Econometrics 1

Lecture 12

ARMA and ARIMA Model and Time Series Forecasting

Autoregressive Process

$$Y_t = \delta + \theta_1 y_{t-1} + e_t$$

$$E(y_t) = E(y_{t-1}) = ... = E(y_{t-k}) = \mu$$

$$E(Y_y) = E(\delta + \theta_1 y_{t-1} + e_t)$$

$$\mu = \delta + \theta_1 \mu \ ; \ \mu = \frac{\delta}{1-\theta_1}$$

$$\operatorname{var}(y_t) = \operatorname{var}(\theta_1 y_{t-1} + e_t) \ => \ \sigma_y^2 = \frac{\sigma_e^2}{1-\theta_1^2}$$

$$\operatorname{cov}(Y_t Y_{t-1}) = E(y_t - E(y_t))(y_{t-1} - E(y_t)) = \theta_1 \sigma_y^2$$

Some examples:

$$Y_t = 0.8 y_{t-1} + e_t$$
$$Y_t = -0.8 y_{t-1} + e_t$$

Convergence occurs if $|\theta_1| < 1$. The series is called stationary.

ARMA(1,1) Process

$$Y_t = \delta + \theta_1 y_{t-1} + e_t + \alpha_1 e_{t-1}$$

$$\operatorname{var}(y_t) = E\left[(y_t - \mu)^2\right] = E\left[\left(\delta + \theta_1 y_{t-1} + e_t + \alpha_1 e_{t-1}\right)^2\right]$$

$$= \theta_1^2 \gamma_0 + \sigma_e^2 + \alpha_1^2 \sigma_e^2 + 2\theta_1 \alpha_1 E\left[y_{t-1} e_{t-1}\right]$$

Moving Average-MA Process

$$Y_t = \mu + e_t + \alpha_1 e_{t-1}$$

$$E(y_t) = \mu$$

$$\text{var}(y_t) = \text{var}(\mu + e_t + \alpha_1 e_{t-1}) = \sigma_e^2 \left(1 + \alpha_1\right)$$

$$\text{cov}(Y_t Y_{t-1}) = E(y_t - \mu)(y_{t-1} - \mu) = \sigma_e^2 \alpha_1$$

Autocorrelation function: it tapers off after k lags

$$\rho_k = \frac{\text{cov}\left(y_t y_{t-1}\right)}{\text{var}\left(y_t\right)} = \frac{\alpha_1 \sigma_e^2}{\sigma_e^2 \left(1 + \alpha_1^2\right)}$$

Some examples of MA (1) process:

$$Y_t = \mu + e_t + 0.8 e_{t-1}$$

$$Y_t = \mu + e_t - 0.8 e_{t-1}$$

MA(2) Process

$$Y_t = \mu + e_t + \alpha_1 e_{t-1} + \alpha_2 e_{t-2}$$
$$E(y_t) = \mu$$

$$\text{var}(y_t) = \text{var}(\mu + e_t + \alpha_1 e_{t-1} + \alpha_2 e_{t-2}) = \sigma_e^2\left(1 + \alpha_1^2 + \alpha_2^2\right)$$

$$\text{cov}(Y_t Y_{t-1}) = E(y_t - \mu)(y_{t-1} - \mu) = \sigma_e^2\left(\alpha_1 + \alpha_1 \alpha_2\right)$$

$$\text{cov}(Y_t Y_{t-2}) = E(y_t - \mu)(y_{t-2} - \mu) = \alpha_2 \sigma_e^2$$
$$\text{cov}(Y_t Y_{t-3}) = 0$$

$$\rho_1 = \frac{\text{cov}\left(y_t y_{t-1}\right)}{\text{var}(y_t)} = \frac{\alpha_1\left(1 + \alpha_2\right)}{\left(1 + \alpha_1^2 + \alpha_2^2\right)}$$

$$\rho_2 = \frac{\text{cov}\left(y_t y_{t-2}\right)}{\text{var}(y_t)} = \frac{\alpha_1}{\left(1 + \alpha_1^2 + \alpha_2^2\right)} \quad ; \quad \rho_k = 0$$

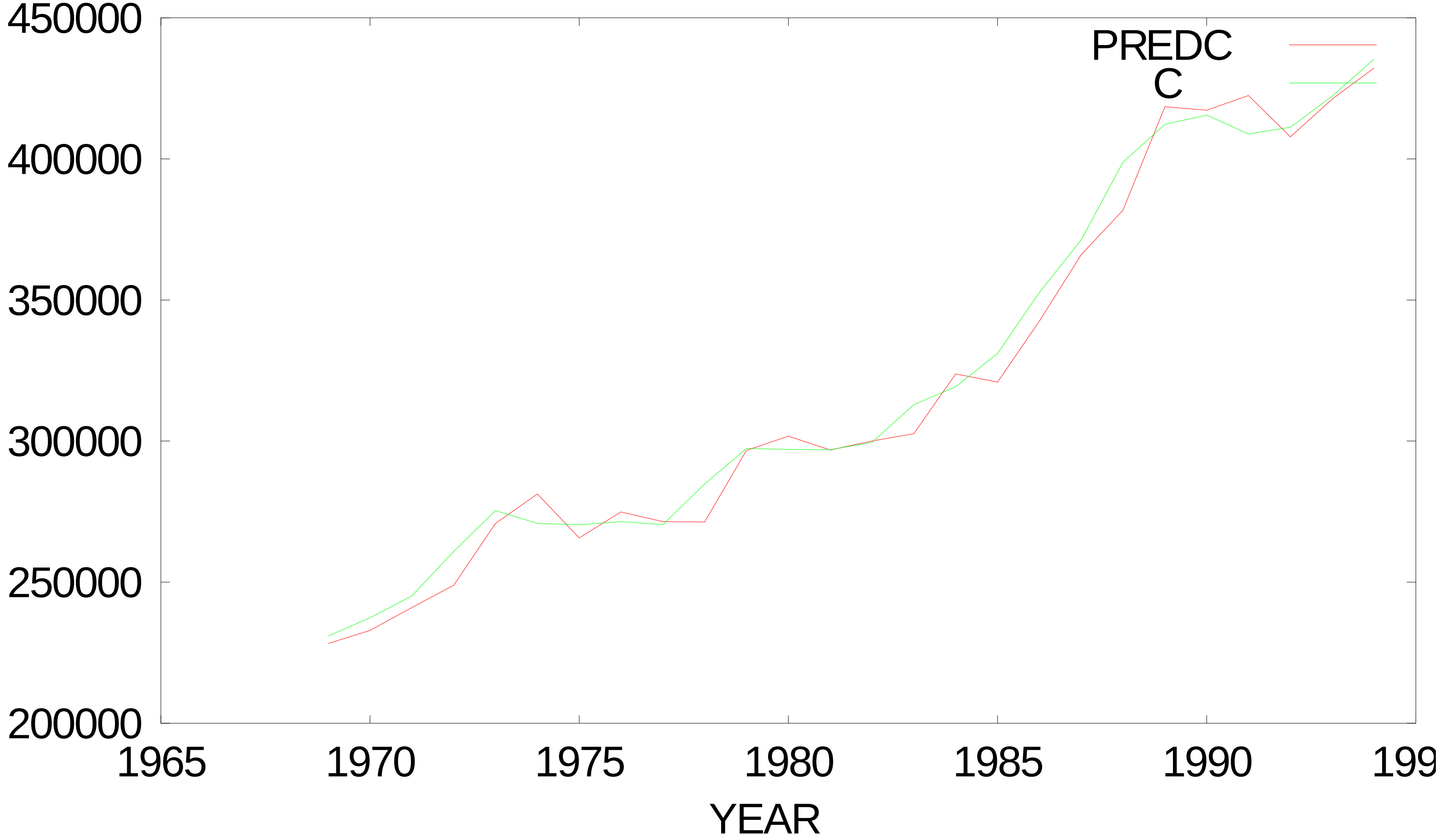
MA(2) process has tow period long memory.

Commands for ARMA (2,2) forecasting model

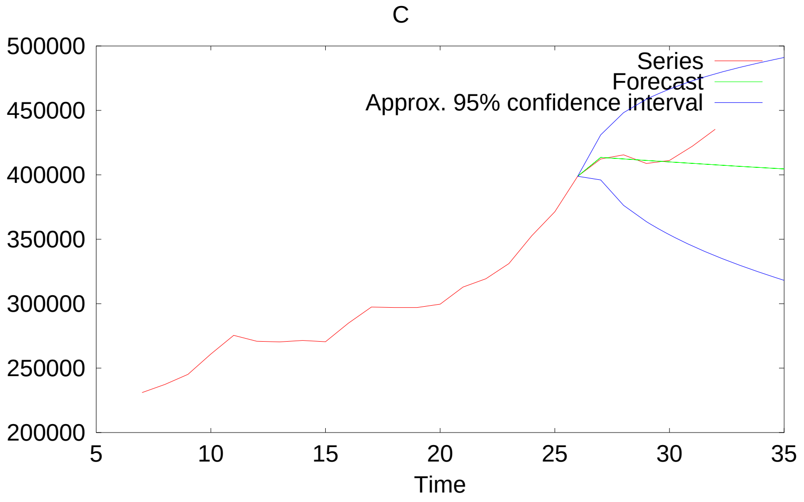
```
Arima C / NAR=2 NMA=2  Predict=predc plotac plotpac acf=cacf

plot predc c year /gnu lineonly

dim alpha 3
gen1 alpha:1=0.5
gen1 alpha:2=-0.2
gen1 alpha:3=100

arima c /NAR=1 NMA=1 coef=beta start=alpha
gen1 S=sqrt($sig2)
arima c/NAR=1 NMA=1 coef=beta fbeg=26 fend=30 sigma=s gnu
arima c/NAR=1 NMA=1 coef=beta fbeg=26 fend=35 sigma=s gnu

arima c/NAR=1 NMA=1 coef=beta fbeg=26 fend=40 sigma=s gnu

arima c/NAR=1 NMA=1 coef=beta fbeg=26 fend=50 sigma=s gnu

arima c/NAR=1 NMA=1 coef=beta fbeg=26 fend=100 sigma=s gnu
```

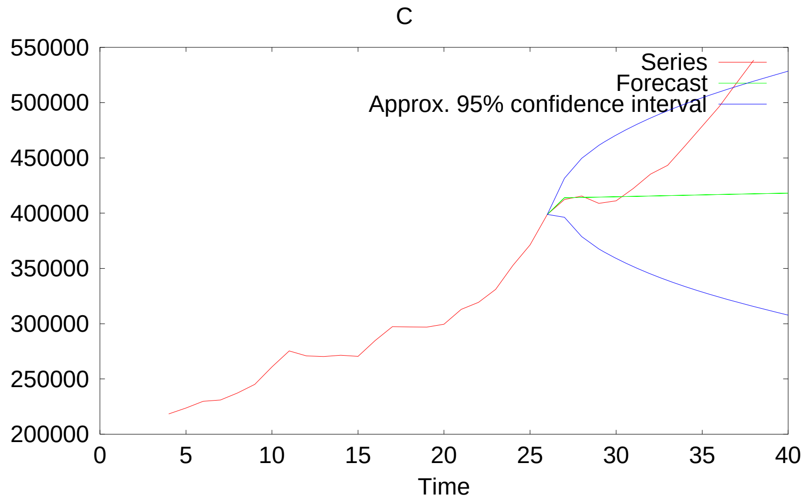
Plot of predected and actual investment series. It is a very good prediction.

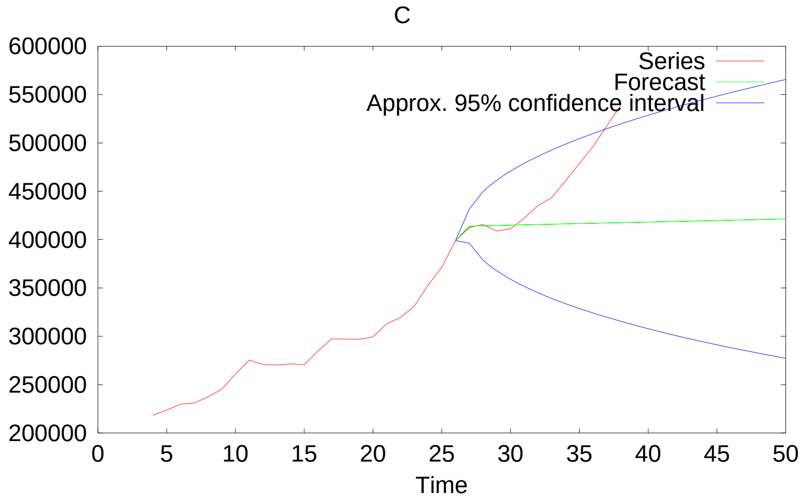
Prediction with an ARIMA Model

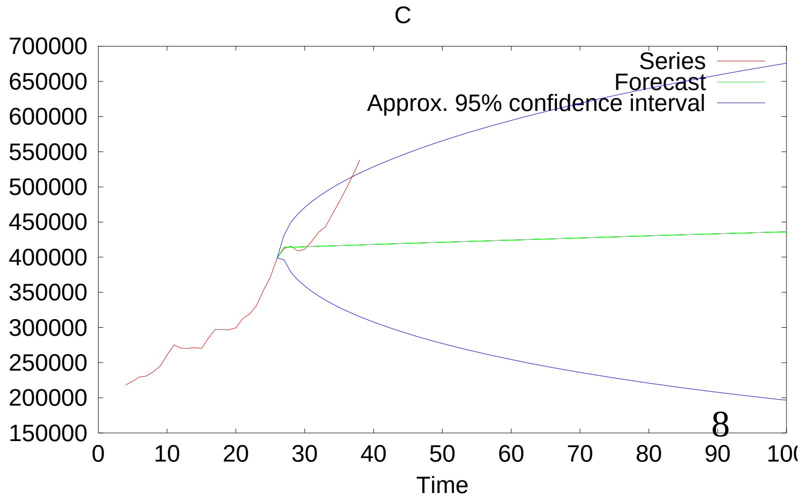


Forecast from an ARIMA Model

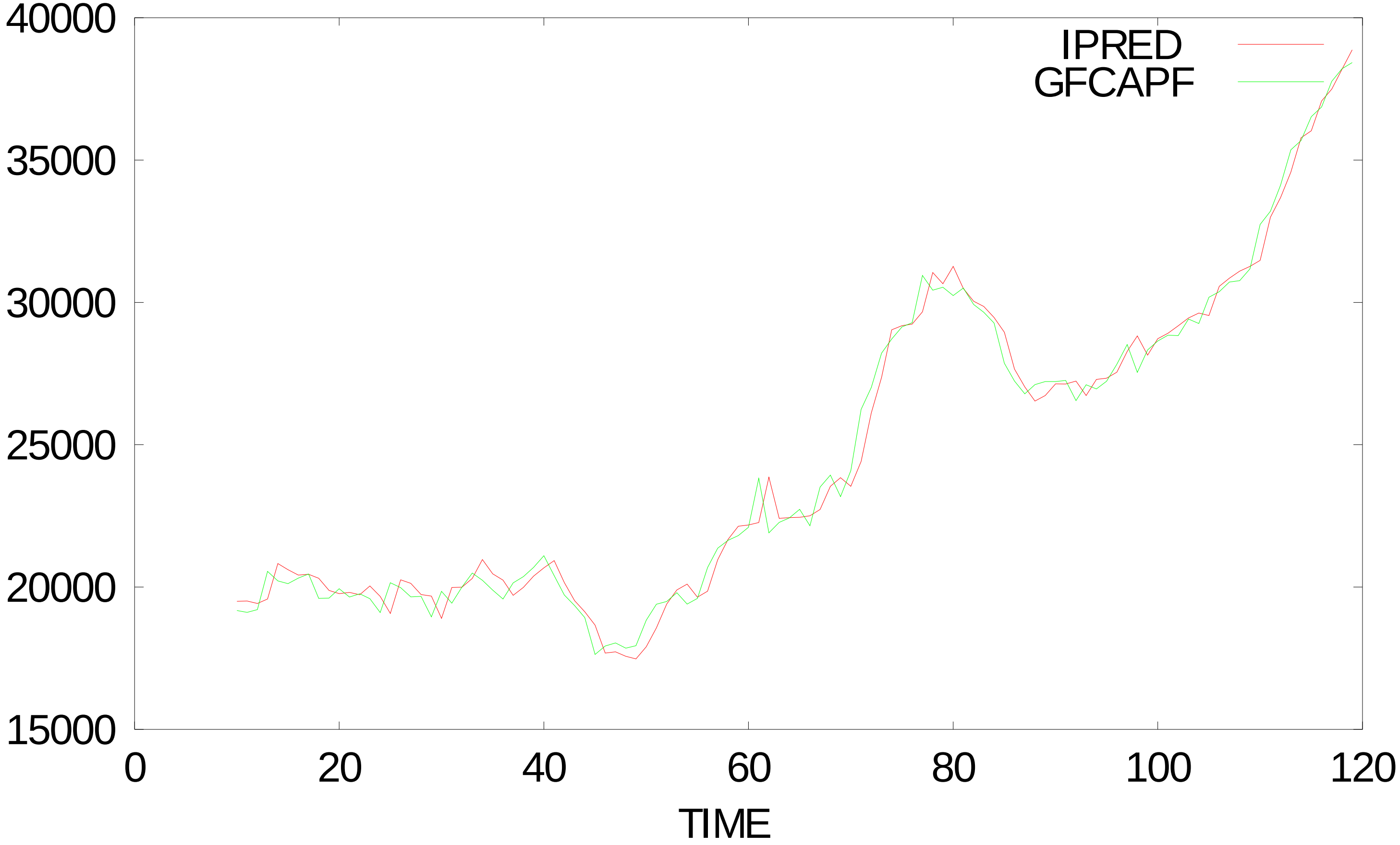




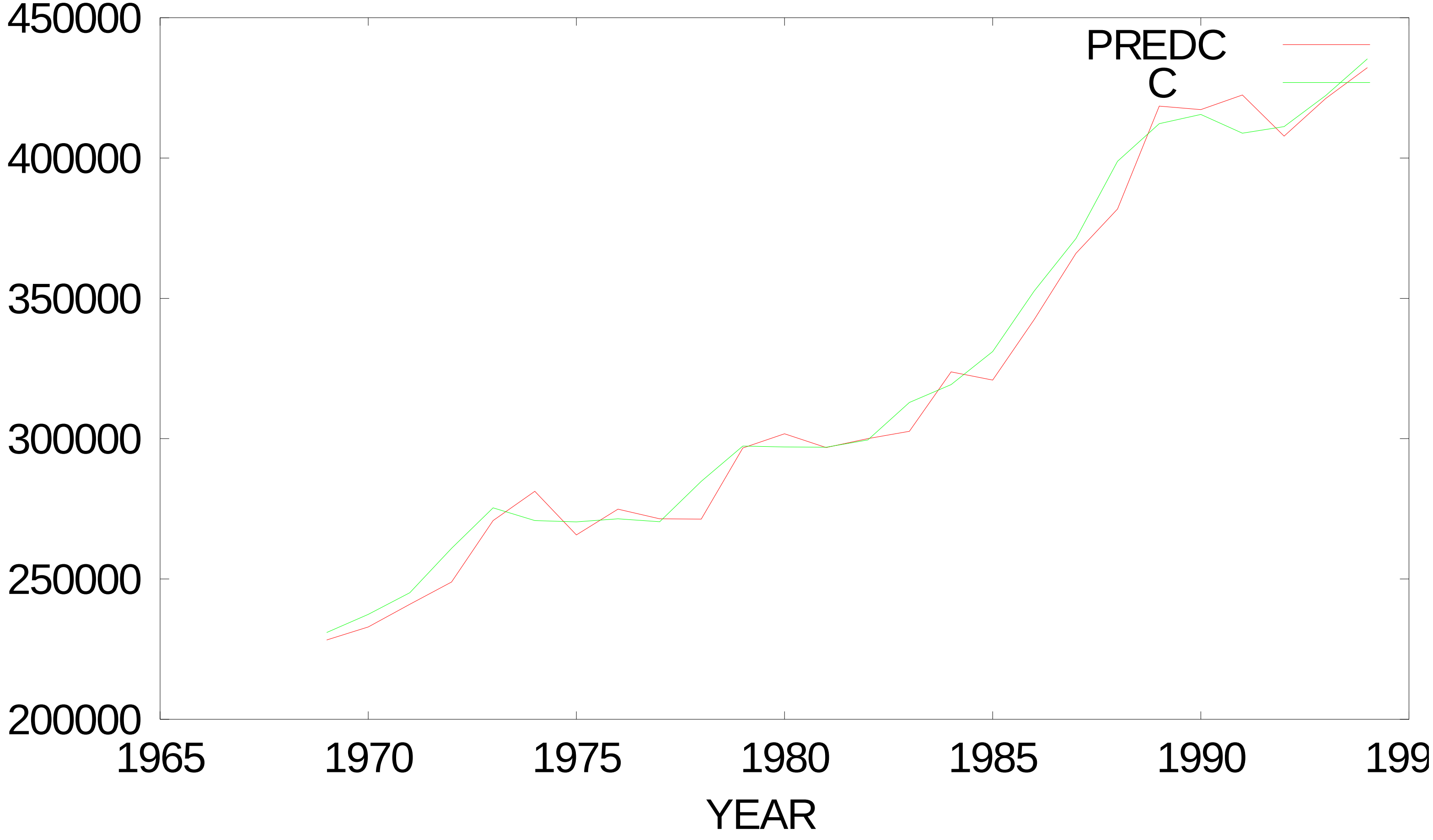




Prediction with AR or MA or ARMA Processes



Prediction with an ARIMA Model



ARIMA of original C series

arima c /all Nlag=12 nlagp=12 plotac plotpac

*ARIMA of log of c

arima clog /all Nlag=12 nlagp=12 plotac plotpac

*Arima C / NAR=2 NMA=2 plotac plotpac Predict=predc acf=cacf gnu
*Arima C / NAR=2 NMA=2 plotac plotpac Predict=predc acf=cacf gnu

*ARIMA of the first difference of c

Arima Cdf1 / Nlag=10 nlagp=14 plotac plotpac

*ARIMA of the second difference of c

arima cdf2/ all plotac plotpac

*ARIMA on log laf second difference of c
*coefficients are not significant for the second differences

arima cllgdf2 /all plotac plotpac

Autocorrelation and Ljung and Box statistics:

Non-Stationary Series

```
arima c /all Nlag=12 nlagp=12 plotac plotpac

IDENTIFICATION SECTION - VARIABLE=C
 NUMBER OF AUTOCORRELATIONS =  12
 NUMBER OF PARTIAL AUTOCORRELATIONS =  12

              0     0 0
 SERIES  (1-B) (1-B  )  C

   NET NUMBER OF OBSERVATIONS =   35
 MEAN=   0.34307E+06   VARIANCE=   0.86418E+10   STANDARD DEV.=    92961.

   LAGS                      AUTOCORRELATIONS                          STD ERR
   1 -12    0.90 0.80 0.71 0.62 0.54 0.47 0.40 0.34 0.28 0.21 0.13 0.05   0.17

  MODIFIED BOX-PIERCE (LJUNG-BOX-PIERCE) STATISTICS  (CHI-SQUARE)
     LAG     Q    DF  P-VALUE       LAG     Q    DF  P-VALUE
       1   30.81   1  .000            7  122.02   7  .000
       2   55.98   2  .000            8  127.54   8  .000
       3   76.29   3  .000            9  131.41   9  .000
       4   92.36   4  .000           10  133.74  10  .000
       5  104.93   5  .000           11  134.70  11  .000
       6  114.66   6  .000           12  134.81  12  .000
```

Autocorrelation and Ljung and Box statistics: Stationary Series

NET NUMBER OF OBSERVATIONS = 35

MEAN= 0.79279E-03 VARIANCE= 0.59320E-03 STANDARD DEV.= 0.24356E-01

LAGS	AUTOCORRELATIONS												STD ERR
1 -12	-.10	-.25	-.07	-.27	0.15	0.06	0.03	-.03	-.21	0.25	0.10	-.26	0.17
13 -24	-.01	-.07	0.20	0.12	-.02	0.01	-.20	0.02	0.11	-.03	-.05	0.01	0.22

MODIFIED BOX-PIERCE (LJUNG-BOX-PIERCE) STATISTICS (CHI-SQUARE)

LAG	Q	DF	P-VALUE	LAG	Q	DF	P-VALUE
1	0.40	1	.527	13	17.02	13	.198
2	2.81	2	.246	14	17.33	14	.239
3	3.00	3	.392	15	20.02	15	.171
4	6.07	4	.194	16	20.95	16	.180
5	7.08	5	.215	17	20.99	17	.227
6	7.23	6	.300	18	21.00	18	.280
7	7.26	7	.402	19	24.22	19	.188
8	7.30	8	.504	20	24.26	20	.231
9	9.40	9	.401	21	25.33	21	.233
10	12.61	10	.246	22	25.40	22	.279
11	13.12	11	.286	23	25.62	23	.319
12	17.02	12	.149	24	25.63	24	.372

Autocorrelation function (AC): Correlogram

All autocorrelation coefficients are significant as Q-statistic for each of them is significant.

```
   LAGS                  PARTIAL AUTOCORRELATIONS                         STD ERR
   1 -12    0.90 -.04 -.02 -.03 -.01 -.01 -.02 0.00 -.05 -.07 -.12 -.11   0.17

                                                     0     0 0
  AUTOCORRELATION FUNCTION OF THE SERIES    (1-B) (1-B   )  C

   1 0.90 .                      +          RRRRRRRRRRRRRRRRRRRRRRRRRRRRRRRR   .
   2 0.80 .              +                  RRRRRRRRRRRRRRRRRRRRRRRRRRRR       .
   3 0.71 .          +                      RRRRRRRRRRRRRRRRRRRRRRRRR          .
   4 0.62 .        +                        RRRRRRRRRRRRRRRRRRRRRR   +         .
   5 0.54 .      +                          RRRRRRRRRRRRRRRRRRR        +       .
   6 0.47 .    +                            RRRRRRRRRRRRRRRRR            +     .
   7 0.40 .   +                             RRRRRRRRRRRRRRR               +    .
   8 0.34 .   +                             RRRRRRRRRRRRR                 +    .
   9 0.28 .  +                              RRRRRRRRRR                     +   .
  10 0.21 .  +                              RRRRRRRR                       +   .
  11 0.13 .  +                              RRRRRR                         +   .
  12 0.05 . +                               RRR                             +  .
```

Forecast and Forecast Errors

FROM ORIGIN DATE 26, FORECASTS ARE CALCULATED UP TO 9 STEPS AHEAD

FUTURE DATE	LOWER	FORECAST	UPPER	ACTUAL	ERROR
27	396219.	413884.	431550.	412276.	-1608.49
28	378792.	414220.	449648.	415557.	1336.98
29	367723.	414555.	461386.	408865.	-5689.65
30	358976.	414888.	470801.	411204.	-3684.39
31	351549.	415221.	478894.	422273.	7051.77
32	345004.	415553.	486103.	435350.	19796.8
33	339103.	415884.	492665.	443367.	27482.8
34	333701.	416214.	498728.	460760.	44545.6
35	328698.	416544.	504389.	478738.	62194.3

STEPS AHEAD	STD ERROR	PSI WT
1	9013.	1.0000
2	0.1808E+05	1.7384
3	0.2389E+05	1.7337
4	0.2853E+05	1.7291
5	0.3249E+05	1.7244
6	0.3599E+05	1.7198
7	0.3917E+05	1.7152
8	0.4210E+05	1.7106
9	0.4482E+05	1.7060