Economic Modelling
Lecture 12

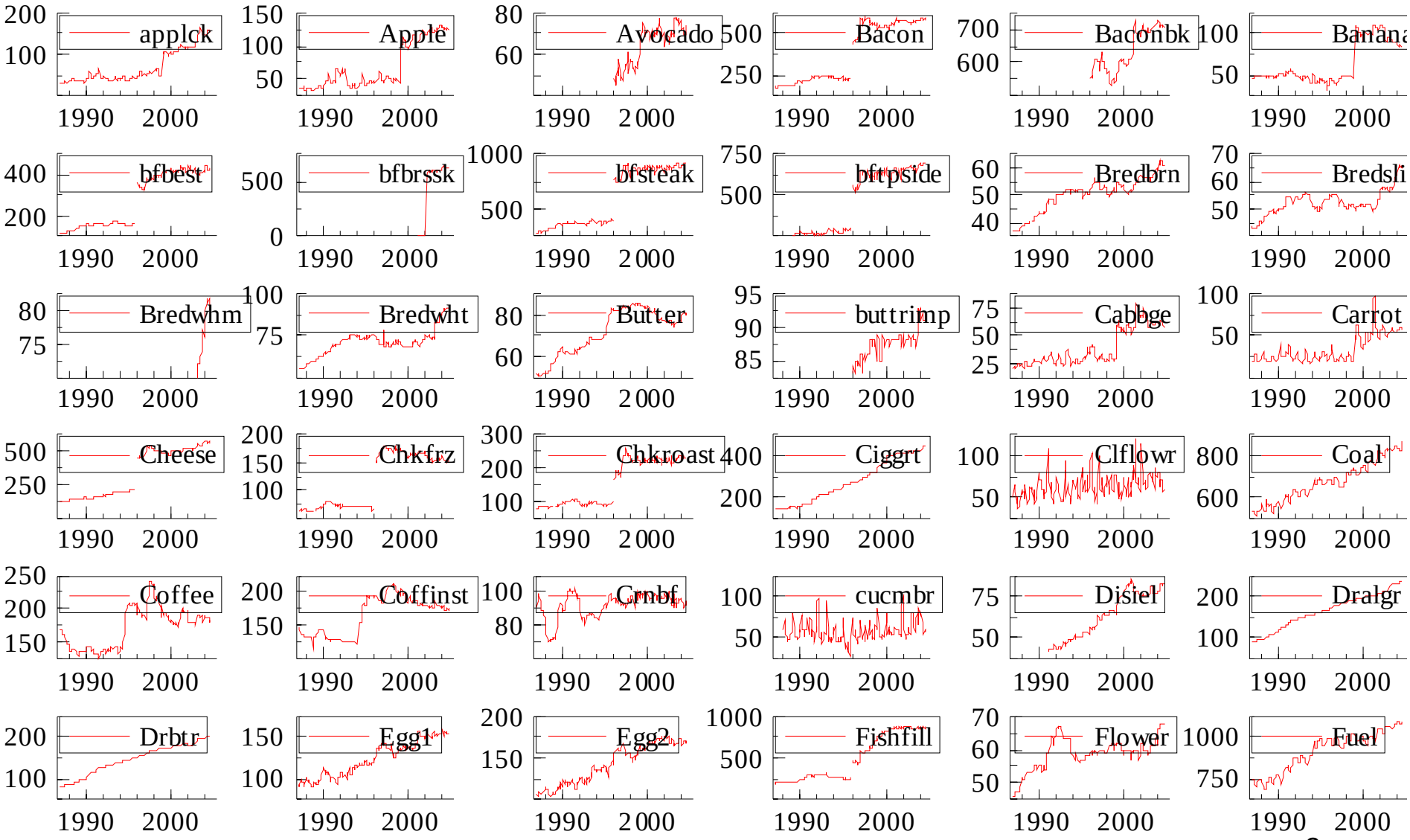
General Equilibrium Model

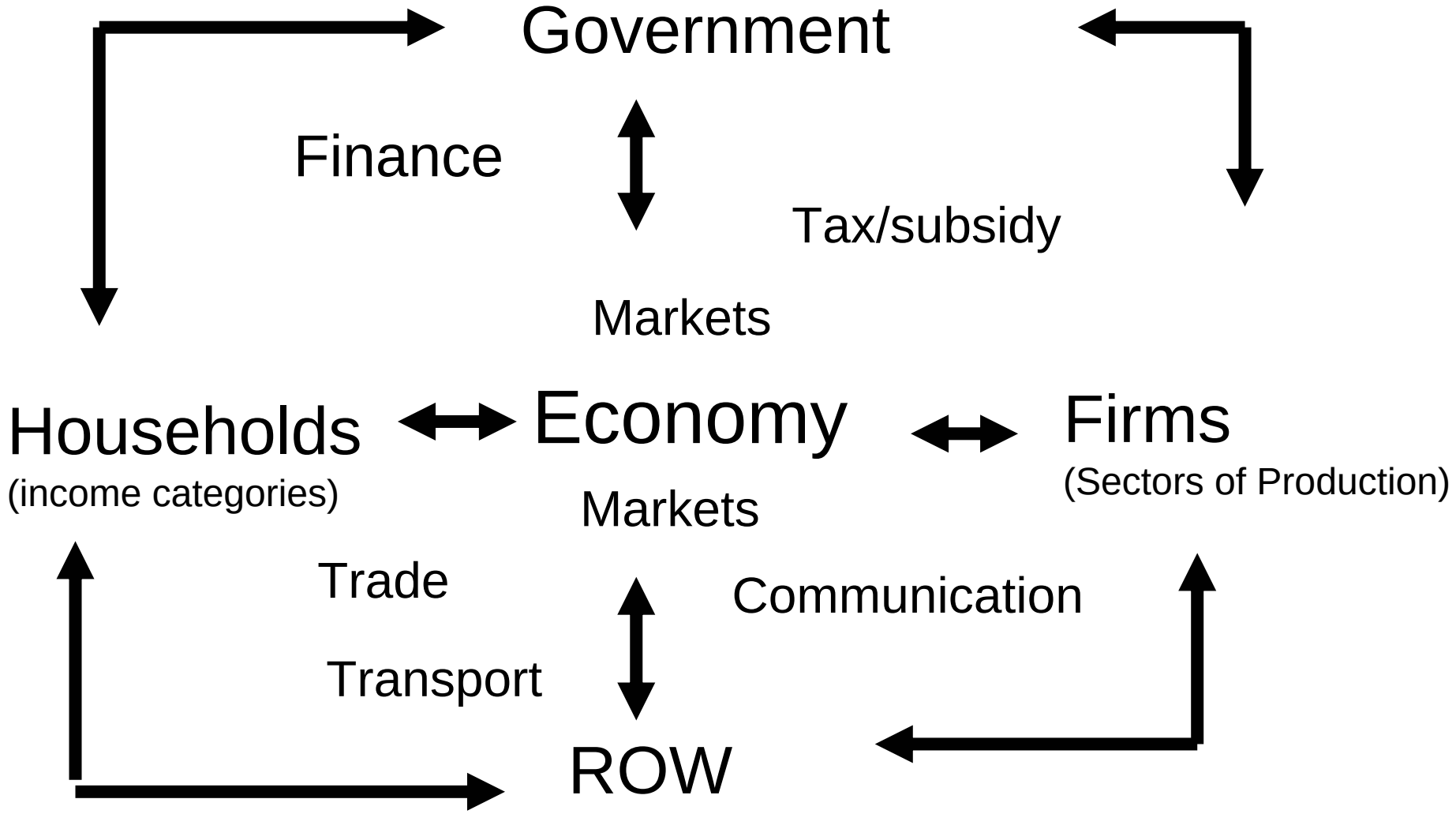
Price System and Optimal allocation in an Economy

Understanding Knock-on Effects in the markets

Two Person Two Goods Pure Exchange Economy

Movement in Some of the Prices in the UK





General Equilibrium Models Assume Clearing of all Markets

But has no trade-off between unemployment and inflation.

Main Features of A General Equilibrium Model

- Three conditions
  - Demand = supply ; n markets, n-1 relative prices
  - Income = expenditure
  - Firms maximise profit: zero economic profit in competitive markets
- Relative Prices
  - Preferences and technology parameters determine relative prices in equilibrium.
  - Relative prices are determined by forces of demand and supply.
  - Numeraire or anchor price; normalised to 1.
- Markets allocations depend on relative prices.
  - Demand for a commodity depends on preferences and income.
  - Income of a household is determined by her endowment and price of that endowment.
- Exchange or trade of goods is mutually beneficial.
  - Each consumer/ producer optimises in equilibrium.

Basics of A General Equilibrium Model: An Example of a Pure Exchange Economy

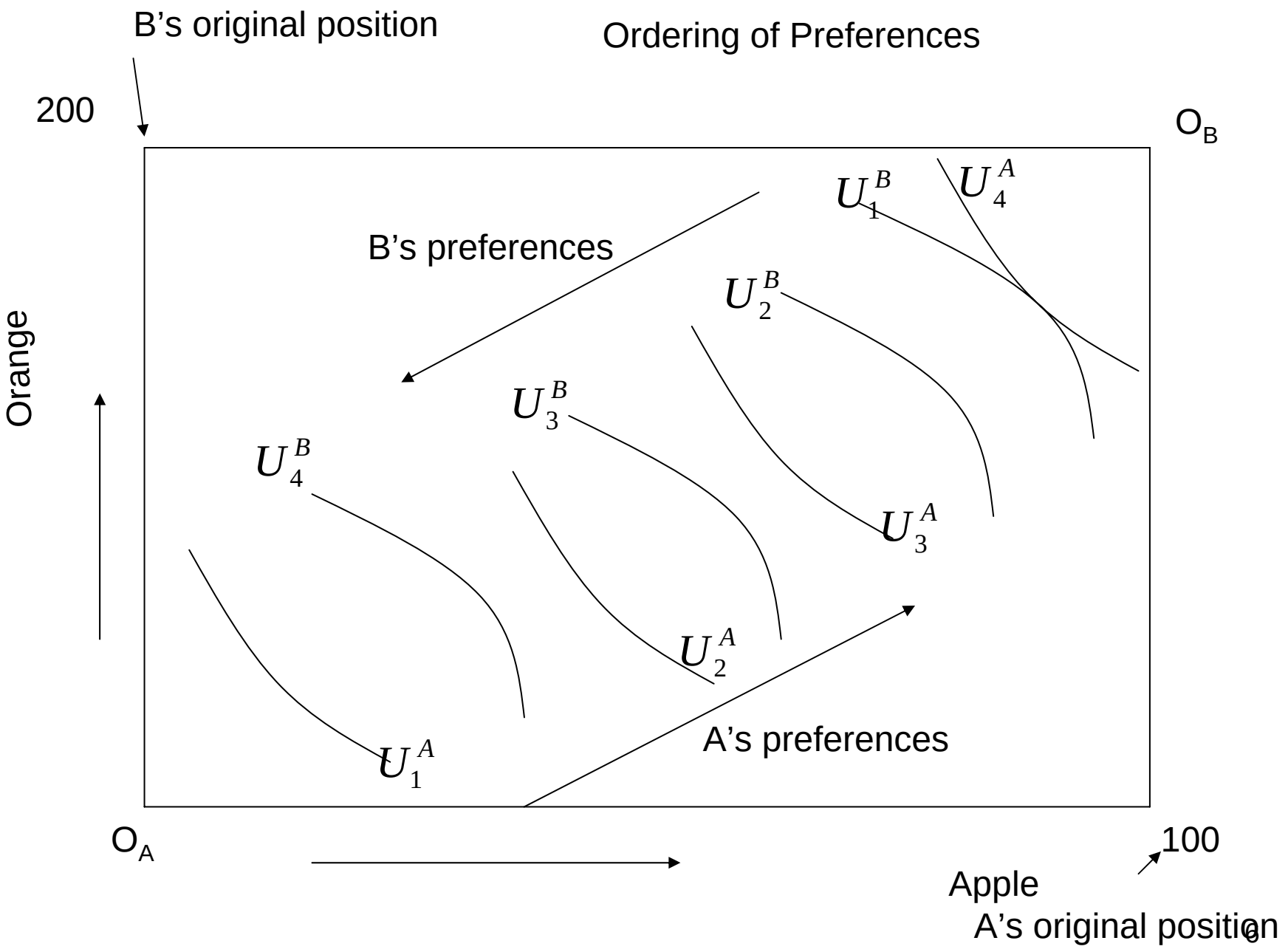
- Two goods: apples and oranges.
- Households **A and B**
- Production or **Supply (Endowments)**

	A	B
apples	100	0.00
oranges	0.00	200

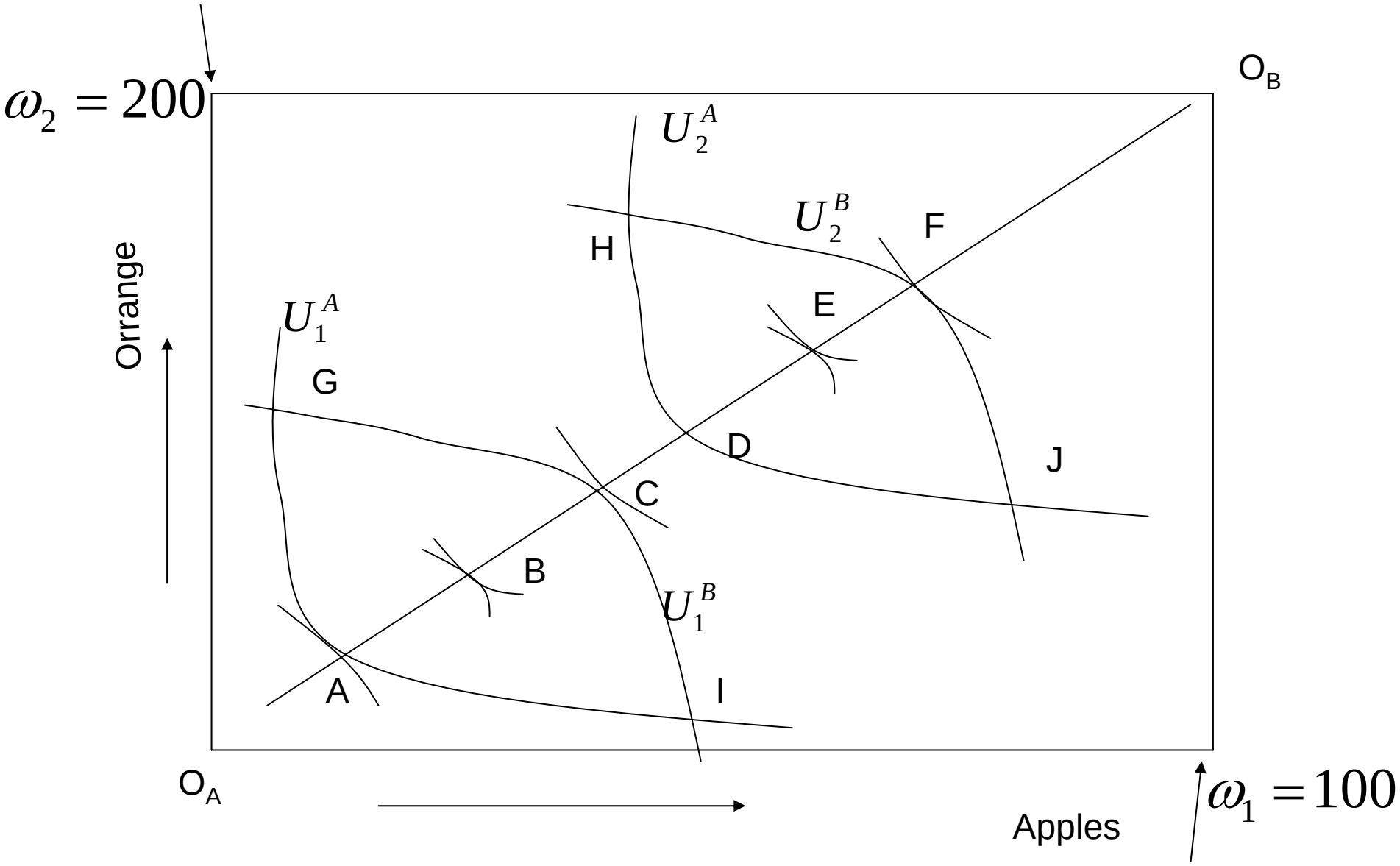
- Both have Cobb-Douglas **preferences**.
- **Allocation** of income between apples and oranges

	A	B
apples	40%	60%
oranges	60%	40%

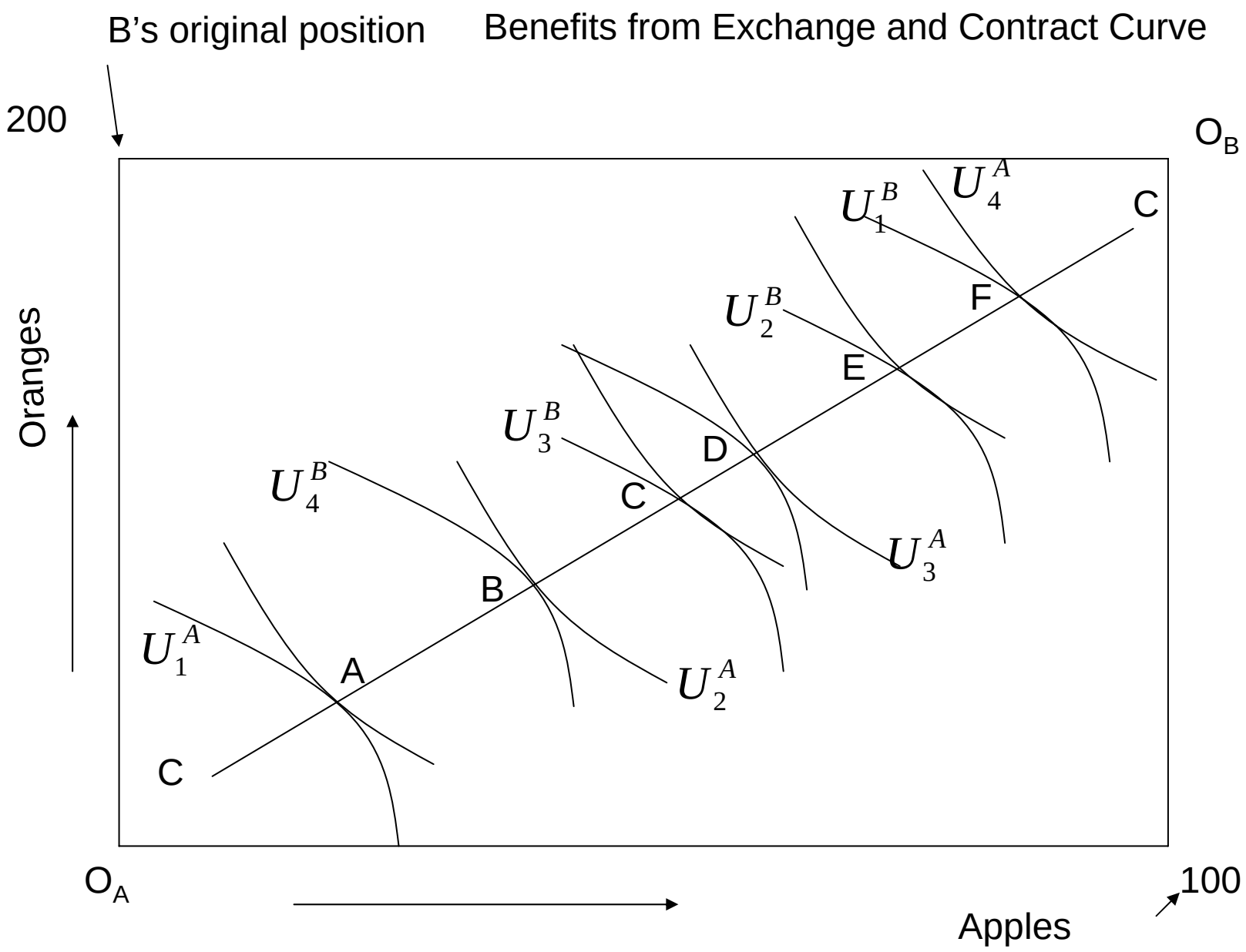
- **Market structure is competitive** and everything is consumed.
- Let apple be X1 and orange be X2.

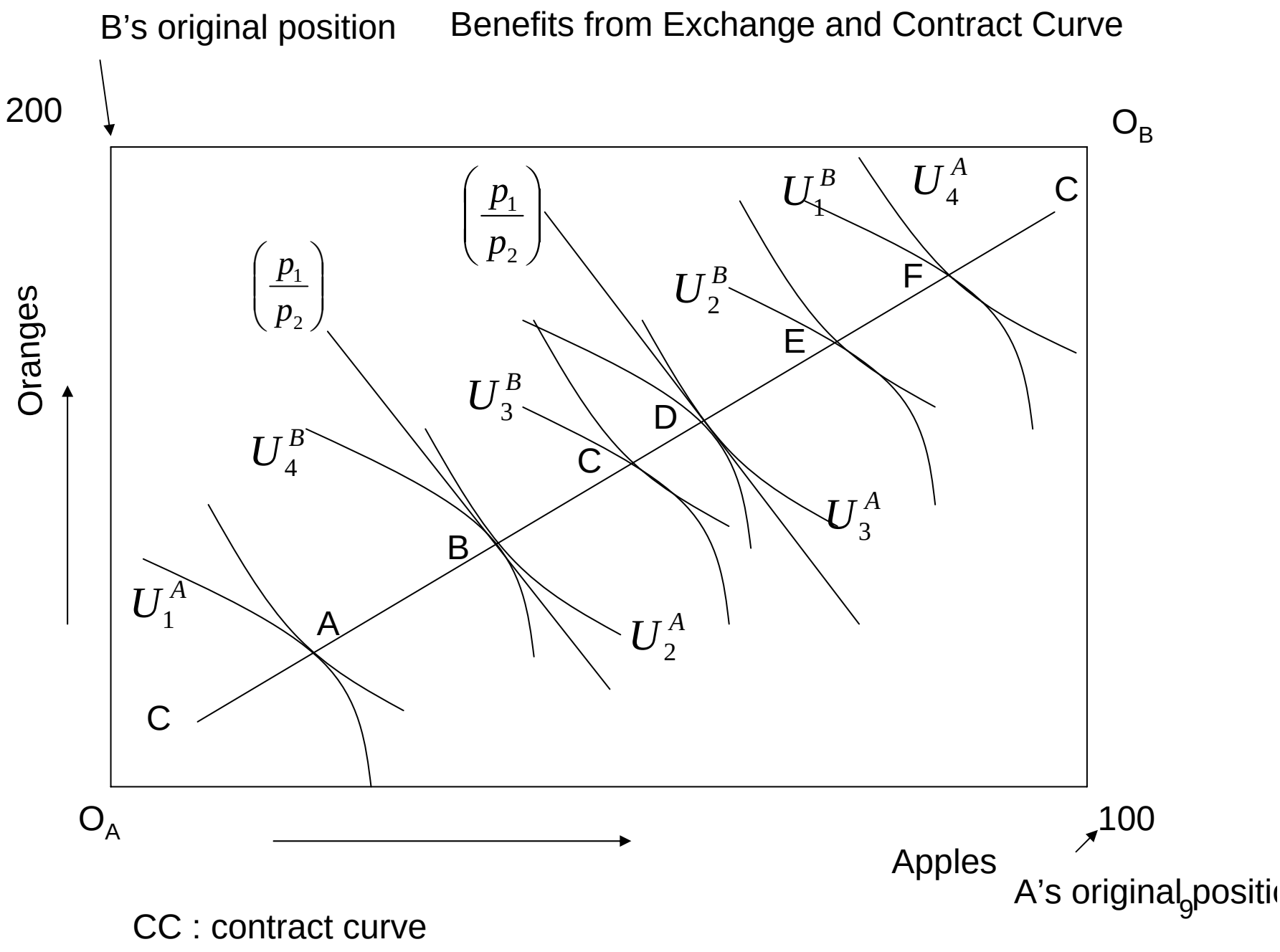

B's original position
Ordering of Preferences
200
O_B
U_1^B
U_4^A
B's preferences
U_2^B
Orange
U_3^B
U_4^B
U_3^A
U_2^A
U_1^A
A's preferences
O_A
100
Apple
A's original position

Pareto Optimality Conditions in Two good Two person Economy



Any allocation is possible within this Edgeworth box.


B's original position
Benefits from Exchange and Contract Curve
200
O_B
Oranges
U_1^B
U_4^A
C
F
U_2^B
E
U_3^B
D
U_4^B
C
U_3^A
B
U_1^A
A
U_2^A
C
O_A
100
Apples
CC : contract curve
A's original position


B's original position
Benefits from Exchange and Contract Curve
200
O_B
Oranges
$\left(\frac{p_1}{p_2}\right)$
$\left(\frac{p_1}{p_2}\right)$
U_1^B
U_4^A
C
U_2^B
F
E
U_3^B
D
U_4^B
C
U_3^A
B
U_1^A
A
U_2^A
C
O_A
100
Apples
A's original positi
CC : contract curve

Household A's Problem

$$\underset{X_1^A, X_2^A}{Max} \quad U^A = \left(X_1^A\right)^{\alpha}\left(X_2^A\right)^{(1-\alpha)}$$

$$Subj\ to \quad I^A = P_1\omega_1$$

$$\omega_1^A = 100 \qquad \alpha = 0.4$$

Household B's Problem

$$\underset{X_1^B, X_2^B}{Max} \quad U^B = \left(X_1^B\right)^{\beta}\left(X_2^B\right)^{(1-\beta)}$$

$$Subj\ to \quad I^B = P_2\omega_2$$

$$\omega_2^B = 200 \qquad \beta = 0.6$$

Question

What are the relative prices $\left(\frac{p_1}{p_2}\right)$ that bring optimal allocations in this economy or solves above problems?

What are the demands? $X_1^A \quad X_2^A \quad X_1^B \quad X_2^B$

What are income and welfare? $I^A \quad I^B \quad U^B \quad U^A$

What are the prices? $P_1 \quad P_2$ **Numeraire:** $P_1 = 1$

Demand, Supply and Preferences

$$X_1^A = \frac{\alpha I^A}{P_1} \qquad X_2^A = \frac{(1-\alpha) I^A}{P_2}$$

$$X_1^B = \frac{\beta I^B}{P_1} \qquad X_2^B = \frac{(1-\beta) I^B}{P_2}$$

$$\omega_1 = \omega_1^A + \omega_1^B \qquad \omega_1^A = 100 \qquad \omega_1^B = 0$$

$$\omega_2 = \omega_2^A + \omega_2^B \qquad \omega_2^B = 200 \qquad \omega_2^A = 0$$

$$\alpha = 0.4 \qquad \beta = 0.6$$

Market Clearing Conditions

$$X_1^A + X_1^B = \omega_1 \longrightarrow \frac{\alpha I^A}{P_1} + \frac{\beta I^B}{P_1} = \omega_1$$

$$X_2^A + X_2^B = \omega_2 \longrightarrow \frac{(1-\alpha) I^A}{P_2} + \frac{(1-\beta) I^B}{P_2} = \omega_2$$

Where:

$$I^A = P_1 \omega_1 \qquad I^B = P_2 \omega_2$$

Prices and Quantities in Equilibrium

$$\frac{\alpha P_1 \omega_1}{P_1} + \frac{\beta P_2 \omega_2}{P_1} = \omega_1 \quad P_1 = 1 \quad \alpha_1 \omega_1 + \beta P_2 \omega_2 = \omega_1$$

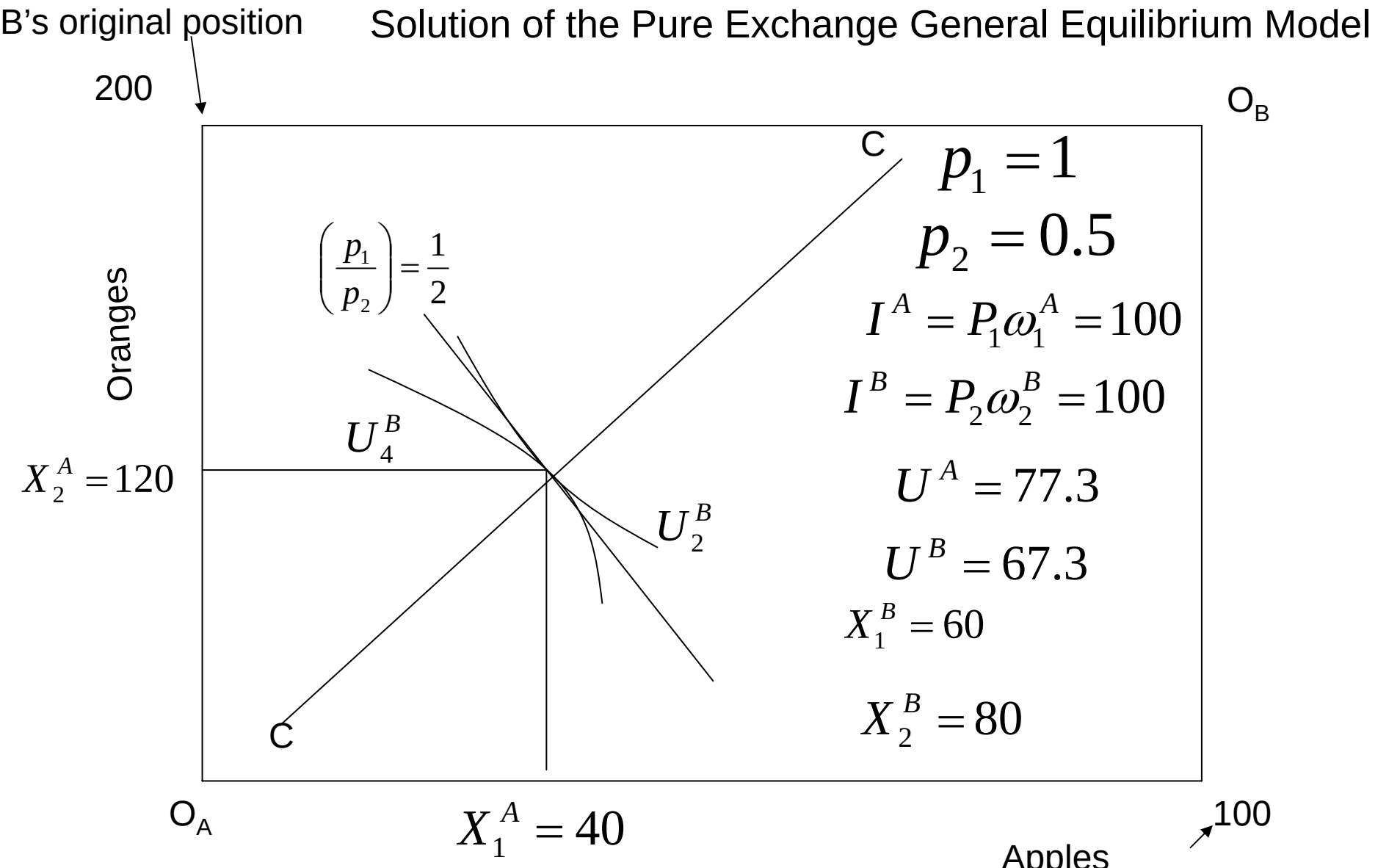
$$100(0.4) + 200(0.6)P_2 = 100 \qquad 0.4 + 1.2P_2 = 1$$

$$P_2 = \frac{0.6}{1.2} = 0.5$$

$$X_1^A = \frac{\alpha I^A}{P_1} = \frac{0.4 \times 100}{1} = 40 \qquad X_2^A = \frac{(1-\alpha) I^A}{P_2} = \frac{0.6 \times 100}{0.5} = 120$$

$$X_1^B = \frac{\beta I^B}{P_1} = \frac{0.6 \times 100}{1} = 60 \qquad X_2^B = \frac{(1-\beta) I^B}{P_2} == \frac{0.4 \times 100}{0.5} = 80$$

Solution of the Pure Exchange General Equilibrium Model



CC : contract curve

Theoretical Observations on Role of Preferences Income and Welfare of Households

- Relative prices of goods, income and amounts of consumption change when preferences change.
- Change in the relative income affects the level of utility and welfare of households
- Household A can make household B worse off by increasing the demand of good 1 that he owns (or supplying less to the market).
- Household B can increase his relative income and reduce the relative price of good 1 by increasing the demand for good 2 (reducing its supply).
- Relative prices and allocations depend on preferences and endowments.

Sensitivity Analysis -1

$0 < \alpha < 1$ Share of consumption of good 1 by consumer A

alpha	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Beta	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
Income A	100	100	100	100	100	100	100	100	100
Income B	150	133.3333	116.6667	100	83.33333	66.66667	50	33.33333	16.66667
P1	1	1	1	1	1	1	1	1	1
P2	0.75	0.666667	0.583333	0.5	0.416667	0.333333	0.25	0.166667	0.083333
X1A	10	20	30	40	50	60	70	80	90
X1B	90	80	70	60	50	40	30	20	10
X2A	120	120	120	120	120	120	120	120	120
X2B	80	80	80	80	80	80	80	80	80
UA	93.59726	83.85925	79.17047	77.32728	77.45967	79.17047	82.28544	86.75774	92.62674
UB	85.85814	80	73.84053	67.31731	60.34176	52.78032	44.41286	34.82202	22.97397

$0 < \beta < 1$ Share of consumption of good 1 by consumer B

Relative price of good 2 falls as demand for good 1 by consumer A rises.

Sensitivity Analysis -2

alpha	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
Beta	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Income A	100	100	100	100	100	100	100	100	100
Income B	600	300	200	150	120	100	85.714	75	66.667
P1	1	1	1	1	1	1	1	1	1
P2	3	1.5	1	0.75	0.6	0.5	0.4286	0.375	0.3333
X1A	40	40	40	40	40	40	40	40	40
X1B	60	60	60	60	60	60	60	60	60
X2A	20	40	60	80	100	120	140	160	180
X2B	180	160	140	120	100	80	60	40	20
UA	26.39	40	51.017	60.629	69.314	77.327	84.82	91.896	98.625
UB	161.27	131.5	108.58	90.943	77.46	67.317	60	55.326	53.758

A simple input-output table for a two sector economy

	Primary	***Secondary***	***C***	***I***	***G***	***X***	***M***	***Total***
Primary	x_{11}	x_{12}	C_1	I_1	G_1	X_1	M_1	GY_1
Secondary	x_{21}	x_{22}	C_2	I_2	G_2	X_2	M_2	GY_2
Labour	L_1	L_2						***wL***
Lab-tax	$T_{l,k}$	$T_{l,k}$						***TL***
Capital	K_1	K_2						***rK***
Cap taxes	$T_{1,k}$	$T_{2,k}$						***TK***
Total	GY_1	GY_2	***C***	***I***	***G***	***X***	***M***	

Three Sector Input-Output Table of UK, 2002 (in Million £)
Derived from 123 sector IO Table

	Manufacturing	Distribution	Services	Consumption	Government	Investment	Exports	Total demand
Manufacturing	173863	22397	48669	79745	0	70629	135961	531265
Distribution	23676	5667	10841	116237	0	4611	20789	181822
Services	63622	48036	186526	160908	141031	11088	39238	650448
Import	73133	10542	34791	51083	0	29947	7555	207052
Tax	6715	6559	22744	51875	0	5564	-33	93424
Wage	114014	60487	212218	0	0	0	0	386718
Capital	76241	28134	134659	0	0	0	0	239034
Total	531265	181822	650447	459848	141031	121839	203511	

http://www.statistics.gov.uk economy/input-output

Tax	Spending	balance	Income	Consumption	saving
93424	141031	-47607	625752	459848	165904
Investment	Sav - Inv	National saving	Exorts	imports	Tradebal
121839	44065	-3542	203511	207052	-3541

Leontief Coefficients in Input-Output Model

$$X_1 = X_{1,1} + X_{1,2} + X_{1,3} + C_1 + I_1 + G_1 + X_1$$

$$X_2 = X_{2,1} + X_{2,2} + X_{2,3} + C_2 + I_2 + G_2 + X_2$$

$$X_3 = X_{3,1} + X_{3,2} + X_{3,3} + C_3 + I_3 + G_3 + X_3$$

$a_{i,j}$ share of input from row sector i to sector j

$$a_{1,1} = \frac{X_{1,1}}{X_1} \quad a_{1,2} = \frac{X_{1,2}}{X_2} \quad a_{1,3} = \frac{X_{1,3}}{X_3} \quad a_{2,1} = \frac{X_{2,1}}{X_1} \quad a_{2,2} = \frac{X_{2,2}}{X_2} \quad a_{2,3} = \frac{X_{2,3}}{X_3}$$

$$X_1 = a_{1,1}X_1 + a_{1,2}X_2 + +a_{1,3}X_3 + F_1$$

$$X_2 = a_{2,1}X_1 + a_{2,2}X_2 + a_{2,3}X_3 + F_2$$

$$X_3 = a_{3,1}X_1 + a_{3,2}X_2 + a_{3,3}X_3 + F_3$$

Analysis of Structure of Aggregate Supply

	Manf	Dis	Serv	Cons	Gov	In v	Exp
Manf	0.33	0.12	0.07	0.17	0.00	0.58	0.67
Dis	0.04	0.03	0.02	0.25	0.00	0.04	0.10
Serv	0.12	0.26	0.29	0.35	1.00	0.09	0.19
Import	0.14	0.06	0.05	0.11	0.00	0.25	0.04
Tax	0.01	0.04	0.03	0.11	0.00	0.05	0.00
wage	0.21	0.33	0.33	0.00	0.00	0.00	0.00
capital	0.14	0.15	0.21	0.00	0.00	0.00	0.00
Gross outp	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Analysis of Structure of aggregate Demand

	Manf	Dis	Serv	Cons	Gov	In v	Exp	al demand
Manf	0.33	0.04	0.09	0.15	0.00	0.13	0.26	1.00
Dis	0.13	0.03	0.06	0.64	0.00	0.03	0.11	1.00
Serv	0.10	0.07	0.29	0.25	0.22	0.02	0.06	1.00
Import	0.35	0.05	0.17	0.25	0.00	0.14	0.04	1.00
Tax	0.07	0.07	0.24	0.56	0.00	0.06	0.00	1.00
wage	0.29	0.16	0.55	0.00	0.00	0.00	0.00	1.00
capital	0.32	0.12	0.56	0.00	0.00	0.00	0.00	1.00
Gross outp	0.23	0.08	0.28	0.20	0.06	0.05	0.09	1.00

References

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