Role of Labour Demand Elasticities in Tax Incidence Analysis

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Heterogenous Labour and Impact of Taxes

$$U^h = \left[\alpha^h C^{h\frac{\sigma^h - 1}{\sigma^h}} + (1-\alpha^h) L^{h\frac{\sigma^h - 1}{\sigma^h}} \right]^{\frac{\sigma^h}{\sigma^h - 1}}$$

$$I^h = (1 - t_I^h) w^h \overline{L}^h + RH^h$$

$$C^h = \left[\frac{\alpha^h}{P(1+t_C^h)} \right]^{\sigma^h} \left[\frac{I^h}{\alpha^h (P(1+t_C^h))^{1-\sigma^h} + (1-\alpha^h)(w^h(1-t_I^h))^{1-\sigma^h}} \right]$$

$$L^h = \left[\frac{(1-\alpha^h)}{w^h(1-t_I^h)} \right]^{\sigma^h} \left[\frac{I^h}{\alpha^h (P(1+t_C^h))^{1-\sigma^h} + (1-\alpha^h)(w^h(1-t_I^h))^{1-\sigma^h}} \right]$$

Income and Production in Heterogeneous Labour Model

$$I^h = P(1 + t_c^h)C^h + w^h(1 - t_l^h)L^h$$

$$LS^h = \overline{L}^h - L^h$$

$$Y = \lambda\left(\sum_h \delta^h LS^{h\frac{\sigma_p - 1}{\sigma_p}}\right)^{\frac{\sigma_p}{\sigma_p - 1}}$$

$$\Pi = PY - \sum_h w^h LS^h$$

$$LD^h = \frac{1}{\lambda}\left(\delta^h + \sum_{hh} \delta^{hh}\left(\frac{\prod_{hh \neq h} w^{hh}}{w^h}\right)^{\sigma_{p-1}}\right)^{\frac{\sigma_p}{\sigma_p - 1}}$$

Revenue and Market Clearing Conditions

$$R = \sum_h t_I^h w^h LS^h + \sum_h t_c^h PC^h$$

$$Y = \sum_h C^h$$

$$LS^h = LD^h$$

$$R = \sum_h RH^h$$

Homogenous labour case

$$Y = \lambda \sum_h LS^h$$

$$P = \frac{w}{\lambda}$$

Table1

Model admissible household data set by deciles of income for non-retired households, UK 1994/95[1]

Households[2]	(a) Gross of tax labor income[a]	(b) Transfers[a]	(c) Income and other direct taxes paid[a,][3]	(d) Indirect taxes paid[a,][4]	(e) Consumption gross of indirect taxes [a,][5]	(f) Leisure [a,][6]	(g) Income tax rate[7]	(h) Indirect tax rate[8]
Decile 1 (Poor)	3079	14638	930	2139	16787	14491	0.05	0.13
Decile 2	5918	12880	1194	2183	17604	17085	0.06	0.12
Decile 3	11021	11040	1880	2759	20181	14076	0.09	0.14
Decile 4	16111	9867	2815	3213	23163	11934	0.11	0.14
Decile 5	21184	8559	3831	3572	25912	10895	0.13	0.14
Decile 6	26161	8205	4745	3957	29621	10455	0.14	0.13
Decile 7	30140	6858	5531	4266	31467	12293	0.15	0.14
Decile 8	34614	5645	6576	4362	33683	13895	0.16	0.13
Decile 9	41918	5166	8175	4537	38909	15449	0.17	0.12
Decile 10 (Rich)	71147	4440	15082	5551	60505	6871	0.20	0.09

Notes: All figures in this table noted with superscript **a** are millions of £, for the tax year 1994/95.

[1] See Appendix 1 for more detail.
[2] These households are grouped by “original” household income as in Economic Trends (1995). Original income is pre tax / pre transfer income.
[3] This includes all social insurance contributions.
[4] This includes VAT and all excises (especially on petrol, tobacco, drink).
[5] This is gross of indirect taxes.
[6] This is from UK time use survey data; leisure time is valued at the net of tax wage.
[7] This includes income tax and social insurance contributions.
[8] This includes the VAT plus specific excise taxes.

$$e_{LD}^{h} = \frac{\partial LD^{h}}{\partial w^{h}} \frac{w^{h}}{LD^{h}}$$

$$e_{LD}^{h} = \frac{-\sigma_{p} \sum_{hh \neq h} \delta^{hh} \left(\prod_{h \neq h} w^{hh} \right)^{\sigma_{p}-1}}{w^{h^{\sigma_{p}-1}} \left[\delta^{h} + \sum_{hh} \delta^{hh} \left(\frac{\prod_{h \neq h} w^{hh}}{w^{h}} \right)^{\sigma_{p}-1} \right]}$$

$$\frac{\partial LS^{h}}{\partial w^{h}} \frac{w^{h}}{LS^{h}} = \frac{\partial LS^{h}}{\partial L^{h}} \frac{\partial L^{h}}{\partial w^{h}} \frac{w^{h}}{L^{h}} \frac{L^{h}}{LS^{h}} = (-1)\eta_{LE} \frac{L^{h}}{LS^{h}}$$

$$\eta_{LE}^{h} = -\left(\sigma^{h} + \frac{(\sigma^{h}-1)(1-\alpha^{h}) w^{h^{1-\sigma^{h}}}}{\alpha^{h} P^{1-\sigma^{h}} + (1-\alpha^{h}) w^{h^{1-\sigma^{h}}}} \right)$$

$$e_{LS}^{h} = \left(\sigma^{h} + \frac{(\sigma_{p} - 1)(1-\alpha^{h}) w^{h^{1-\sigma^{h}}}}{\alpha^{h} P^{1-\sigma^{h}} + (1-\alpha^{h}) w^{h^{1-\sigma^{h}}}} \right) \frac{L^{h}}{LS^{h}}$$

$$e_{LS}^{h} = \left(\sigma^{h} + (\sigma_{p} - 1)(1-\alpha^{h}) w^{h^{1-\sigma^{h}}} \right) \frac{L^{h}}{LS^{h}}$$

$$\left(1-\alpha^{h}\right)$$

$$e_{LS}^{h} \approx \sigma^{h} \frac{L^{h}}{LS^{h}}$$

$$EV = E(U^{N}, P^{0}) - E(U^{0}, P^{0})$$

$$EV = \frac{U^{N} - U^{0}}{U^{0}} I^{0}$$

Table 2

Model production and consumption side elasticities, and literature justification

A. Range of labour supply elasticities based on those reported in Killingsworth (1983)

Range of values	Labour supply elasticity assumed for each household	Range of elasticities of substitution in consumption implied for household deciles
High	1.0	0.52-10.5
Mid (central case)	0.3	0.38-3.50
Low	0.15	0.32-1.57

B. Range of labour demand elasticities based on those reported in Hamermesh (1993)

Range	Range of labour demand elasticities by decile	Elasticity of substitution used in production
High	-1.81 to -2.10	1.93
Mid Range (central case)	-1.05 to -1.24	1.32
Low	-0.58 to -0.67	0.71

Welfare gains/losses by decile in terms of Hicksian EV as a fraction of base income (with low labour supply elasticity (0.3))

Decile	Homogeneous Labour model	Heterogeneous Labour model Labour demand elasticities ranges as specified in Table 2 Low (-0.58 to -0.67)	Middle (-1.05 to -1.24)	High (-1.8 to-2.1)
1 poor	-0.0581	-0.0134	-0.0501	-0.0591
2	-0.0485	-0.0173	-0.0434	-0.0493
3	-0.0424	-0.0334	-0.0401	-0.0430
4	-0.0299	-0.0306	-0.0296	-0.0303
5	-0.0143	-0.0211	-0.0153	-0.0146
6	-0.0064	-0.0159	-0.0078	-0.0066
7	0.0044	-0.0059	0.0028	0.0042
8	0.0173	0.0060	0.0155	0.0171
9	0.0272	0.0154	0.0253	0.0270
10 rich	0.0670	0.0562	0.0660	0.0666

Welfare gains/losses by decile in terms of Hicksian EV as a fraction of base income (with low labour supply elasticity (1.0))

Decile	Homogeneous Labour model	Heterogeneous Labour model Labour demand elasticities ranges as specified in Table 2 Low (-0.58 to -0.67)	Middle (-1.05 to -1.24)	High (-1.8 to-2.1)
1 poor	-0.0577	-0.0376	-0.0503	-0.0585
2	-0.0481	-0.0709	-0.0442	-0.0486
3	-0.042	-0.2191	-0.0420	-0.0421
4	-0.0295	-0.2944	-0.0312	-0.0294
5	-0.014	-0.3300	-0.0150	-0.0140
6	-0.0061	-0.1114	-0.0060	-0.0061
7	0.0047	0.3908	0.0066	0.0045
8	0.0176	0.0739	0.0221	0.0172
9	0.0276	0.1476	0.0349	0.0270
10 rich	0.0676	-0.0025	0.0737	0.0670

Welfare gains/losses by households, Hicksian EV as a fraction of base income

Decile	Labour supply elasticity (0.15)		Labour supply elasticity (0.3)		Labour supply elasticity (1.0)		
	Homogeneous labour Model	Heterogeneous labour model	Homogeneous labour Model	Heterogeneous labour model	Homogeneous labour Model	Heterogeneous labour model	
1 poor	-0.0582	-0.0500	-0.0581	-0.0501	-0.0577	-0.0585	
2	-0.0486	-0.0431	-0.0485	-0.0434	-0.0481	-0.0486	
3	-0.0426	-0.0395	-0.0424	-0.0401	-0.0420	-0.0421	
4	-0.0300	-0.0290	-0.0299	-0.0296	-0.0295	-0.0294	
5	-0.0144	-0.0149	-0.0143	-0.0153	-0.0140	-0.0140	
6	-0.0065	-0.0075	-0.0064	-0.0078	-0.0061	-0.0061	
7	0.0043	0.0028	0.0044	0.0028	0.0047	0.0045	
8	0.0172	0.0150	0.0173	0.0155	0.0176	0.0172	
9	0.0270	0.0244	0.0272	0.0253	0.0276	0.0270	
10 rich	0.0668	0.0650	0.0670	0.0660	0.0676	0.0670	

Welfare gains/losses by households, Hicksian EV as a fraction of base income

	Only income tax		Income and sales tax		Only sales tax	
Decile	Homogeneous labour Model	Heterogeneous labour model	Homogeneous labour Model	Heterogeneous labour model	Homogeneous labour Model	Heterogeneous labour model
1 poor	-0.0581	-0.0501	-0.0551	-0.0489	0.0029	0.0026
2	-0.0485	-0.0434	-0.0478	-0.0434	0.0006	0.0006
3	-0.0424	-0.0401	-0.0324	-0.0307	0.0101	0.0096
4	-0.0299	-0.0296	-0.0167	-0.0166	0.0132	0.0129
5	-0.0143	-0.0153	-0.0008	-0.0014	0.0134	0.0134
6	-0.0064	-0.0078	0.0039	0.0032	0.0100	0.0102
7	0.0044	0.0028	0.0164	0.0155	0.0116	0.0118
8	0.0173	0.0155	0.0235	0.0225	0.0058	0.0061
9	0.0272	0.0253	0.0217	0.0208	-0.0057	-0.0054
10 rich	0.0670	0.0660	0.0314	0.0307	-0.0337	-0.0338

Appendix 2
Solution Method of the Model

Both homogeneous and heterogeneous labour models discussed in this paper are set up as a mixed complementarity problems and solved in GAMS software using the PATH solver.

Dirkse and Ferris (1995) state the basic idea behind the PATH solver in terms of a "zero finding problem". For any function F: $\Re^n \rightarrow \Re^n$ with lower bound $-\infty \leq l$ and an upper bound $\leq u \leq +\infty$ the problem *is to* find $z \in \Re^n$ such that

$$\begin{array}{llll} \text{either} & z_i = l_i & \text{and} & F_i(z) \geq 0 \\ \text{or} & z_i = l_i & \text{and} & F_i(z) \leq 0 \\ \text{or} & l_i \leq z_i \leq u_I & \text{and} & Fi(z) = 0 \end{array}$$

PATH constructs a solution using a damped Newton method such as

$$0 = F_{B_{(x)}} = F_{x_{(B)}} + (x - x_B)$$

where x_B is the Euclidean projection of x onto the Box $B := [l, u]$. A vector x solves this non-linear equation only if $z = x_B$ solves the MCP. A more detailed explanation of this algorithm is beyond the scope of this paper, many technical papers on the topic are available in Ferris's homepage : http://www.cs.wisc.edu/~ferris/.

GAMS syntax (Brook, Kendrick and Meeraus (1992)) permits us to generate a non linear mixed complemetarity model by declaring and assigning sets, data, parameters, variables, equations in the model. PATH is invoked by the "OPTION MCP = PATH" statement in the GAMS code and a command line "solve <model name> using MCP" instructs GAMS to solve the model using the PATH solver. We use batch files to compute incidence profiles across various scenarios for different values of elasticities and tax rates for households.

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