Basic Skills Needed for Economic Forecasting

Mathematics

Statistics

Econometrics

Six rules of approximation for small numbers

I. Log

$$\ln(1+r)=r$$

II. Product

$$(1+\pi)(1+g)\cong(1+\pi+g) \qquad (1+g)^n\cong 1+ng$$

III. Division

$$(1+r)=\frac{1+i}{1+\pi^e}\approx\left(1+i-\pi^e\right)$$

IV. Growth of a product

$$Y=LW \qquad g_Y=g_L+g_W$$

V. Growth of a Ratio

$$y=\frac{Y}{L} \qquad g_y=g_Y-g_L$$

VI. Sum of a geometric Series

$$s=1+X+X^2+....+X^n=\frac{1-X^{n+1}}{1-X}$$

Five Rules of log

I. Power Rule $Y = K^{\alpha}$ $\ln(Y) = \alpha \ln(K)$

II. Product Rule $R = PY$ $\ln(R) = \ln(Y) + \ln(P)$

III. Quotient Rule $y = \frac{Y}{L}$ $\ln(y) = \ln(Y) - \ln(L)$

IV. Log of Exponentials $Y_t = Y_0 e^{gt}$ $\ln(Y_t) = \ln(Y_0) + gt$

V. Differentiation of variable in log with respect to time:

$$\frac{d(\ln Y_t)}{dt} = \frac{dY_t}{Y_t} = g_y$$

Four Rules of Differentiation

I. Power Rule $$Y = K^{\alpha} \qquad \frac{\partial Y}{\partial K} = \alpha K^{\alpha-1}$$

II. Product Rule $$R = PY \qquad dR = Y \times dP + P \times dY$$

III. Quotient Rule $$y = \frac{Y}{L} \qquad dy = \frac{L \times dY - dL \times Y}{L^2} = \frac{dY}{L} - \frac{dL}{L}\frac{Y}{L}$$

IV. Chain Rule $$Y = L^{0.5} \quad L = 50\left(\frac{w}{p}\right)^2 \qquad Y = \left[50\left(\frac{w}{p}\right)^2\right]^{0.5}$$

$$\frac{\partial Y}{\partial (w/p)} = \frac{\partial Y}{\partial L}\frac{\partial L}{\partial (w/p)} = 0.5L^{-0.5}50 \times 2(w/p) = \left(\frac{50}{L}\right)(w/p)$$

Some Example of Matrix Operation

1. Find the determinant of the following matrix.

a) $A=\begin{bmatrix} 2 & 1 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ b) $B=\begin{bmatrix} -7 & 0 & 3 \\ 9 & 1 & 4 \\ 0 & 6 & 5 \end{bmatrix}$

(a)

$$|A| = \begin{vmatrix} 2 & 1 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$ = (2.5.9)+(1.6.7)+(3.8.4)-(7.5.3)-(8.6.2)-(9.4.1)=90+42+96-105-96-36=228-237=-9

(b)

$$|B| = \begin{vmatrix} -7 & 0 & 3 \\ 9 & 1 & 4 \\ 0 & 6 & 5 \end{vmatrix}$$ =(-7.1.5)+(0.4.0)+(3.6.9)-(0.1.3)-(6.4.(-7))-(5.9.0)=-35+0+162-0+168-0=

-35+330=295.

Sum and Multiplication of Vectors

$$e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{bmatrix} \text{ (T rows and 1 column) } e'e = \begin{bmatrix} e_1 & e_2 & e_3 & e_4 & e_5 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{bmatrix} = \sum_i^5 e_i^2 \;;$$

$$ee' = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{bmatrix} \begin{bmatrix} e_1 & e_2 & e_3 & e_4 & e_5 \end{bmatrix} = \begin{bmatrix} e_1^2 & e_1e_2 & e_1e_3 & e_1e_4 & e_1e_5 \\ e_2e_1 & e_2^2 & e_2e_3 & e_2e_4 & e_2e_5 \\ e_3e_1 & e_3e_2 & e_3^2 & e_3e_4 & e_3e_5 \\ e_4e_1 & e_4e_2 & e_4e_3 & e_4^2 & e_4e_5 \\ e_5e_1 & e_5e_2 & e_5e_3 & e_5e_4 & e_5^2 \end{bmatrix}$$

Matrix Inversion

- If the determinant of a matrix is zero then that matrix is called a singular matrix. Singularity reflects linear dependence among explanatory variables.

1. Find the inverse of the following matrix

$$A=\begin{bmatrix} 4 & 1 & -1 \\ 0 & 3 & 2 \\ 3 & 0 & 7 \end{bmatrix}$$

Determinant of this matrix = $|A|$ =99≠0 => the inverse A^{-1} exists.

The cofactor matrix $C_{i,j} = (-1)^{i+j}\left|M_{i,j}\right|$ where $M_{i,j}$ are minors of each element.

$$C = \begin{bmatrix} \begin{vmatrix} 3 & 2 \\ 0 & 7 \end{vmatrix} & -\begin{vmatrix} 0 & 2 \\ 3 & 7 \end{vmatrix} & \begin{vmatrix} 0 & 3 \\ 3 & 0 \end{vmatrix} \\ -\begin{vmatrix} 1 & -1 \\ 0 & 7 \end{vmatrix} & \begin{vmatrix} 4 & -1 \\ 3 & 7 \end{vmatrix} & -\begin{vmatrix} 4 & 1 \\ 3 & 0 \end{vmatrix} \\ \begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 4 & -1 \\ 0 & 2 \end{vmatrix} & \begin{vmatrix} 4 & 1 \\ 0 & 3 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 21 & 6 & -9 \\ -7 & 31 & 3 \\ 5 & -8 & 12 \end{bmatrix} \quad C' = Adj(A) = \begin{bmatrix} 21 & -7 & 5 \\ 6 & 31 & -8 \\ -9 & 3 & 12 \end{bmatrix}$$

The desired inverse matrix is $A^{-1} = \frac{1}{|A|}Adj(A) = \frac{1}{99}\begin{bmatrix} 21 & -7 & 5 \\ 6 & 31 & -8 \\ -9 & 3 & 12 \end{bmatrix}$

1. Solve following equations system using Cramer's rule

$$x_1 + 2x_2 + 2x_3 = 1 \quad (1)$$
$$2x_1 + 2x_2 + 3x_3 = 3 \quad (2)$$
$$x_1 - x_2 + 3x_3 = 5 \quad (3)$$

Answer: First trite this system of three equations and three unknowns in matrix format as following.

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 3 \\ 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} \quad ;$$

A x b

$$|A| = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 2 & 3 \\ 1 & -1 & 3 \end{vmatrix}$$ =1.2.3+2.3.1+2(-1).2-2.2.1-(-1).3.1-3.2.2=6+6-4-4+3-12=12-8+3-12=-5

The solutions for x1, x2, and x3 is defined as following: $x_1 = \dfrac{\begin{vmatrix} 1 & 2 & 2 \\ 3 & 2 & 3 \\ 5 & -1 & 3 \end{vmatrix}}{|A|} = \dfrac{-5}{-5} = 1$

$$x_2 = \frac{\begin{vmatrix} 1 & 1 & 2 \\ 2 & 3 & 3 \\ 1 & 5 & 3 \end{vmatrix}}{|A|} = \frac{5}{-5} = -1 \quad x_3 = \frac{\begin{vmatrix} 1 & 2 & 1 \\ 2 & 2 & 3 \\ 1 & -1 & 5 \end{vmatrix}}{|A|} = \frac{-5}{5} = -1 \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

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Answer: First trite this system of three equations and three unknowns in matrix format as following.

$$\underset{A}{\begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 3 \\ 1 & -1 & 3 \end{bmatrix}} \underset{x}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \underset{b}{\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}} \quad ;$$

$$|A| = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 2 & 3 \\ 1 & -1 & 3 \end{vmatrix}$$ =1.2.3+2.3.1+2(-1).2-2.2.1-(-1).3.1-3.2.2=6+6-4-4+3-12=12-8+3-12=-5

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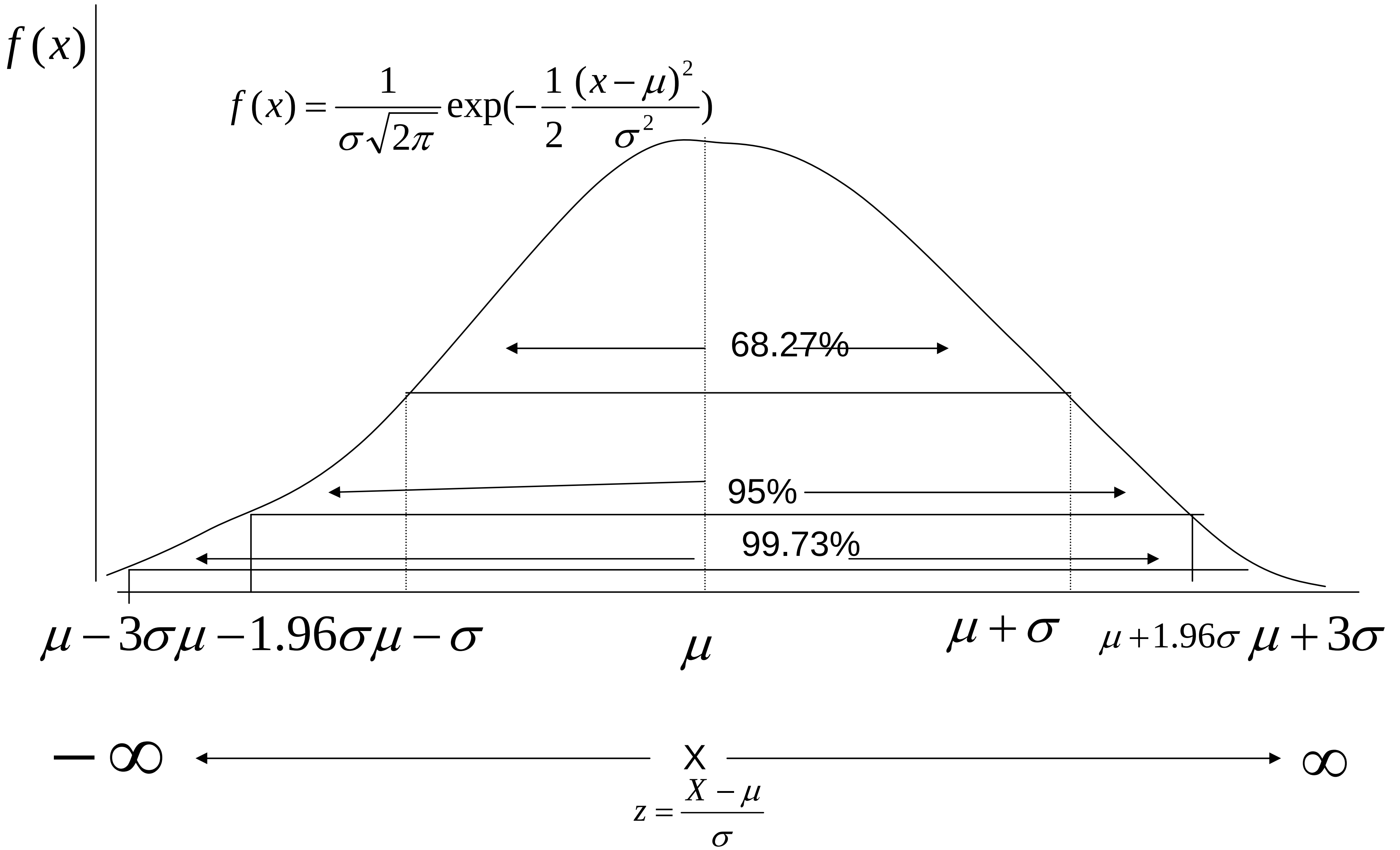
$$x_2 = \frac{\begin{vmatrix} 1 & 1 & 2 \\ 2 & 3 & 3 \\ 1 & 5 & 3 \end{vmatrix}}{|A|} = \frac{5}{-5} = -1 \quad x_3 = \frac{\begin{vmatrix} 1 & 2 & 1 \\ 2 & 2 & 3 \\ 1 & -1 & 5 \end{vmatrix}}{|A|} = \frac{-5}{5} = -1 \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

Normal Distribution of X: Bell Shaped Distribution



Mean

$$\overline{X} = \frac{\sum X_i}{N}$$

Variance

$$\operatorname{var}(X) = \frac{\sum \left(X_i - \overline{X}\right)^2}{N-1}$$

Standard Dev

$$s = \sqrt{\frac{\sum \left(X_i - \overline{X}\right)^2}{N-1}}$$

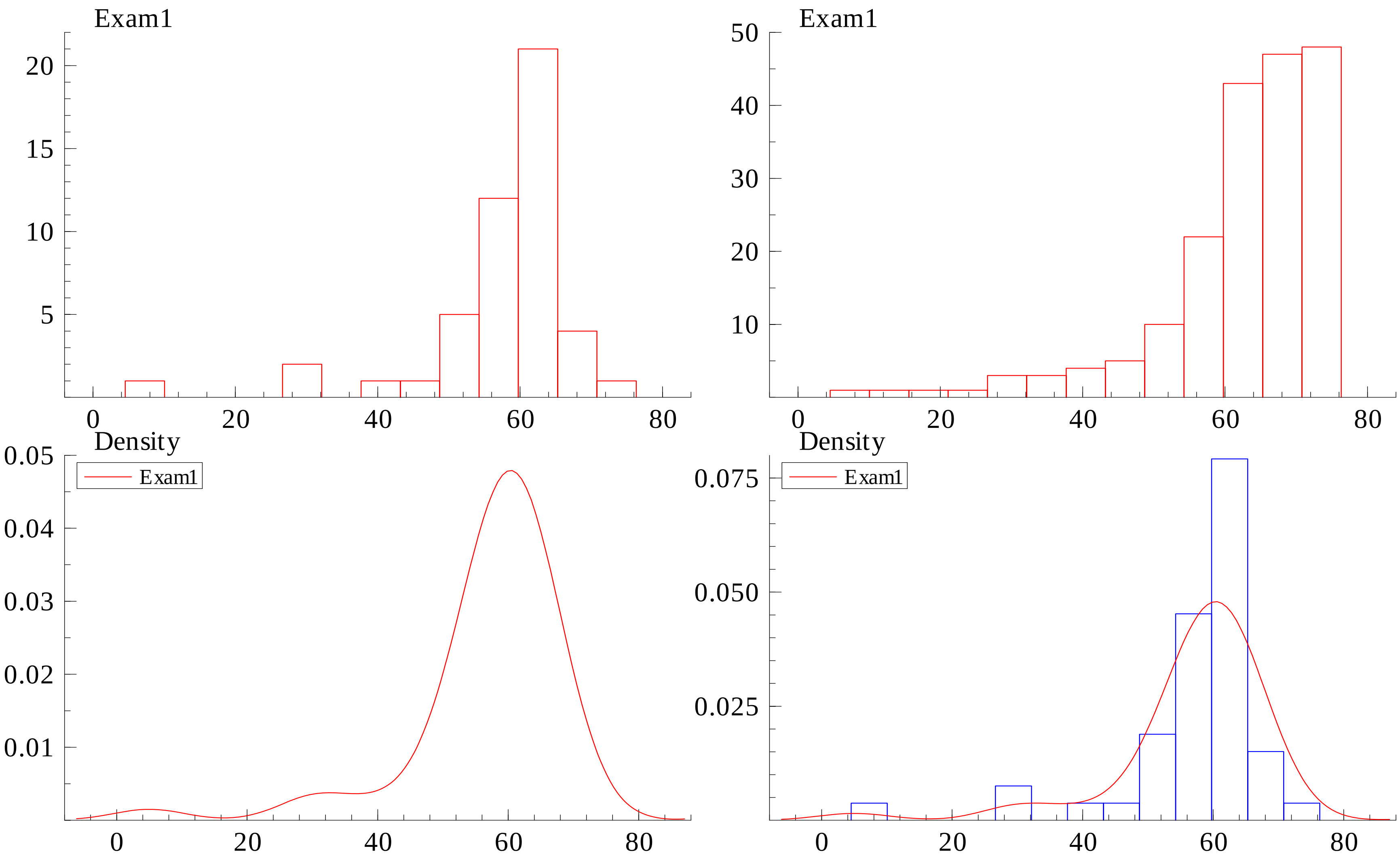
Covariance Correlation and Regression

$$Cov(X,Y) = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{N}$$

$$\rho_{x,y} = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\mathrm{var}(X)\sqrt{Var(Y)}}}$$

$$-1 \leq \rho_{x,y} \leq 1$$

$$Y_i = \beta_1 + \beta_2 K_i + \beta_3 L_i + e_i$$



Exam1
Exam1
Density
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Exam1

Properties of a Discrete Random Variable

$$P(x_i) \geq 0$$

$$\sum_{i=1}^{n} P(x_i) = 1$$

$$E[X_i] = \sum_i X_i P(x_i)$$

$$\text{var}[X_i] = \sum_i (X_i - E(X_i))^2 P(x_i)$$

Continuous Random variable: IQ scores in a certain Exam

Profits of firms
Age of individuals in a country
Income of individuals and households
Consumer's surplus for a given product
Distance travelled by cars in between two destinations
Amount of blood in ones body
Amount of water in a bath tub

$$f(x) = P[X = x_1] \geq 0$$

$$\int_{\infty}^{\infty} f(x).dx = 1$$

$$\int_{a}^{b} f(x).dx = P(a \leq x \leq b)$$

Conditional Distributions

Mutually Exclusive and Exhaustive events vs. Dependent Events

$$f(x,y)$$

$$f(x)=\sum_y f(x,y)$$

$$f(y)=\sum_x f(x,y)$$

$$f(x/y)=\frac{f(x,y)}{f(y)}$$

$$f(y/x)=\frac{f(x,y)}{f(x)}$$

Type II Error: Accepting a Null Hypothesis When it is False

Type I error is more serious than type II error
But when type I error is minimised type II error raises