Economic Forecasting

Mean and Variance Forecast in AR, MA and ARMA Model

(see more on CGH Chapter 20)

h =1 period ahead Forecast in AR(1) Model

$$y_t = \delta + \theta_1 y_{t-1} + e_t$$

$$y_{T+1} = \delta + \theta_1 y_T + e_{T+1} \qquad e_{T+1} \sim N(0,1)$$

$$\hat{Y}_{T+1} = E(Y_{T+1}) = \delta + \theta_1 y_T$$

Error of Forecast

$$\hat{e}_{T+1} = Y_{T+1} - \hat{Y}_{T+1} = \delta + \theta_1 y_T + e_{T+1} - \delta - \theta_1 y_T = e_{T+1}$$

$$\operatorname{var}\left(\hat{e}_{T+1}\right) = \sigma_e^2$$

h =2 periods ahead Forecast in AR(1) Model

$$Y_{T+2} = \delta + \theta_1 y_{T+1} + e_{T+2} \qquad e_{T+2} \sim N(0,1)$$

$$\hat{Y}_{T+2} = E(Y_{T+1}) = \delta + \theta_1 \hat{y}_{T+1}$$

$$\hat{e}_{T+2} = y_{T+2} - \hat{y}_{T+2} = \delta + \theta_1 y_{T+1} + e_{T+2} - \delta - \theta_1 \hat{y}_{T+1} = e_{T+1} + \theta_1 \left(y_{T+1} - \hat{y}_{T+1} \right)$$

$$= e_{T+1} + \theta_1 \left(e_{T+1} \right)$$

$$\operatorname{var}\left(\hat{e}_{T+1} \right) = \sigma_e^2 \left(1 + \theta_1^2 \right)$$

h period ahead Forecast in AR(1) Model

$$y_{T+h} = \delta + \theta_1 y_{T+h-1} + e_{T+h}$$

$$\hat{y}_{T+h} = E(y_{T+h}) = \delta + \theta_1 \hat{y}_{T+h-1}$$

$$\hat{e}_{T+h} = y_{T+h} - \hat{y}_{T+h} = \delta + \theta_1 y_{T+h-1} + e_{T+h} - \delta - \theta_1 \hat{y}_{T+h-1} = e_{T+h} + \theta_1 \left(y_{T+h-1} - \hat{y}_{T+h-1} \right)$$

$$\operatorname{var}\left(\hat{e}_{T+h} \right) = \sigma_e^2 \left(1 + \theta_1^2 + \theta_1^4 + \ldots + \theta_1^{2(h-1)} \right)$$

h =1 period ahead Forecast in MA(1) Model

$$y_t = \mu + e_t + \alpha_1 e_{t-1}$$

$$y_{T+1} = \mu + e_{T+1} + \alpha_1 e_T$$

$$E\left(y_{T+1}\right) = \hat{y}_{T+1} = \mu + \alpha_1 e_T$$

$$\left(y_{T+1} - \hat{y}_{T+1}\right) = \mu + e_{T+1} + \alpha_1 e_T - \mu - \alpha_1 e_T = e_{T+1}$$

$$E\left(y_{T+1} - \hat{y}_{T+1}\right)^2 = \mathrm{var}\left(e_{T+1}\right)^2 = \sigma_e^2$$

h =2 periods ahead Forecast in MA(1) Model

$$y_{T+2} = \mu + e_{T+2} + \alpha_1 e_{T+1}$$

$$E\left(y_{T+2}\right) = \hat{y}_{T+2} = \mu$$

$$\hat{e}_{T+2} = \left(y_{T+2} - \hat{y}_{T+2}\right) = \mu + e_{T+2} + \alpha_1 e_{T+1} - \mu = e_{T+2} + \alpha_1 e_{T+1}$$

$$\text{var}\left(\hat{e}_{T+2}\right) = \text{var}\left(e_{T+2} + \alpha_1 e_{T+1}\right) = \sigma_e^2\left(1 + \alpha_1^2\right)$$

h period ahead forecasts

$$E\left(y_{T+h}\right) = \hat{y}_{T+h} = \mu$$

$$\text{var}\left(\hat{e}_{T+h}\right) = \text{var}\left(e_{T+h} + \alpha_1 e_{T+h-1}\right) = \sigma_e^2\left(1 + \alpha_1^2\right)$$

h =1 period ahead Forecast in ARMA(1,1) Mode

$$Y_t = \delta + \theta_1 y_{t-1} + e_t + \alpha_1 e_{t-1}$$

$$y_{T+1} = \delta + \theta_1 y_{t-1} + e_{T+1} + \alpha_1 e_T$$

$$E\left(y_{T+1}\right) = \hat{y}_{T+1} = \delta + \theta_1 y_{t-1} + \alpha_1 e_T$$

$$\hat{e}_{T+1} = \left(y_{T+1} - \hat{y}_{T+1}\right) = \delta + \theta_1 y_{t-1} + e_{T+1} + \alpha_1 e_T - \delta - \theta_1 y_{t-1} - \alpha_1 e_T = e_{T+1}$$

$$\operatorname{var}\left(\hat{e}_{T+1}\right) = E\left(y_{T+1} - \hat{y}_{T+1}\right)^2 = \operatorname{var}\left(e_{T+1}\right) = \sigma_e^2$$

h =2 period ahead Forecast in ARMA(1,1) Mode

$$y_{T+2} = \delta + \theta_1 y_{t+1} + e_{T+2} + \alpha_1 e_{T+1}$$

$$E\left(y_{T+2}\right) = \hat{y}_{T+2} = \delta + \theta_1 \hat{y}_{t+1}$$

$$\hat{e}_{T+2} = \left(y_{T+2} - \hat{y}_{T+2}\right) = \delta + \theta_1 y_{t+1} + e_{T+2} + \alpha_1 e_{T+1} - \delta - \theta_1 \hat{y}_{t+1}$$

$$\hat{e}_{T+2} = \theta_1\left(y_{t+1} - \hat{y}_{t+1}\right) + e_{T+2} + \alpha_1 e_{T+1} = \left(\theta_1 + \alpha_1\right)e_{T+1} + e_{T+2}$$

$$\operatorname{var}\left(\hat{e}_{T+1}\right) = \operatorname{var}\left[\left(\theta_1 + \alpha_1\right)e_{T+1} + e_{T+2}\right] = \sigma_e^2\left[\left(\theta_1 + \alpha_1\right)^2 + 1\right]$$

h =3 periods ahead Forecast in ARMA(1,1) Model

$$y_{T+3} = \delta + \theta_1 y_{t+2} + e_{T+3} + \alpha_1 e_{T+2}$$

$$E\left(y_{T+3}\right) = \hat{y}_{T+3} = \delta + \theta_1 \hat{y}_{t+2}$$

$$\hat{e}_{T+3} = \left(y_{T+3} - \hat{y}_{T+3}\right) = \delta + \theta_1 y_{t+2} + e_{T+3} + \alpha_1 e_{T+2} - \delta - \theta_1 \hat{y}_{t+2}$$

$$\hat{e}_{T+3} = e_{T+3} + \alpha_1 e_{T+2} + \theta_1\left(y_{t+2} - \hat{y}_{t+2}\right) = e_{T+3} + \alpha_1 e_{T+2} + (\theta_1 + \alpha_1) e_{T+1} + e_{T+2}$$

$$\hat{e}_{T+3} = e_{T+3} + \alpha_1 e_{T+2} + \theta_1\left(y_{t+2} - \hat{y}_{t+2}\right) = e_{T+3} + \alpha_1 e_{T+2} + (\theta_1 + \alpha_1) e_{T+1} + e_{T+2}$$

$$\mathrm{var}\left(\hat{e}_{T+3}\right) = \mathrm{var}\left(e_{T+3} + \alpha_1 e_{T+2} + (\theta_1 + \alpha_1) e_{T+1} + e_{T+2}\right) = \sigma_e^2\left[1 = \left((1+\alpha_1)^2 + (\theta_1 + \alpha_1)^2\right)\right]$$

Estimates, Tests and Forecasts using AR(1) Model

Modelling TBILLS by OLS (using uk_r.xls)
The estimation sample is: 1970 (3) to 2000 (1)

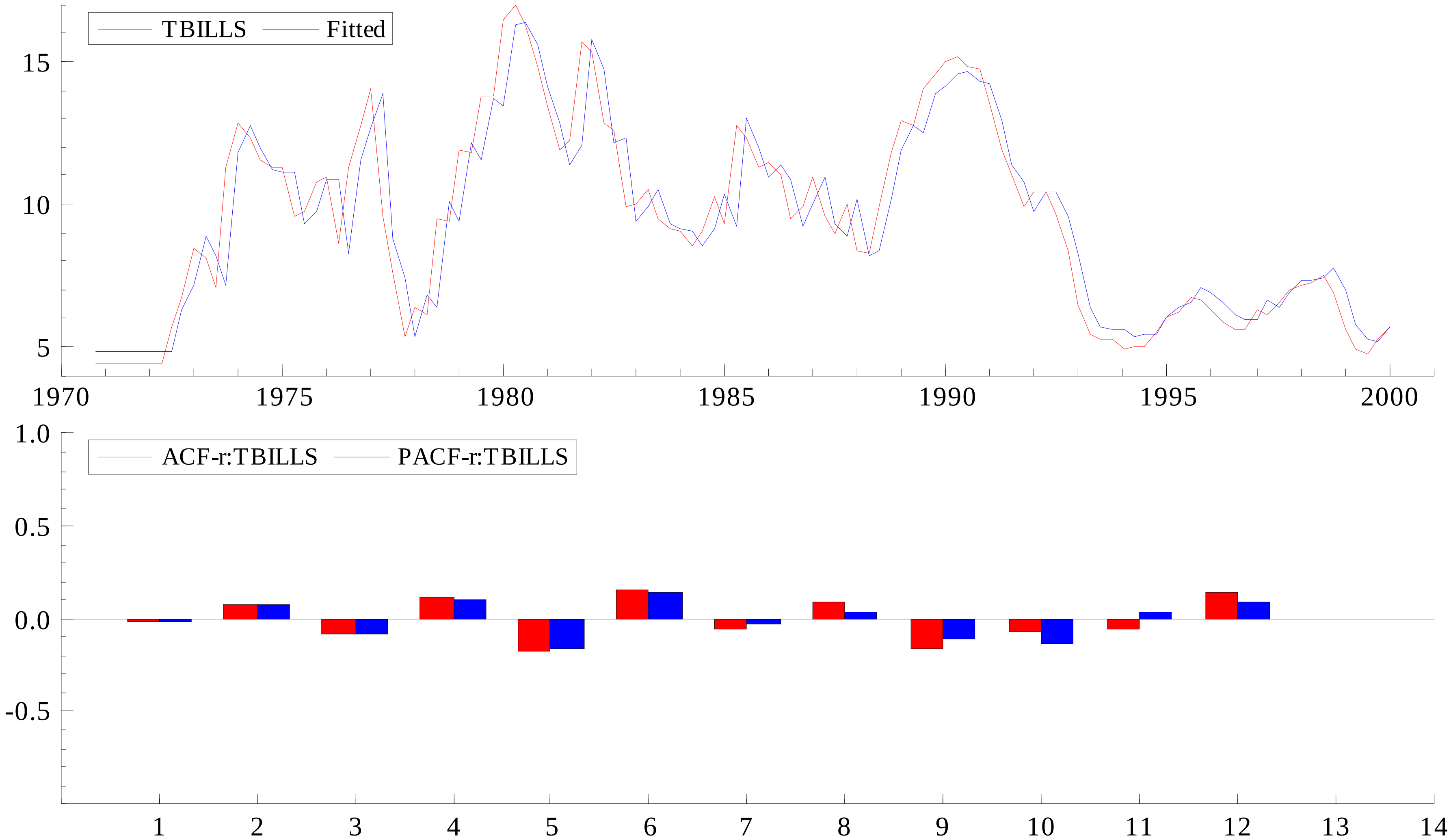
TBILLS = + 0.9232*TBILLS_1 + 0.7311
(SE) (0.0345) (0.344)

sigma	1.26921	RSS	188.473476
R^2	0.859749	F(1,117) =	717.2 [0.000]**
log-likelihood	-196.214	DW	1.69
no. of observations	119	no. of parameters	2
mean(TBILLS)	9.38748	var(TBILLS)	11.2927

AR 1-5 test: F(5,112) = 1.6483 [0.1531]
ARCH 1-4 test: F(4,109) = 1.3557 [0.2541]
Normality test: Chi^2(2) = 12.114 [0.0023]**
hetero test: F(2,114) = 2.5382 [0.0835]
hetero-X test: F(2,114) = 2.5382 [0.0835]
RESET test: F(1,116) = 0.96693 [0.3275]

Horizon	Forecast	(SE)
2000-2	5.99349	1.269
2000-3	6.26445	1.727
2000-4	6.51460	2.038
2001-1	6.74555	2.270
2001-2	6.95877	2.450
2001-3	7.15562	2.594
2001-4	7.33736	2.710
2002-1	7.50515	2.806

Autocorrelation and Partial Autocorrelation in an AR(1) Model



Forecast of Treasury Bills using AR(1) Model