taxes	930	1194	1880	2815	3831	4745	5531	6576	8175	15082	50759
Total	16787	17604	20181	23163	25912	29621	31467	33683	38909	60505	297013

The transfers presented in column (b) of Table 1 in the text include direct cash benefits, in kind transfers, and consumption of publicly provided goods services such as national defence. Direct cash benefits consist of retirement pension contributions, unemployment benefit, invalidity pension and allowance, sickness and industrial injury benefit, widow's benefits, and statutory maternity pay/allowance. Non-contributory benefits include income support, child benefit, housing benefit, invalid care allowances, attendance allowance, disability living allowance, industrial injury disablement benefit, student maintenance awards, government training schemes, family credit and other non-contributory benefits. Benefits in kind consist of education, national health service, housing subsidy, rail travel subsidy, bus travel subsidy, school meals and welfare milk.

The gross consumption of each household, included in column (e) of Table 1 in the text, is derived by adding cash, in kind and non-contributory benefits to original income and subtracting the direct and indirect taxes paid by the household. Consumption thus is gross of indirect taxes that include taxes on final goods and services, VAT, duty on tobacco, beer and cider, wines and spirits, hydrocarbon oils, vehicle excise duty, TV licences, stamp duty on house purchase, customs duties, betting taxes, fossil fuel levy, and Camelot national lottery fund. It also includes intermediate taxes such as commercial and industrial rates, employer's NI contributions, duty on hydrocarbon oils, vehicle excise and other duties.

Table A3

The Value of Leisure Consumption by Households in Table 1 (see text)

	H1	H2	H3	H4	H5	H6	H7	H8	H9	H10	Total
Earnings/week/households (£)	160	210	223	243	272	306	355	403	473	543	319
Working weeks	13	23	41	55	64	70	69	70	71	91	57
Leisure weeks	91	81	63	49	40	34	35	34	33	13	47
Value of leisure by household (£ million)	14491	17085	14076	11934	10895	10455	12293	13895	15449	6871	127444

The value of leisure reported in Table 1 in the text has been obtained by multiplying non-working weeks by the weekly earnings rate. The number of non-working weeks is the difference between the working weeks and 104 weeks. The total working week represents the total labour endowment per household with two working members. Earnings per week for top and bottom deciles, and first and third quartiles are taken from the New Earnings Survey 1995. These are interpolated for other deciles. Working weeks are derived by dividing the original income by the weekly earnings.

Appendix 2
Solution Method of the Model

Both homogeneous and heterogeneous labour models discussed in this paper are set up as a mixed complementarity problems and solved in GAMS software using the PATH solver.

Dirkse and Ferris (1995) state the basic idea behind the PATH solver in terms of a "zero finding problem". For any function F: $\Re^n \rightarrow \Re^n$ with lower bound $-\infty \leq l$ and an upper bound $\leq u \leq +\infty$ the problem *is to* find $z \in \Re^n$ such that

$$\begin{array}{llll} \text{either} & z_i = l_i & \text{and} & F_i(z) \geq 0 \\ \text{or} & z_i = l_i & \text{and} & F_i(z) \leq 0 \\ \text{or} & l_i \leq z_i \leq u_I & \text{and} & Fi(z) = 0 \end{array}$$

PATH constructs a solution using a damped Newton method such as

$$0 = F_{B_{(x)}} = F_{x_{(B)}} + (x - x_B)$$

where x_B is the Euclidean projection of x onto the Box $B := [l, u]$. A vector x solves this non-linear equation only if $z = x_B$ solves the MCP. A more detailed explanation of this algorithm is beyond the scope of this paper, many technical papers on the topic are available in Ferris's homepage : http://www.cs.wisc.edu/~ferris/.

GAMS syntax (Brook, Kendrick and Meeraus (1992)) permits us to generate a non linear mixed complemetarity model by declaring and assigning sets, data, parameters, variables, equations in the model. PATH is invoked by the "OPTION MCP = PATH" statement in the GAMS code and a command line "solve <model name> using MCP" instructs GAMS to solve the model using the PATH solver. We use batch files to compute incidence profiles across various scenarios for different values of elasticities and tax rates for households.