

Economic Modelling

Lecture 2: Endogenous Growth Models and Role of Human Capital

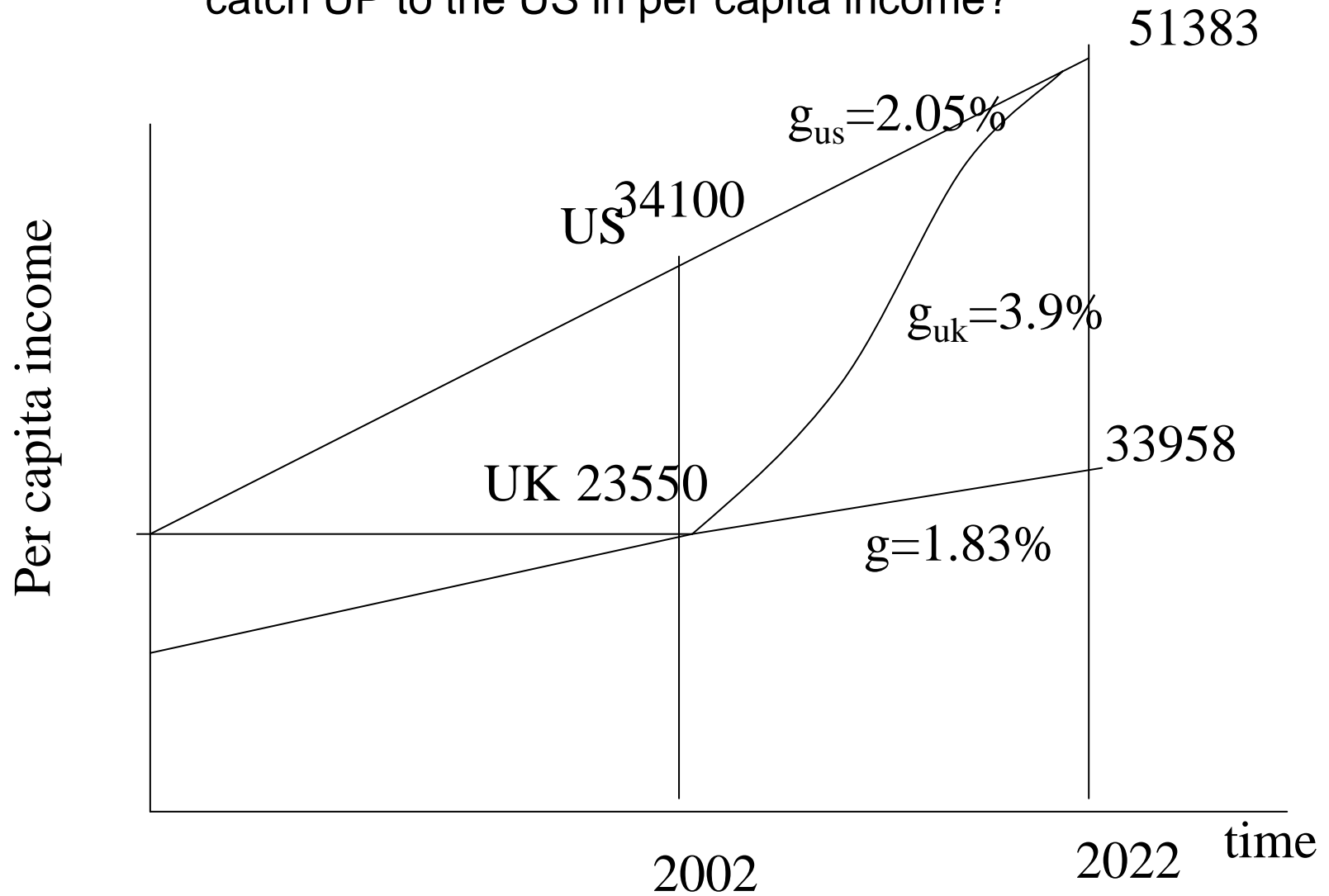
- Idea, Technology and Human Capital
- How Does Solow Model Include Technology?
- Lucas' Model of Study, Human Capital and output
- Romer's Model on Research, Technology Output
- R& D and Economic Growth and Patent Rights

Catching Up or Falling Behind in the Long Run: An Arithmetic

		Real GDP per capita	
Argentina	1950	7090.65	
Argentina	2003	10172.06	0.6809%
India	1950	741.9	
India	2003	2989.77	2.6297%
Japan	1950	2391.68	
Japan	2003	24036.44	4.3539%
United Kingdom	1950	8245.56	
United Kingdom	2003	26044.01	2.1700%
USA	1950	11086.6	
USA	2003	34875.41	2.1623%

Data Source: Penn World Table http://pwt.econ.upenn.edu/php_site/pwt_index.php
Accessed on Feb 1, 2007.

Why does UK need higher rate of growth per capita income to catch UP to the US in per capita income?



Time Taken to Bridge the GAP in Per Capita Income

$$Y_{02}^{UK} = 23550; Y_{02}^{India} = 2340$$

Growth rate of Per Capita Income

$$\text{India } g^{India} = 3.81\%$$

$$\text{UK } g^{UK} = 1.83\%$$

Ratio of UK to Indian Income in 2002

$$\frac{Y_0^{UK}}{Y_0^{India}} = (23550/2340) = 10.06$$

Equalisation of Income in year t

$$Y_0^{UK} e^{g^{UK}t} = 10.06 * Y_0^{India} e^{g^{UK}t} = Y_0^{India} e^{g^{India}t}$$

Take logs both side and solve for t

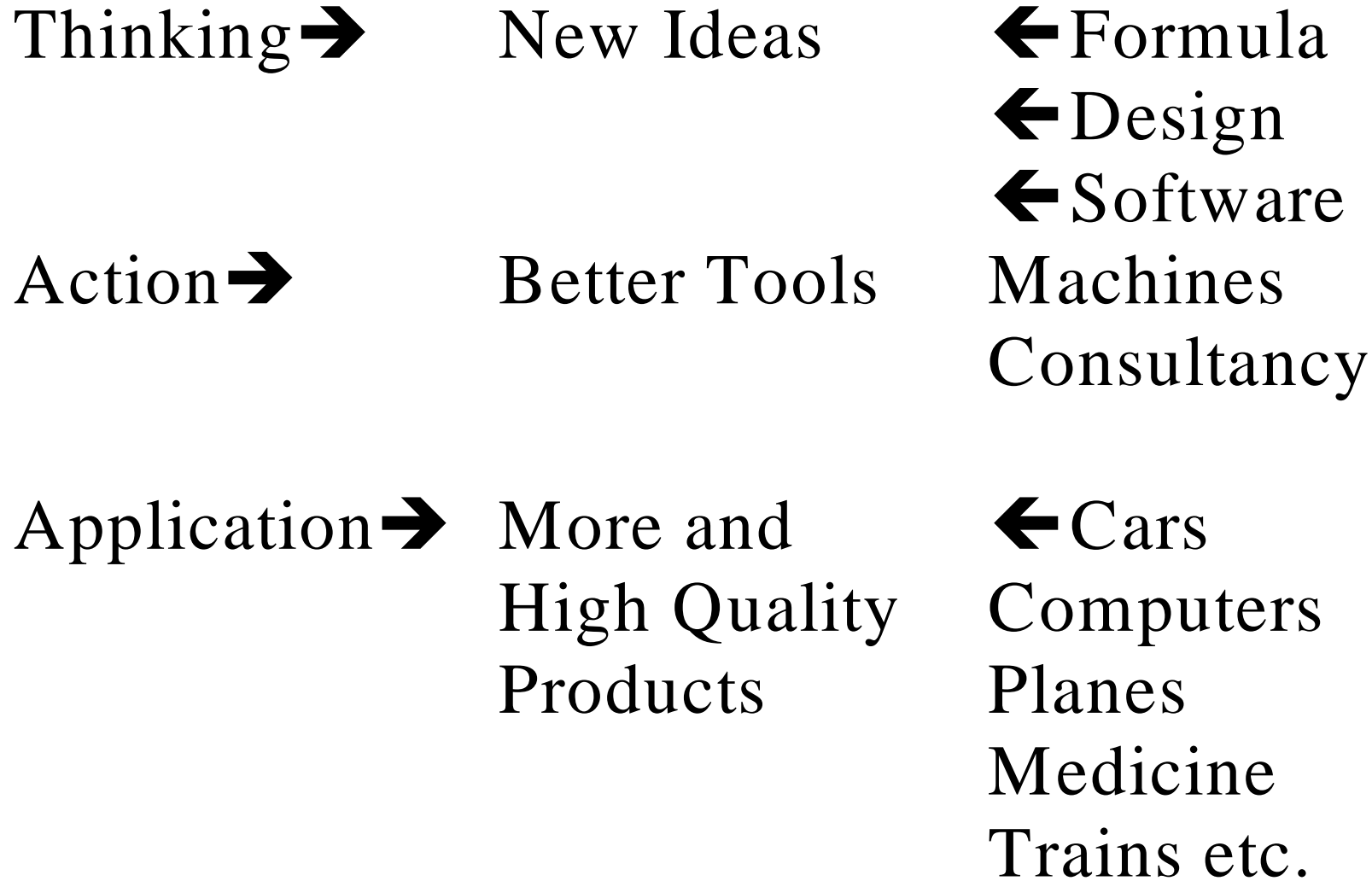
$$t = \frac{\ln(10.06)}{(g^{India} - g^{UK})} = \frac{2.31}{(0.0381 - 0.0183)} = \frac{2.31}{0.0198} = 116.6$$

Years.

Endogenous Growth Model: Role of Human Capital

- Ideas come from skilled and trained people.
- These ideas are translated into tools.
- Better tools allow production of more and high quality goods
 - Examples
 - Rockets, Cars, computers, trains, planes, medicine, TV, Phone Internet; high yielding varieties of crops, cloning (?)
- Ideas are non-rivalrous; Many people can use it at the same time can be found in books, journals, manuals and papers and reports.

How Does Human Capital Contributes to the Economic Growth?



General Indicators of human capital

- Education: literacy, numeracy and problem solving ability
- Sound health and long life expectancy
- Productive skills
- Proportion of Independent Thinkers in the population
- Opportunities for on the job training

Elements of Human Capital

A. General Human capital Index:

1. Literacy
2. Innumeracy
3. Health
4. Income
5. HDI (total of 1-4)

B. Information processing aspect of human capital:

6. Gathering information
7. Summarising the information
8. Analysing links among variables.
9. Constructing an analytical model
10. testing the model

C. Application of human capital for production:

entrepreneurship:

11. planning
12. programming
13. organising
14. implementing
15. monitoring

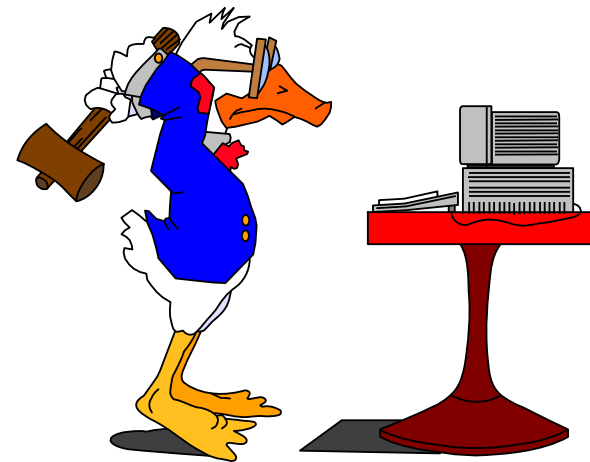
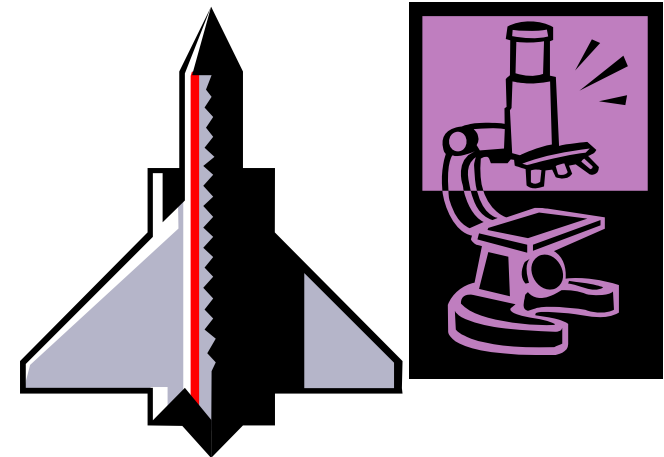
D. Generating new ideas: Research and Innovations

16. Generating new ideas
17. Creating a new product
18. innovation
19. Generalising the idea.

Types of Technical Progress and Opinions on its Role

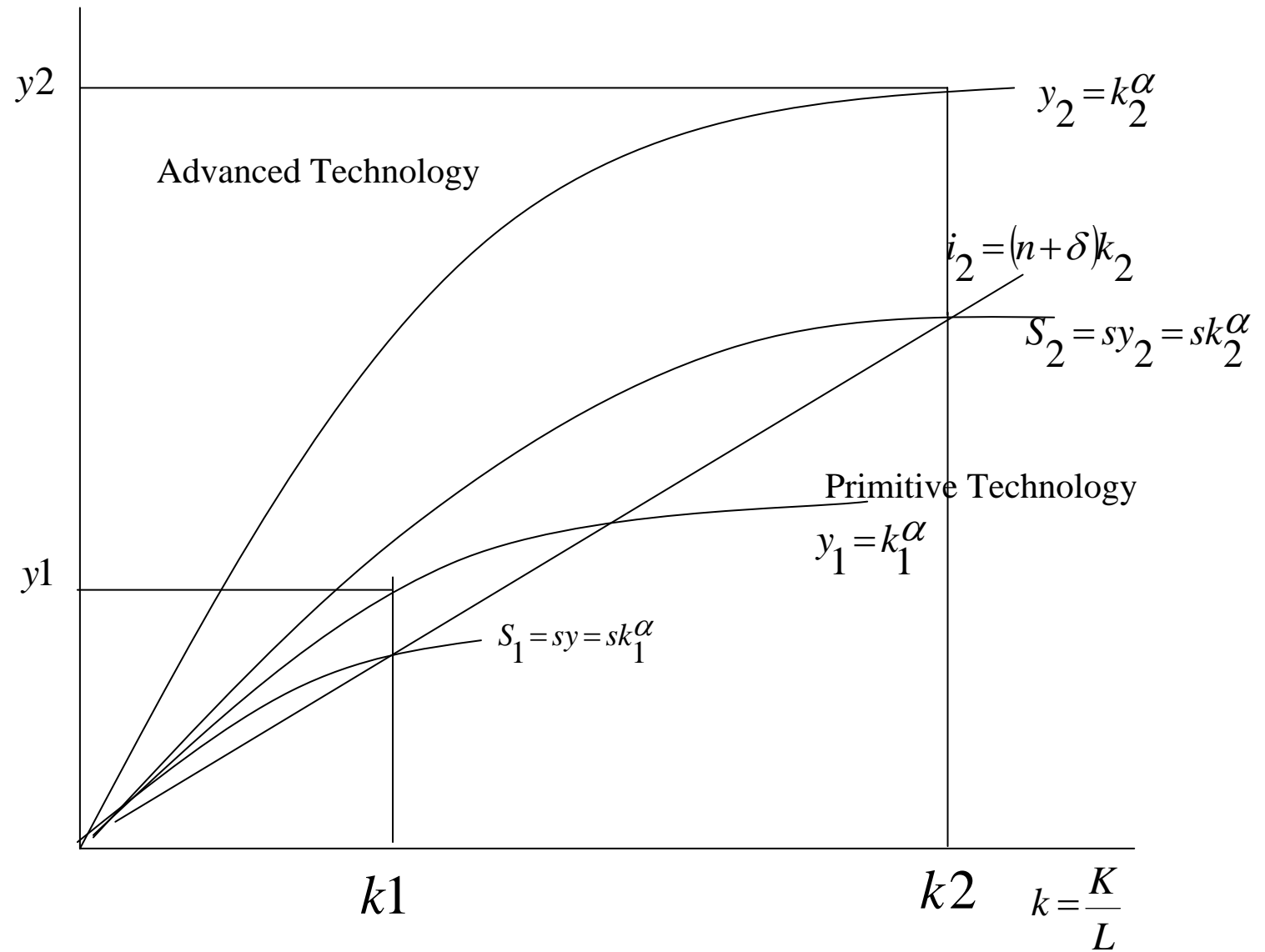
- Three Types
 - Labour Augmenting Technology : skilled workers
 - Capital Augmenting Technology - better Machines
 - Hicks Neutral Technical Progress

- Three Views
 - Optimistic (rising wage rate)
 - Pessimistic (falling wage rate)
 - Evidence based (mixed)

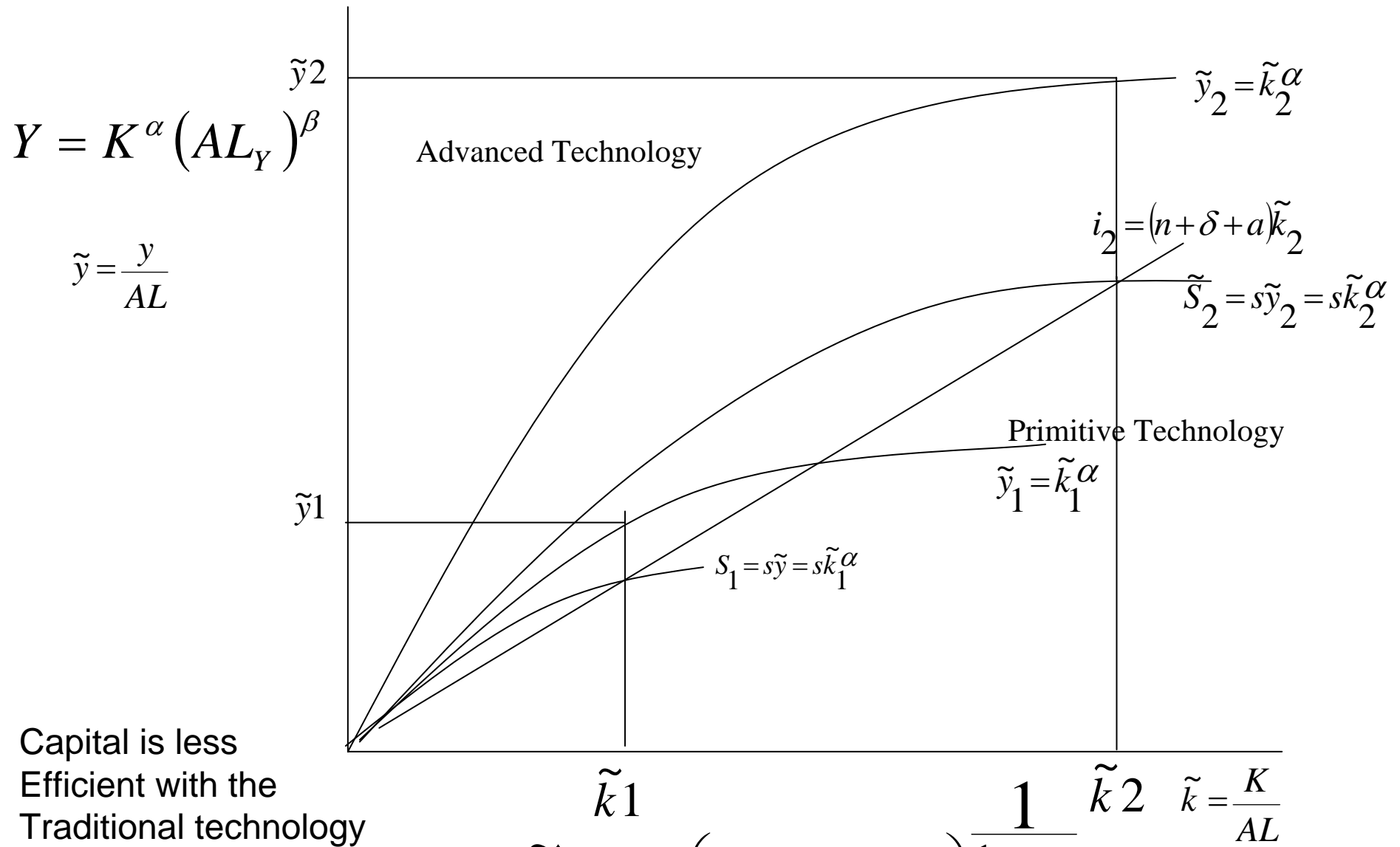


Lauddites

How Does Solow Model Include Technology and Human Capital ?



How Does Solow Model Include Technology and Human Capital ?



Capital is less Efficient with the Traditional technology

Capital is more efficient with Modern technology

$$\tilde{k}^{ss} = \left(\frac{s}{\delta + n + a} \right)^{\frac{1}{1-\alpha}}$$

Accounting of Growth with Human Capital in Solow Model

$$Y = AK^\alpha L^\beta H^\beta$$

$$\alpha + \beta + \gamma = 1$$

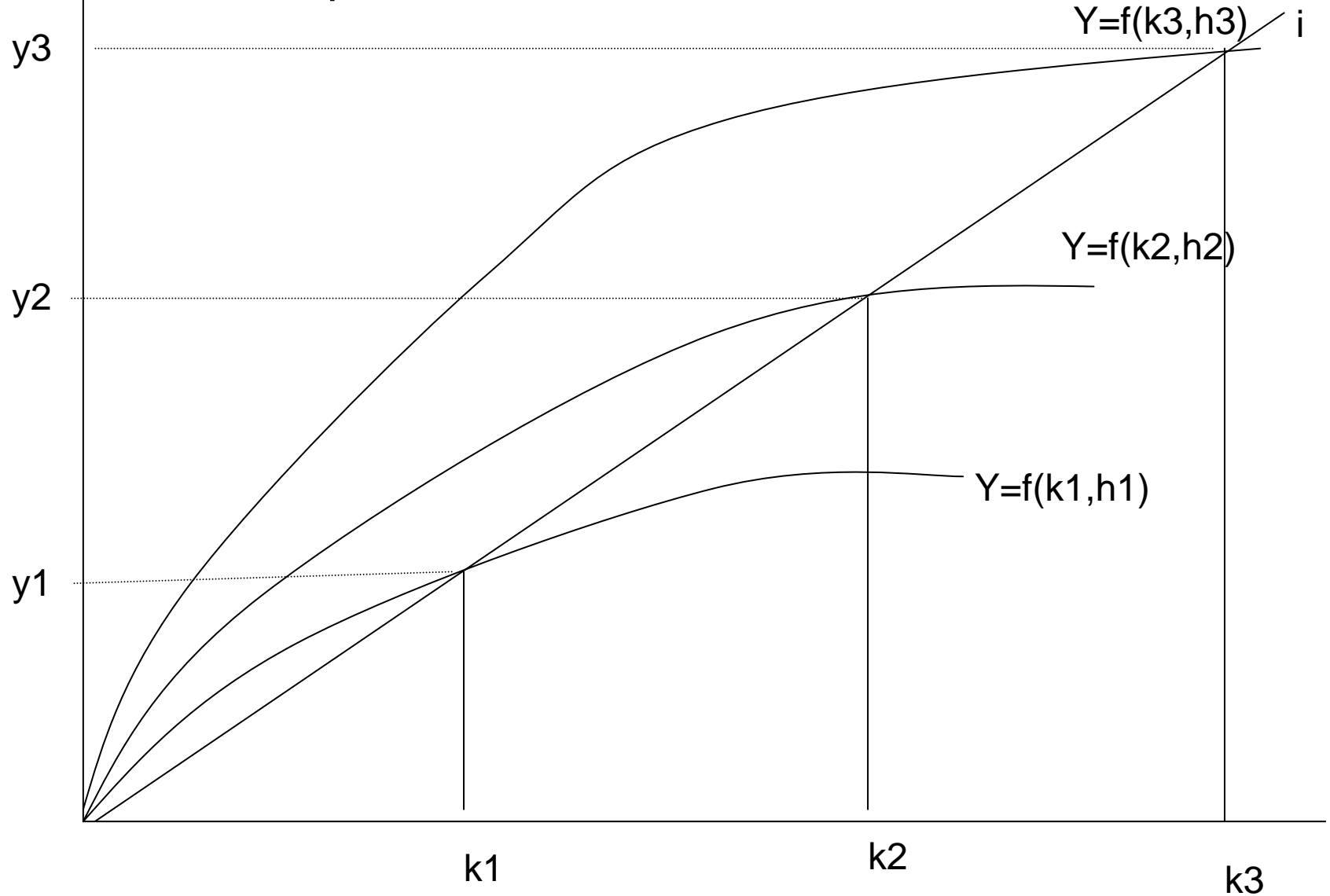
$$\frac{\Delta y}{y} = g_y = g_a + \alpha g_k + (1 - \alpha - \gamma)g_n + \gamma g_h$$

$$g_y - g_n = g_a + \alpha(g_k - g_n) + \gamma(g_h - g_n)$$

$$(g_y - g_n) - \gamma(g_h - g_n) = g_a + \alpha(g_k - g_n)$$

Per capita growth rate adjusted for human capital this depends on technological growth rate as well as per capita growth rate of capital stock.¹²

Human Capital Shifts the Production Function



Role of Human Capital in Production

Complimentarily Between Physical and Human Capital

Solow model with human capital (H):

$$Y = AK^\alpha L^\beta H^\gamma$$

Marginal product of human capital:

$$MPH = \gamma AK^\alpha L^\beta H^{\gamma-1} = \gamma Y / H$$

Marginal product of physical capital:

$$MPK = \alpha AK^{\alpha-1} L^\beta H^\gamma = \alpha Y / K$$

If the Human capital is proportion of physical capital:

$$H = \psi K \quad \text{and} \quad \gamma + \alpha = 1$$

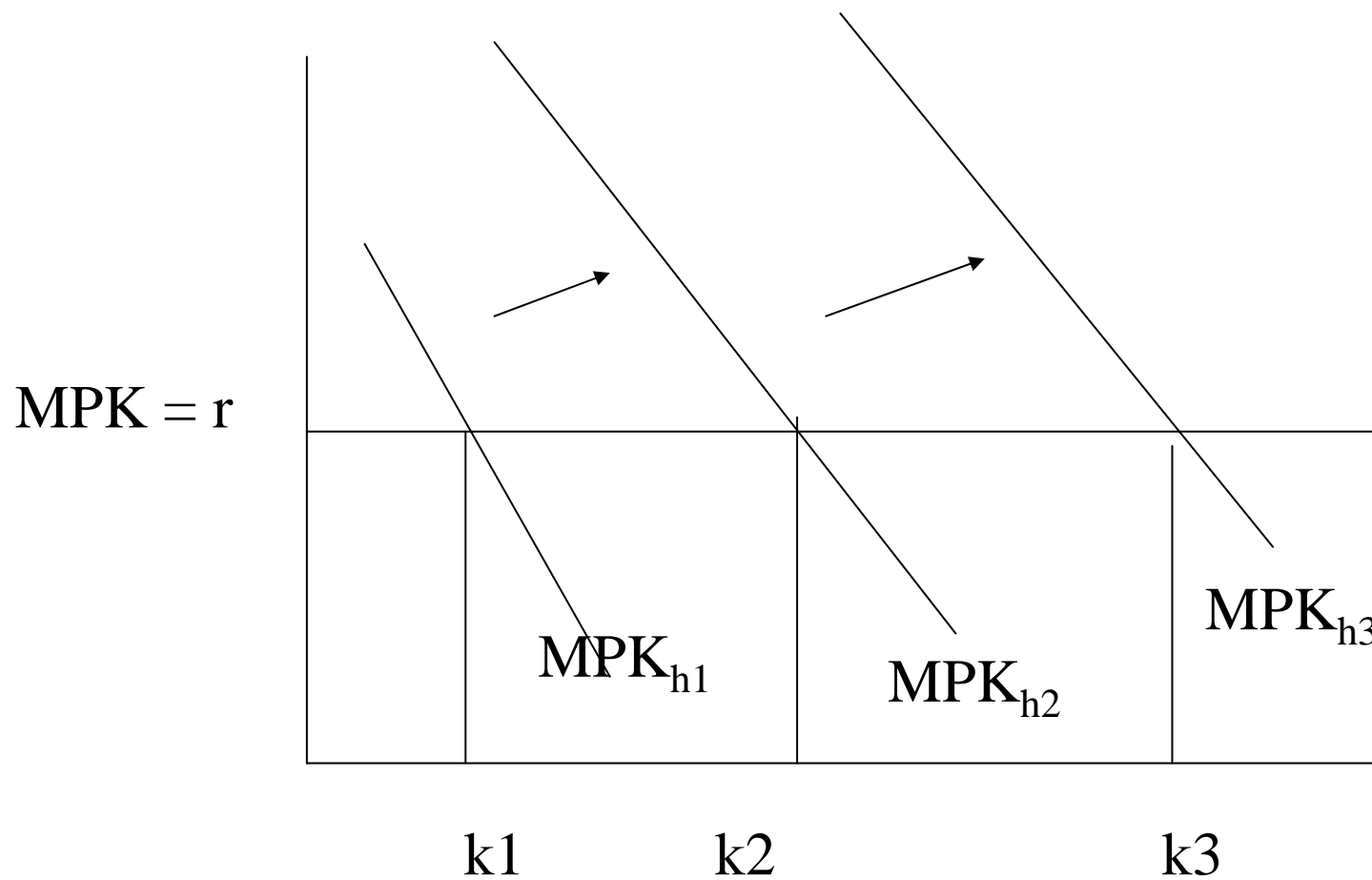
$$Y = AK^\alpha L^\beta = A(\psi)^\gamma K^{\alpha+\gamma} L^\beta \quad \text{.or} \quad Y = A(\psi)^\gamma KL^\beta .$$

Constant Marginal product of capital:

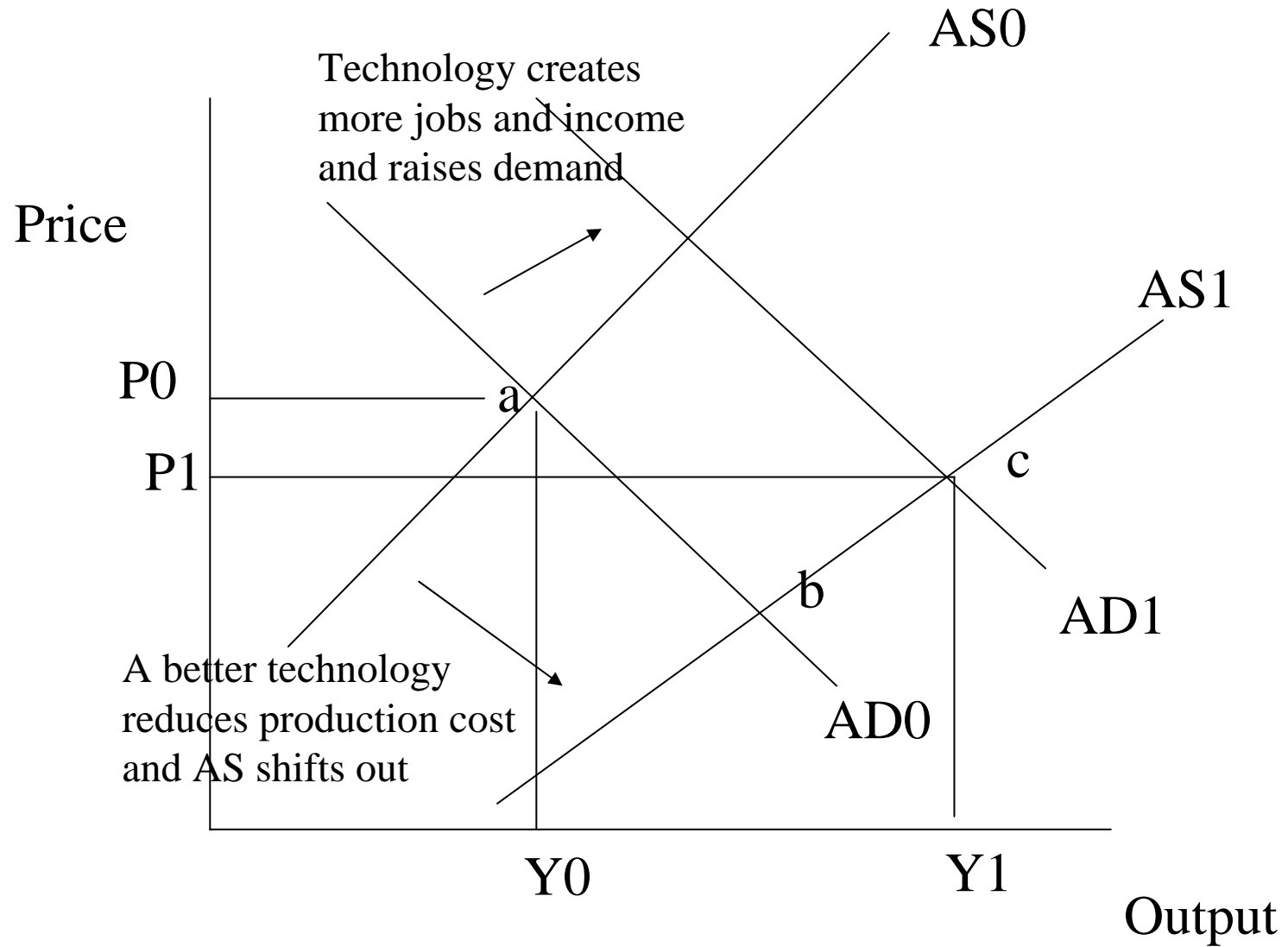
$$MPK = \frac{\partial Y}{\partial K} = A(\psi)^\gamma L^\beta$$

More saving \rightarrow More Capital \rightarrow Steadily Rising Output

Constant Marginal Product of Capital with Human Capital



How Does Aggregate Supply Demand Model Explain the Role of Technology ?



Role of Human Capital in Lucas (1988) Model

Production function

$$Y = K^{\alpha} (\theta h L)^{1-\alpha}$$

h = is human capital per worker

θ = fraction of time spent on working

$1 - \theta$ = fraction of time spent on studies

L = labour supply –(assume this as given)

Example :

If $K=100$, $L=100$ $h=3$ $\theta=0.8$, $\alpha =0.3$

$$Y = 100^{0.3} (0.8 * 3 * 100)^{0.7} = 100(2.4)^{0.7}$$

$$= 185 \text{ where with } Y = K^{\alpha} L^{1-\alpha} = 100.$$

Growth of Human Capital and Study Time

$$g_h = \frac{\dot{h}}{h} = \phi(1-\theta)$$

where ϕ is the rate of creation of human capital per unit of time spent on studying
($1-\theta$) is the fraction of time spent on studying.

Stock of human capital for time t : $h_t = h_0 e^{\phi(1-\theta)t}$

if $h_0 = 1$, $\phi = 0.4$, $(1-\theta) = 0.2$, time $(t) = 20$

$$h_t = 1 \cdot e^{0.4(0.2)20} = 4.95$$

Output, Fraction of Time Spent on Studying and the Physical Capital

$$y = \theta^{1-\alpha} k^\alpha$$

Capital stock per effective labour $k = \frac{K}{hL}$

Output per effective Labour $y = \frac{Y}{hL}$

h Human Capital

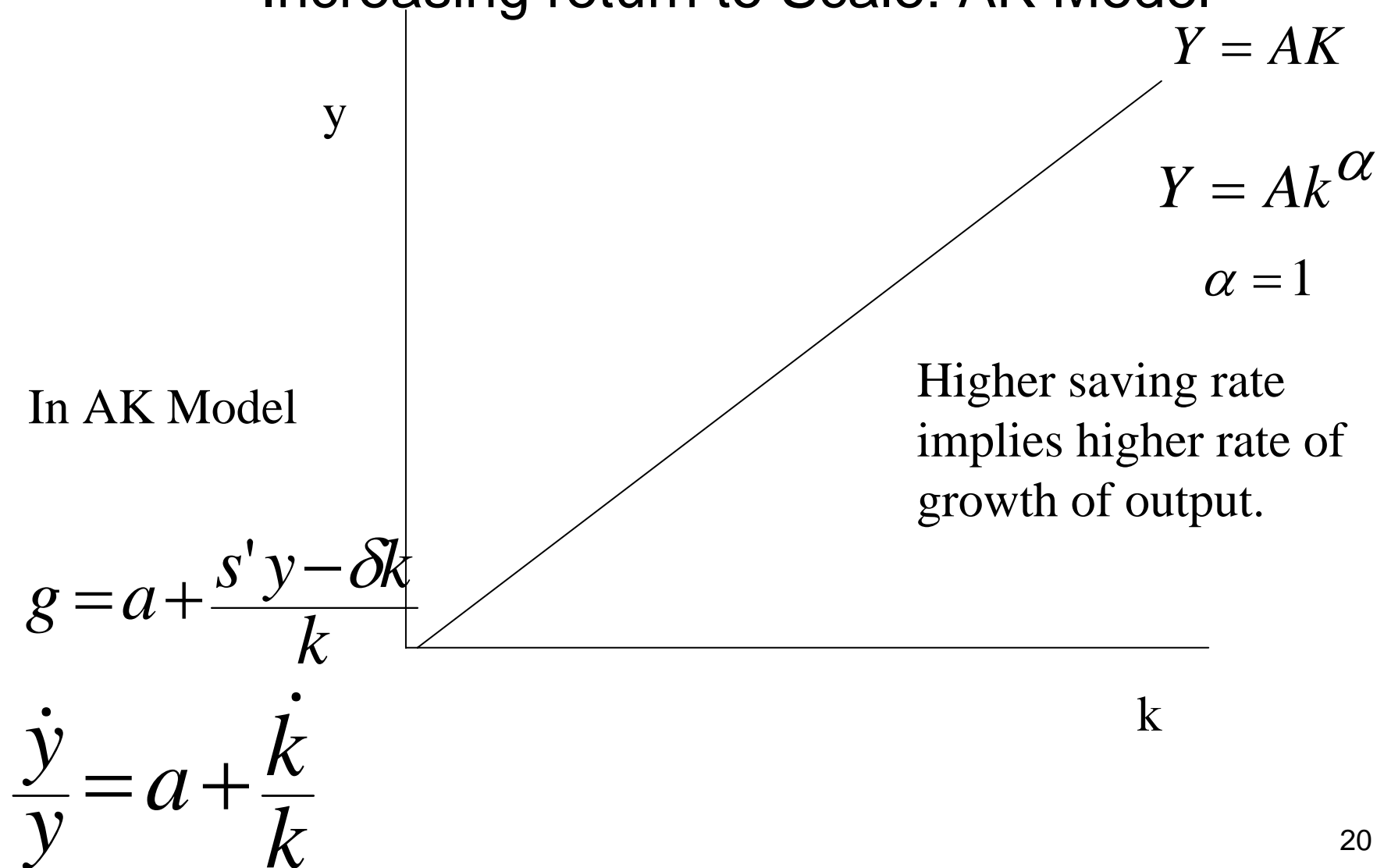
L Labour

hL Effective labour

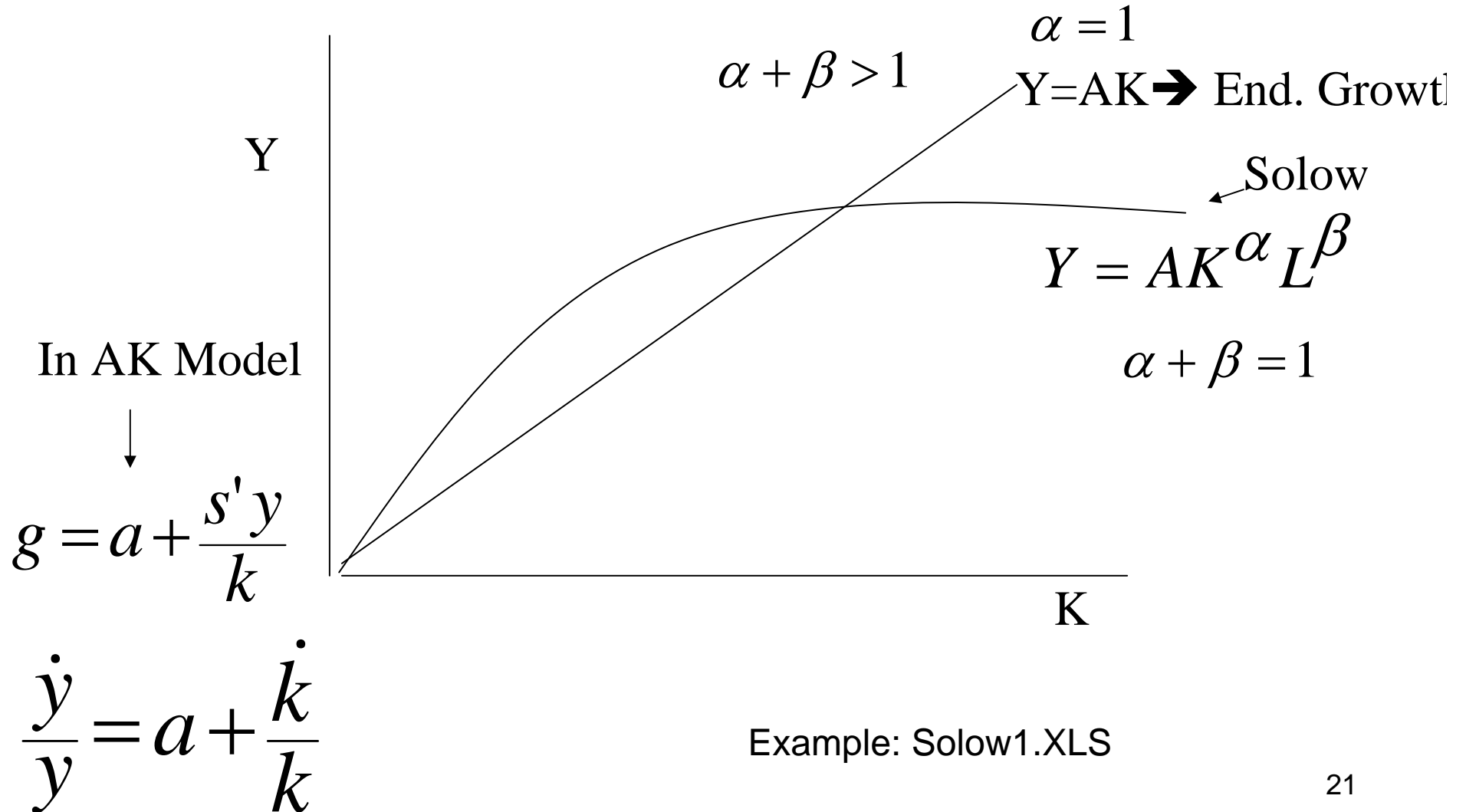
Derivation:

$$\frac{Y}{hL} = \frac{K^\alpha (\theta hL)^{1-\alpha}}{hL}$$

Saving, Capital Accumulation and Output with Increasing return to Scale: AK Model

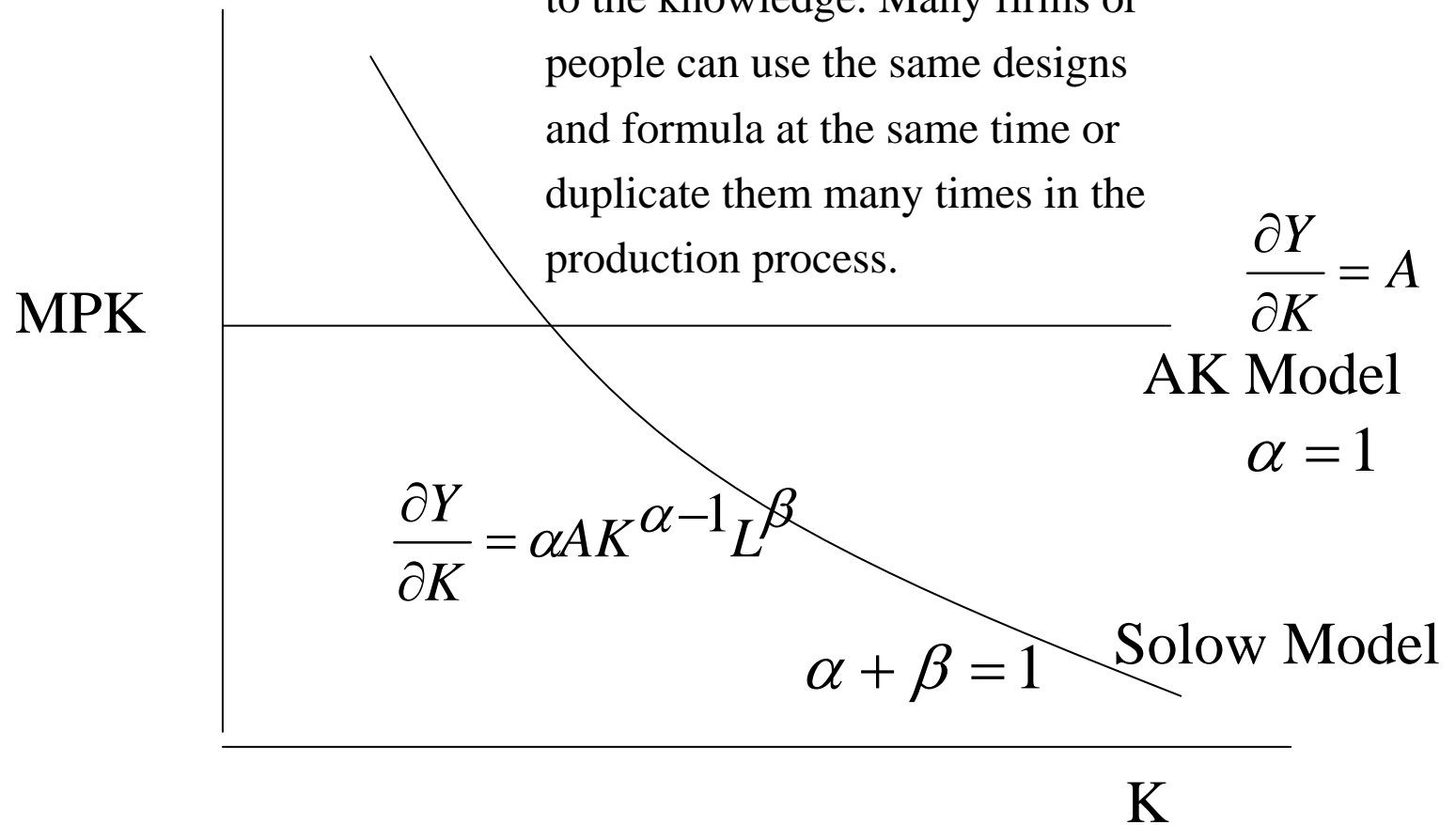


Comparison of Production Technology in Endogenous and Neoclassical Growth Models

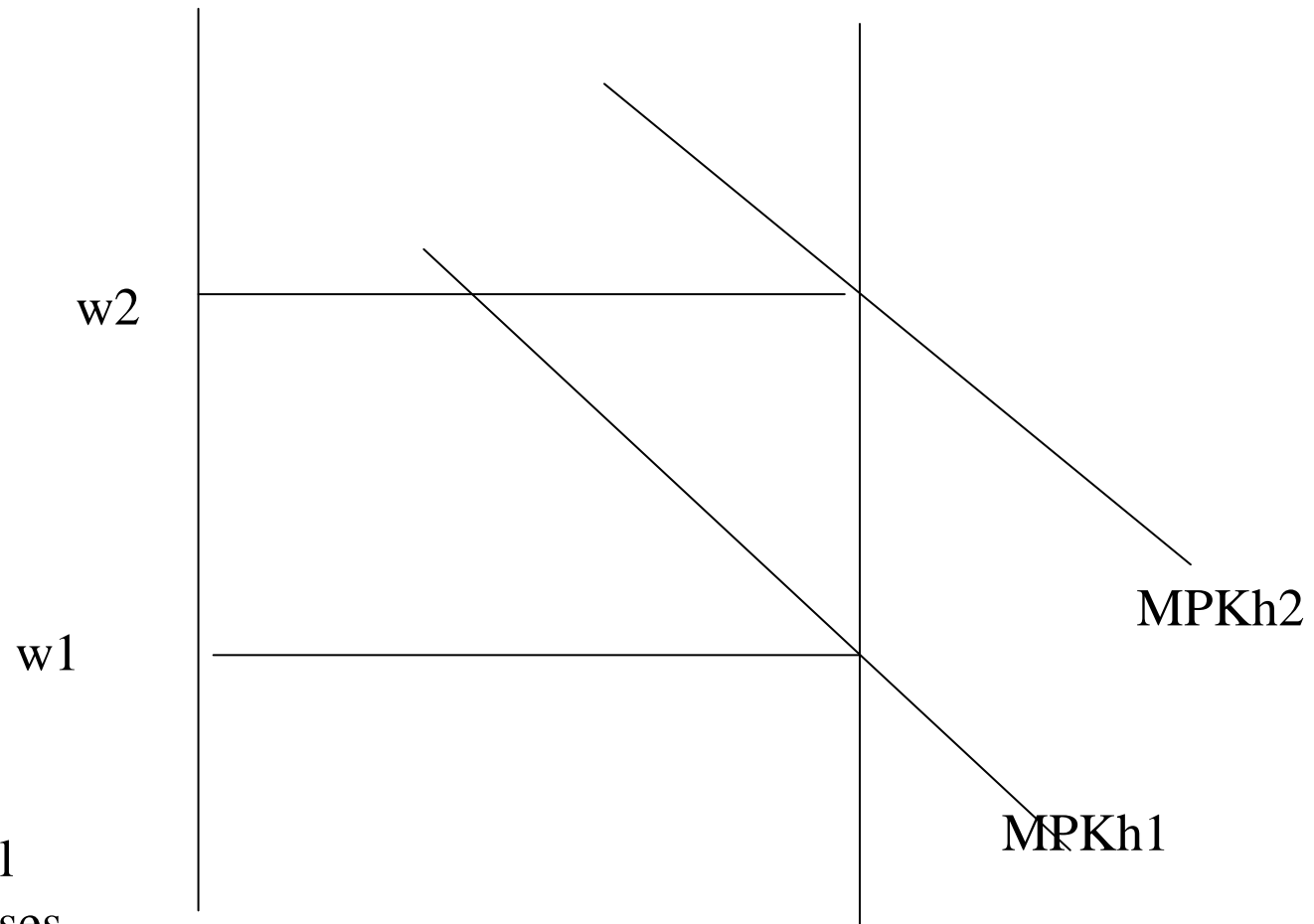


Marginal Product of Capital

How is AK Technology possible?
There is an increasing return scale to the knowledge. Many firms or people can use the same designs and formula at the same time or duplicate them many times in the production process.



Increase in Real Wage Rate with Human Capital



Technological advancement raises wage rate but reduces Work hours.

Three Sectors in the Romer's Endogenous Growth Model

- Research Sector: Universities/ research labs produce ideas
- Intermediate sector: Takes those ideas to make tools and machines
- Final Goods sector used those ideas to produce consumer goods.

Endogenous Growth Model

Output:
$$Y = K^\alpha (AL_Y)^\beta$$

A = Stock of knowledge

Labour use:
$$L = L_y + L_A$$

The stock of knowledge rises if more people do research:

$$\dot{A} = \bar{\delta} L_A^\lambda = \delta A^\phi L_A^\lambda$$

Growth rate of knowledge:
$$g_A = \frac{\dot{A}}{A} = \frac{\delta A^\phi L_A^\lambda}{A}$$

Capital Accumulation:

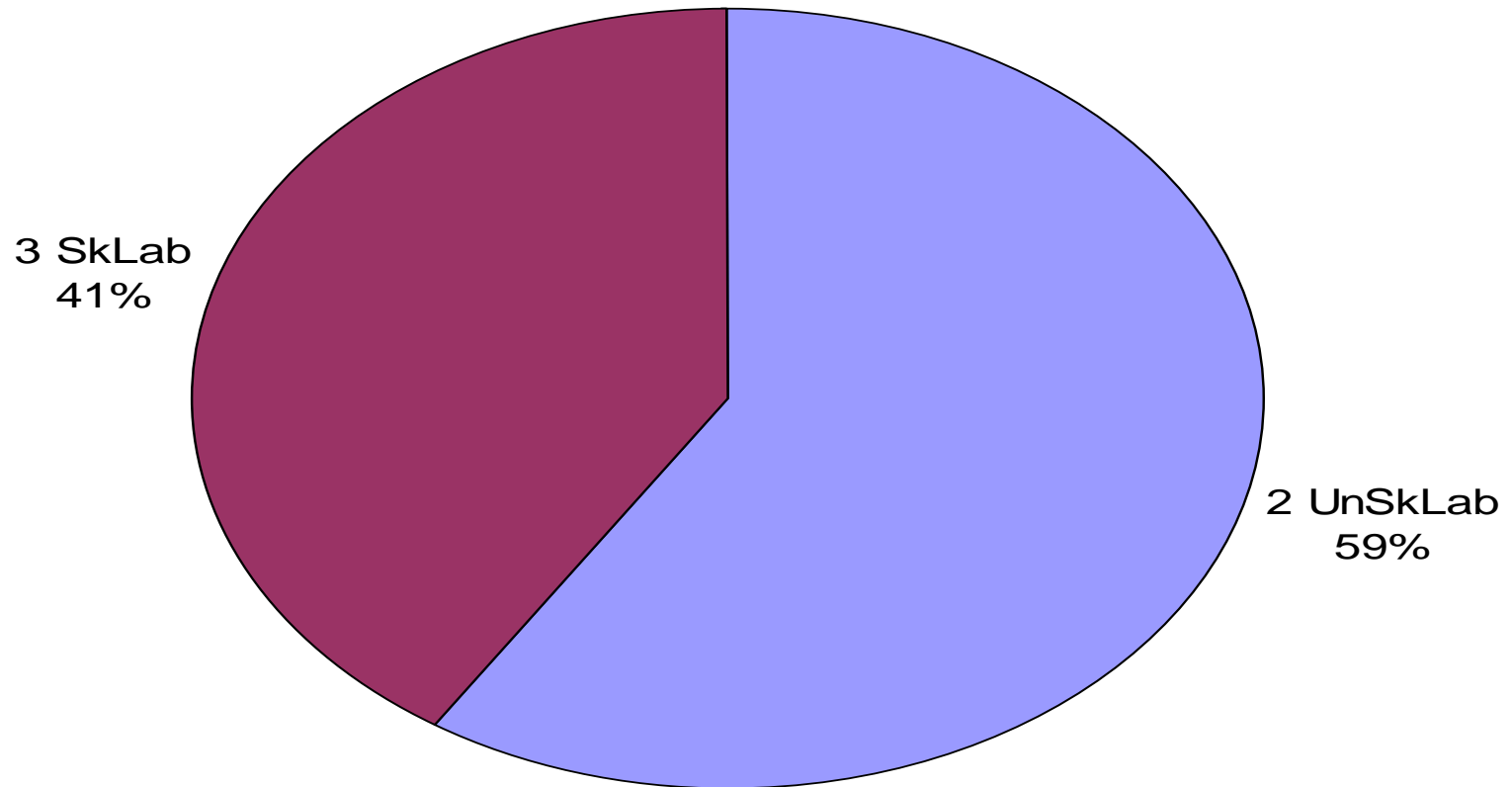
$$K_t = K_{t-1} (1 - \delta) + I_t$$

Market clearing:
$$Y_t = C_t + I_t$$

Here technology is endogenous to efforts in production and application of research.

What Can Policy Do to Improve the Human Capital and Technology?

Skilled and Unskilled Labour In Production, UK 2000



Scientific Products of Research and Innovation
(Economist, December 31, 1999)

- Algebra - Arabs, India (0)
- Printing Press - Gutenberg 1440
- Calculus - Newton (1684)
- Steam Engine - James Watt (1765)
- Electricity - Edison (1879)
- Computer- Babbage (1984)
- Radium
- Radio
- TV
- Photography and Cinematography - Daguerre (1839)
- Telephone Bells (1876)
- Wireless (1896)
- Electro Magnetic Telegraph (1833)
- Powered flight (1903)
- X-ray
- Jet Engine
- PC
- Internet

Why Market Under Provides Research?

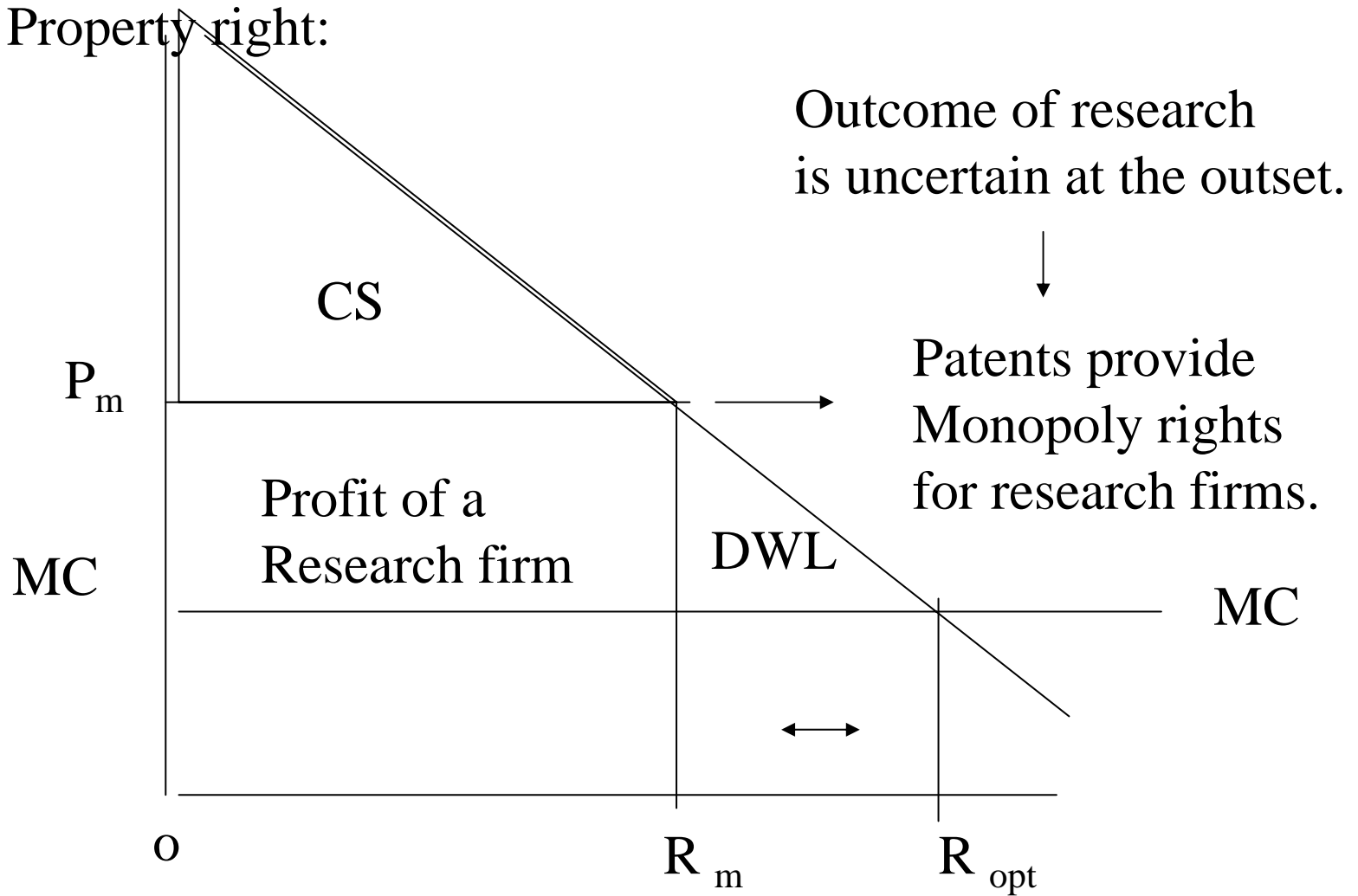
Intellectual Property right:

Patents

Designs

Trademark

Copyright



<http://www.intellectual-property.gov.uk/>

Marginal cost pricing is not profitable for a research firm
Economic reason for granting a patent right or subsidy
to a research firm

Output: $Y = 100(L - F)$

$F =$ fixed labour

Cost: $C = wL$

Cost function: $C = w\left(\frac{Y}{100} + F\right)$

Constant marginal cost pricing: $\frac{\partial C}{\partial Y} = \frac{w}{100} = P$

Declining average cost: $\frac{C}{Y} = \left(\frac{w}{100} + \frac{F}{Y}\right)$

Negative profit: $\pi = R - C = \frac{w}{100}Y - w\left(\frac{Y}{100} + F\right) < 0$

Government need to subsidise to produce optimal amount of research.

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