

Cointegration, ARCH (p), GARCH(p,q) and
Vector Autoregressive Models

Importance of Error Terms in Regressions
Estimation, Impulse Response Analysis

Engle-Granger Approach to Co-integration

$$Y_t = \beta_1 + \beta_2 X_t + e_t$$

$$e_t = Y_t - \beta_1 - \beta_2 X_t$$

If Y_t and X_t $I(1)$ but e_t $I(0)$

$$e_t = \alpha_0 + \gamma e_{t-1} + v_t$$

$\gamma = 0 \longrightarrow$ Cointegration

Long-Run Multiplier in ARDL(1,1)

$$Y_t = \beta_1 + \beta_2 Y_{t-1} + \beta_3 X_t + \beta_4 X_{t-1} + \varepsilon_t$$

$$\varepsilon_t \sim N(0,1)$$

$$Y_t = Y_{t-1} = \bar{Y} \qquad X_t = X_{t-1} = \bar{X}$$

$$\bar{Y} - \beta_2 \bar{Y} = \beta_1 + \beta_3 \bar{X} + \beta_4 \bar{X}$$

$$\bar{Y} = \frac{\beta_1}{(1 - \beta_2)} + \left(\frac{\beta_3 + \beta_4}{(1 - \beta_2)} \right) \bar{X}$$

Adjustment towards the long run in ARDL(1,1)

$$Y_t - Y_{t-1} = \beta_1 + \beta_2 Y_{t-1} - Y_{t-1} + \beta_3 X_t - \beta_3 X_{t-1} + \beta_3 X_{t-1} + \beta_4 X_{t-1} + \varepsilon_t$$

$$\Delta Y_t = \beta_1 + (\beta_2 - 1)Y_{t-1} + \beta_3(X_t - X_{t-1}) + (\beta_3 + \beta_4)X_{t-1} + \varepsilon_t$$

$$\Delta Y_t = (\beta_2 - 1) \left[Y_{t-1} - \frac{\beta_1}{(1 - \beta_2)} - \frac{(\beta_3 + \beta_4)}{(1 - \beta_2)} X_{t-1} \right] + \beta_3 \Delta X_t + \varepsilon_t$$

Deviation from the long-term trend:

$$\left[Y_{t-1} - \frac{\beta_1}{(1 - \beta_2)} - \frac{(\beta_3 + \beta_4)}{(1 - \beta_2)} X_{t-1} \right]$$

$(\beta_2 - 1)$ Adjustment to the long run.

Analysis of Volatility using ARCH(p)

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \alpha_3 \varepsilon_{t-2}^2 + \dots + \alpha_p \varepsilon_{t-p}^2$$

Analysis of Volatility using GARCH(p,q)

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \alpha_3 \varepsilon_{t-3}^2 + \dots + \alpha_p \varepsilon_{t-p}^2 + \phi_1 \sigma_{t-1}^2 + \phi_2 \sigma_{t-2}^2 + \phi_3 \sigma_{t-3}^2 + \dots + \phi_q \sigma_{t-q}^2$$

Steps for Granger Causality Test

Does increase in money supply increase GDP or an increase in money supply increase GDP?

$$Y_t = \sum_{i=1}^n \alpha_i M_{t-i} + \sum_{j=1}^n \beta_j Y_{t-j} + u_{1,t}$$

$$M_t = \sum_{i=1}^n \lambda_i M_{t-i} + \sum_{j=1}^n \delta_j Y_{t-j} + u_{2,t}$$

Four possible cases for Granger Causality

1. Uni-directional causality from money and GDP

$$\sum_i^n \alpha_i \neq 0 \quad \sum_i^n \delta_i = 0$$

2. Uni-directional causality from GDP to money

$$\sum_i^n \alpha_i = 0 \quad \sum_i^n \delta_i \neq 0$$

3. Bilateral causality

$$\sum_i^n \alpha_i \neq 0 \quad \sum_i^n \delta_i \neq 0$$

3. Independence from each other

$$\sum_i^n \alpha_i = 0 \quad \sum_i^n \delta_i = 0$$

Steps for Granger Causality Test

- Regress GDP to all lagged GDP and other variables but not to the lagged money terms and get error some square of the restricted model
- Include lags for GDP as well as money and get the unrestricted residual sum square

$$F_{m,(n-k)} = \frac{(RSS_R - RSS_{UR})/m}{RSS_{UR}/(n-k)}$$

$$H_0 : \sum_i^n \alpha_i = 0 \quad \text{No causality}$$

Specification of Vector Autoregression Model

$$Y_t = A_1 Y_{t-1} + \varepsilon_t$$

 Y_t

vector of variables
interest rate,
output gap
inflation gap

 ε_t

normally and identically distributed random error term

$$\Delta Y_t = (A_1 - I) Y_{t-1} + \varepsilon_t$$

$$\Pi = (A_1 - I)$$

Decomposition of Long and Short Run Responses

Π

Long run response

$$\Pi = \alpha\beta'$$

α

Dynamic process of adjustment

β

Long run Steady State Relation

Trace and Max Test for Cointegration Rank

$$\lambda_{trace(r)} = -T \sum_{i=r+1}^n \ln(1 - \hat{\lambda}_i)$$

$$\lambda_{\max(r,r+1)} = -T \ln(1 - \hat{\lambda}_{r+1})$$

λ_i **Eigenvalues of the characteristic matrix**

rank	Trace test [Prob]	Max test [Prob]	Trace test (T-nm)	Max test (T-nm)
0	56.86 [0.000]**	34.38 [0.000]**	55.43 [0.000]**	33.52 [0.000]**
1	22.48 [0.003]**	12.68 [0.087]	21.91 [0.004]**	12.36 [0.097]
2	9.80 [0.002]**	9.80 [0.002]**	9.55 [0.002]**	9.55 [0.002]**

Application of VAR Analysis

$$y_t - y_t^* = d(i_{t-1} - i_{t-1}^*) + \varepsilon_{1,t}$$

$$\pi_t = \pi_t^* + c(y_{t-1} - y_{t-1}^*) + \varepsilon_{2,t}$$

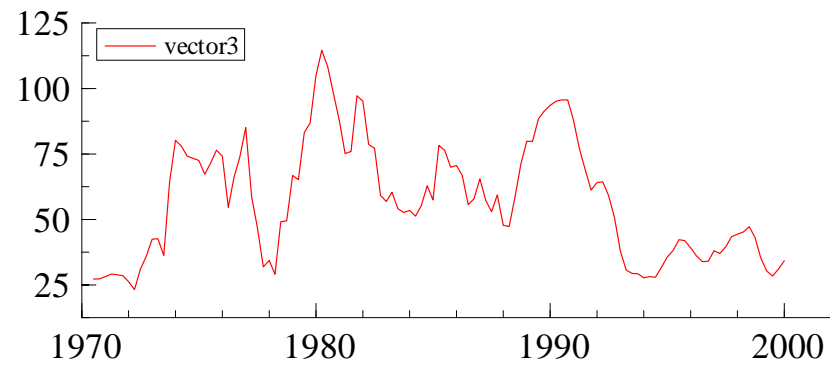
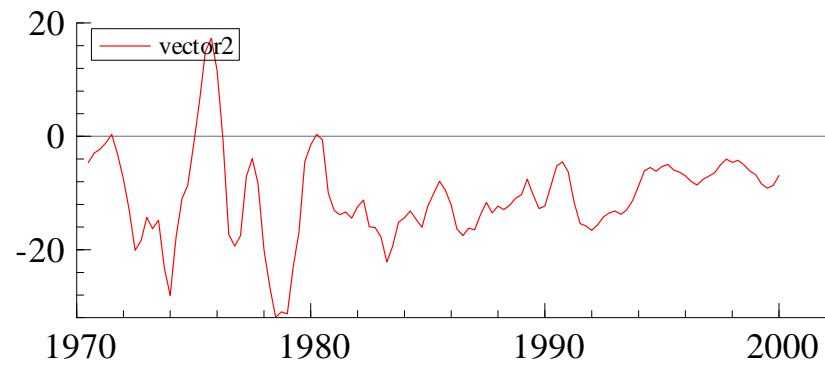
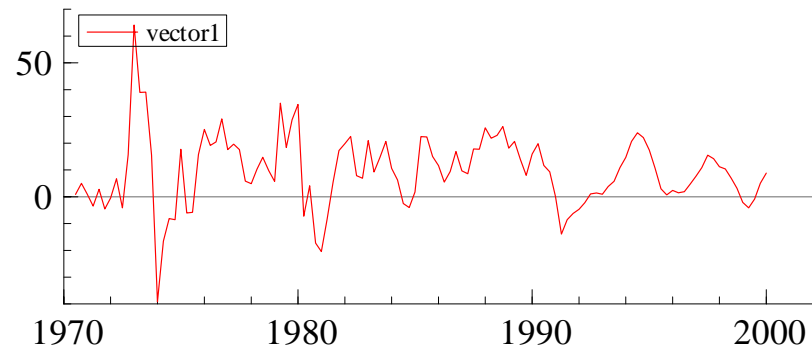
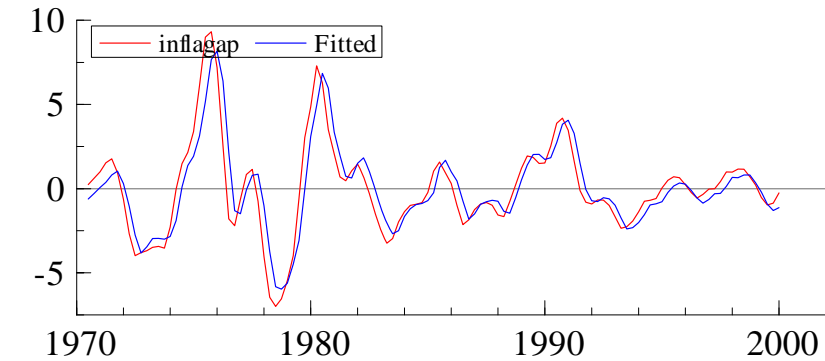
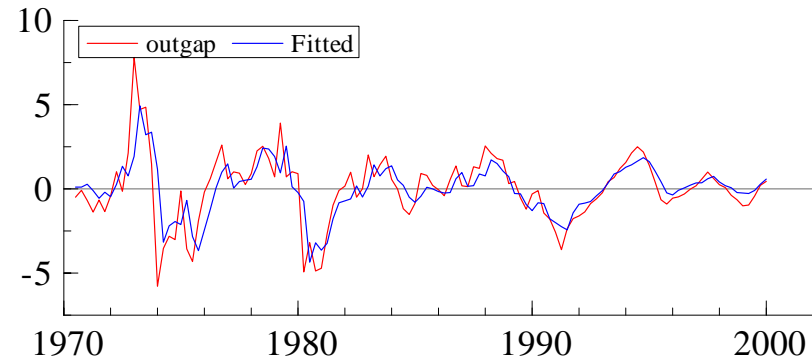
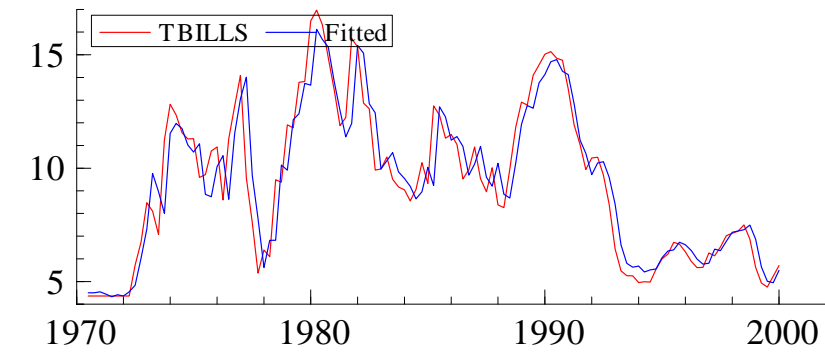
$$i_t = i_t^* + a(y_t - y_t^*) + b(\pi_t - \pi_t^*) + \varepsilon_{3,t}$$

Stationarity of variables in the model

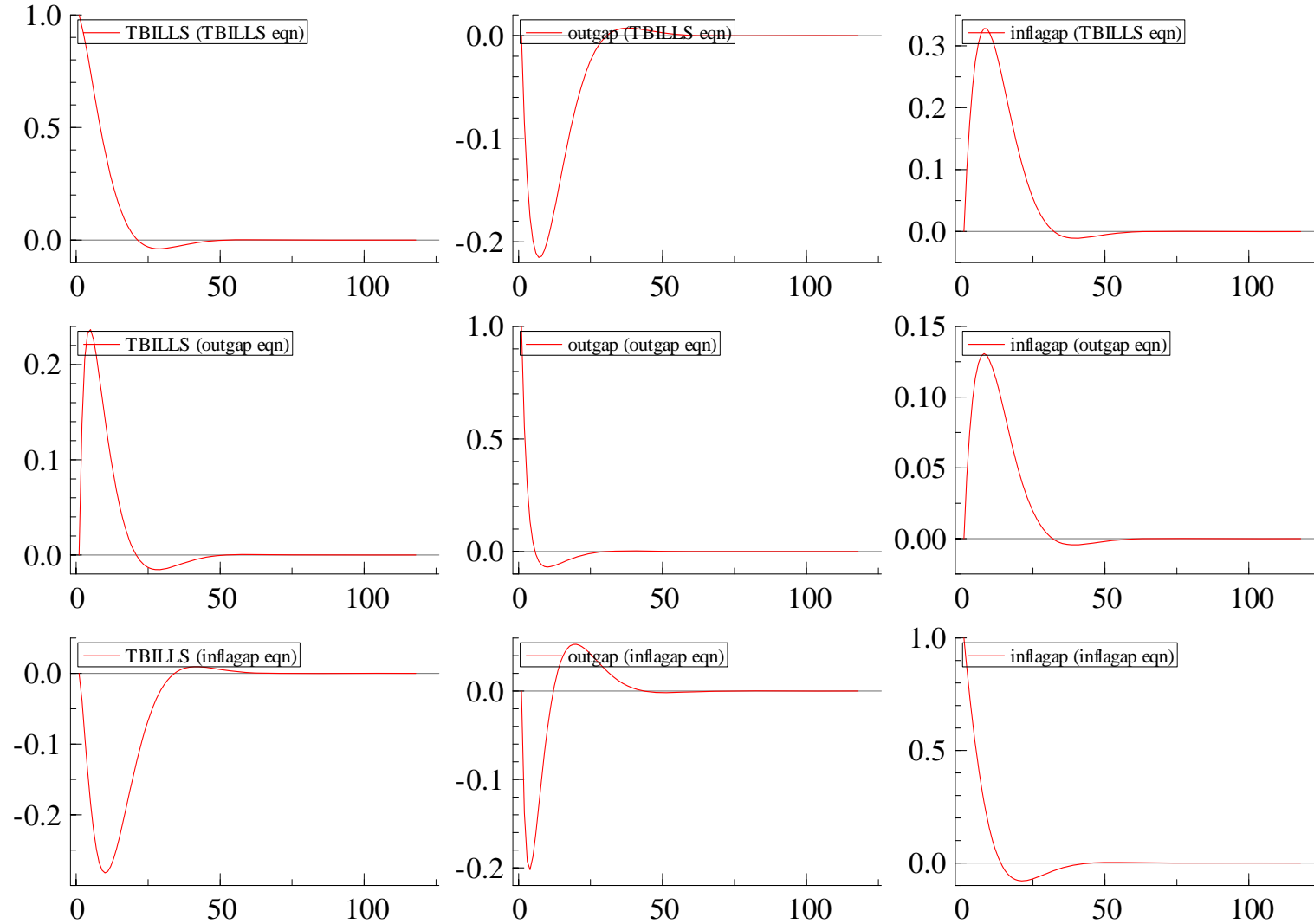
ADF tests (T=116, Constant; 5%=-2.89 1%=-3.49)

	Interest rate	Difference of Interest rate	Output gap	Inflation gap
Coefficient	-2.723	-6.463**	-6.160**	-7.428**
Lags	2	2	3	1

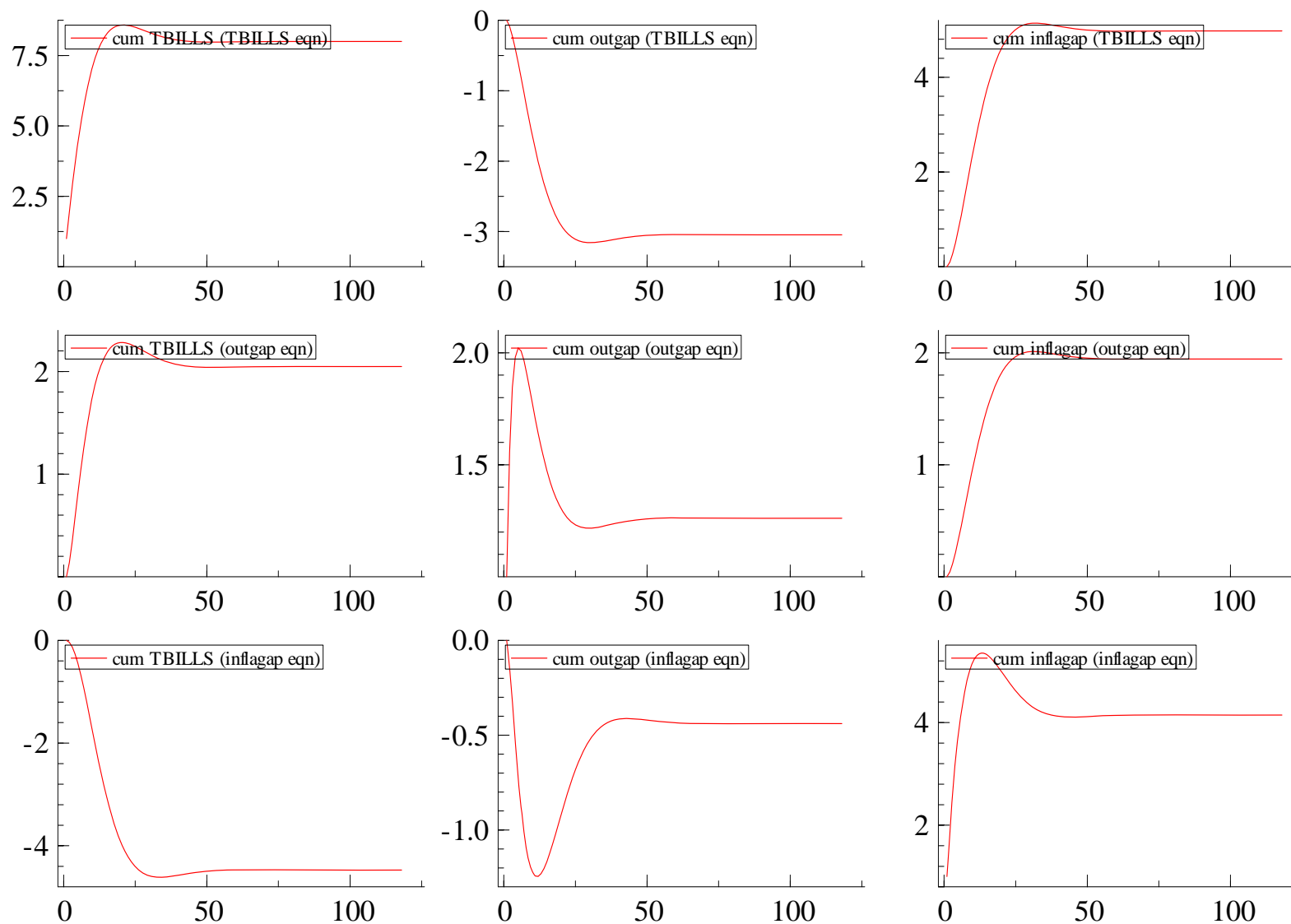
Prediction from Cointegrating VAR Model



Impulse Response Analysis



Cumulative Impulse Response Analysis



References

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