Economic Modelling

Lecture 22

Tax in a General Equilibrium Model

Household Problem in Presence of Consumption and Income Taxes

$$Max \quad U = c^{\phi} l^{1-\phi}$$

$$l + h^{s} = 1$$

$$p(1 + t_c)c = w(1 - t_l)h^{s} + \pi + R$$

$$pt_c c + wt_l h^{s} = R$$

$$c \geq 0; l \geq 0; h^{s} \geq 0 \qquad (1')$$

$$p(1 + t_c)c = w(1 - t_l)h^{s} + \pi + R \qquad wh^{s} + \pi = pc \qquad (2')$$

$$L(c, l, \lambda) = c^{\phi}(1 - h^{s})^{1-\phi} + \lambda[w(1 - t_l)h^{s} + \pi + R - p(1 + t_c)c] \qquad (3')$$

First Order Optimisation Conditions in the Presence of Taxes

$$\frac{\partial L(c,l,\lambda)}{\partial c} = \phi c^{\phi-1}\left(1-h^s\right)^{1-\phi} - \lambda p\left(1+t_c\right) = 0 \qquad (4')$$

$$\frac{\partial L(c,l,\lambda)}{\partial h^s} = (1-\phi)c^{\phi}\left(1-h^s\right)^{-\phi}(-1) + \lambda w\left(1-t_l\right) = 0 \qquad (5')$$

$$\frac{\partial L(c,l,\lambda)}{\partial \lambda} = w\left(1-t_l\right)h^s + \pi + R - p\left(1+t_c\right)c = 0 \quad wh^s + \pi = pc \qquad (6')$$

$$\frac{\frac{\partial L(c,l,\lambda)}{\partial h^s}}{\frac{\partial L(c,l,\lambda)}{\partial c}} = \frac{(1-\phi)c^{\phi}\left(1-h^s\right)^{-\phi}(-1)}{\phi c^{\phi-1}\left(1-h^s\right)^{1-\phi}} = \frac{w\left(1-t_l\right)}{p\left(1+t_c\right)} \qquad (7')$$

$$c = \left(\frac{\phi}{1-\phi}\right)\left(1-h^s\right)\frac{w\left(1-t_l\right)}{p\left(1+t_c\right)} \qquad (8')'$$

$$h^s\frac{w}{p} + \frac{\pi}{p} = c = \left(\frac{\phi}{1-\phi}\right)\left(1-h^s\right)\frac{w}{p}\left(\frac{1-t_l}{1+t_c}\right) \qquad (9')$$

Labour Supply in the Presence of Taxes

$$h^s \frac{w}{p} + \left(\frac{\phi}{1-\phi}\right)\frac{w}{p}\left(\frac{1-t_l}{1+t_c}\right)h^s = c = \left(\frac{\phi}{1-\phi}\right)\frac{w}{p}\left(\frac{1-t_l}{1+t_c}\right) - \left(\frac{\pi}{p}\right)$$

$$h^s = \frac{\left(\frac{\phi}{1-\phi}\right)\frac{w}{p}\left(\frac{1-t_l}{1+t_c}\right) - \left(\frac{\pi}{p}\right)}{\frac{w}{p}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right)}$$

$$h^s = \frac{\left(\frac{\phi}{1-\phi}\right)\frac{w}{p}\left(\frac{1-t_l}{1+t_c}\right) - \left(\left(\frac{w}{p}\right)^{\frac{\alpha}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]\right)}{\frac{w}{p}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right)}$$

Determination of Real Wage Rate in the Presence of Taxes

$$h^d = \left(\frac{1}{\alpha}\frac{w}{p}\right)^{\frac{1}{\alpha-1}} = h^s = \frac{\left(\frac{\phi}{1-\phi}\right)\frac{w}{p}\left(\frac{1-t_l}{1+t_c}\right) - \left(\left(\frac{w}{p}\right)^{\frac{\alpha}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]\right)}{\frac{w}{p}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right)}$$

$$\left(\frac{w}{p}\right)^{\frac{1}{\alpha-1}} = \frac{\left(\frac{\phi}{1-\phi}\right)\left(\frac{1-t_l}{1+t_c}\right)}{\left[\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right) + \left\{\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right\}\right]}$$

$$\frac{w}{p} = \left[\frac{\left(\frac{\phi}{1-\phi}\right)\left(\frac{1-t_l}{1+t_c}\right)}{\left[\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right) + \left\{\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right\}\right]}\right]^{\alpha-1}$$

Labour Supply and Output in the Presence of Taxes

$$h^s = h^d = \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}} \left[\frac{\left(\frac{\phi}{1-\phi}\right)\left(\frac{1-t_l}{1+t_c}\right)}{\left[\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right)+\left\{\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right\}\right]} \right]$$

$$\hat{y} = \left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} \left[\frac{\left(\frac{\phi}{1-\phi}\right)\left(\frac{1-t_l}{1+t_c}\right)}{\left[\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right)+\left\{\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right\}\right]} \right]^{\alpha}$$

Leisure and Consumption in the Presence of Taxes

$$\hat{l} = 1 - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}} \left[\frac{\left(\frac{\phi}{1-\phi}\right)\left(\frac{1-t_l}{1+t_c}\right)}{\left[\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right) + \left\{\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right\}\right]} \right]$$

$$\hat{c} = \left(\frac{\phi}{1-\phi}\right)\left[1 - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}} \left[\frac{\left(\frac{\phi}{1-\phi}\right)\left(\frac{1-t_l}{1+t_c}\right)}{\left[\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right) + \left\{\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right\}\right]} \right]\right] \frac{w(1-t_l)}{p(1+t_c)}$$

$$\hat{U} = \hat{c}^{\phi}\,\hat{l}^{1-\phi}$$

Table 1

Parameters of the model in the base scenario

Parameter of the model	Numerical value in the base model
Utility weight on consumption (ϕ)	0.6
Utility weight on leisure (1-ϕ)	0.4
Elasticity of output to labour input (α)	0.6
Value of Endowment	68 hours
Consumption tax rate in the base model	0.17
Income tax rate in the base case	0.35
Normalisation of price $w+p=1$	

Experiments for the tax reform

Use of both consumption and income taxes and lump sum transfers (base case)
Elimination of all taxes and no transfer
Only labour income tax and lump sum transfers
Only consumption tax and lump sum transfers

Table 2

Parameter of the model	Numerical value in the base model
Share of spending on consumption (ϕ)	0.25 to 0.6 with steps size of 0.05
Share of spending on leisure (1-ϕ)	0.75 to 0.4 with steps size of 0.05
Elasticity of output to labour input (α)	0.3 to .65 with steps size of 0.05
Value of Endowment	68 to 108 hours with steps size of 5
Consumption tax rate in the base model	0.17 to 0.67 with steps size of 0.05
Income tax rate in the base case	0.40 to 0.85 with steps size of 0.05

Table 3

Overall Welfare Impacts of Tax Changes in the General Equilibrium Model of Taxes

	Equivalent variation	Compensating variation
Elimination of all taxes	3.2%	-3.1%
Labour tax only	-6.2%	6.7%
Consumption tax only	-0.05%	0.05%

Table 4
Macroeconomic Impacts of Alternative Taxes

Variables	both taxes	not tax	labor income tax	Consumption tax
Utility	14.142	14.601	13.689	14.526
Output	6.505	8.032	5.849	7.431
Leisure	45.333	35.789	49.012	39.703
Labour Supply	22.667	32.211	18.988	28.297
Consumption	6.505	8.032	5.849	7.431
Revenue	2.109		1.687	1.687
Wage	0.147	0.13	0.156	0.136
Price	0.853	0.87	0.844	0.864
Profit	14.142	14.601	13.689	14.526
Consumption tax	0.17			0.263
Labour income tax	0.35		0.57	

Table 5

Impact of Alternative Taxes: Percentage changes compared to the base case

	Base two tax case	Labour only tax	Consumption tax
Output	23.471	-27.174	-7.478
Leisure	-21.053	36.946	10.936
Labour supply	42.105	-41.051	-12.151
Consumption	23.471	-27.174	-7.478
Revenue	-100		
Wage rate	-11.406	19.868	4.595
Price	1.964	-2.972	-0.687
Profit	25.896	-29.339	-8.114
Utility	3.246	-6.246	-0.511

Table 6

Sensitivity of Welfare cost to tax changes

Scenarios	EV	Consumption tax rate	Labour Income tax rate
1	4.727	0.22	0.4
2	6.567	0.27	0.45
3	8.83	0.32	0.5
4	11.607	0.37	0.55
5	15.03	0.42	0.6
6	19.289	0.47	0.65
7	24.685	0.52	0.7
8	31.71	0.57	0.75
9	41.246	0.62	0.8
10	55.092	0.67	0.85

Table 7

Sensitivity of model results in comparison to the base case

Endowment	Change in utility	alpha	Change in utility	Phi	Change in utility
73	5.54	0.3	-44.054	0.25	106.269
78	10.99	0.35	-38.744	0.3	81.531
83	16.357	0.4	-32.729	0.35	61.117
88	21.647	0.45	-25.937	0.4	44.153
93	26.865	0.5	-18.286	0.45	29.991
98	32.016	0.55	-9.677	0.5	18.144
103	37.104	0.6	-9.40E-12	0.55	8.241

Efficiency Gains in the UK from elimination of all taxes and transfers (Measured as a percent of benchmark utility level of a representative household)

Equivalent Variation	=	3.715
Compensating Variation	=	-3.582

Efficiency Gains from Switching to Labour income Taxes

Equivalent Variation	=	-0.693
Compensating Variation	=	0.697

Efficiency Gains from Switching to Consumption Taxes

Equivalent Variation	=	2.967
Compensating Variation	=	-2.882

Table 8

Macroeconomic Impacts of Consumption and Labour Income Taxes in the UK

Variable	Benchmark Both taxes	Labour Income tax only	Consumption tax only
Utility	190.073	188.757	195.713
Output	86.85	84.687	100.008
Leisure	615.385	628.104	535.791
Labour supply	384.615	371.896	464.209
Consumption	86.85	84.687	100.008
Revenue	32.122	25.698	25.698
wage rate	0.145	0.146	0.139
Price	0.855	0.854	0.861
Profit	190.073	188.757	195.713
Optimal Consumption tax rate	0.17		0.298
Optimal labour income tax rate	0.35	0.474	

Key Results of Tax Reform Analysis

The efficiency gains from switching to only consumption taxes are about 80 percent the gains of eliminating all the taxes.

Optimal consumption tax rate given the revenue constraint set equal to 80 percent of the benchmark revenue level is 2.9 percent.

Labour income tax is highly distortionary in this model for various reasons. As before 47 percent tax rate of labour income is optimal to meet the required revenue target.

Our first result shows that the net deadweight loss of the current tax and transfer system is about 4 percent of GDP

References

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