Game theory:
Repeated Game
Strategic Behaviour Under Uncertainty
Moral Hazard and Adverse Selection

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What is a repeated game?

	E	SO
S	0,0	5,1
A	2,2	5,1

	E	SO
S	0,0	5,1
A	2,2	5,1

	E	SO
S	0,0	5,1
A	2,2	5,1

	E	SO
S	0,0	5,1
A	2,2	5,1

	E	SO
S	0,0	5,1
A	2,2	5,1

E = enter
SO = Stay out

S=smash
A =Accommodate

Nash Solution: Cournot

	A	Ch
A	1600,1600	1600,1600
CH	1600,1600	1600,1600

	A	Ch
A	1600,1600	1600,1600
CH	1600,1600	1600,1600

	A	Ch
A	1600,1600	1600,1600
CH	1600,1600	1600,1600

	E	SO
A	1600,1600	1600,1600
CH	1600,1600	1600,1600

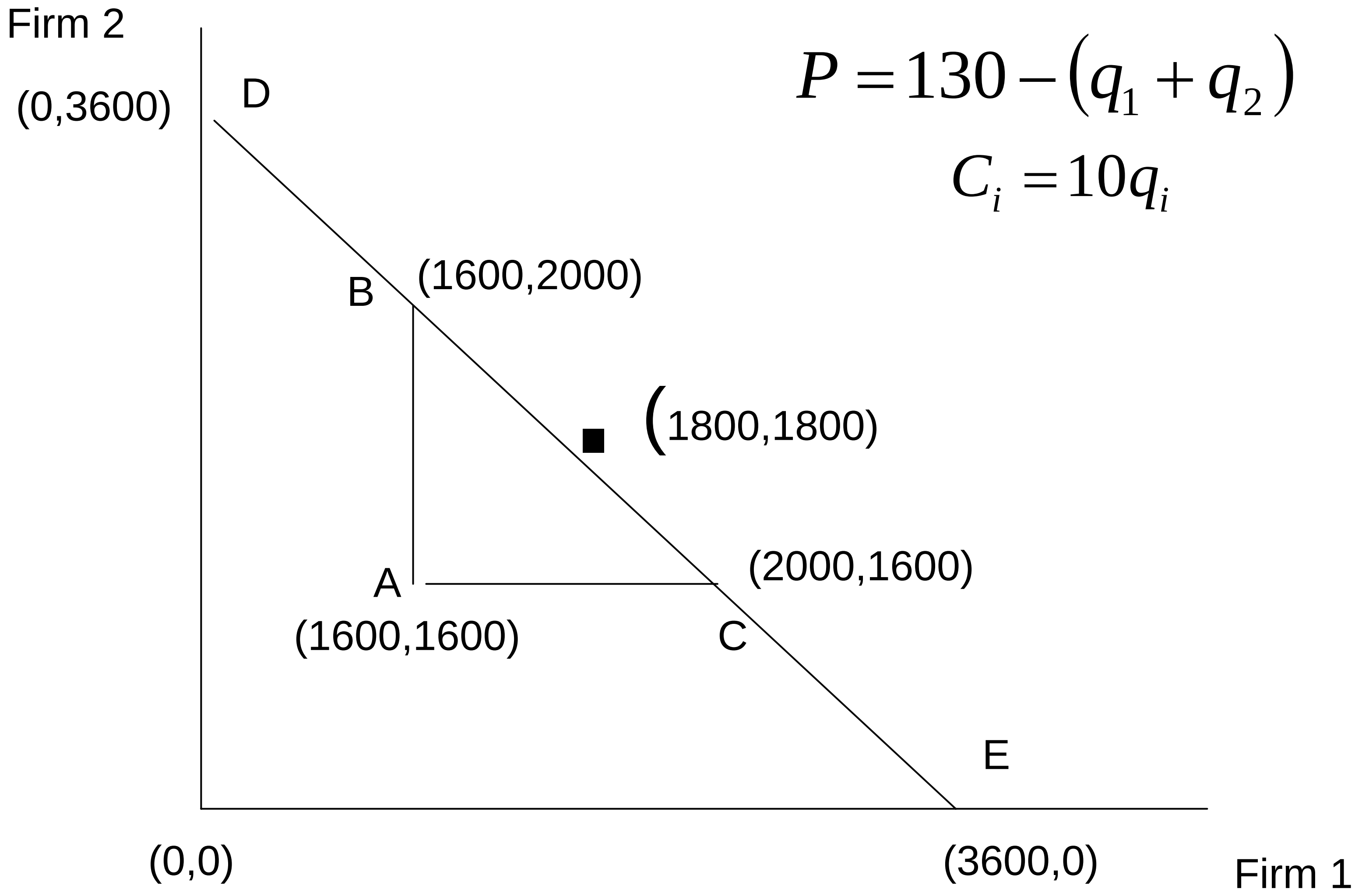
	A	Ch
A	1600,1600	1600,1600
CH	1600,1600	1600,1600

A = stick to agreement
Ch = Cheat

Infinitely Repeated Game in a Duopoly
Profits for firm 1 and 2

$$P = 130 - (q_1 + q_2)$$

$$C_i = 10q_i$$



Cooperative Solution in Infinitely Played Repeated Game

Market demand for a product is $P = 130 - (q_1 + q_2)$

Cost of production for each of two firms is $C_i = 10q_i$.

If played infinite number of time two firms form a cartel and monopolise the market.

Each will supply only 30, set market price to monopoly level at £70 and divide total profit £3600 equally; each getting £1800.

This is shown by (1800,1800) point in the diagram.

It pays to cooperate in the long run; it is sub-game perfect equilibrium.

$$\Pi = (130 - Q)Q - 10Q \; ; \; \frac{\partial \Pi}{\partial Q} = 130 - 2Q - 10 = 0 ; \quad Q = \frac{120}{2} = 60 ;$$

$$P = 130 - Q = 130 - 60 = 70 ; \quad C = 10Q = 600 ;$$

$$\Pi = PQ - C = 70 \times 60 - 10 \times 60 = 4200 - 600 = 3600$$

Non-Cooperative Nash Equilibrium

If any one firm cheats and tries to supply more in order to get more profit; it will be found out by another firm.

It will react to this.

Game will be non-cooperative with resulting in a Cournot Nash equilibrium.

with each firm producing 40 units, market price of 50 and each getting £1600 profits.

$$\Pi_1 = (130 - (q_1 + q_2))q_1 - 10q_1 \text{ and } \Pi_2 = (130 - (q_1 + q_2))q_2 - 10q_2$$

with reaction functions $2q_1 + q_2 = 120$ and $q_1 + 2q_2 = 120$

Total supply is 80, each supplying 40 and making profit is 1600 and market price 50.

Trigger Strategy and Perpetual Punishment

If firm 1 plays Cournot game but firm 2 still plays cartel and supply just 30.

Then from the firm 1' reaction function $2q_1 + q_2 = 120$

$$q_1 = 60 - \frac{1}{2}q_2 = 60 - \frac{1}{2}(30) = 45.$$

If firm 1 supplies 45, market price will be $P = 130 - (q_1 + q_2) = 130 - 45 - 30 = 55$.

This makes profit margin of firm 1 to be 45 and its profit $\Pi_1 = 45 \times 45 = 2025$.

Firm 2 will find out that firm 1 has cheated.

It will also produce according to its reaction curve.

Thus the Nash equilibrium will result with each firm producing 40 and earning 1600 profit for the rest of the periods and the market price will be 50.

Cooperation or Cheating?

Cartel Solution and Agreement

	A	Ch
A	1800,1800	1800,1800
CH	1800,1800	1800,1800

	A	Ch
A	1800,1800	1800,1800
CH	1800,1800	1800,1800

	A	Ch
A	1800,1800	1800,1800
CH	1800,1800	1800,1800

	E	SO
A	1800,1800	1800,1800
CH	1800,1800	1800,1800

	A	Ch
A	1800,1800	1800,1800
CH	1800,1800	1800,1800

A = stick to agreement
Ch = Cheat

Trigger strategy and perpetual punishment for all

	A	Ch
A	1600,1600	1600,1600
CH	1600,1600	1600,1600

	A	Ch
A	1600,1600	1600,1600
CH	1600,1600	1600,1600

	A	Ch
A	1800,1800	1600,2000
CH	2000,1600	1600,1600

	E	SO
A	1600,1600	1600,1600
CH	1600,1600	1600,1600

	A	Ch
A	1600,1600	1600,1600
CH	1600,1600	1600,1600

A = stick to agreement
Ch = Cheat

For whom is it profitable to Cheat?

Does firm 1 gain or lose by deviation from the agreement. For this evaluate the infinite series of profits in deviation and in compliance with agreement.

Present value of profit in case of cheating

$\Pi_1 = (1-\delta)\left[2025 + 1600\delta + 1600\delta^2 + ... + ...\right]$ by adding and subtracting 1600 and applying the formula for infinite series

$$\Pi_1 = (1-\delta)\left[2025 - 1600 + 1600 + 1600\delta + 1600\delta^2 + ... + ...\right] = (1-\delta)\left[425 + \frac{1600}{1-\delta}\right]$$

$$\Pi_1 = 425(1-\delta) + 1600 = 425 - 425\delta + 1600 = 2025 - 425\delta$$

By comparing profits with and without cheating

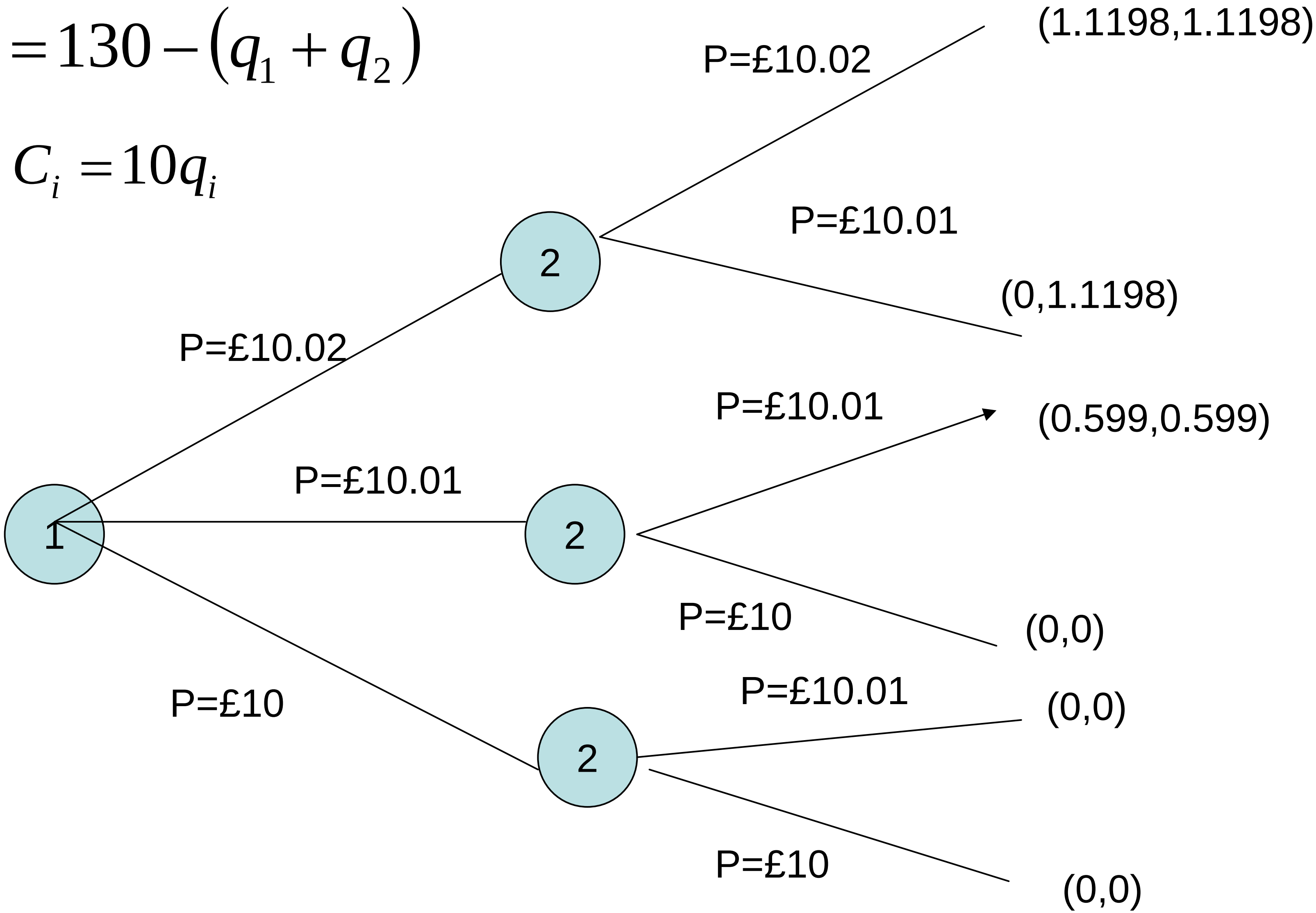
$$2025 - 425\delta < 1800 \text{ or } 425\delta > 2025 - 1800; \quad \delta > \frac{225}{425}; \ \delta > \frac{9}{17}$$

Whether the firm 1 will stick to agreement or not depends on whether its discount factor if greater than $\delta > \frac{9}{17}$. For discount factor $\delta < \frac{9}{17}$ it benefits from sticking to the agreement.

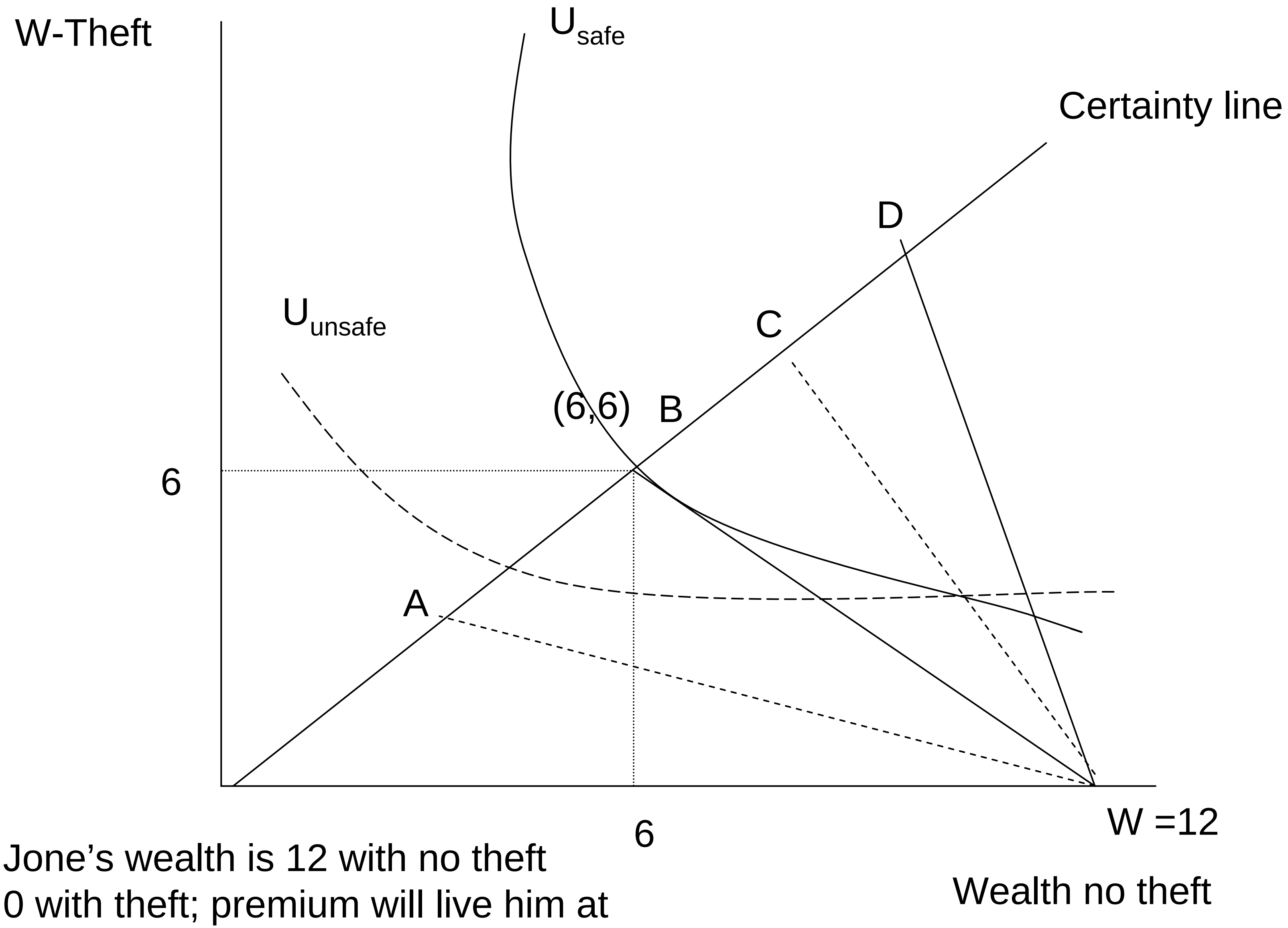
Bertrand-Stackelberg Cut-Throat Competition

$$P = 130 - (q_1 + q_2)$$

$$C_i = 10 q_i$$



Theft Insurance



Jone’s wealth is 12 with no theft
0 with theft; premium will live him at
(6,6) in both states.

Insuring Safe and Unsafe Customers?

Probability of theft reduces with safety precautions of individual.

Theft insurance company does not know who is safe and who is unsafe customer, therefore it aims to maximise its expected profit from the business.

Nature decides 0.5 chances of theft for safe and 0.75 for unsafe.

Property value: 12   Pooling insurance: 6

Safe customers with full insurance get expected utility:

0.5 [12-x] +0.5 [12-x] = 6

Unsafe customer with full insurance get:

0.25 [12-x] +0.75 [12-x] =6

Expected Profit of the Insurance Company

Insurance company's expected profit with safe customer:

$$0.5\,[x] + 0.5\,[x-12] = 3 - 3 = 0$$

Insurance company's expected profit with unsafe customer:

$$0.25\,[x] + 0.75\,[x-12] = 1.5 - 4.5 = -3.0$$

Insurance company believes that 60 percent of consumers are safe and 40 percent are unsafe.

Adjusting for probability of safe and unsafe customer,
insurance companies expected profit is:

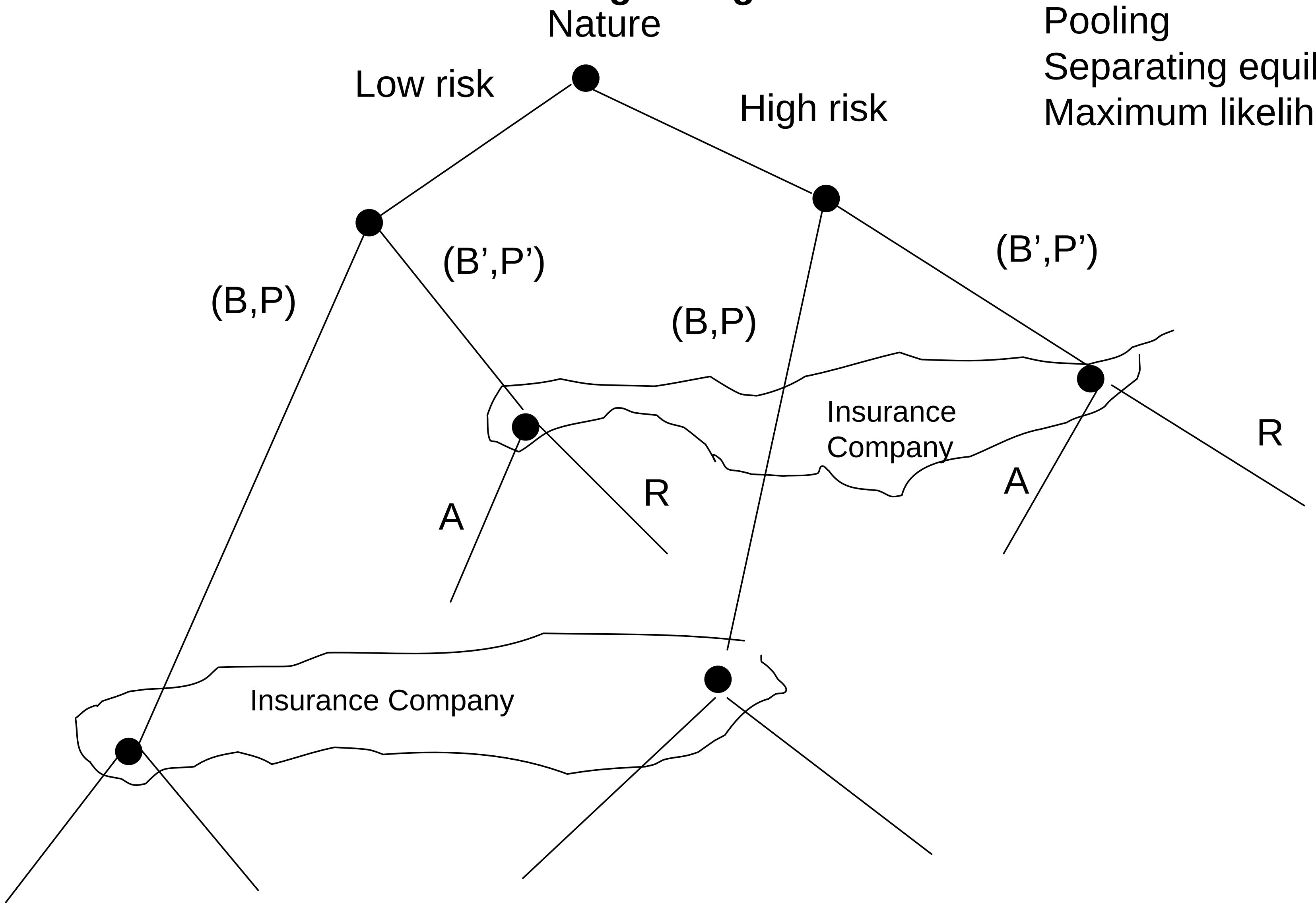
$$0.6\{0.5\,[x] + 0.5\,[x-12]\} + 0.4\{0.25\,[x] + 0.75\,[x-12]\}$$

$$0.6\,(0) - 0.4(-3) = -1.2$$

Calculate for separating insurance: x1 =6 for safe and x2 =8 for unsafe.

Insurance Signalling Game

Pooling
Separating equilibrium
Maximum likelihood ratio



Nature decides whether a customer is high or low risk type but the insurance company does not know which is of which type. It offers insurance schemes suitable for low and high risk individual. Choice of the customer signals the insurance company of its type. Then the insurance company can make an incentive mechanism so that it is beneficial to buy high risk type insurance for high risks and low risk type for low risks.

Insurance Game with Moral Hazard

Let p be insurance premium.

$\pi(e)$ probability of accident with effort e, this diminishes with greater care (higher e).

Level of benefit offered in case of accident is B_L specific to losses L =1,2,….L .

The Moral hazard problem is for insurance company to set the premium according to efforts

$$\underset{e,p,B_0....B_L}{Max} \quad p-\sum_{l=0}^{L}\pi_l(e)B_l \qquad (1)$$

subject to participation constraint:

$$\sum \pi_l(e)u(W-p-l+B_l)-d(e)\geq \bar{u} \qquad (2)$$

Lagrangian function

$$L = p-\sum_{l=0}^{L}\pi_l(e)B_l+\lambda\left[\sum_{l}\pi_l(e)u(W-p-l+B_l)-d(e)-\bar{u}\right] \quad (3)$$

Solving Moral Hazard for Insurance

$$\frac{\partial L}{\partial p} = 1 - \lambda\left[\sum \pi_l(e) u'(W - p - l + B_l)\right] = 0 \quad (4)$$

$$\frac{\partial L}{\partial B_l} = -\pi_l(e) + \lambda \pi_l(e) u'(W - p - l + B_l) = 0 \qquad (5)$$

$$\frac{\partial L}{\partial \lambda} = \sum_l \pi_l(e) u(W - p - l + B_l) - d(e) - \bar{u} = 0 \qquad (6)$$

From (5) $u(W - p - l + B_l) = d(e) + \bar{u}$

Under full insurance $B_l = l$

this implicitly defines the insurance premium for effort level

$$u(W - p(e)) = d(e) + \bar{u} \; .$$

Since low effort is less costly than more effort for the costumer
$d(0) \leq d(1)$;

the premium under lower effort must be set higher than for the higher effort: $p(0) \geq p(1)$ f or profit maximisation

$$p - \sum_{l=0}^{L} \pi_l(e) \cdot l$$

This is the prediction of moral hazard with complete information
but uncertainty with consumer’s hidden action.

Insurance company cannot observe the consumer's choice of accident prevention efforts.

But the insurance company continues to seek maximize he expected profit.

It now need to add incentive compatibility constraint.

$$\underset{e,p,B_0....B_L}{Max} \quad p-\sum_{l=0}^{L}\pi_l(e)B_l \qquad (1)$$

subject to participation constraint:

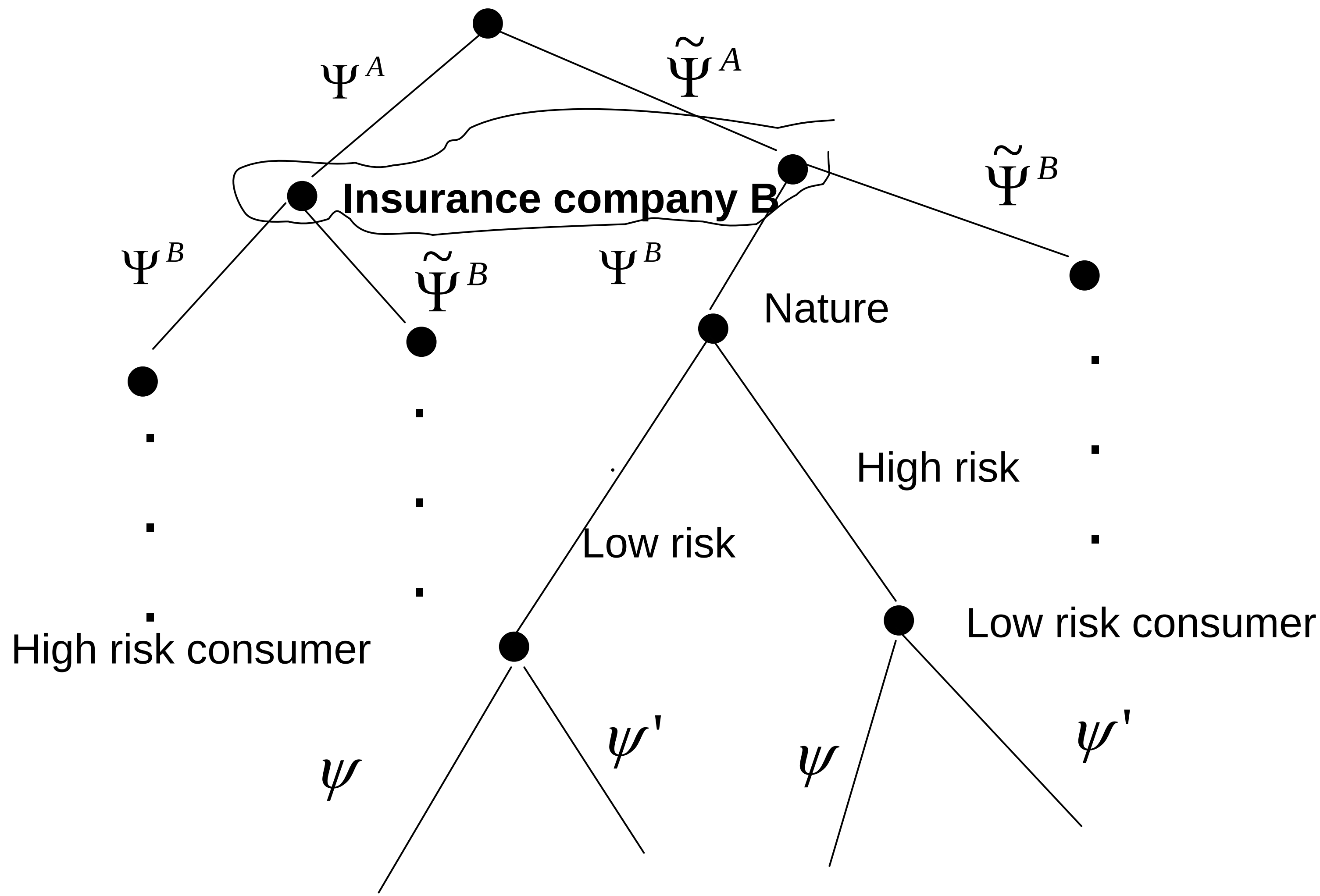
$$\sum_l \pi_l(e)u(W-p-l+B_l)-d(e) \geq \sum_l \pi_l(e')u(W-p-l+B_l)-d(e')$$

$$\sum \pi_l(e)u(W-p-l+B_l)-d(e) \geq \bar{u} \qquad (2)$$

Incentive compatibility constraint

$$\sum_l \pi_l(e)u(W-p-l+B_l)-d(e) \geq \sum_l \pi_l(e')u(W-p-l+B_l)-d(e')$$

Solving Screening Game for an Insurance Company



Insurance company A and B move simultaneously. They do not know who is risky and who is not risky consumer. Consumers reveal their choice by choosing the insurance policy. Then companies guess who is risky and who is safe.

Optimisation Conditions

$$L = p - \sum_{l=0}^{L} \pi_l(e) B_l + \lambda\left[\sum_l \pi_l(e) u(W - p - l + B_l) - d(e) - \bar{u}\right]$$
$$+ \beta\left[\left\{\sum_l \pi_l(e) u(W - p - l + B_l) - d(e)\right\} - \left\{\sum_l \pi_l(e') u(W - p - l + B_l) - d(e')\right\}\right] \quad (3)$$

$$\frac{\partial L}{\partial p} = 1 - \left[\sum \left(\lambda \pi_l(1) + \beta(\pi_l(1) - \pi_l(0))(u'(W - p - l + B_l))\right)\right] = 0 \quad (4)$$

$$\frac{\partial L}{\partial B_l} = -\pi_l(1) + [\lambda \pi_l(1) + \beta(\pi_l(1) - \pi_l(0))] u'(W - p - l + B_l) = 0 \quad (5)$$

$$\frac{\partial L}{\partial \lambda} = \sum_l \pi_l(e) u(W - p - l + B_l) - d(e) - \bar{u} \geq 0$$

$$\frac{\partial L}{\partial \beta} = \left\{\sum_l (\pi_l(1) - \pi_l(0)) u(W - p - l + B_l) + d(0) - d(1)\right\} \geq 0$$

$$\frac{1}{u'(W - p - l + B_l)} = \lambda + \beta\left[1 - \frac{\pi_l(0)}{\pi_l(1)}\right]$$

$$\frac{1}{u'\left(W - p - l + B_l\right)} = \lambda + \beta\left[1 - \frac{\pi_l(0)}{\pi_l(1)}\right]$$

$$\sum_l \pi_l(e)u\left(W - p - l + B_l\right) - d(e) \geq \sum_l \pi_l(e')u\left(W - p - l + B_l\right) - d(e')$$

since $\beta > 0$ the RHS is strictly decreasing, this implies that $u'\left(W - p - l + B_l\right)$ must be strictly increasing for this to happen $l - B_l$ be must increase with effort levels and losses
$l = 0,1,2,......L$. Optimal high policy does not provide full insurance but the deductible payment increases size of loss (Jehle and Reny (2001, Chapter 8).

Optimal policy is crafted so that the utility of benefit from high efforts equals higher costs.

Incentive System: "How can I get someone to do something for me?" :Spence Model

- If a worker puts *x* amount of effort, the land produces $y = f(x)$
- Then the land owner pays worker s(y) .
- The land owner wants to maximise profit

$$\pi = f(x) - s(y) = f(x) - s(f(x))$$

- Worker has cost of putting effort *c*(*x*) and has a reservation utility, $\overline{u}$
- The participation constraint is given by .

$$s(f(x)) - c(x) \geq \overline{u}$$

- Including this constraint, the maximisation problem becomes $Max \quad f(x) - s(f(x))$
- subject to $s(f(x)) - c(x) \geq \overline{u}$

Incentive compatible contract (Varian Chapter 36)

- (a) renting the land where the workers pays a fixed rent R to the owner and takes the residual amount of output, at equilibrium $f\left(x^*\right)-c\left(x^*\right)-R=\overline{u}$
- (b) Take it or leave it contract where the owner gives some amount such as,, $B^*-c\left(x^*\right)=\overline{u}$
- (c) hourly contract $s\left(f\left(x\right)\right)=wx+K$
- (d) sharecropping, in which both worker and owner divide the output in a certain way.
- In (a)-(c) burden of risks due to fluctuations in the output falls on the worker but it is shared by both owner and worker in (d).
- Which of these incentives work best depends on the situation.