

# Simulation Models of an Economy

1. Input-Output Model
2. Lorenz Curve and Gini Coefficient
3. Two sector Model General Equilibrium Model
4. A Sixteen Sector Model of the UK Economy

## A simple input-output table for a two sector economy

	<i>Primary</i>	<i>Secondary</i>	<i>C</i>	<i>I</i>	<i>G</i>	<i>X</i>	<i>M</i>	<i>Total</i>
<i>Primary</i>	$x_{11}$	$x_{12}$	$C_1$	$I_1$	$G_1$	$X_1$	$M_1$	$GY_1$
<i>Secondary</i>	$x_{21}$	$x_{22}$	$C_2$	$I_2$	$G_2$	$X_2$	$M_2$	$GY_2$
<i>Labour</i>	$L_1$	$L_2$						$wL$
<i>Lab-tax</i>	$T_{1,k}$	$T_{2,k}$						$TL$
<i>Capital</i>	$K_1$	$K_2$						$rK$
<i>Cap taxes</i>	$T_{1,k}$	$T_{2,k}$						$TK$
<i>Total</i>	$GY_1$	$GY_2$	<i>C</i>	<i>I</i>	<i>G</i>	<i>X</i>	<i>M</i>	

## Benchmark Data Set for a Two Sector Economy

	Sector 1	Sector 2	C	I	G	GY
Sector 1	15	15	40	10	20	100
Sector 2	10	20	30	10	30	100
Labour	30	24				54
Labour income tax	20	16				36
Capital	15	20				35
Capital income tax	10	5				15
<b>Total</b>	<b>100</b>	<b>100</b>	<b>70</b>	<b>20</b>	<b>50</b>	

**Intermediate demand: 60**

**Final demand: 140**

	Sector 1	Sector 2	C	I	G	GY
Sector 1	0.15	0.15	0.40	0.10	0.20	1
Sector 2	0.10	0.20	0.30	0.10	0.30	1
Labour	0.30	0.24				
Labour income tax	0.20	0.16				
Capital	0.15	0.20				
Capital income tax	0.10	0.05				
Total	1	1				

## Liontief Coefficients in Input-Output Model

$$X_1 = X_{1,1} + X_{1,2} + F_1$$

$$X_2 = X_{2,1} + X_{2,2} + F_2$$

$a_{i,j}$  share of input from row sector  $i$  to sector  $j$

$$a_{1,1} = \frac{X_{1,1}}{X_1}$$

$$a_{1,2} = \frac{X_{1,2}}{X_2}$$

$$a_{2,1} = \frac{X_{2,1}}{X_1}$$

$$a_{2,2} = \frac{X_{2,2}}{X_2}$$

$$X_1 = a_{1,1}X_1 + a_{1,2}X_2 + F_1$$

$$X_2 = a_{2,1}X_1 + a_{2,2}X_2 + F_2$$

## Input-Output Model

$$(1 - a_{1,1})X_1 - a_{1,2}X_2 = F_1$$

$$-a_{2,1}X_1 + (1 - a_{2,2})X_2 = F_2$$

$$\begin{bmatrix} (1 - a_{1,1}) & -a_{1,2} \\ -a_{2,1} & (1 - a_{2,2}) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} (1 - a_{1,1}) & -a_{1,2} \\ -a_{2,1} & (1 - a_{2,2}) \end{bmatrix}^{-1} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$$X = (I - A)^{-1} F$$

## Input-Output Model

$$(I - A)^{-1} = \frac{Adj(A)}{|A|} = \frac{[C_{i,j}]}{|A|} = \frac{\begin{bmatrix} (1 - a_{2,2}) & a_{2,1} \\ a_{1,2} & (1 - a_{1,1}) \end{bmatrix}}{(1 - a_{1,1})(1 - a_{2,2}) - a_{1,2}a_{2,1}}$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \frac{\begin{bmatrix} (1 - a_{2,2}) & a_{2,1} \\ a_{1,2} & (1 - a_{1,1}) \end{bmatrix}}{(1 - a_{1,1})(1 - a_{2,2}) - a_{1,2}a_{2,1}} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$$a_{1,1} = \frac{X_{1,1}}{X_1} = \frac{15}{100} = 0.15$$

$$a_{1,2} = \frac{X_{1,2}}{X_2} = \frac{15}{100} = 0.15$$

$$a_{2,1} = \frac{X_{2,1}}{X_2} = \frac{10}{100} = 0.1$$

$$a_{2,2} = \frac{X_{2,2}}{X_2} = \frac{20}{100} = 0.2$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} (1-0.15) & -0.15 \\ -0.1 & (1-0.2) \end{bmatrix}^{-1} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$$(I - A)^{-1} = \frac{Adj(A)}{|A|} = \frac{[C_{i,j}]}{|A|} = \frac{\begin{bmatrix} (1-a_{2,2}) & a_{2,1} \\ a_{1,2} & (1-a_{1,1}) \end{bmatrix}}{(1-a_{1,1})(1-a_{2,2}) - a_{1,2}a_{2,1}}$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \frac{\begin{bmatrix} 0.8 & 0.15 \\ 0.1 & 0.85 \end{bmatrix}}{0.8 \times 0.85 - 0.15 \times 0.1} \begin{bmatrix} 70 \\ 70 \end{bmatrix}$$

$$Multiplier = \frac{1}{0.8 \times 0.85 - 0.15 \times 0.1} = \frac{1}{0.665} = 1.50376$$

## Input-Output Model

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \frac{\begin{bmatrix} 0.8 & 0.1 \\ 0.15 & 0.85 \end{bmatrix}}{(0.8)(0.85) - 0.15 \times 0.1} \begin{bmatrix} 80 \\ 70 \end{bmatrix} = \frac{\begin{bmatrix} 0.8 & 0.15 \\ 0.1 & 0.85 \end{bmatrix}}{0.68 - 0.015} \begin{bmatrix} 70 \\ 70 \end{bmatrix} = \begin{bmatrix} \frac{66.5}{0.665} \\ \frac{66.5}{0.665} \end{bmatrix} = \begin{bmatrix} 100 \\ 100 \end{bmatrix}$$

Model is calibrated

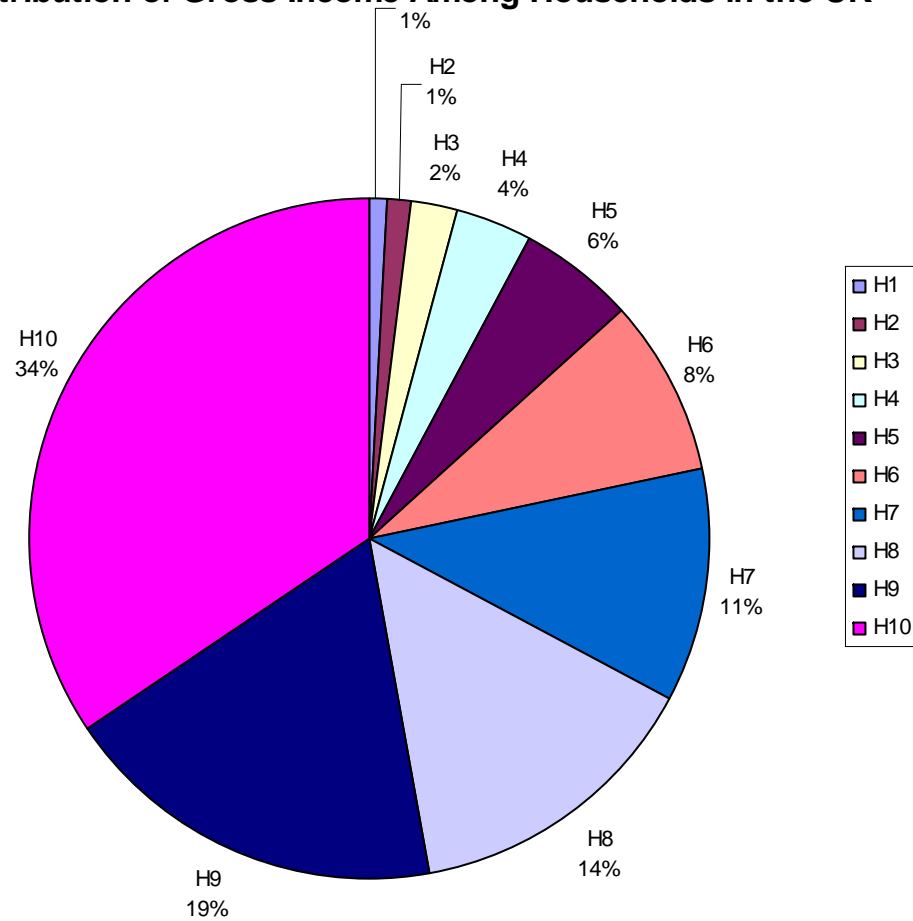
Impact of 15 percent increase in final demand

$$\begin{bmatrix} \Delta X_1 \\ \Delta X_2 \end{bmatrix} = \frac{\begin{bmatrix} 0.8 & 0.1 \\ 0.15 & 0.85 \end{bmatrix}}{0.68 - 0.015} \begin{bmatrix} 0.15 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{0.12}{0.665} \\ \frac{0.0225}{0.665} \end{bmatrix} = \begin{bmatrix} 0.1805 \\ 0.0338 \end{bmatrix}$$

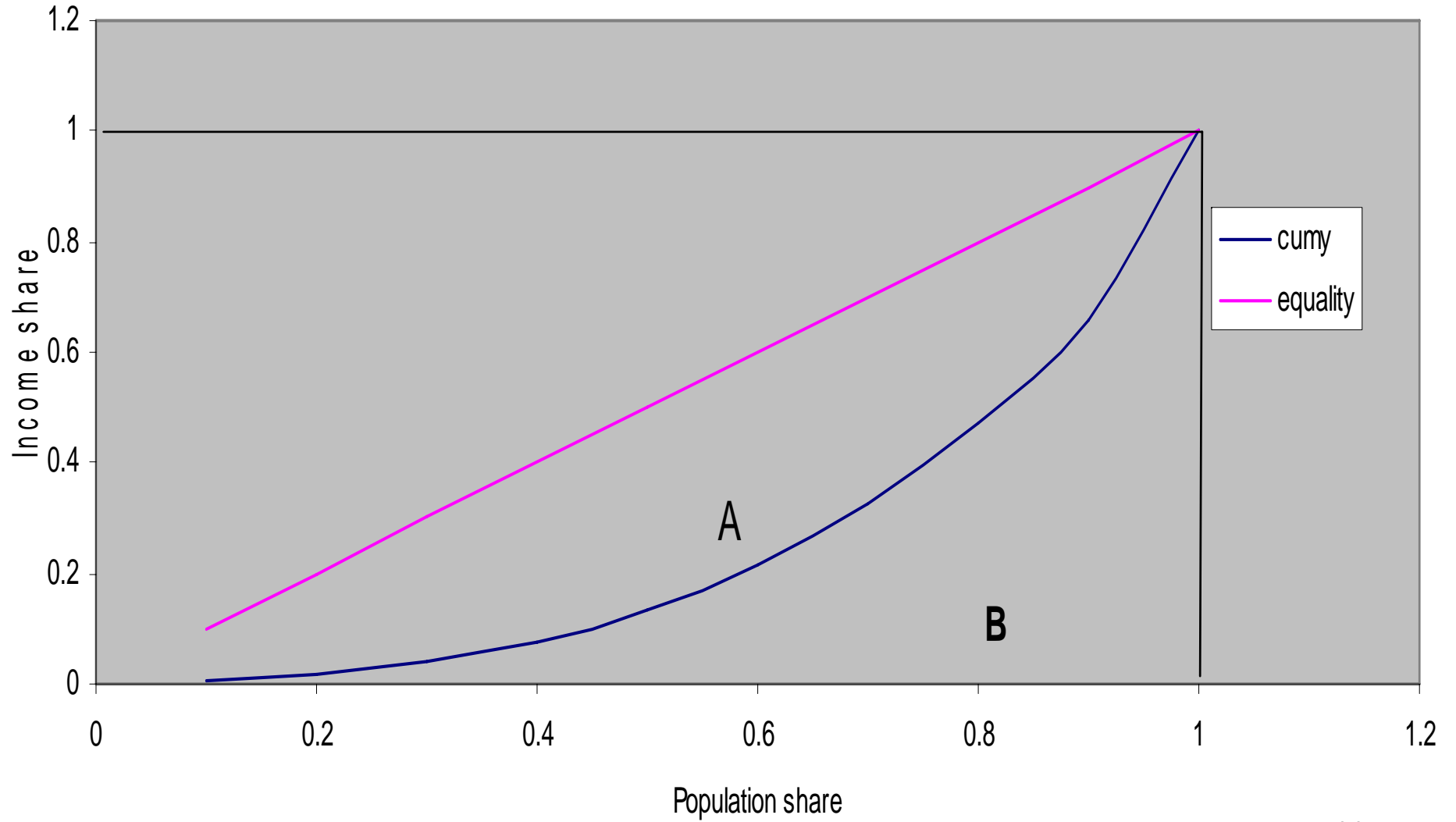
Do Excel Exercise on 16 sector Input-Output Model of the UK Economy.

	Gross Income	Consumption	Transfers
H1	2587	10286	7699
H2	3803	15935	12132
H3	6699	18247	11548
H4	11796	18395	6599
H5	18627	18631	4
H6	26837	21032	-5805
H7	36106	25662	-10444
H8	46189	31639	-14550
H9	60047	39886	-20161
H10	111296	72299	-38997
Total	323987	272012	-51975

Distribution of Gross Income Among Households in the UK



Lorenz Curve of Income Distribution in the UK, 2002



# Calculation of a Gini Coefficient

	Consumpti	Share by H	Cumshare	Difference	Population	Triangle	Rectangle	Area	equality
H1	10286	0.037814508	0.037815		0.1	0.001891		0.001891	0.1
H2	15935	0.058581974	0.096396	0.058582	0.2	0.002929	0.003781	0.006711	0.2
H3	18247	0.067081599	0.163478	0.067082	0.3	0.003354	0.00964	0.012994	0.3
H4	18395	0.067625693	0.231104	0.067626	0.4	0.003381	0.016348	0.019729	0.4
H5	18631	0.068493302	0.299597	0.068493	0.5	0.003425	0.02311	0.026535	0.5
H6	21032	0.077320118	0.376917	0.07732	0.6	0.003866	0.02996	0.033826	0.6
H7	25662	0.094341426	0.471259	0.094341	0.7	0.004717	0.037692	0.042409	0.7
H8	31639	0.116314721	0.587573	0.116315	0.8	0.005816	0.047126	0.052942	0.8
H9	39886	0.146633237	0.734207	0.146633	0.9	0.007332	0.058757	0.066089	0.9
H10	72299	0.265793421	1	0.265793	1	0.01329	0.073421	0.08671	1
Total	272012					0.05	0.299835	0.349835	

$$G = \frac{A}{A+B} = \frac{0.15012}{0.5} = 0.30024$$

$$B = 0.3498$$

$$A = (A+B) - B = 0.5000 - 0.3498 = 0.150165$$

# Two Sector General Equilibrium Model of Taxes

Consumer's Problem

$$\underset{X_1, X_2}{Max} \quad U_1 = X_1^\alpha X_2^\beta$$

$$I = P_1(1 + t_1)X_1 + P_2(1 + t_2)X_2$$

$$I = (1 - t_k)r(K_1 + K_2) + (1 - t_w)w(L_1 + L_2)$$

Firms' Problem

$$\underset{L_1, K_1}{Max} \quad \Pi_1 = P_1Y_1 - wL_1 - rK_1 \quad Y_1 = K_1^{\gamma_1} L_1^{(1-\gamma_1)} \quad GY_1 = F(Y_1, X_{i,1})$$

$$\underset{L_2, K_2}{Max} \quad \Pi_2 = P_2Y_2 - wL_2 - rK_2 \quad Y_2 = K_2^{\gamma_2} L_2^{(1-\gamma_2)} \quad GY_2 = F(Y_2, X_{i,2})$$

Government Budget:

$$R = t_k r(K_1 + K_2) + t_w w(L_1 + L_2) + t_1 P_1 X_1 + t_2 P_2 X_2 = G$$

## Preparation of the Benchmark Data Set for General Equilibrium Model

Declaration of benchmark parameters								
y10	c10	TL10	TxL10	x11				
y20	c20	Tl20	Txl20	x12				
L10	i10	TK10	TxK10	x21				
L20	i20	Tk20	Txk20	x22				
K10	g10			PL10				
K20	g20			PL20				
				PK10				
				Pk20				
				grev				
Assignments of base year data								
y10 =100;	c10=40;	TL10=20;	TxL10=TL10/L10;	x11=15;	PL10 =(1+txl10);			
y20 =100;	c20=30;	Tl20=16;	Txl20=TL20/L20;	x12=15;	PL20=(1+txl20);			
L10 =30;	i10=10;	TK10=10;	TxK10=TK10/K10;	x21=10;	PK10=(1+txK10);			
L20=24;	i20=10;	Tk20=5;	Txk20=TK20/K20;	x22=20;	Pk20=(1+txK20);			
K10=15;	g10=20;							
K20=20;	g20=30;							
grev =txl10*L10+txl20*L20+txk10* K10+txk20*k20;								

# An MPSGE Program for the Static General Equilibrium Tax Model

<pre> \$ontext  \$model:tax  \$sectors: y1 y2  \$commodities: p1 p2 pl pk pt  \$consumers: RA gov  \$prod:y1 s:0 va:8 o:p1 q:y10 i:p1 q:x11 i:p2 q:x21 i:pl q:L10 p:PL10 A:gov T:txl10 va: i:pk q:K10 p:PK10 A:gov T:txK10 va:  \$prod:y2 s:0 va:8 o:p2 q:y20 i:p1 q:x12 i:p2 q:x22 i:pl q:L20 p:PL10 A:gov T:txl10 va: i:pk q:K20 p:PK20 A:gov T:txK20 va: </pre>	<pre> \$demand:RA E:pl q:(L10+L20) E:pk q:(K10+K20) D:p1 q:y10 D:p2 q:y20 E:PT q:grev  \$report:  v:out1 o:p1 prod:y1 v:out2 o:p2 prod:y2 V:welf w:ra  \$offtext  \$sysinclude mpsgeset tax \$include tax.gen  tax.iterlim=0; solve tax using mcp;  display p1.l,p2.l,pl.l,pk.l;  *Txl10=0; Txl20=0; TxK10=0; Txk20=0;  \$include tax.gen  tax.iterlim=1000; solve tax using mcp;  display p1.l,p2.l,pl.l,pk.l; </pre>
--	--

## Benchmark Replication in the MPSGE Programme

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR Y1	.	1.0000	+INF	.
---- VAR Y2	.	1.0000	+INF	.
---- VAR P1	.	1.0000	+INF	.
---- VAR P2	.	1.0000	+INF	.
---- VAR PL	.	1.0000	+INF	.
---- VAR PK	.	1.0000	+INF	.
---- VAR PT	.	1.0000	+INF	.
---- VAR RA	.	140.0000	+INF	.
---- VAR GOV	.	51.0000	+INF	.
---- VAR OUT1	.	100.0000	+INF	.
---- VAR OUT2	.	100.0000	+INF	.
---- VAR WELF	.	0.7000	+INF	.
**** REPORT SUMMARY : 0 NONOPT				
0 INFEASIBLE				
0 UNBOUNDED				
0 REDEFINED				
0 ERRORS				
---- 461 VARIABLE P1.L		= 1.000		
VARIABLE P2.L		= 1.000		
VARIABLE PL.L		= 1.000		
VARIABLE PK.L		= 1.000		
MODEL STATISTICS				
BLOCKS OF EQUATIONS	1	SINGLE EQUATIONS		1
BLOCKS OF VARIABLES	12	SINGLE VARIABLES		12
NON ZERO ELEMENTS	12	NON LINEAR N-Z		0
DERIVATIVE POOL	3	CONSTANT POOL		0
CODE LENGTH	1			

## Counter Factual Scenarios

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR Y1	.	1.0187	+INF	.
---- VAR Y2	.	1.0187	+INF	.
---- VAR P1	.	0.9460	+INF	.
---- VAR P2	.	1.0173	+INF	.
---- VAR PL	.	0.9904	+INF	.
---- VAR PK	.	1.4533	+INF	.
---- VAR PT	.	0.6991	+INF	.
---- VAR RA	.	140.0000	+INF	.
---- VAR GOV	.	35.6531	+INF	.
---- VAR OUT1	.	101.8670	+INF	.
---- VAR OUT2	.	101.8670	+INF	.
---- VAR WELF	.	0.7131	+INF	.
**** REPORT SUMMARY : 0 NONOPT				
0 INFEASIBLE				
0 UNBOUNDED				
0 REDEFINED				
0 ERRORS				
---- 624 VARIABLE P1.L		= 0.946		
VARIABLE P2.L		= 1.017		
VARIABLE PL.L		= 0.990		
VARIABLE PK.L		= 1.453		

## Benchmark and Tax Reform Scenarios

	Benchmark	Counterfactual
LTAX1	0.667	0.667
LTAX2	0.667	0
KTAX1	0.667	0
KTAX2	0.25	0
WAGE	1	1.047
RENT	1	1.536
PRICE1	1	1 (numeraire)
PRICE2	1	1.075
WELF	0.7	0.713
OUTPUT1	100	101.867
OUTPUT2	100	101.867
GROSSINC	140	147.99
TRANSFER	1	0.739

## Explanation or Results of tax Reform

- Both wage rate and rental rates have increased because reduction of taxes increases their demand, given their fixed supplies, their prices have increased
- Relative price of good 2 has increased
- Output of both sectors have increased
- Gross income of household has increased too
- Amount of transfer has reduced

# Multisectoral General Equilibrium Tax Model for the UK Economy

With Multiple Capital Asset

## Demand Side of the Economy

$$U_h = \left( \sum_i \alpha_{i,h} C_{i,h}^{\rho_h} + \beta_h L_h^{\rho_h} \right)^{\frac{1}{\rho_h}} \quad C = \psi \left( \sum_i \delta_i^c C C_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

$$H = \sum_j \sum_i r_j (1 - t_{j,i}) \theta_{j,i} \bar{K}_j + (1 - t_l) w \bar{L} + TR$$

$$P(1 + t_v)C + w(1 - t_l)L = H$$

$$C = \left( \frac{\alpha H}{(P(1 + t_v))^{1-\sigma} (\alpha(P(1 + t_v)))^{1-\sigma} + \beta (w(1 - t_l))^{1-\sigma}} \right)$$

$$L = \left( \frac{\beta H}{(w(1 - t_l))^{1-\sigma} (\alpha(P(1 + t_v)))^{1-\sigma} + \beta (w(1 - t_l))^{1-\sigma}} \right)$$

$$LS = \bar{L} - L$$

$$P \cdot C = \sum_i P_i \cdot CC_i$$

## Production and Government Sector

$$VA_i = \Omega_i \left( (1 - \delta_i)(K_i)^{\gamma_i} + \delta_i(LS_i)^{\gamma_i} \right)^{\frac{1}{\gamma_i}}$$

$$PY_i Y_i = PV_i VA_i + \sum_j PA_i (1 + t_{i,j}^d) DI_{i,j} + \sum_j PM_i (1 + t_{i,j}^m) MI_{i,j}$$

$$\Pi_i = PY_i Y_i - wL_i - \sum_{j,i} r_j K_{j,i} - \sum_j PA_j (1 + t_{i,j}^m) MI_{j,i} - \sum_j PA_j (1 + t_{i,j}^d) DI_{j,i}$$

$$G + TR = \sum_j \sum_i t_{j,i}^k r_j K_{j,i} + \sum_i t_i^{vc} P_i CC_i + \sum_i t_i^{vg} P_i G_i + \sum_i t_i^{vk} P_i I_i + \sum_i t_i^l w LS_i + \sum_i t_i^m M_i + \sum_j \sum_i PA_j t_{i,j}^m MI_{j,i} + \sum_j \sum_i PA_j t_{i,j}^d DI_{j,i}$$

$$G = \sum_i PA_i GD_i + \sum_i PA_i GM_i$$

$$I = \sum_i PA_i ID_i + \sum_i PA_i IM_i$$

## Specification of Trade

$$A_i = \Phi \left( (1 - \delta_i^m) D_i^{\frac{\sigma_m - 1}{\sigma_m}} + \delta_i^m M_i^{\frac{\sigma_m - 1}{\sigma_m}} \right)^{\frac{\sigma_m}{\sigma_m - 1}}$$

$$A_i = CC_i + G_i + I_i$$

$$PA_i A_i = PD_i D_i + PM_i M_i$$

$$Y_i = \Theta \left( (1 - \delta_i^e) D_i^{\frac{\sigma_y - 1}{\sigma_y}} + \delta_i^e E_i^{\frac{\sigma_y - 1}{\sigma_y}} \right)^{\frac{\sigma_y}{\sigma_y - 1}}$$

$$PY_i Y_i = PD_i D_i + PE_i E_i$$

$$\sum_i PE_i E_i = \sum_i PM_i M_i$$

## Definition of the Competitive Equilibrium

In this model a competitive equilibrium is given by prices of consumption goods,  $P_i$ ; the rental rate of capital assets  $r_j$ ; a wage rate for labour,  $w$ ; levels of gross output,  $Y_i$  (gross of intermediate use); capital use,  $K_i$ ; and sectoral use of labour,  $L_i$ ; imports  $M_i$ , exports  $X_i$ , intermediate inputs  $DI_{i,j}$ ,  $MI_{i,j}$  investment  $I_i$ , government consumption  $G_i$ , private consumption  $C_i$ , such that,

- i) The markets for goods and services, labour and capital clear; and
- ii) budget constraints of households, the government and investors are satisfied.

## Market Clearing Conditions

$$\bar{K}_j = \sum_i K_{i,j}$$

$$\bar{K}_j + FK_j = \sum_i K_{i,j}$$

$$r_j^{UK} \geq r_j^w \Rightarrow FK_j \geq 0$$

$$r_j^{UK} \leq r_j^w \Rightarrow FK_j \leq 0$$

$$Y_i = F_i^d + \sum_{j=1}^N a_{i,j}^d Y_j$$

$$\sum_j a_{ij}^d Y_j$$

$$M_i = F_i^m + \sum_{j=1}^N a_{i,j}^m Y_j$$

$$LS = \sum_i LS_i$$

$$F_i^m = C_i^m + I_i^m + G_i^m + E_i^m$$

$$EV = E(U^N, P^0) - E(U^0, P^0)$$

$$CV = E(U^N, P^N) - E(U^0, P^N)$$

**Table 1**

**Aggregate Welfare Results of Replacing Capital Income Taxes By Uniform Rates in Equal and No-equal-yield Cases (with labour supply elasticity of 0.15)**

A. Equal Yield Case

Tax Experiments	Hicksian equivalent variation as % of GDP	Hicksian compensating variation as % of GDP
<b>Replacing the Existing Capital Income Taxes By Yield Preserving Uniform Rates (Central case)</b>	0.035	-0.036
<b>Replacing the Existing Capital Income Taxes By Yield Preserving Uniform Rates (Unit elasticity case)</b>	0.022	-0.022

B. No equal yield central elasticity case with low and high labour supply elasticities

Tax experiments	Hicksian equivalent variation as % of GDP	Hicksian compensating var as % of GDP
<b>Replacing the Existing Capital Income Taxes By Uniform Rates (Low case)</b>	0.281	-0.279
<b>Replacing the Existing Capital Income Taxes By Uniform Rates (High case)</b>	0.283	-0.281

Note: See Table A6 in the appendix for numerical values of substitution elasticities in central and unit cases.

**Table 2**

Sensitivity of aggregate welfare as a percentage of UK GDP to substitution elasticities between capital and labour, and to substitution elasticities across capital assets

**A. Labour supply elasticity 0.15**

$\sigma_k$	0.75	1.0	3.0	5.0
$\sigma_v$				
0.75	0.01513	0.01705	0.0316	0.04594
1.0	0.01951	0.0223	0.03607	0.05046
3.0	0.04999	0.05252	0.06898	0.08426
5.0	0.07694	0.08039	0.09992	0.11647

**B. Labour supply elasticity 0.3**

$\sigma_k$	0.75	1.0	3.0	5.0
$\sigma_v$				
0.75	0.01496	0.01688	0.03143	0.04576
1.0	0.0193	0.02124	0.03587	0.05026
3.0	0.04947	0.05206	0.06866	0.08399
5.0	0.07616	0.07971	0.09953	0.11618

Note:  $\sigma_v$  is the elasticity of substitution between capital and labour  
 $\sigma_k$  is the elasticity of substitution among capital assets.

**Table 3**

**Capital Asset Reallocation from Equal Yield Replacement of Capital Income Taxes by  
Uniform Tax Rates By Industry ( % Change in Capital Use (By Asset By Sector)**

**a. Central Case elasticity specification****b. Unit elasticity specification**

Asset Class	Buildings	PM Long	PM Short	Vehicles	Asset Class	Buildings	PM Long	PM Short	Vehicles
Agric	2.05		2.76	-5.3	Agric	2.1		2.17	-4.63
Extr	21.17	-0.45	10.26	2.11	Extr	16.21	2.46	9.15	4.06
Min	-1.47		0.3	2.49	Min	-0.6		0.09	1.46
Chem	-8.98	-0.52	-5.79	7.88	Chem	-5.15	-0.46	-3.63	4.58
Metal	-11.62	-4.93	-9.55	4.81	Metal	-7.29	-3.53	-6.41	2.54
Eng	-12.06	-5.88	-8.12	3.06	Eng	-8.76	-5.05	-6.51	0.92
Food	-5.87	-1.08	-3.72	4.64	Food	-5.51	-1.24	-3.82	4.52
OTHMA	-6.74	-1.97	-3.45	4.24	OTHMA	-7.04	-2.28	-3.85	4.69
Power	-6.26	0.38	20.12	6.89	Power	-3.96	-0.02	12.92	4.29
Constr	-8.31	-1.15	0.85	1.92	Constr	-7.91	-1.23	0.86	1.86
Distr	-10.82	-1.66	4.68	5.63	Distr	-6.6	-1.24	2.82	3.31
Trans	-11.56	3.34	9.65	-1.03	Trans	-7	1.99	5.97	-0.76
Fin	14.18	-8.78	-1.34	-2.01	Fin	9.42	-5.6	-0.73	-1.25
PubAD	18.89	-5.02	0.91	2.03	PubAD	12.12	-3.27	0.58	1.19
EducA	20.89		-2.55	1.96	EducA	13.39		-1.54	1.22

Note: The capital income tax rates used here may be different from the capital income tax rates in use in the Inland Revenue.

Equal yield uniform capital tax rate 26.5%

Aggregate Welfare Effect :           £218.1 mill (95)  
  = 0.0347% of UK 1995 GDP

Aggregate Welfare Effect :           +£140 mill (95)  
  = 0.0223% of UK 1995 GDP

**Table 4**  
**% Changes in Employment and Output**  
**Equal Yield Replacement of Capital Income Taxes By Uniform Tax Rates**  
 (Central case specification of elasticities)

Industry	Labour supply elasticity 0.15	
	% change in employment (labour use)	% change in output
Agric	-0.989	-0.065
Extra	-0.843	1.606
Minin	-0.207	-0.367
Chemi	4.758	-0.251
Metal	1.850	-0.731
Engin	0.352	-0.970
Food	2.797	-0.044
Othma	0.951	-0.241
Power	4.084	-0.262
Constr	0.130	-0.040
Distr	2.673	-0.121
Trans	1.818	-0.015
Finan	-4.757	0.124
PubAD	-0.827	0.035
EducA	-0.897	0.107
House		0.01

**Table 5**  
**Aggregate Welfare for other cases**  
 (as % of GDP)

Equal Yield Replacement of non-uniform VAT By Uniform Rates - Central Case Specification of Elasticities	0.0186% of UKGDP
Equal Yield Replacement of all indirect Taxes By Uniform indirect tax Rates	0.01704% of UKGDP
Equal Yield Replacement of all indirect Taxes By equal yield lump-sum tax	1.723% of UKGDP
Equal Yield Replacement of household income taxes By equal yield lump-sum tax	3.67% of UKGDP

# Marginal Excess Burden of Taxes

$$MEB_t = \frac{\Delta W_t}{g \cdot \Delta R_t}$$

Table 6  
Marginal Excess Burden of Taxes (pence/per £ of additional revenue: low elasticity case)

Tax instrument	Low elasticity case			High elasticity case		
	MEB	Change in revenue	Hicksian money metric EV	MEB	Change in revenue	Hicksian money metric EV
Capital income tax	-0.350	11305	-3962	-0.660	4449	-2936
Production tax	-0.544	6585	-3582	-0.673	876	-590
Labour income tax	-0.435	7984	-3473	-0.580	8182	-4750
Household consumption tax	-0.517	6911	-3574	-0.669	4519	-3025
Indirect tax on government consumption	-0.540	6629	-3578	-0.540	6629	-3578
Indirect tax on Investment goods	-0.542	6609	-3581	-0.614	344	-211

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Table A1

## Aggregation of 123 sectors into 16 sectors from 1990 Input-Output Sectoral Classification

INDUSTRY/ASSET	1990 I-O Sectors	1990 sectoral code	1995 sectoral code
Agriculture	Agriculture, Forestry, Fishing	1,2,3	1-3
Extraction	Extraction – oil and gas	5	5
Other mining & quarrying	Coal extraction, stone, clay, sand, gravel, metal ores and minerals	4,14, 10	4,6,7
Chemicals	Coke ovens, oil proc, nuclear fuel, inorganic chemicals, organic chemicals, fertilisers, synthetic resins, paints, dyes, printing ink, special chemical for industry, pharmaceutical products, soap and toilet preparations, chemical products, man-made fibres	6, 20-29	35-46
Metals and mineral products	Iron and Steel, Aluminium, other non-ferrous metals, structural clay products, Cement, lime and plaster, concrete, asbestos, abrasive prods, glass, refractory and ceramic goods, metal casting, metal doors, windows, packaging products of metals, industrial plant and steel work, engineers small tools	11-13, 15-19, 30-34, 37	49-61
Engineering	Agricultural machinery and tractors, metal working machine tools, textile etc machinery, process machinery and contractors, mining equipment, mech power transmission equipment, other machinery, ordnance samll arms and ammunition, insulated wires and cables, basic electrical equipment, industrial electrical equipment, telecommunications etc. equipment, electronic components, electronic consumer goods, demestic electric appliances, electric lighting equipment, instrument engineering	35,36,38-52,57	62-76
Food, drinks and tobacco	Oils and fats, slaughtering and meat processing, milk and products, fruit vegetable and fish processing, grain milling and starch, bread, biscuits, sugar, confectionary, animal feeding stuffs, miscellaneous foods, alcoholic drink soft drinks, tobacco	58-70	8-20
Other manufacturing	Motor vehicles and parts, shipbuilding and repairing, aerospace etc, other vehicles, woollen and worsted, cotton spinning and weaving, hosiery and other knitted goods, textile finishing, carpets, jute, leather and leather goods, footwear, clothing furs, household and other textiles, timber and wood products, wooden furniture, pulp, paper and board, paper and board products, printing and publishing, rubber products, processing of plastics, jewellery and coins, sports goods and toys, other goods	53-56, 71-90	21-34, 47-48,77-84
Electricity, gas and water	Electricity production, gas, water supply	7,8,9	85-87
Construction	Construction	91	88
Distribution, hotels, etc.	Wholesale distribution, retail distribution, distribution and vehicles repairs, hotels catering, pubs etc.	92,93,94,95	89-92
Transport, storage, and communication	Railways, road and other inland transport, sea transport, air transport, transport services, postal services, telecommunication	96-102	93-99
Financial sector	Banking and finance, insurance, auxiliary financial services, estate agents, legal services, accountancy services, other professional services, advertising, computing services, other business services, renting of movables, owning and dealing in real estate, research and development	103-114, 118	100-103, 105-114
Public administration	Public administration	115	115
Education, health and social work	Sanitary services, education, health services, recreation and welfare services, personal services, domestic services	116, 117, 119-122	116-123
Housing services	Ownership of dwelling	123	104

Source: General equilibrium model of the UK economy.

Table A2  
A 16 Sector Industry by Industry Input-Output Table of the United Kingdom 1995

Industry	Agriculture	Extraction	Other Mining	Chemicals	Metals	Engineering	Food, drink	Other Manuf.	Utilities	Construction	Distribution	Transport	Financial	Public Admin	Educ. Health,	Housing	Total intermediate	Consumers' expenditure	GGFC	GDFCF	Stocks	Exports	Total final demand	Total
<b>Agriculture</b>	2,096	0	14	27	7	5	12,132	435	0	4	564	48	15	0	148	0	<b>15,495</b>	6,730	42	0	0	1,942	8,713	<b>24,208</b>
<b>Extraction</b>	0	2,439	0	4,697	3	0	0	0	3,622	0	0	0	0	0	0	0	<b>10,762</b>	0	0	0	0	6,942	6,942	<b>17,704</b>
<b>Other Mining</b>	20	0	353	218	846	26	45	130	1,897	401	105	17	8	0	57	0	<b>4,124</b>	339	47	0	0	983	1,369	<b>5,493</b>
<b>Chemicals</b>	1,433	10	37	3,899	433	546	571	1,484	466	737	1,299	1,254	913	0	3,204	19	<b>16,304</b>	3,764	3,116	0	261	28,663	35,804	<b>52,108</b>
<b>Metals</b>	110	162	192	1,225	7,249	6,320	1,831	5,197	50	7,074	503	389	5	0	84	0	<b>30,392</b>	346	588	7,158	779	10,230	19,101	<b>49,493</b>
<b>Engineering</b>	0	576	317	682	1,254	5,705	528	2,432	634	788	848	1,808	1,018	0	1,567	36	<b>18,192</b>	0	1,589	2,613	332	50,923	55,457	<b>73,649</b>
<b>Food, drink</b>	2,797	52	25	356	82	120	6,382	350	64	51	6,589	650	1,058	0	1,796	4	<b>20,377</b>	25,904	411	0	153	10,270	36,737	<b>57,114</b>
<b>Other Manuf.</b>	583	80	134	1,781	1,839	3,005	2,816	16,404	474	4,242	6,702	4,139	8,242	0	3,340	283	<b>54,064</b>	18,082	3,872	8,933	1,185	39,858	71,928	<b>125,992</b>
<b>Utilities</b>	279	0	160	1,330	1,596	1,189	931	1,980	12,273	272	1,201	857	1,184	0	705	23	<b>23,981</b>	16,353	1,323	0	0	62	17,738	<b>41,719</b>
<b>Construction</b>	172	0	122	109	32	56	0	31	0	21,085	603	151	1,985	0	146	3,929	<b>28,420</b>	3,521	4,414	47,764	285	0	55,983	<b>84,404</b>
<b>Distribution</b>	1,005	200	206	1,479	2,489	4,115	1,647	3,724	355	1,371	4,164	2,470	2,276	0	790	0	<b>26,289</b>	111,181	1,229	2,586	0	13,701	128,698	<b>154,987</b>
<b>Transport</b>	245	704	335	1,232	2,047	1,415	1,583	3,614	183	887	14,871	15,642	17,082	0	3,175	198	<b>63,216</b>	19,715	2,637	779	0	12,194	35,324	<b>98,540</b>
<b>Financial</b>	1,949	671	471	4,070	2,781	6,194	4,205	9,177	1,884	10,483	22,425	12,387	50,836	0	13,435	15,221	<b>156,189</b>	25,373	8,458	8,483	0	12,545	54,859	<b>211,047</b>
<b>Public Admin</b>	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	<b>0</b>	0	63,843	0	0	0	63,843	<b>63,843</b>
<b>Educ. Health,</b>	378	1	41	520	253	581	496	2,618	179	242	1,001	1,369	4,031	0	7,756	67	<b>19,535</b>	43,653	46,265	0	0	4,504	94,422	<b>113,957</b>
<b>Housing</b>	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	<b>0</b>	53,269	0	0	0	0	53,269	<b>53,269</b>
<b>Total intermediate</b>	<b>11,067</b>	<b>4,895</b>	<b>2,410</b>	<b>21,626</b>	<b>20,912</b>	<b>29,276</b>	<b>33,168</b>	<b>47,576</b>	<b>22,081</b>	<b>47,638</b>	<b>60,876</b>	<b>41,182</b>	<b>88,652</b>	<b>0</b>	<b>36,201</b>	<b>19,781</b>	<b>487,339</b>	<b>328,229</b>	<b>137,832</b>	<b>78,316</b>	<b>2,995</b>	<b>192,816</b>	<b>740,188</b>	<b>1,227,527</b>
<b>Imports</b>	1,630	989	425	10,639	7,613	15,965	8,827	30,336	3,612	5,151	3,532	4,895	3,949	0	2,960	19	<b>100,541</b>	52,021	9,995	28,174	1,563	2,494	94,248	<b>194,789</b>
<b>Duty on imports</b>	34	6	5	136	101	214	171	405	48	66	51	26	2	0	9	0	<b>1,273</b>	547	91	382	20	32	1,073	<b>2,346</b>
<b>VAT</b>	0	0	0	0	0	0	0	0	0	0	0	218	3,259	0	1,181	0	<b>4,658</b>	33,257	3,915	3,731	0	0	40,902	<b>45,561</b>
<b>Duties and levies</b>	211	2	103	1,175	344	176	460	331	1,378	130	1,275	2,026	896	0	344	36	<b>8,887</b>	22,713	434	0	0	0	23,147	<b>32,034</b>
<b>Other taxes and subsidies</b>	-265	-25	-10	-50	-53	-46	-1,454	-212	-10	-34	-443	-404	-409	0	-186	-6	<b>-3,607</b>	4,559	-577	-45	4	-556	3,384	<b>-223</b>
<b>Value added – Labour</b>	7,143	1,409	1,822	10,151	15,790	18,529	9,691	36,483	5,492	29,947	61,877	35,191	70,149	60,316	69,067	0	<b>433,059</b>	0	0	0	0	0	0	<b>433,059</b>
<b>Value added – Gross profits etc</b>	4,388	10,428	738	8,432	4,786	9,536	6,250	11,074	9,118	1,505	27,820	15,406	44,549	3,527	4,381	33,440	<b>195,376</b>	0	0	0	0	0	0	<b>195,376</b>
<b>Total inputs</b>	<b>24,208</b>	<b>17,704</b>	<b>5,493</b>	<b>52,108</b>	<b>49,493</b>	<b>73,649</b>	<b>57,114</b>	<b>125,992</b>	<b>41,719</b>	<b>84,404</b>	<b>154,987</b>	<b>98,540</b>	<b>211,047</b>	<b>63,843</b>	<b>113,957</b>	<b>53,269</b>	<b>1,227,526</b>	<b>441,325</b>	<b>151,691</b>	<b>110,558</b>	<b>4,582</b>	<b>194,786</b>	<b>902,942</b>	<b>2,130,468</b>

Source: ONS, Input-Output Tables of the United Kingdom, 1995, Siddorn (1999).

Table A4  
Effective Tax rates on capital income by assets for year 1995 used in the UK tax model

INDUSTRY/ASSET	Buildings	P&M long life	P&M short life	Vehicles	Dwellings	P&M long life (new life '95)
Agriculture	46.2	14.6	25.3	16.9	0.0	25.3
Extraction	51.1	15.9	27.8	21.3	0.0	27.8
Other mining & quarrying	44.3	14.6	23.3	21.3	0.0	23.3
Chemicals	39.9	13.0	17.9	21.3	0.0	17.9
Metals and mineral products	39.7	12.0	17.1	21.3	0.0	17.1
Engineering	39.7	12.0	18.3	21.3	0.0	18.3
Food, drinks and tobacco	39.7	12.4	17.8	21.3	0.0	17.8
Other manufacturing	39.7	12.9	19.1	22.7	0.0	19.1
Electricity, gas and water	40.8	13.6	30.0	21.3	0.0	30.0
Construction	39.7	14.6	23.5	21.3	0.0	23.5
Distribution, hotels, etc.	39.7	13.3	23.9	21.3	0.0	23.9
Transport, storage, and communication	39.7	16.4	26.5	18.5	0.0	26.5
Financial sector	50.7	13.3	24.7	21.3	0.0	24.7
Public administration	50.7	13.3	23.8	21.3	0.0	23.8
Education, health and social work	51.3	13.3	22.2	21.3	0.0	22.2
Housing services	0.0	0.0	0.0	0.0	0.0	0.0

Source: P-Tax calculator, Inland Revenue 1998.

Table A5  
Composite indirect tax rates on final demand expressed as percent of net prices for year  
1995

	Tax on household consumption		Tax on investment		Tax in government consumption	
	Domestic sales	Imports	Domestic sales	Imports	Domestic sales	imports
Agric	-10.9	4.9			-7.1	2.6
Minin	5.4	5.4			16.0	16.0
Chemi	163.1	167.2		1.5	14.7	16.3
Metal	17.3	17.3	3.3	4.7	17.5	17.5
Engin	15.4	16.9	5.0	6.6	16.6	18.2
Foodd	47.5	49.0			2.0	3.1
Othma	14.6	16.0	5.4	6.8	10.8	12.3
Power	9.0	9.0			16.9	16.9
Const	13.9	14.0	2.4	2.5	17.0	17.1
Distr	12.8	12.8			7.9	7.9
Trans	5.9	8.7	-2.2	0.3	6.7	9.3
Finan	1.1	1.1	0.3	0.3	6.1	6.1
Educa	6.3	7.5			-0.8	0.3
House	-2.0					

Source: GE data set, Inland Revenue 1998.

## A6

## Elasticity Parameters used in the UK tax model (central case)

INDUSTRY	(a) Elasticity of substitution between labour and capital	(b) Elasticity of substitution among capital assets	(c) Elasticity of substitution among domestic supplies and imports	(d) Elasticity of transformation between domestic supplies and exports	(e) Central tendency values of own price elasticities of household demand functions
Agriculture	0.90	0.90	2.12	1.46	-0.475
Extraction	1.19	1.19	2.82	1.93	-0.305
Other mining & quarrying	1.11	1.11	2.63	1.80	-0.362
Chemicals	1.18	1.18	2.80	1.92	-0.542
Metals and mineral products	1.22	1.22	2.89	1.98	-0.503
Engineering	1.11	1.11	2.63	1.80	-0.275
Food, drinks and tobacco	0.74	0.74	1.75	1.20	-0.795
Other manufacturing	1.18	1.18	2.80	1.92	-0.530
Electricity, gas and water	1.11	1.11	2.63	1.80	-0.000
Construction	0.74	0.74	1.75	1.20	-0.000
Distribution, hotels, etc.	1.18	1.18	2.80	1.92	-0.489
Transport, storage, and communication	1.18	1.18	2.80	1.92	-0.321
Financial sector	1.22	1.22	2.89	1.98	-0.000
Public administration	0.67	0.67	1.58	1.08	-0.481
Education, health and social work	1.18	1.18	2.80	1.92	-0.253
Housing services	0.74	0.74	1.75	1.20	-0.000

Source: These values are comparable to those in Piggott and Whalley (1985).