

Discreteness and the Welfare Cost of Labour Supply Tax Distortions

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- 1. Motivation : Article with revise and resubmit status with the International Economic Review (joint work)**
- 2. Issue: Taxes distort prices and affect reallocation of labour and leisure. Evaluation of welfare cost of tax distortion with discrete and continuous labour supply choices in single or two member households**
- 3. Economy: Households and government. Households with CES preferences in consumption and leisure, linear production technology, general equilibrium in product and factor markets. Revenue from sales and income taxes returned to households either on the basis of contribution or in equal per head basis.**

4. Models:

- a. Single member household continuous labour supply model**
- b. Two member household continuous labour supply model**
- c. Heterogeneous 100 single member household continuous labour supply model**
- d. Single member household discrete labour supply model**
- e. Two member household discrete labour supply model**

5. Components of presentation

- a. Graphical illustration**
- b. Algebraic explanation**
- c. Computational techniques /GAMS**

6. Parametric specification and results

7. Conclusion

Discreteness and the Welfare Cost of Labour Supply Tax Distortions

Relevant literature

Rosen (1980)

Killingsworth

(1983) and Hausman (1984),

Blinder and Rosen (1985),

MaCurdy, Green, and Paarsch (1990),

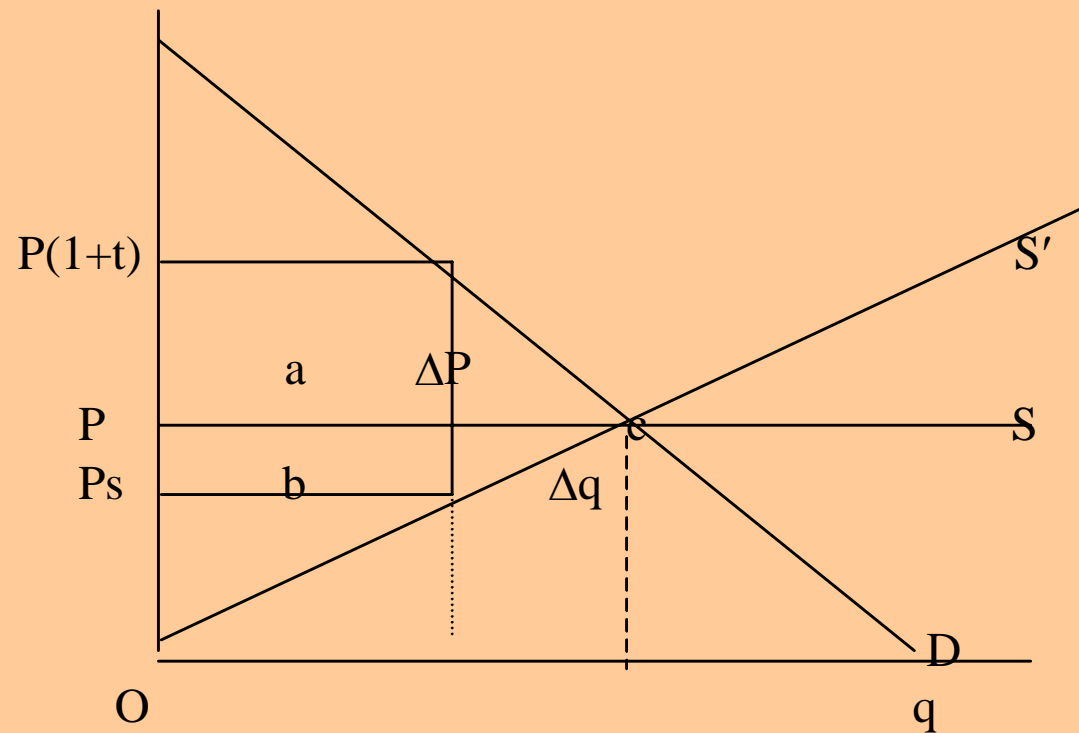
Tummers, M. P. and Isolde W. (1991),

Blundell (1992),

Preston and Walker (1992), Stewart, M.B. and Swaffield J. K. (1996),

Tummers, M. P. and Isolde W. (1991), Blundell R. and T. MaCurdy (2000)

Simple Microeconomic view on Welfare cost of taxes



But this analysis is silent about the implication of resources transferred.

Simple Micro-founded macro model on welfare impact of taxes

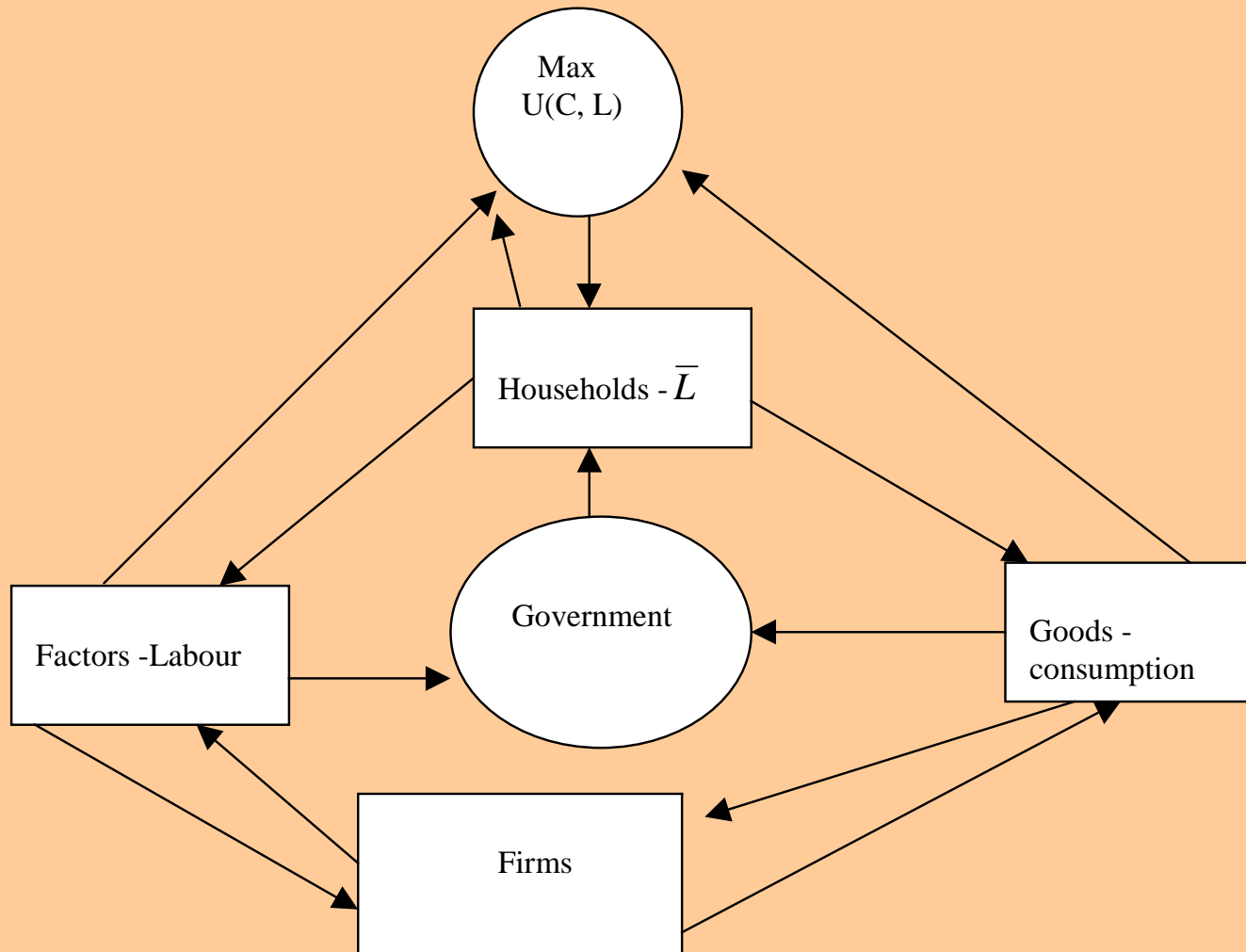


Figure 1

The Effects of an Income Tax on Labour Supply: Continuous Case

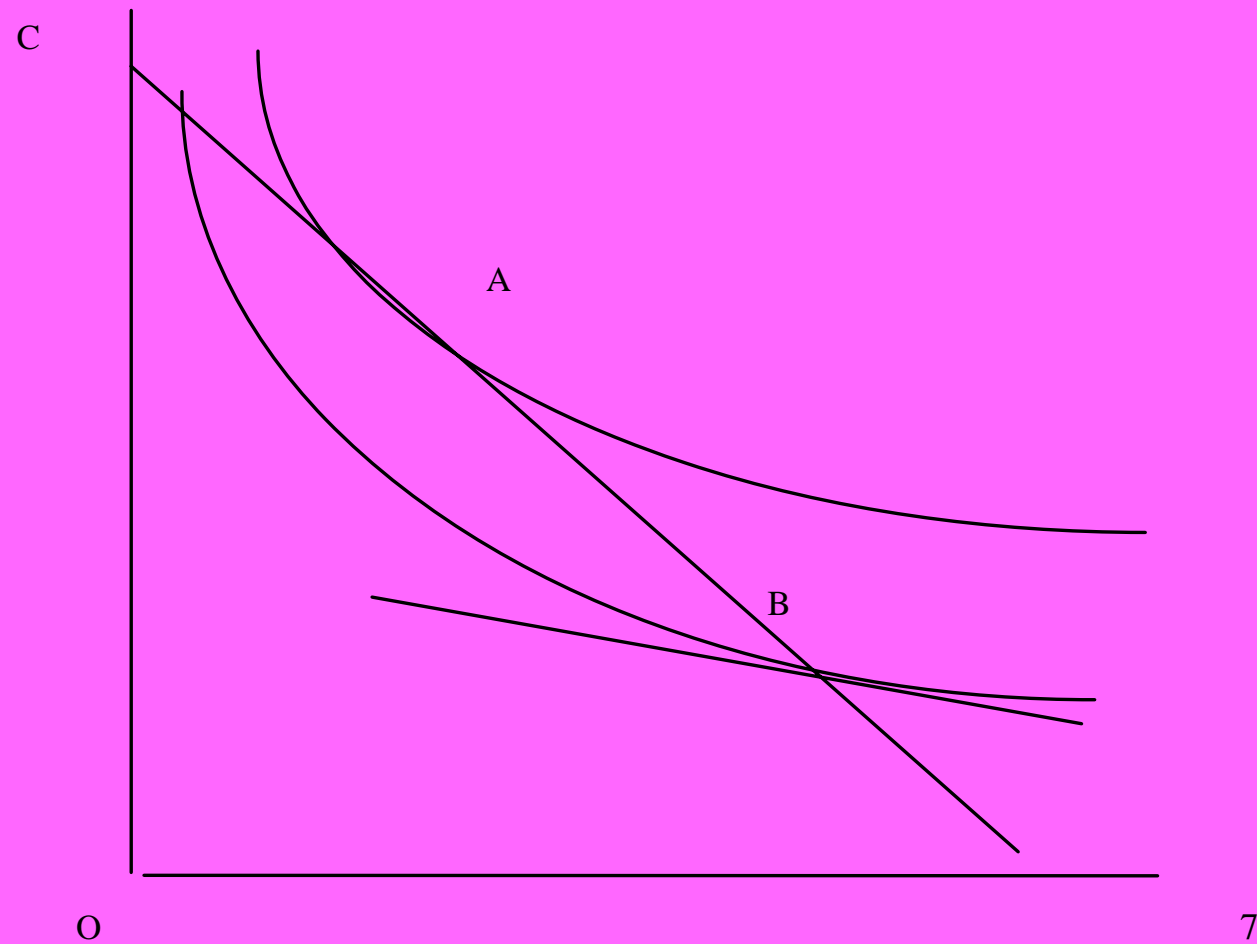
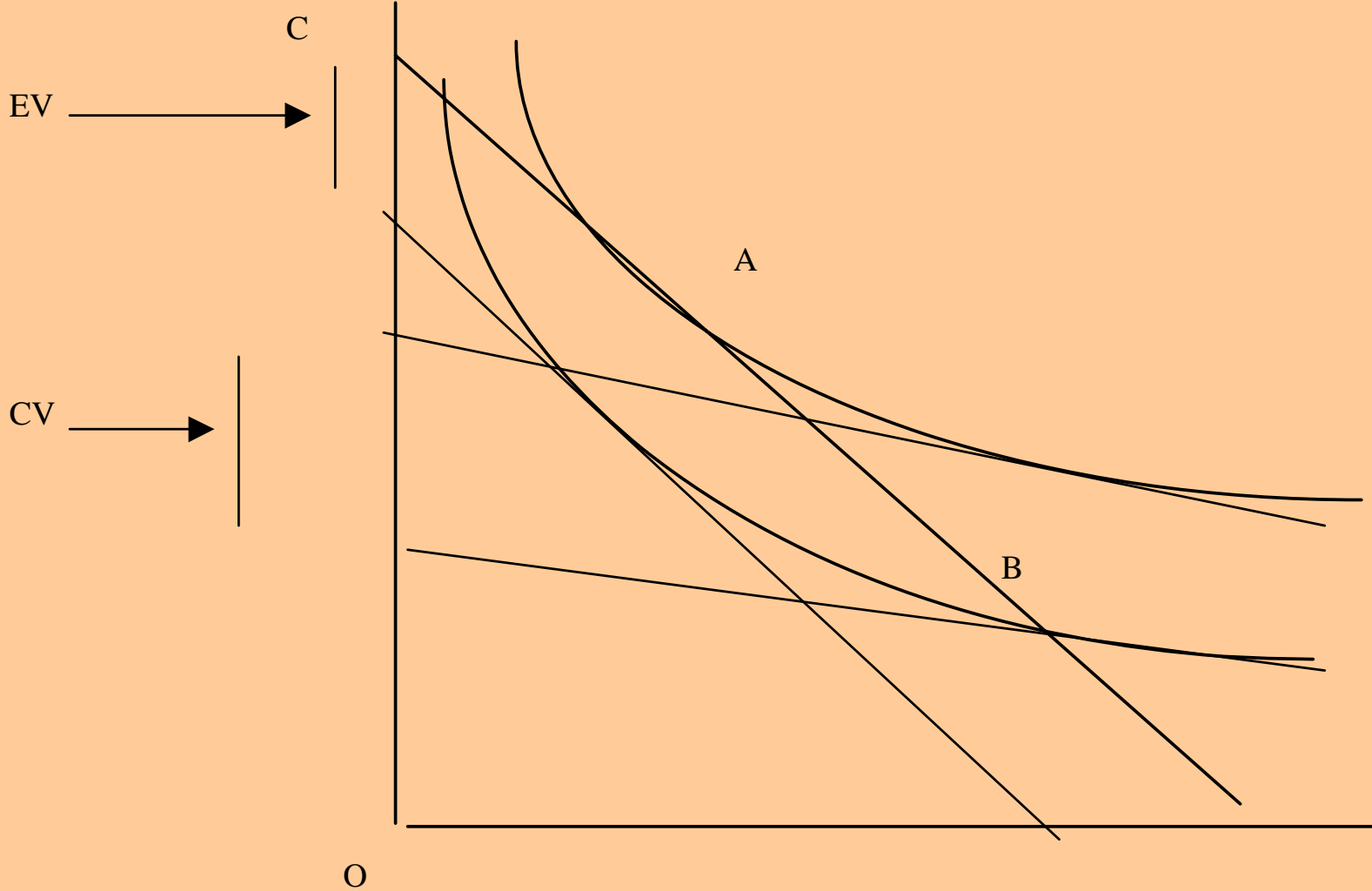


Figure 1.a

Compensating and Equivalent variation in money metric utility after tax changes: Continuous Case



Main Features of the model

Households: Representative and heterogenous cases

Fixed endowment of time

CES preferences on consumption and leisure

Pay income and sales taxes and receive transfers

Utility maximisers

Discrete 0.5-40 hours choice and continuous choice

Firms

Profit maximisers

Linear technology

Pay gross of tax wages

Government

Collects sales and income taxes

**Distributes revenue either to those who pay taxes or
on equity basis**

Markets:

Competitive equilibrium

Goods market clears –through price and quantity adjustment

Labour market clears through labour-leisure choice decision

Assumes a closed economy or global economy

No money illusion

Policy evaluation criteria

Comparative static analysis with equivalent or compensating variation among comparable discrete and continuous models

Efficiency effect of taxes by change in P and W and reallocation C, L, LS, Y, I, R

Welfare cost of taxes in a continuous case

$$\max U(C, L) \quad (1)$$

$$s.t. \quad PC = w(1-t)(\bar{L} - L) + R$$

$$t.w.(\bar{L} - L) = R$$

where C is consumption of goods, L is leisure, \bar{L} is the labour endowment, w is the wage rate,

One way to proceed is to add a tax distorted first order condition

$$\frac{U_c}{U_L} = \frac{P}{w(1-t)}$$

Figure 2

Optimizing Labour Supply Behaviour, No Taxes: Discrete Case

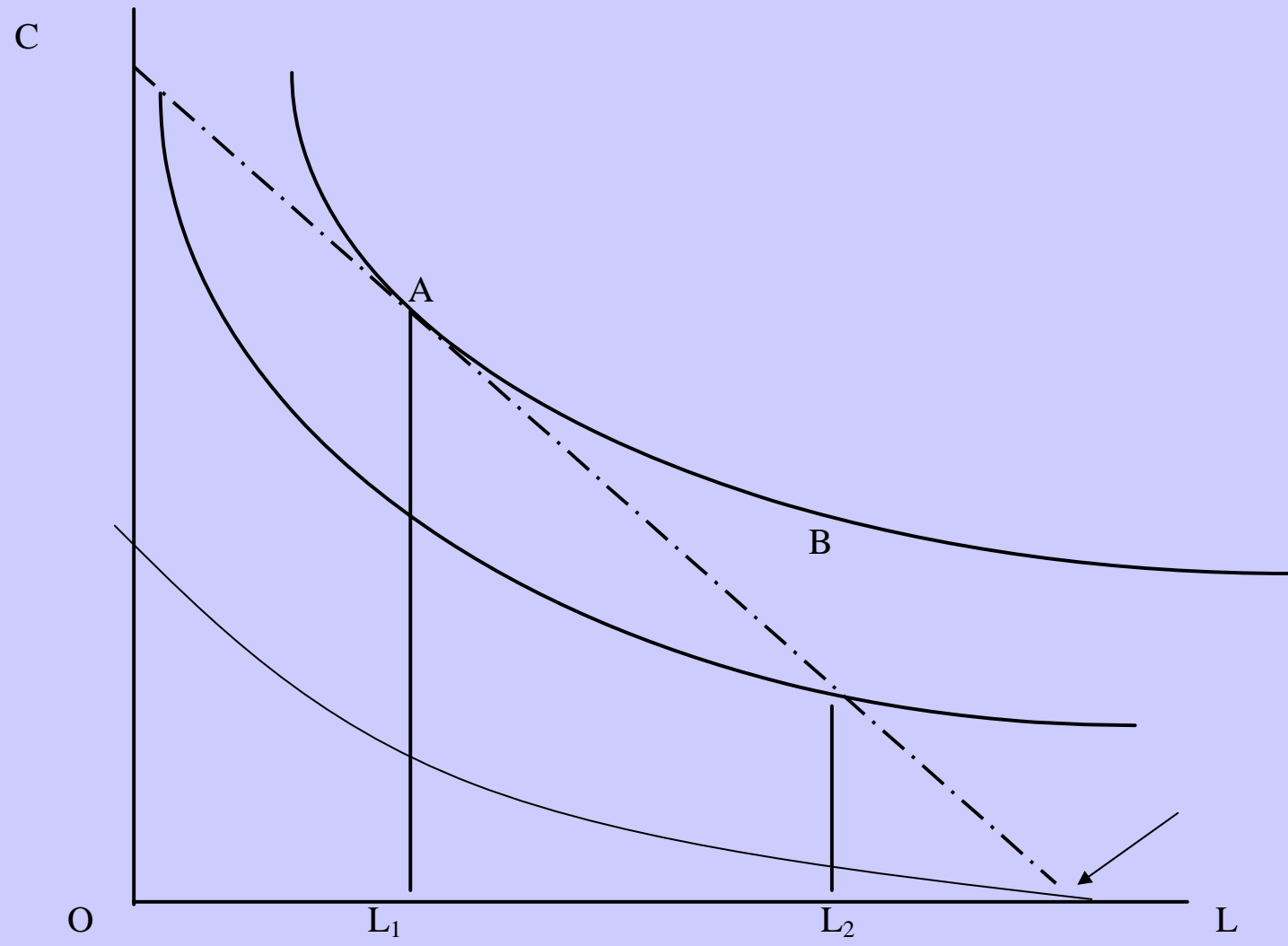


Figure 3

First Discrete Labour Supply Model; with Taxes and Revenue Recycled to Those Who Pay Taxes

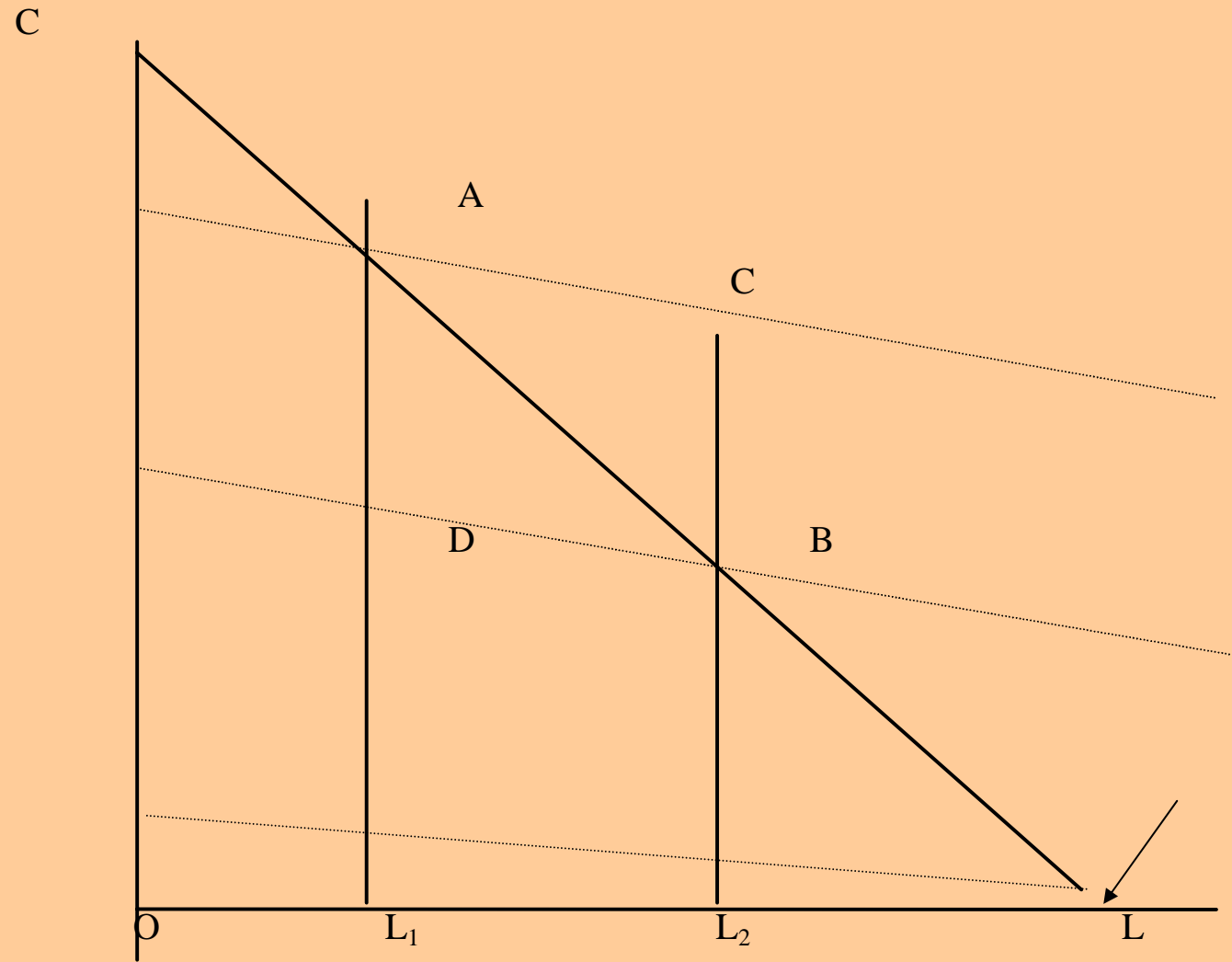
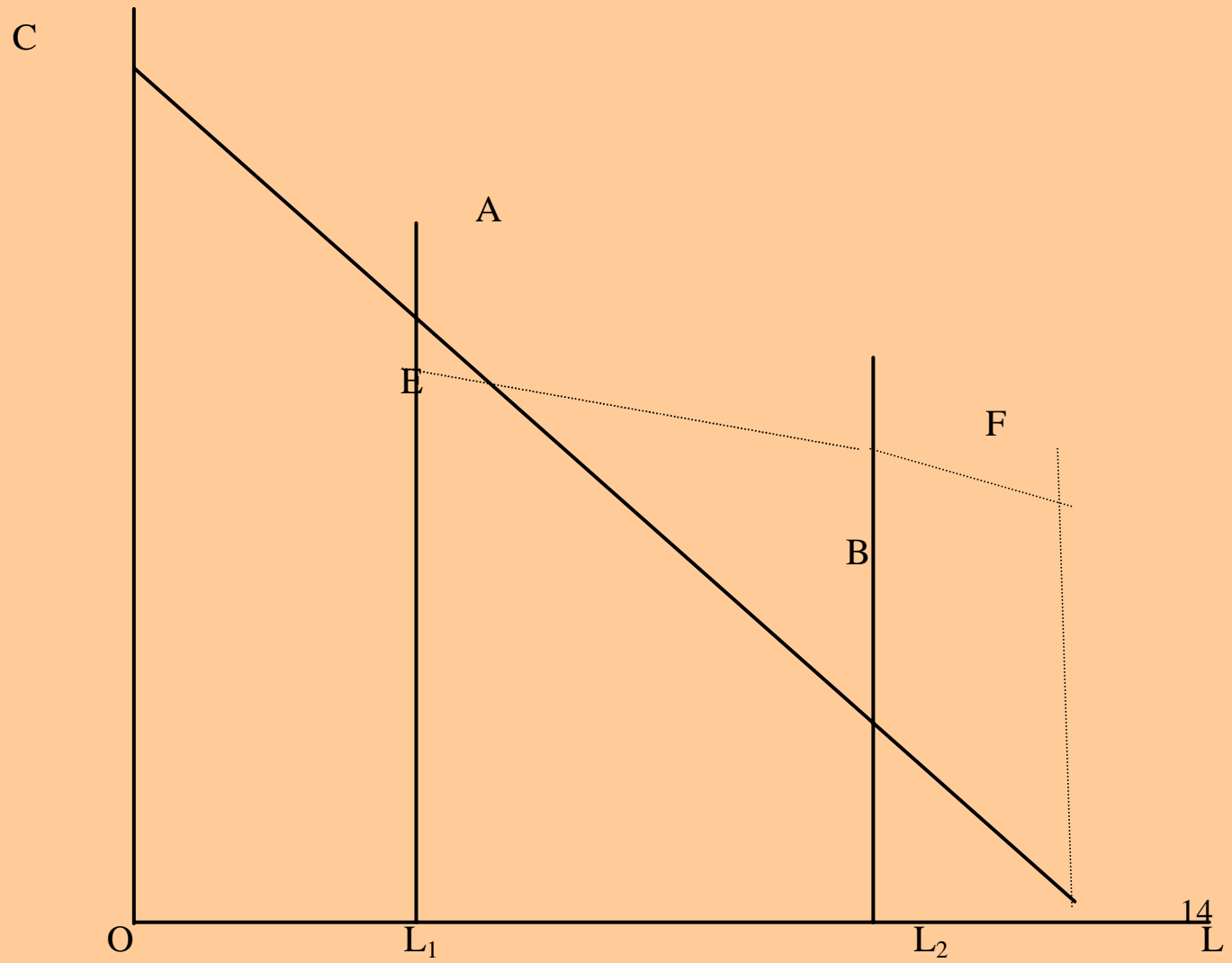


Figure 4

**Discrete Labour Supply Model with Tax Revenues Redistributed
on an Equal Per Capita Basis**



Discrete Labour supply model

$$U_A^h = \left[\delta^h (C_{S,FC}^h)^{\frac{\sigma^h-1}{\sigma^h}} + (1-\delta^h) (L_S^h)^{\frac{\sigma^h-1}{\sigma^h}} \right]^{\frac{\sigma^h}{\sigma^h-1}}$$

$$U_B^h = \left[\delta^h (C_{B,FC}^h)^{\frac{\sigma^h-1}{\sigma^h}} + (1-\delta^h) (L_B^h)^{\frac{\sigma^h-1}{\sigma^h}} \right]^{\frac{\sigma^h}{\sigma^h-1}}$$

$$U_C^h = \left[\delta^h (C_{S,OC}^h)^{\frac{\sigma^h-1}{\sigma^h}} + (1-\delta^h) (L_S^h)^{\frac{\sigma^h-1}{\sigma^h}} \right]^{\frac{\sigma^h}{\sigma^h-1}}$$

$$U_D^h = \left[\delta^h (C_{B,OC}^h)^{\frac{\sigma^h-1}{\sigma^h}} + (1-\delta^h) (L_B^h)^{\frac{\sigma^h-1}{\sigma^h}} \right]^{\frac{\sigma^h}{\sigma^h-1}}$$

Switching Criteria

$$UR^h = \frac{U_C^h}{U_A^h}$$

$$C_{B,OC}^h = C_{S,FC}^h + (L_S^h - L_B^h) \left(\frac{1-t_1}{1+t} \right)$$

Using the utility ratio comparison above, total consumption summed across households (given the linear technology) is

$$C = \sum_{h=1}^N (70 - L_S^h) \text{ if } (U_A^h \geq U_B^h \text{ and } U_C^h \leq U_A^h) + \sum_{h=1}^N (70 - L_B^h) \text{ (if } U_B^h \geq U_A^h \text{ or } U_C^h \geq U_A^h)$$

Tax revenues, R , (given that $p = w = 1$) are

$$R = \sum_{h=1}^N [(p t + t_l w)(70 - L_S^h)] \text{ if } (U_A^h \geq U_B^h \text{ and } U_C^h \leq U_A^h) + \sum_{h=1}^N [(p t + t_l w)(70 - L_B^h)] \text{ if } (U_B^h \geq U_A^h \text{ or } U_C^h \geq U_A^h) \quad (12)$$

$S^h = 1$ if $(U_A^h \geq U_B^h)$ and $(U_C^h \geq U_A^h)$; and $= 0$ otherwise.

The total number of households who change their behaviour as a result of a tax change is then given as

$$T = \sum_{h=1}^N S^h,$$

$$LS = \sum_{h=1}^N C^h$$

$$\Delta LS = (L_B - L_S)T.$$

$$EV^h = \left(\frac{U_B^h - U_A^h}{U_A^h} \right) I_n^h \text{ for those households, } h, \text{ who switch}$$

and

$$CV^h = \left(\frac{U_A^h - U_B^h}{U_B^h} \right) I_0^h \text{ for those households, } h, \text{ who switch}$$

$$AEV = \frac{\sum_{h=1}^N EV^h}{\sum_{h=1}^N I_0^h}$$

and

$$ACV = \frac{\sum_{h=1}^N CV^h}{\sum_{h=1}^N I_n^h}.$$

Continuous Labour Supply model

$$U = \left[\delta C^{\frac{\sigma-1}{\sigma}} + (1-\delta) L^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

These we maximize, subject to the with tax budget constraint

$$P(1+t)C = w(1-t_1)(\bar{L} - L) + R$$

where revenue collected by the government is

$$R = t_1 w(\bar{L} - L) + tPC$$

The first order condition for utility maximisation is

$$\frac{L}{C} = \left[\frac{(1-\delta)P(1+t)}{\delta w(1-t_1)} \right]^{\sigma}$$

We again assume a linear technology with constant marginal product of labour

$$Y = \bar{L} - L,$$

where household labour supply is given by

$$LS = \bar{L} - L.$$

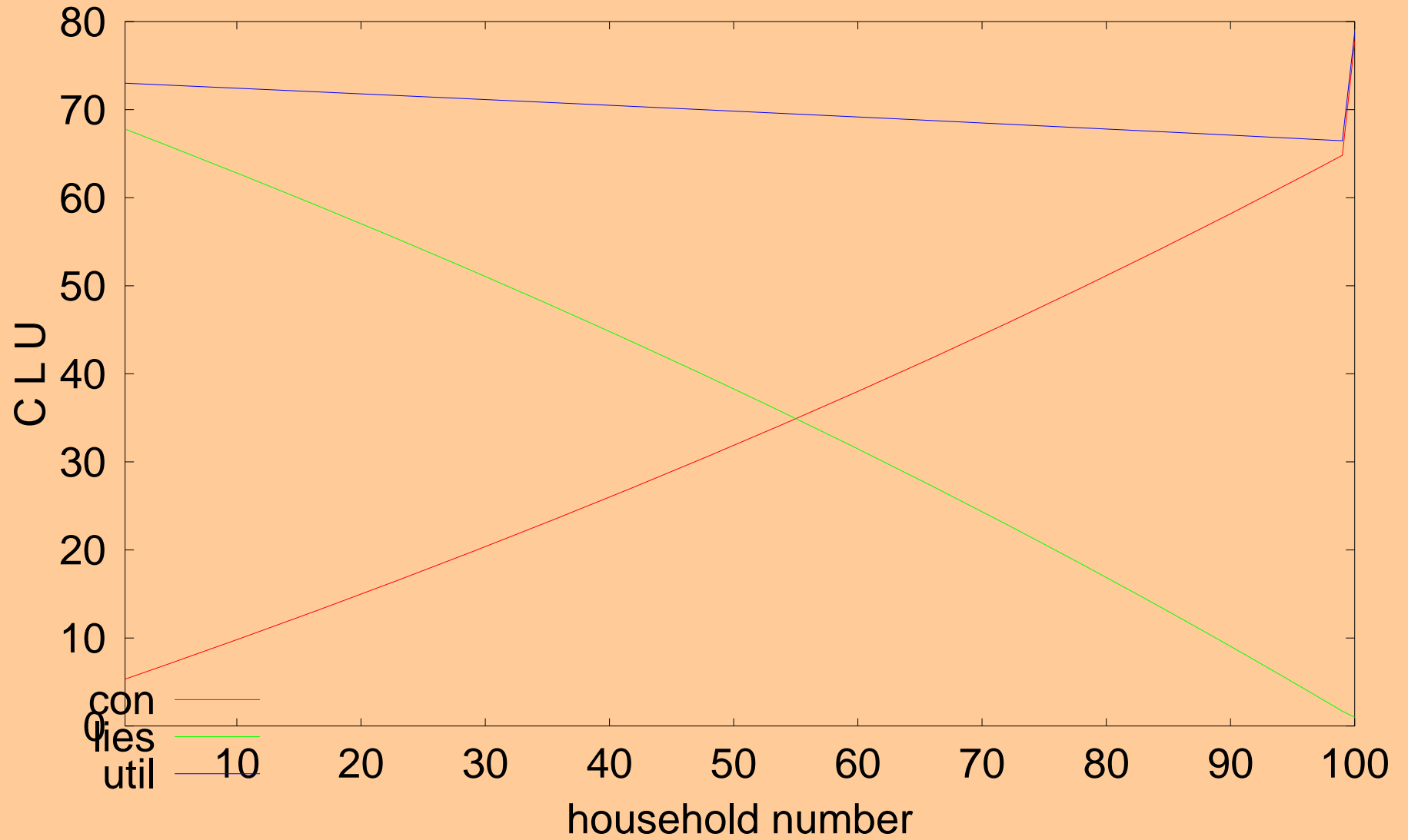
Table 1**Specifications Used in No Tax Base Case for Simple Numerical Examples Showing the Welfare Cost of Taxes in Comparable Discrete and Continuous Models.**

	Continuous Model 1	Discrete Model 1 ¹	Discrete Model 2 ²	Continuous model 2: with heterogeneity
Treatment of tax revenues	Returned to single consumer	Returned to those who pay the tax	Redistributed equally per capita	Redistributed equally per capita
Number of households	1	100	100	100
Labour Endowment in Hours per Week	7000	7000 in aggregate, 70 per household. High and low discrete leisure consumption values for each household are 40 and 1;	7000 in aggregate, 70 per household. High and low discrete leisure consumption values for each household are 40 and 1;	7000 in aggregate, 70 per household.
Share parameter on leisure in utility function, δ	0.5	Uniform distribution across households over the range 0.01 to 0.99	Uniform distribution across households over the range 0.01 to 0.99	Uniform distribution across households over the range 0.1 to 0.91
Net of tax price of consumption goods, P	1.0	1.0	1.0	1
Gross of tax wage, W	1.0	1.0	1.0	1
Point estimate of aggregate uncompensated labour supply elasticity (evaluated at no tax equilibrium)	0.302	0.3	0.305	0.33
Elasticity of substitution in consumption, σ	0.67	0.525	1.5	0.67

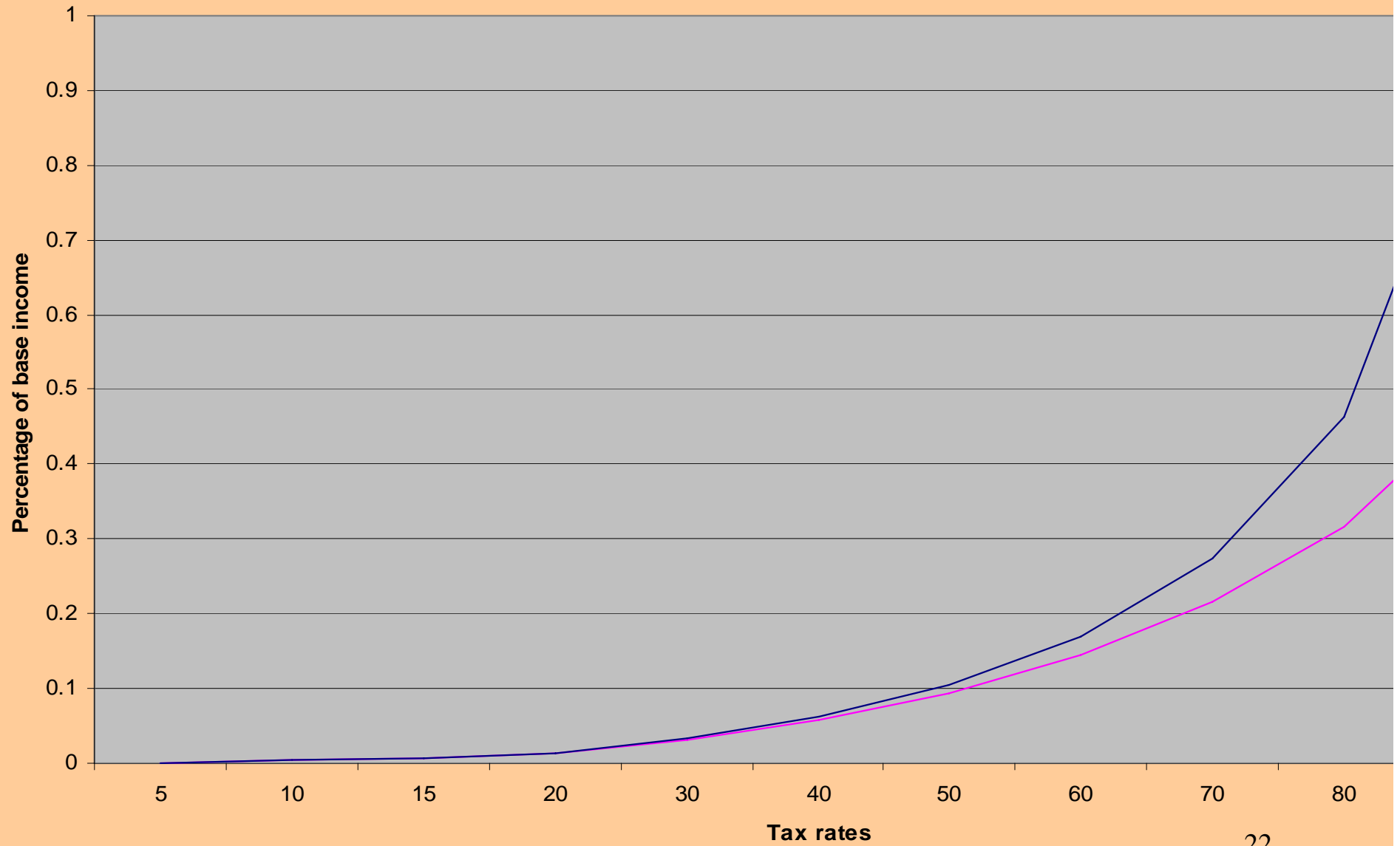
Notes:

¹ This model returns tax revenues to those who pay the tax in lump sum form; as in Figure 3.² This model redistributes tax revenues among households using a fixed proportions distribution scheme; as in Figure 4.³ These are obtained by repeated iteration on various model parameters, and hence the values are only approximately equal across the models rather than exactly so.

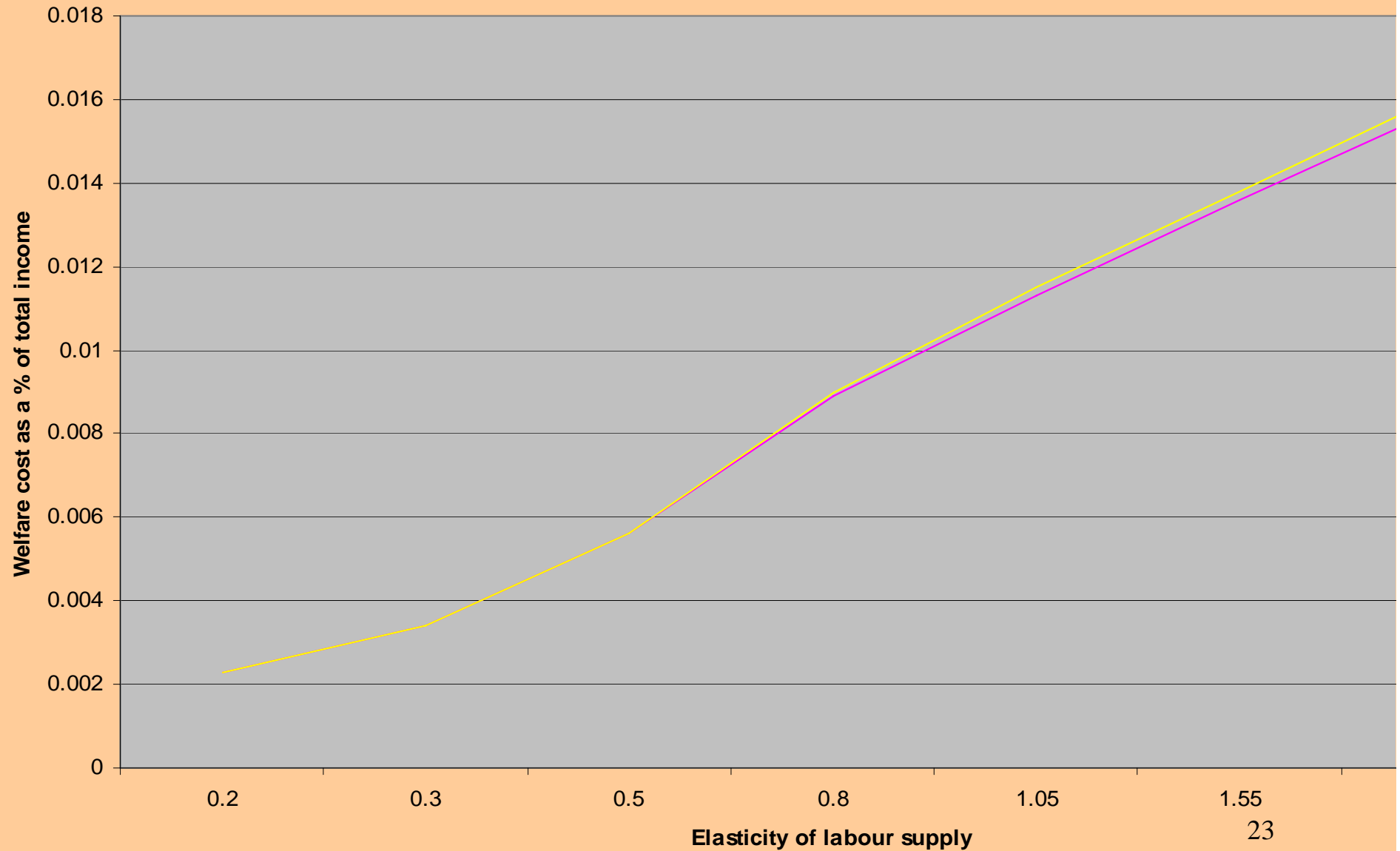
utility consumption and leisure



Welfare cost of income and sales taxes in the continuous labour supply model



Sensitivity of welfare cost to the elasticity of labour supply



Utilities in four different states

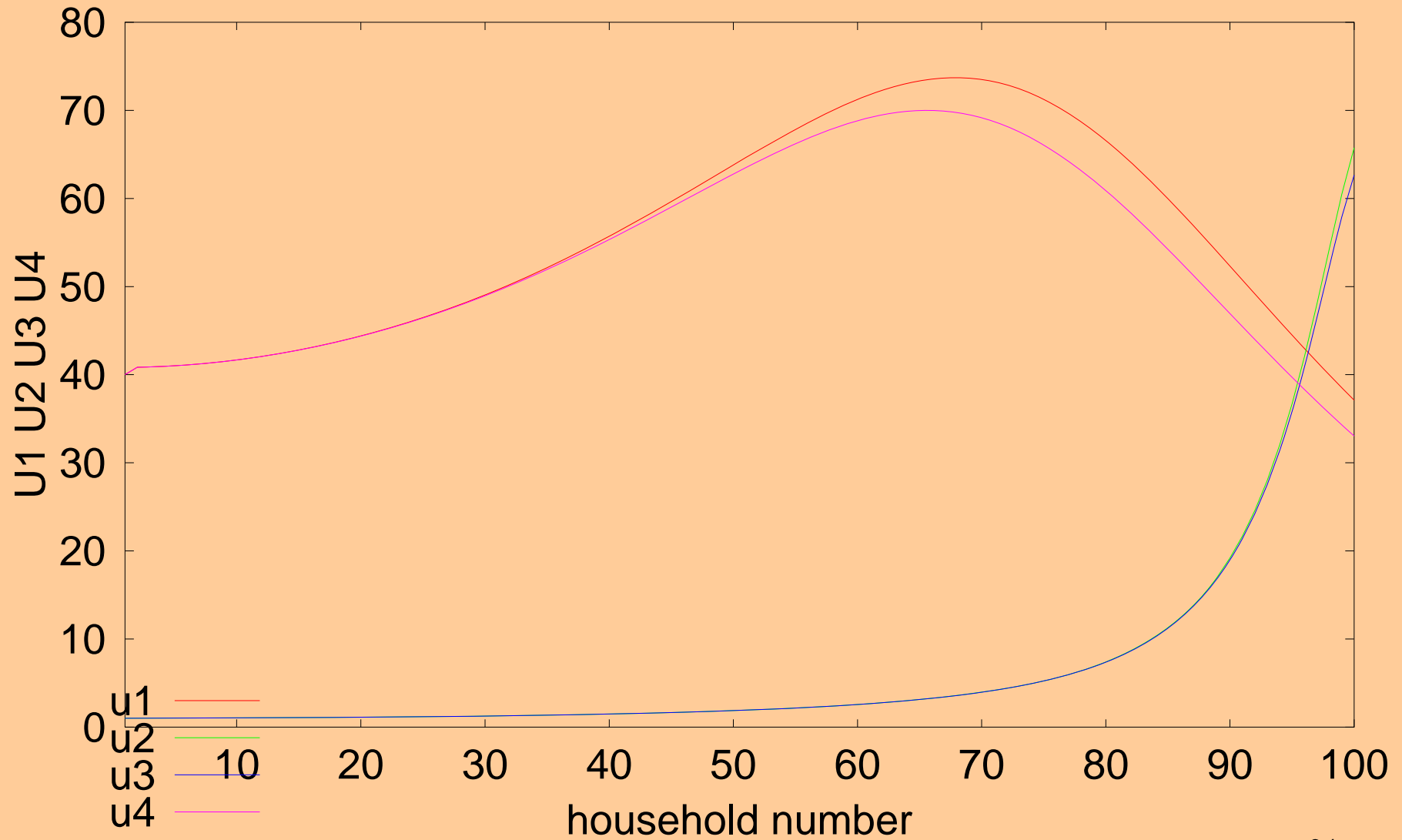


Table 2a

Welfare Cost of Estimates of Different Taxes in the Discrete and Continuous Models Specified in Table 1

	Welfare cost of income and sales taxes in continuous and discrete labour supply models				Welfare cost of income taxes in continuous and discrete labour supply models			
Tax rates	Continuous model 1		Discrete model 1		Continuous model 1		Discrete model 1	
	EV	CV	EV	CV	EV	CV	EV	CV
0.05	-0.0008	0.0008	-0.0009	0.0010	-0.0002	0.0002	-0.0009	0.0000
0.10	-0.0034	0.0034	-0.0009	0.0010	-0.0009	0.0009	-0.0009	0.0010
0.15	-0.0076	0.0077	-0.0032	0.0040	-0.0022	0.0022	-0.0009	0.0010
0.20	-0.0136	0.0138	-0.0032	0.0040	-0.0042	0.0042	-0.0032	0.0010
0.30	-0.0314	0.0324	-0.0066	0.0092	-0.0106	0.0107	-0.0032	0.0040
0.40	-0.0578	0.0613	-0.0066	0.0092	-0.0215	0.022	-0.0066	0.0040
0.50	-0.0946	0.1045	-0.0110	0.0169	-0.0392	0.0408	-0.0110	0.0092
0.60	-0.1451	0.1698	-0.0110	0.0169	-0.0671	0.072	-0.0110	0.0169
0.70	-0.2151	0.274	-0.0160	0.0268	-0.1122	0.1264	-0.0160	0.0169
0.80	-0.3162	0.4624	-0.0160	0.0268	-0.1891	0.2331	-0.0160	0.0268
0.90	-0.4801	0.9236	-0.0160	0.0268	-0.3397	0.5145	-0.0160	0.0268

Notes: Discrete models have 1-40 hours of discrete labour supply choices.

Tax revenue is given back to households.

EVs and CVs are as a proportion of base case aggregate income.

This model involves taxes being returned to those who pay the tax; as in Figure 3.

Table 2.a

Welfare Cost of Estimates of Different Taxes in the Discrete and Continuous Models Specified in Table 1

	Welfare cost of income and sales taxes in continuous and discrete labour supply models				Welfare cost of income taxes in continuous and discrete labour supply models			
Tax rates	Continuous model 1		Discrete model 1		Continuous model 1		Discrete model 1	
	EV	CV	EV	CV	EV	CV	EV	CV
0.05	-0.0008	0.0008	-0.0009	0.0010	-0.0002	0.0002	-0.0009	0.0000
0.10	-0.0034	0.0034	-0.0009	0.0010	-0.0009	0.0009	-0.0009	0.0010
0.15	-0.0076	0.0077	-0.0032	0.0040	-0.0022	0.0022	-0.0009	0.0010
0.20	-0.0136	0.0138	-0.0032	0.0040	-0.0042	0.0042	-0.0032	0.0010
0.30	-0.0314	0.0324	-0.0066	0.0092	-0.0106	0.0107	-0.0032	0.0040
0.40	-0.0578	0.0613	-0.0066	0.0092	-0.0215	0.022	-0.0066	0.0040
0.50	-0.0946	0.1045	-0.0110	0.0169	-0.0392	0.0408	-0.0110	0.0092
0.60	-0.1451	0.1698	-0.0110	0.0169	-0.0671	0.072	-0.0110	0.0169
0.70	-0.2151	0.274	-0.0160	0.0268	-0.1122	0.1264	-0.0160	0.0169
0.80	-0.3162	0.4624	-0.0160	0.0268	-0.1891	0.2331	-0.0160	0.0268
0.90	-0.4801	0.9236	-0.0160	0.0268	-0.3397	0.5145	-0.0160	0.0268

Notes: Discrete models have 1-40 hours of discrete labour supply choices.

Tax revenue is given back to households.

EVs and CVs are as a proportion of base case aggregate income.

This model involves taxes being returned to those who pay the tax; as in Figure 3.

Table 3a

Welfare Costs of 10 percent Income and Sales Taxes under Alternative Values of Labour Supply Elasticities to which both Models are Jointly Calibrated

Elasticity	Welfare cost of Income and sales taxes in continuous and discrete labour supply models				Welfare cost of Income taxes in continuous and discrete labour supply models			
	Continuous 100 household model 1		Discrete two member family model 1		Continuous 100 household model 1		Discrete two member family model 1	
	EV	CV	EV	CV	EV	CV	EV	CV
0.20	-0.0023	0.0023	-0.0015	0.0017	-0.0006	0.0006	-0.0002	0.0002
0.30	-0.0034	0.0034	-0.0032	0.0036	-0.0009	0.0009	-0.0007	0.0007
0.50	-0.0056	0.0056	-0.0030	0.0033	-0.0016	0.0016	-0.0011	0.0011
0.80	-0.0089	0.009	-0.0035	0.0038	-0.0025	0.0025	-0.0016	0.0017
0.105	-0.0113	0.0115	-0.0040	0.0043	-0.0032	0.0032	-0.0014	0.0014
1.550	-0.0136	0.0138	-0.0044	0.0047	-0.0038	0.0039	-0.0018	0.0018
2.050	-0.0158	0.0161	-0.0044	0.0047	-0.0045	0.0045	-0.0018	0.0018

Note: This table gives the sensitivity of welfare costs reported in Table 2.a to various labour supply elasticities.

4. A TWO MEMBER HOUSEHOLD DISCRETE LABOUR CHOICE MODEL

$$U^h = U^h(C^h, L_1^h, L_2^h)$$

regimes IH^h and IL^h , are given by

$$IH^h = w_1 \bar{L}_1^h + w_2 \bar{L}_2^h + RH^h$$

$$IL^h = w_1 \bar{L}_1^h + w_2 \bar{L}_2^h + RL^h$$

$$CH^h = w_1 (\bar{L}_1^h - LH_1^h) + w_2 (\bar{L}_2^h - LH_2^h)$$

$$CL^h = w_1 (\bar{L}_1^h - LL_1^h) + w_2 (\bar{L}_2^h - LL_2^h) \quad (32)$$

$$INH^h = IH^h - w_1 LH_1^h$$

$$INL^h = IL^h - w_1 LL_1^h$$

$$CH_{OC}^h = CL^h + w_1 (LH_1^h - LL_1^h) \left(\frac{1-t_1}{1+t} \right)$$

$$U = \left[\delta_c C^{\frac{\sigma-1}{\sigma}} + \delta_1 L_1^{\frac{\sigma-1}{\sigma}} + \delta_2 L_2^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

Income is derived from the two labour endowments and revenue.

$$I = w_1 (1-t_1)L_1 + w_2 (1-t_2)L_2 + R$$

Revenue

$$R = t PC + t_1 w_1 LS_1 + t_2 w_2 LS_2 \tag{44}$$

Demand functions for goods and leisure are

$$C = \left[\frac{\delta_c}{(1+t)} \right]^\sigma \left[\frac{I}{\delta_c^\sigma (P(1+t))^{1-\sigma} + \delta_1^\sigma (w_1(1-t_1))^{(1-\sigma)} + \delta_2^\sigma (w_2(1-t_2))^{1-\sigma}} \right]$$

$$L_1 = \left[\frac{\delta_1}{w_1 (1+t)} \right]^\sigma \left[\frac{I}{\delta_c^\sigma (P(1+t))^{1-\sigma} + \delta_1^\sigma (w_1(1-t_1))^{(1-\sigma)} + \delta_2^\sigma (w_2(1-t_2))^{1-\sigma}} \right]$$

and,

$$L_2 = \left[\frac{\delta_2}{w_2 (1+t)} \right]^\sigma \left[\frac{I}{\delta_c^\sigma (P(1+t))^{1-\sigma} + \delta_1^\sigma (w_1(1-t_1))^{(1-\sigma)} + \delta_2^\sigma (w_2(1-t_2))^{1-\sigma}} \right]$$

Labour supply of each labour type is

$$LS_1 = \bar{L}_1 - L_1$$

$$LS_2 = \bar{L}_2 - L_2$$

Technology is again assumed to be linear,

$$Y = LS_1 + LS_2$$

Table 4**No Tax Base Case Specification Used in A Numerical Example Showing the Welfare Cost of Taxes in Comparable Discrete and Continuous Two Member Household Models.**

	Continuous Model	Discrete Model Formulation	Continuous heterogeneous model
Number of households	1	100	100
Hour Endowment in hours per week.	14000	14000 in aggregate, 140 per household,	14000 in aggregate, 140 per household,
Hourly consumption of primary worker	Continuous variable	Constrained to high and low discrete values for each household ; $L^H = 40$; $L^L = 1$;	Continuous variable
Hourly consumption of secondary worker	Continuous variable	Continuous variable	Continuous variable
Share parameters, δ , in the utility function	$\delta_0 = 0.462$ $\delta_1 = 0.345$ $\delta_2 = 0.129$	share parameters at both levels of preferences are distributed across the 100 households	share parameters at both levels of preferences are distributed across the 100 households
Price of tax price of consumption goods,	1.0	1.0	1.0
Price of tax wage, W	1.0	1.0	1.0
Point estimates of aggregate uncompensated labour supply elasticities (evaluated at no tax equilibrium)	0.144 ¹ 0.510 ¹	0.146 ¹ 0.517 ¹	0.144 ¹ 0.510 ¹
Elasticity of substitution in consumption, σ	$\sigma = 0.5$ $\sigma_1 = 0.5$	$\sigma = 0.5$ σ , has a distribution of values across households, with mean value 1.025	$\sigma = 0.5$ $\sigma_1 = 0.5$

Note:

¹ Numerical difficulties in calibrating the discrete two labour type model imply that elasticities across the two models are close, but not identical.

Table 5.a

Welfare cost as a proportion of base case income in two member single household continuous labour supply models

Tax rates	Welfare cost of income and sales taxes in single household continuous labour supply models		Welfare cost of income taxes in single household continuous and discrete labour supply models	
	EV	CV	EV	CV
0.05	-0.0010	0.0010	-0.0003	0.0003
0.10	-0.0040	0.0040	-0.0011	0.0011
0.15	-0.0090	0.0091	-0.0026	0.0026
0.20	-0.0161	0.0163	-0.0049	0.0049
0.30	-0.0368	0.0382	-0.0125	0.0126
0.40	-0.0671	0.0719	-0.0253	0.0259
0.50	-0.1247	0.1425	-0.0457	0.0479
0.60	-0.2241	0.2889	-0.0777	0.0843
0.70	-0.3325	0.4981	-0.1273	0.1458
0.80	-0.4574	0.8431	-0.2484	0.3305
0.90	-0.4620	0.8589	-0.3285	0.4893

Notes: Discrete models have 1-40 hours of discrete labour supply choices.

Table 5.b

Welfare Costs of Alternative Tax Distortions of Labour Supply Using the Discrete and Continuous Models Specified in Table 4¹

	Welfare cost of income and sales taxes in two member family 100 household continuous and discrete labour supply models				Welfare cost of income taxes in two member family 100 household continuous and discrete labour supply models			
Tax rates	Continuous model 2		Discrete model 2		Continuous model 2		Discrete model 2	
	EV	CV	EV	CV	EV	CV	EV	CV
0.05	-0.0006	0.0013	-0.0019	0.0019	-0.0002	0.0003	-0.0005	0.0005
0.10	-0.0024	0.0052	-0.0097	0.0110	-0.0007	0.0014	-0.0014	0.0014
0.15	-0.0066	0.0127	-0.0106	0.0108	-0.0016	0.0034	-0.0029	0.0029
0.20	-0.0103	0.0188	-0.0166	0.0170	-0.0031	0.0064	-0.0049	0.0050
0.30	-0.0365	0.0601	-0.0313	0.0321	-0.0098	0.0181	-0.0107	0.0109
0.40	-0.0654	0.107	-0.0485	0.0498	-0.0224	0.0386	-0.0189	0.0196
0.50	-0.1085	0.1823	-0.0674	0.0694	-0.0411	0.0689	-0.0298	0.0316
0.60	-0.1614	0.2854	-0.0878	0.0906	-0.0701	0.1174	-0.0438	0.0475
0.70	-0.2293	0.4389	-0.1090	0.1132	-0.111	0.1905	-0.0616	0.0686
0.80	-0.3173	0.6856	-0.1310	0.1367	-0.1752	0.3193	-0.0840	0.0961
0.90	-0.4106	1.0591	-0.1532	0.1612	-0.2576	0.5298	-0.1122	0.1322

Notes: Discrete models have 1-40 hours of discrete labour supply choices.

Table 5.c

Welfare Costs of Alternative Tax Distortions of Labour Supply Using the Discrete and Continuous Models Specified in Table 4¹

	Welfare cost of income and sales taxes in two member family 100 household continuous and discrete labour supply models				Welfare cost of income taxes in two member family 100 household continuous and discrete labour supply models			
Tax rates	Continuous model 2		Discrete model 2		Continuous model 2		Discrete model 2	
	EV	CV	EV	CV	EV	CV	EV	CV
0.05	-0.0009	0.0010	-0.0021	0.0021	-0.0002	0.0003	-0.0003	0.0003
0.10	-0.0036	0.0040	-0.0082	0.0084	-0.0010	0.0011	-0.0011	0.0011
0.15	-0.0080	0.0090	-0.0183	0.0190	-0.0023	0.0026	-0.0025	0.0026
0.20	-0.0142	0.0162	-0.0323	0.0340	-0.0044	0.0049	-0.0047	0.0049
0.30	-0.0329	0.0380	-0.0343	0.0361	-0.0111	0.0125	-0.0120	0.0125
0.40	-0.1336	0.1630	-0.0690	0.0749	-0.0223	0.0256	-0.0243	0.0257
0.50	-0.1009	0.1222	-0.0880	0.0974	-0.0849	0.0983	-0.0565	0.0598
0.60	-0.1504	0.1912	-0.0815	0.0891	-0.1176	0.1428	-0.0713	0.0764
0.70	-0.2131	0.2913	-0.0815	0.0891	-0.1149	0.1410	-0.0604	0.0637
0.80	-0.2957	0.4518	-0.0815	0.0893	-0.1838	0.2421	-0.0532	0.0565
0.90	-0.4174	0.7779	-0.0816	0.0894	-0.3034	0.4669	-0.0537	0.0573

Notes: Discrete models have 1-40 hours of discrete labour supply choices.

Conclusions

- This paper discusses the influence of discrete labour supply (or leisure) choice for estimates of the welfare costs of taxes on labour supply, issues which appear to be little discussed in the literature.
- We show numerically that aggregate observationally equivalent discrete and continuous models of labour supply behaviour can give substantially different estimates of the welfare cost of equivalent taxes. Welfare costs of tax distortions are much lower when households face discrete choices of working hours in the labour market.
- discrete choice matters, and needs further consideration in the evaluation of the costs of taxes.

1. Summer research programme

a. EAGER project

b. Efficiency and marginal excess burden of taxes

a. dynamic model of the UK economy

b. Determinants of wage with BHPS data

c. Dynamics of income with panel data of BHPS

d. Time series models on macroeconomic fluctuations in the UK economy

e. Benefits from network Services

f. Redistributive effects of transfers

g. General equilibrium model of a developing economy

h. Trade liberalisation and global free trade