

# Unit Root Test and Cointegration

Avoiding Spurious Regression

# Stationary Time Series

$y_t$  is stationary when

mean is constant

$$E(y_t) = \mu$$

and

$$\text{var}(y_t) = \sigma^2$$

variance is constant :

or independent of time.

Covariance is time independent

$$\text{cov}(y_t, y_{t-s}) = \text{cov}(y_t, y_{t+s}) = \gamma_s$$

# Non-Stationarity and Test for Unit Roots

- Many economic variables are non-stationary. They include:
  - GDP its components C, I, M, T, G, X
  - Inflation and exchange rates
  - Labour force and unemployment rate
  - Money supply and interest rates
  - Are non-stationary.
- Estimates from non-stationary variable may generate a **spurious regression** unless they are cointegrated.
- Dicky-Fuller and Augmented Dicky Fuller tests for unit root.
- Hypotheses  
Null: Unit root exists  
Alternative:  
Unit root does not exist

# Dicky-Fuller and Augmented Dicky-Fuller Tests

$$y_t = \rho y_{t-1} + v_t$$

$$\Delta y_t = (1 - \rho) y_{t-1} + v_t;$$

Random Walk:

$$\Delta y_t = \gamma y_{t-1} + v_t$$

Random Walk with a drift (intercept):

$$\Delta y_t = \alpha_0 + \gamma y_{t-1} + v_t$$

Trend stationary process

$$\Delta y_t = \alpha_0 + \alpha_1 t + \gamma y_{t-1} + v_t$$

Augmented Dicky Fuller Test

$$\Delta y_t = \alpha_0 + \alpha_1 t + \gamma y_{t-1} + \sum_{i=1}^m a_i \Delta y_{t-i} + v_t$$

Null hypotheses:

There is unit root and time series is non-stationary

$$K=0 \rightarrow (1-\Psi)=0$$

Alternative hypothesis:

There is no unit root and time series is stationary

$$K < 0 \rightarrow (1-\Psi) < 0 \rightarrow \Psi < 1$$

## Unit Root Tests of Unemployment Rate

### Unit root exists in the level of unemployment rate

URT: ADF tests (T=373, Constant; 5%=-2.87 1%=-3.45)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
3	-1.143	0.99595	0.1969	-0.4586	0.6468	-3.237	
2	-1.165	0.99588	0.1967	-0.9016	0.3679	-3.242	0.6468
1	-1.209	0.99573	0.1966	6.955	0.0000	-3.245	0.6005
0	-0.9868	0.99630	0.2088			-3.127	0.0000

### There is no unit root in the first difference

DURT: ADF tests (T=372, Constant; 5%=-2.87 1%=-3.45)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
3	-7.625**	0.40441	0.1948	-3.144	0.0018	-3.258	
2	-10.17**	0.28958	0.1971	0.4874	0.6263	-3.237	0.0018
1	-11.59**	0.30717	0.1969	0.9270	0.3545	-3.242	0.0068
0	-13.52**	0.33903	0.1969			-3.245	0.0125

### Output from the PcGive

# How to make a Non-Stationary Series to a Stationary Series

- Logs
- ratios
- First difference
- Second difference
- Third or higher order difference
- Cointegration
- Error Correction

## Unit Root Test for RPII

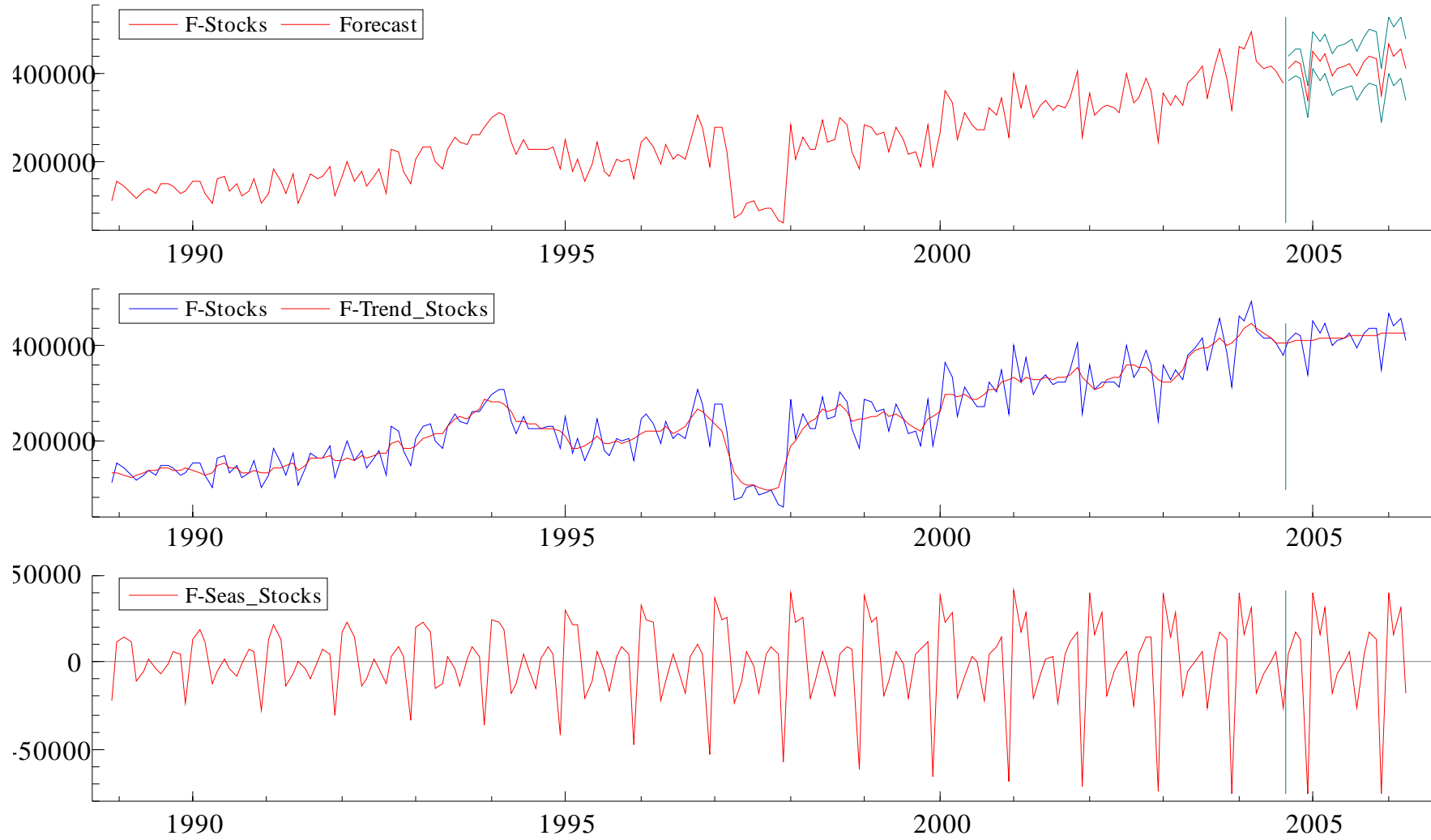
RPI: ADF tests (T=373, Constant; 5%=-2.87 1%=-3.45)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
3	-2.175	0.98866	0.5856	0.8751	0.3821	-1.057	
2	-2.089	0.98918	0.5854	3.513	0.0005	-1.060	0.3821
1	-1.729	0.99095	0.5943	10.04	0.0000	-1.033	0.0016
0	-0.9820	0.99422	0.6695			-0.7971	0.0000

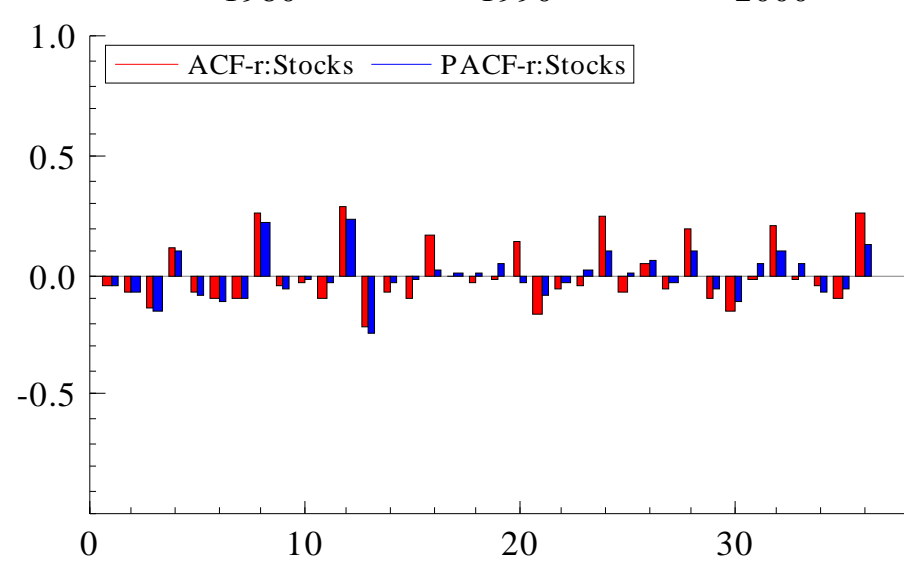
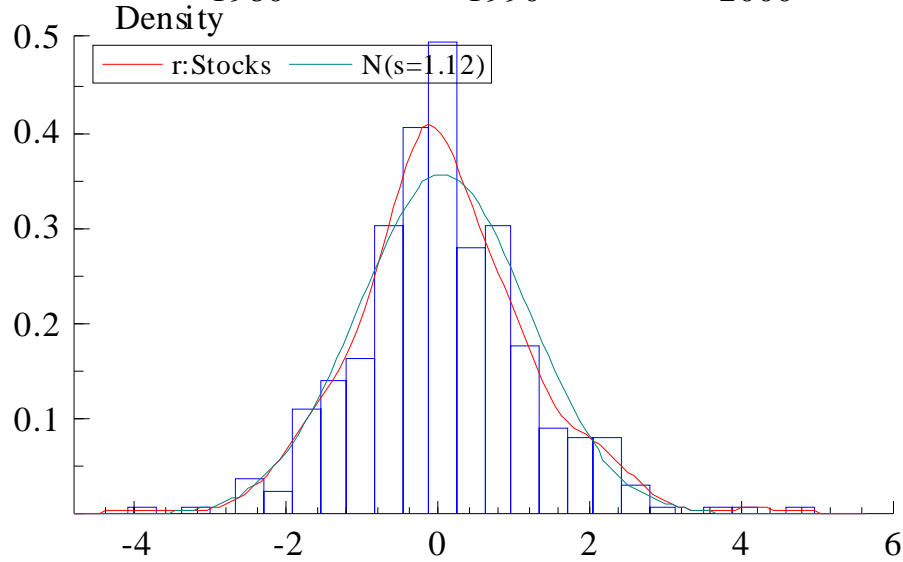
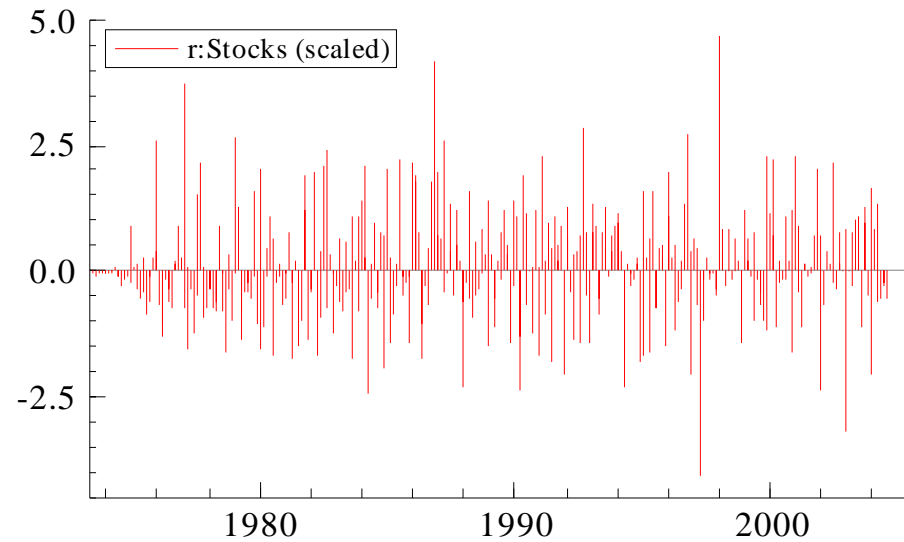
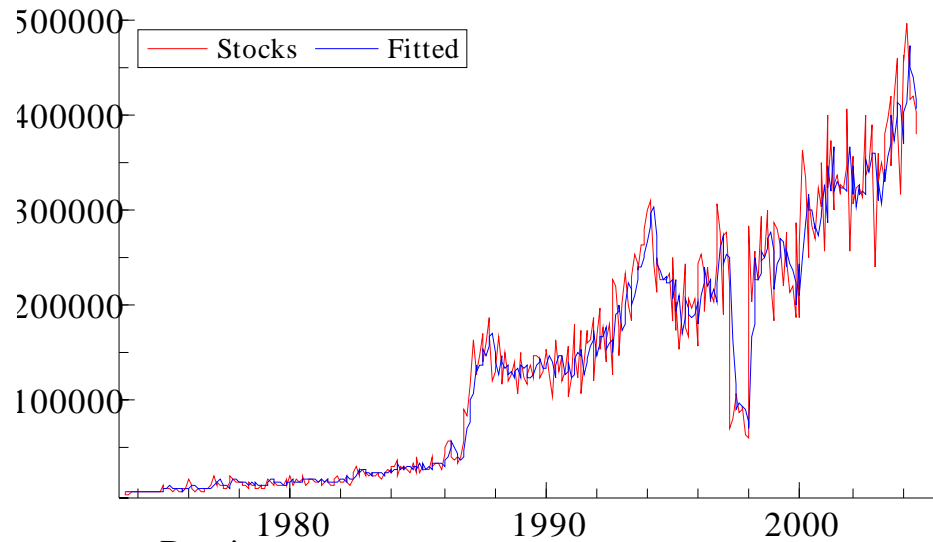
DRPI: ADF tests (T=372, Constant; 5%=-2.87 1%=-3.45)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
3	-6.943**	0.56614	0.5882	-8.517e	-005	0.9999	-1.048
2	-7.458**	0.56614	0.5874	-0.6487	0.5170	-1.054	0.9999
1	-8.435**	0.55099	0.5869	-3.297	0.0011	-1.058	0.8108
0	-11.73**	0.45967	0.5947			-1.034	0.0112

# Trend, Seasonality and Random Components of Stock Price and Forecasts



# Prediction from a GARCH Model



Data source: Navidata

# Confidence Interval for Forecast of Stock Prices from a GARCH Model

