Game theory: Asymmetric information
Signalling and Screening and Sequential Equilibrium
Principal agent model

Economic Modelling, 2007

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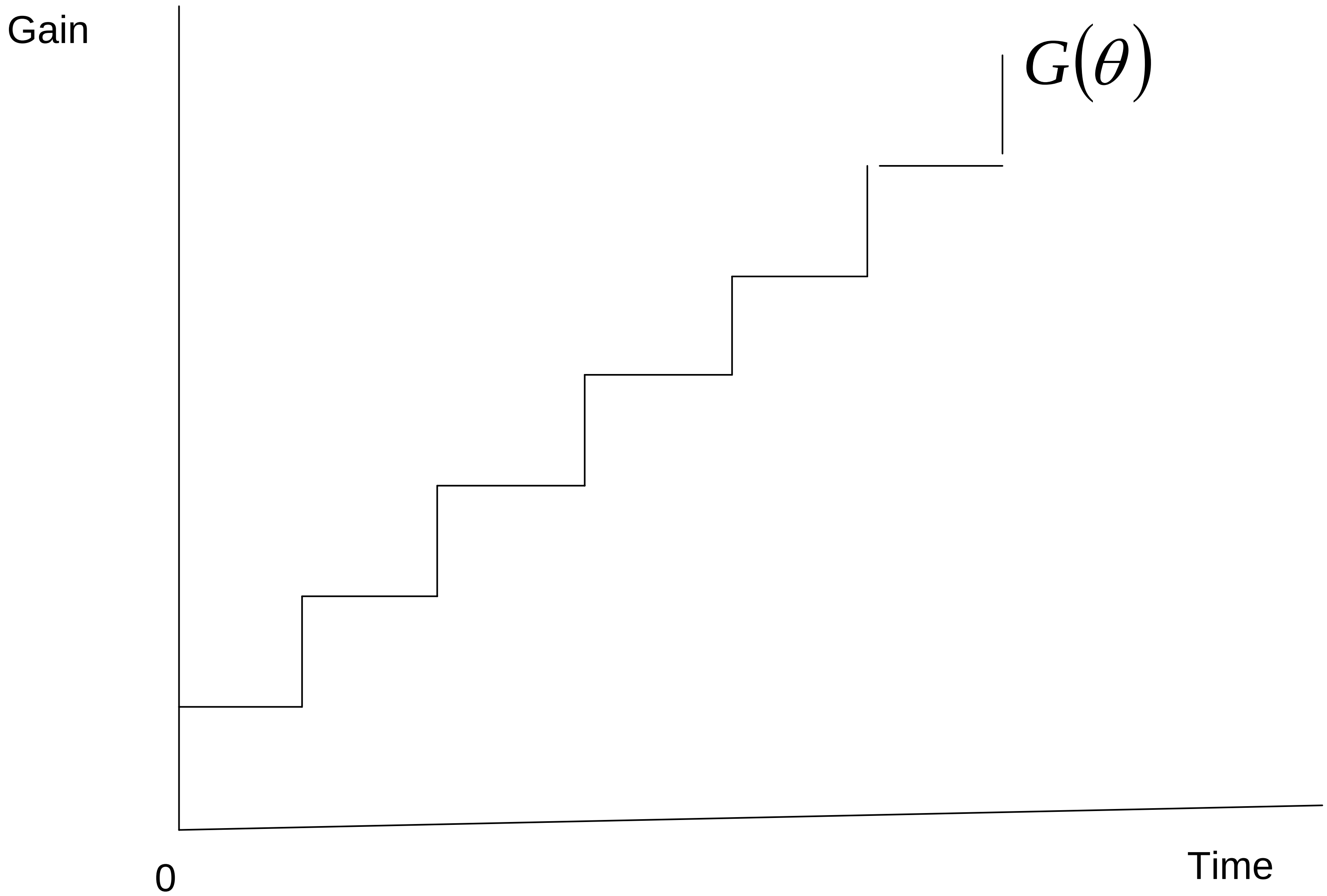
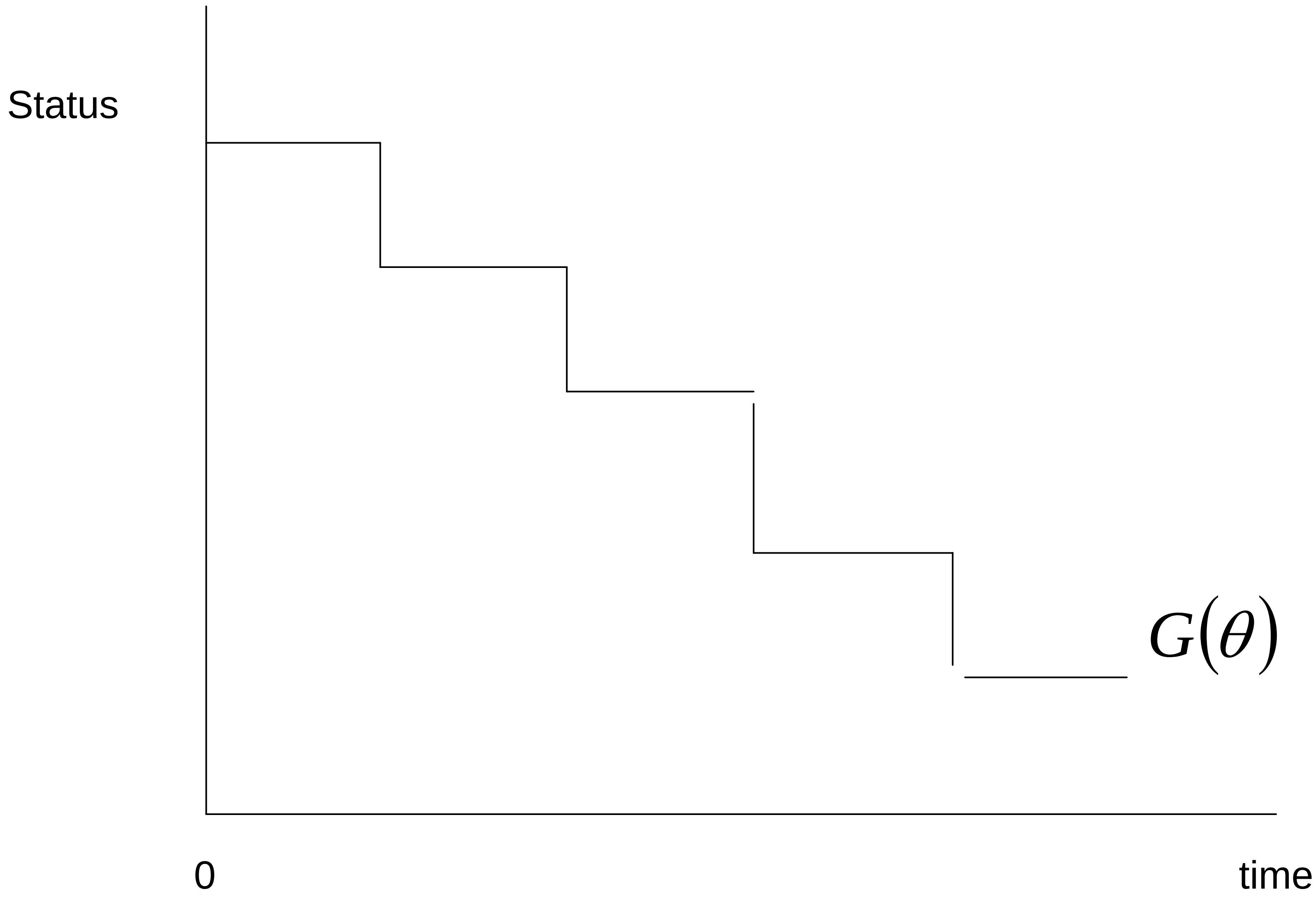
Reference texts: Gardner(2003) and Rasmusen (2007)

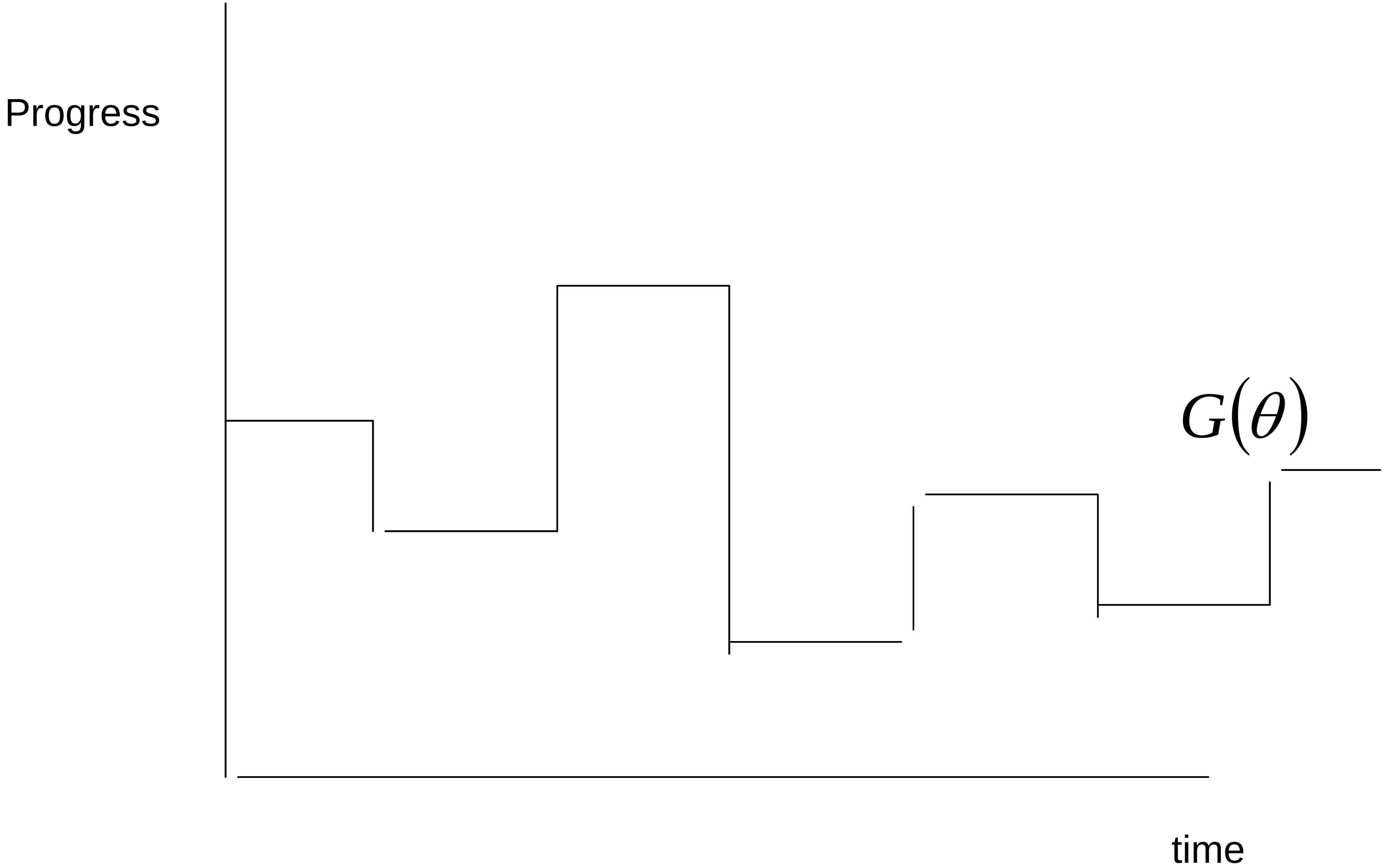
Results of a sequence of right actions following correct signals



Results of a sequence of wrong actions following reading signals incorrectly



Results of sequences of right and wrong actions reading signals correctly and incorrectly

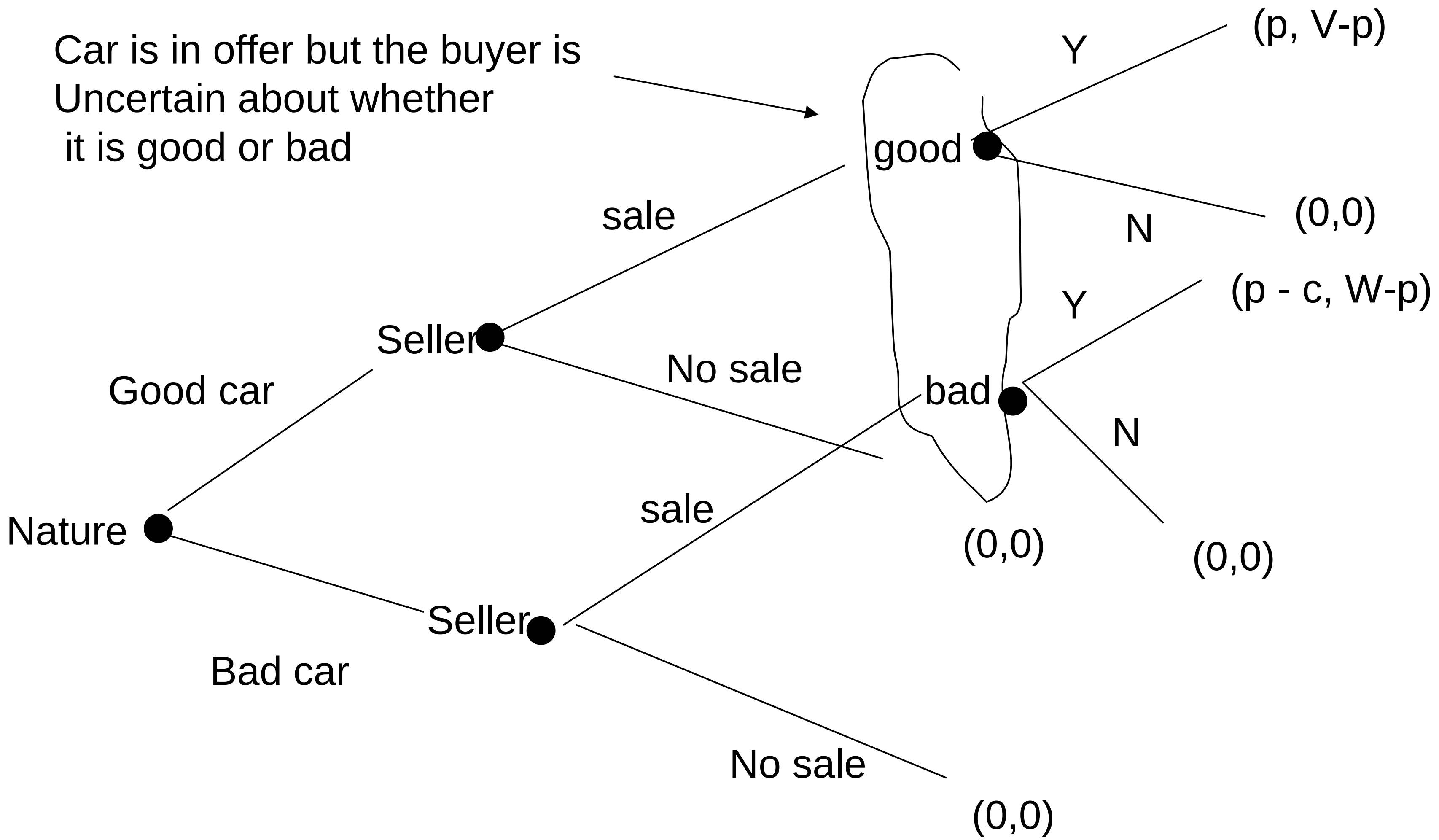


Market Situations in Signalling Game

- Complete market failure
  - pooling equilibrium (same price for good and bad cars; good cars disappear from the market)
- Complete market success
  - Separating equilibrium where players act as they should according to the signal (prices according to quality)
- Partial market success
  - (both good and bad cars are bought, some feel cheated)
- Near Market failure (mixed strategies)

Bayesian updating mechanism at work

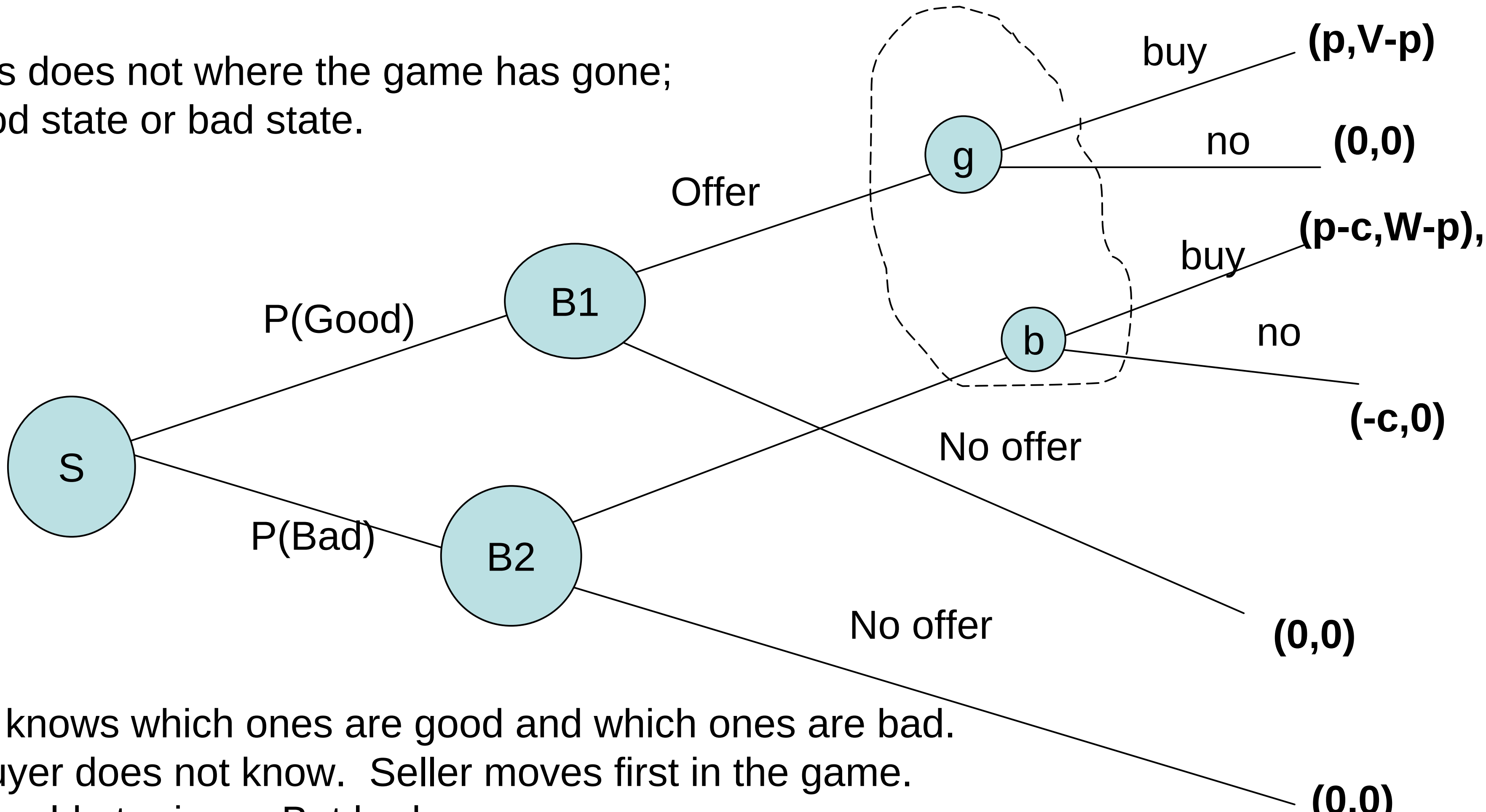
Signalling game in Markets for Used Car (Akerlof)



The buyer is uncertain about where the game is going- Need to resolve this- then the game can be solved by backward induction.

Informed Player Moves First in a Signalling Game (Akerlof)

Buyers does not where the game has gone;
At good state or bad state.



Seller knows which ones are good and which ones are bad.
But buyer does not know. Seller moves first in the game.
Car is sold at price p. But bad cars
need amount c to make repair and look just like a good one.
V is value of good car is V and W for bad car for the buyer.

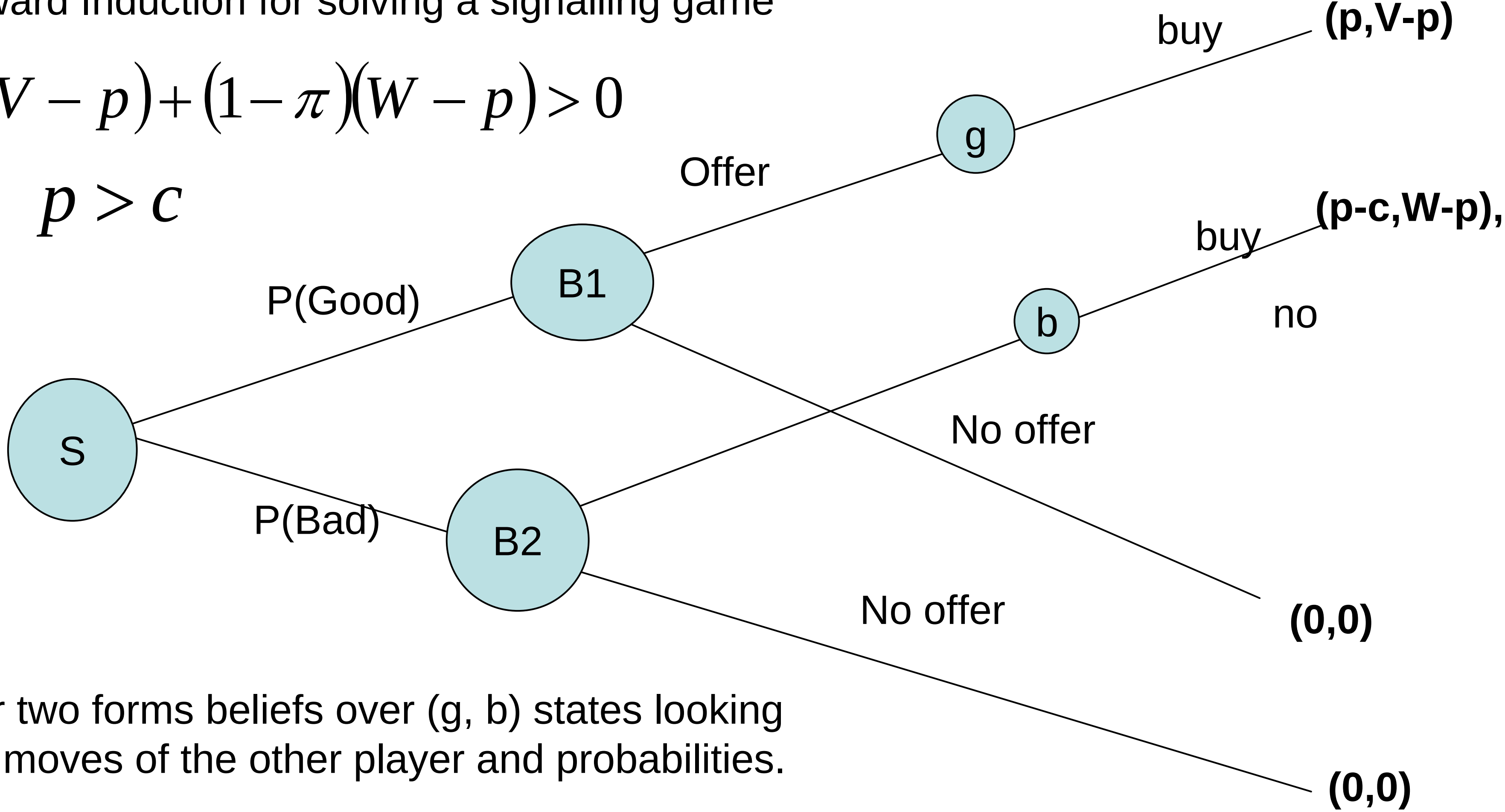
Partial Success of Market with asymmetric information.

Signals need to be credible

Backward Induction for solving a signalling game

$$\pi(V-p)+(1-\pi)(W-p)>0$$

$$p>c$$



Player two forms beliefs over (g, b) states looking at the moves of the other player and probabilities.

Tries to find a sub-game perfect equilibrium maximising expected utility.

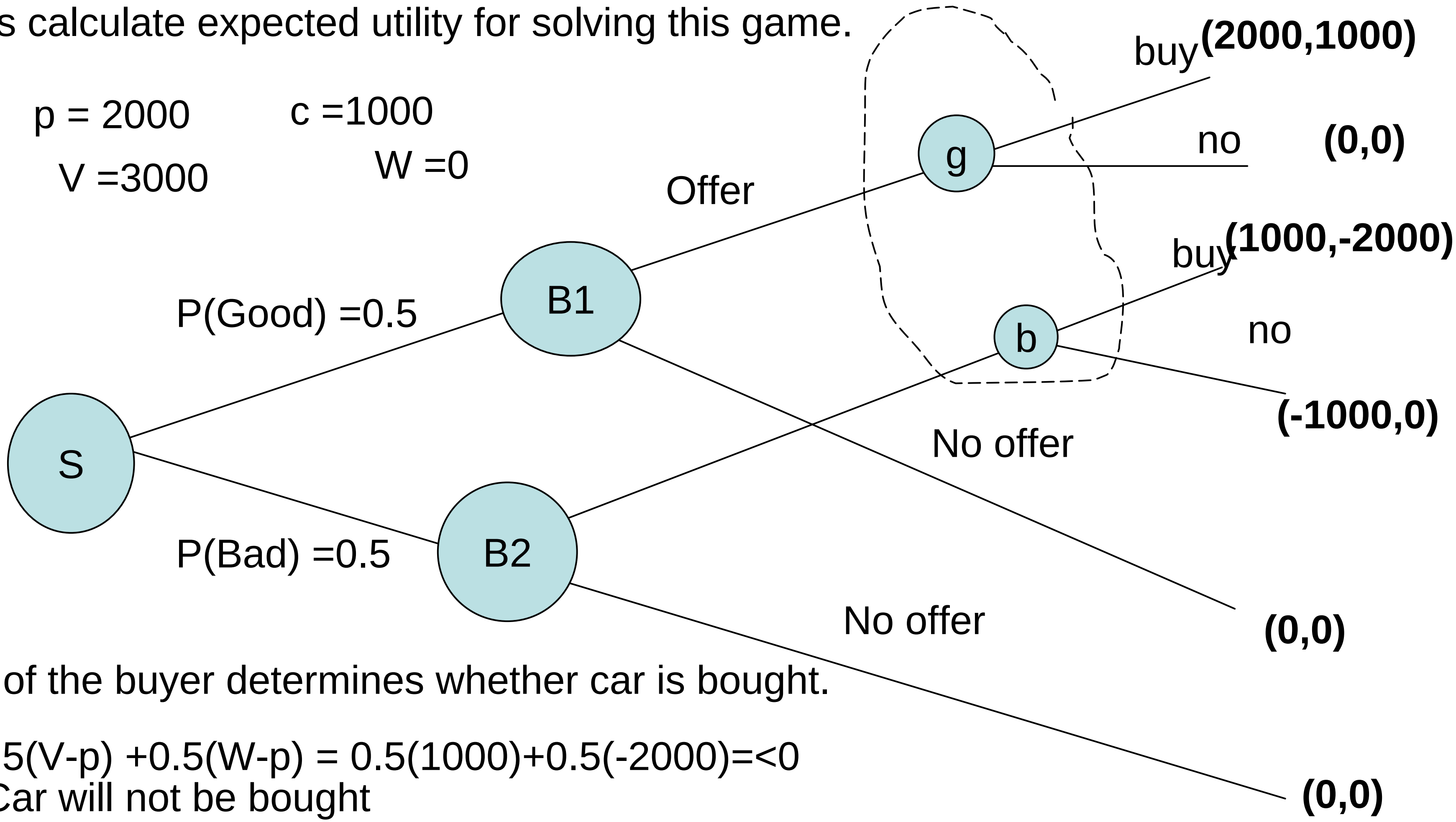
Both good and bad cars will be sold; Some buyers who get bad cars feel cheated

Mixed Strategy Sequential Equilibrium

Buyers calculate expected utility for solving this game.

p = 2000 c =1000

V =3000 W =0



Belief of the buyer determines whether car is bought.

0.5(V-p) +0.5(W-p) = 0.5(1000)+0.5(-2000)=<0
Car will not be bought

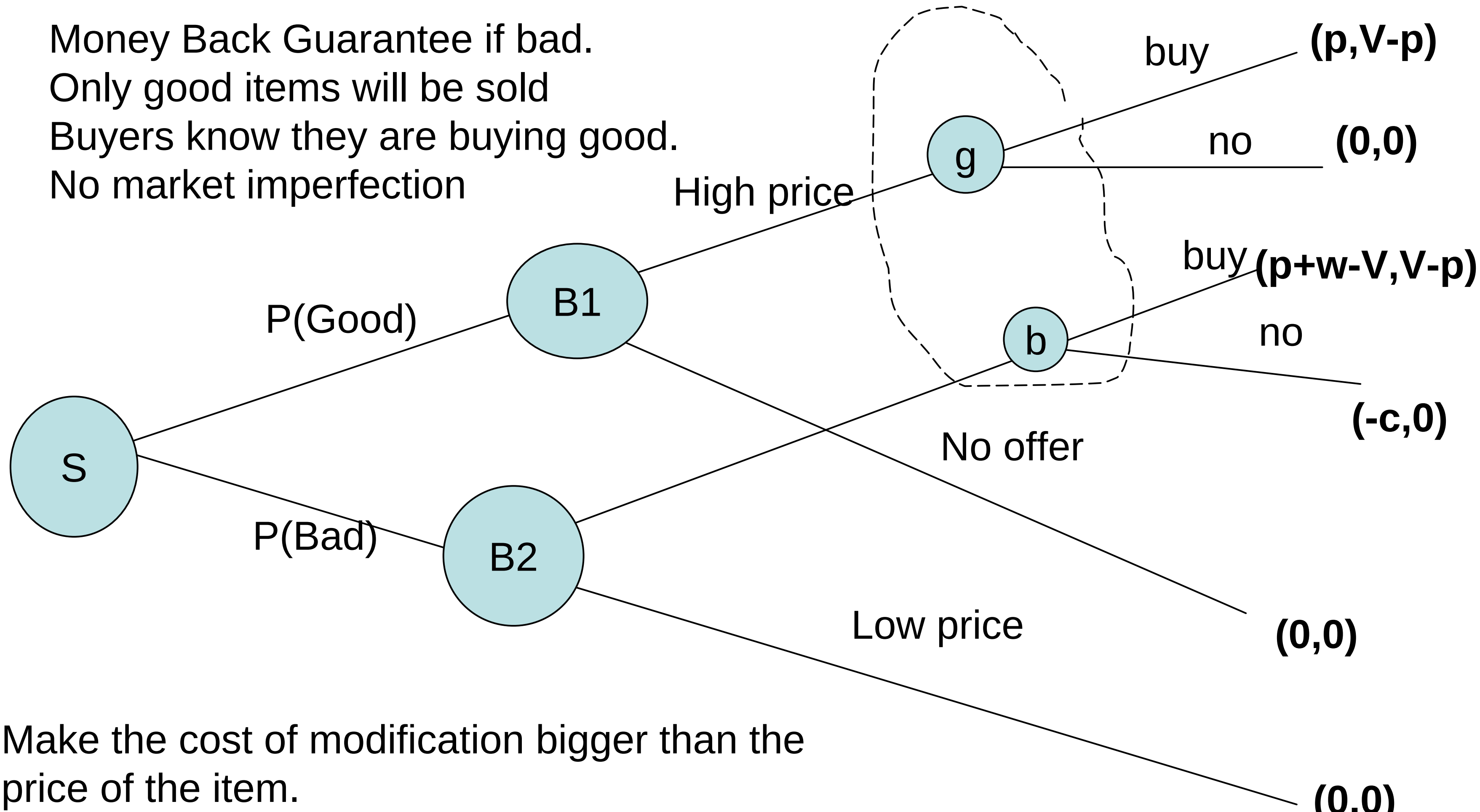
0.5(V-p) +0.5(W-p) = 0.66(1000)+0.33(-2000)=0

EU(good) = 0.5(2000)+0.5(0) = 1000>0

EU(bad) = 0.5(1000)+0.5(-1000) = 0

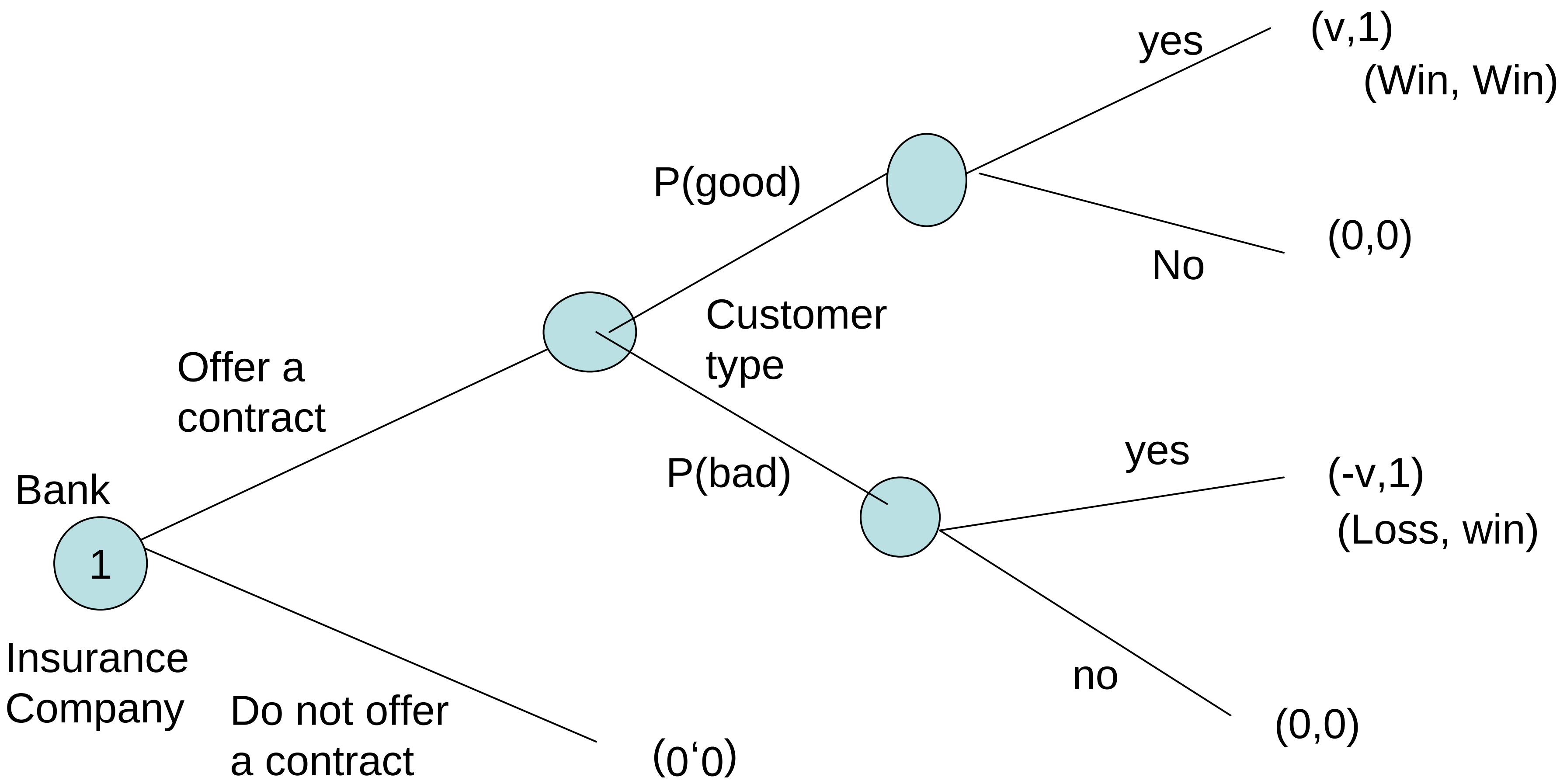
Costly Commitment for Separating Equilibrium: Complete Market Success

Money Back Guarantee if bad.
Only good items will be sold
Buyers know they are buying good.
No market imperfection



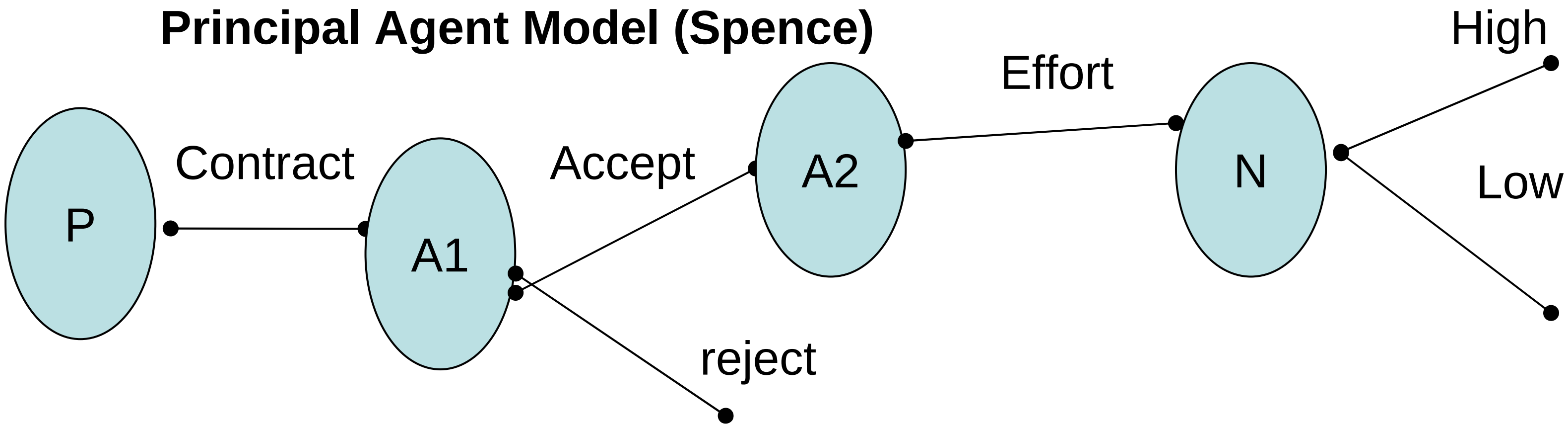
Make the cost of modification bigger than the price of the item.
Higher punishment for wrong Standards.

A Screening Game (Stiglitz)
Uninformed Players Moves First in

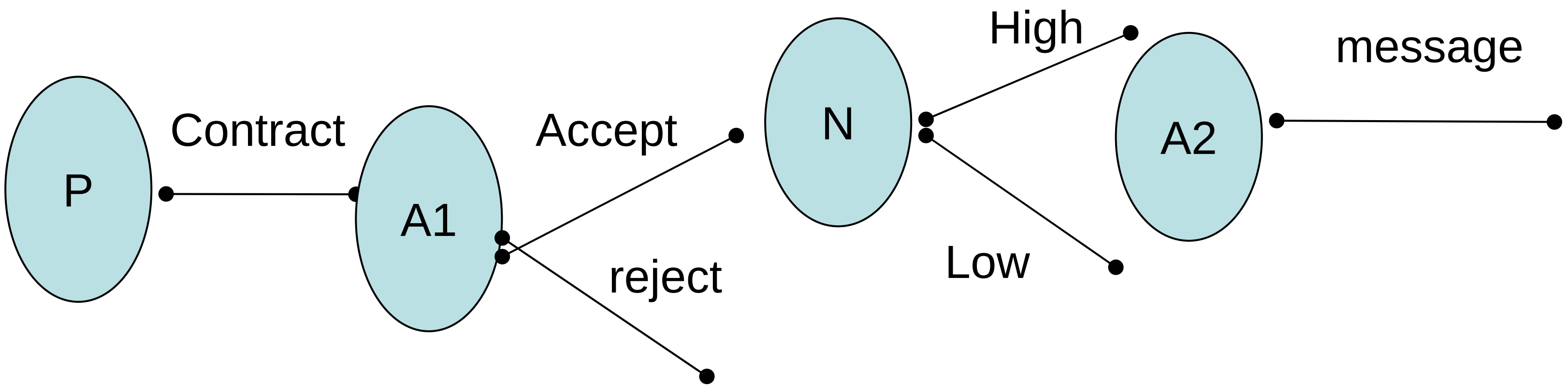


Credit Rationing or Insurance Market

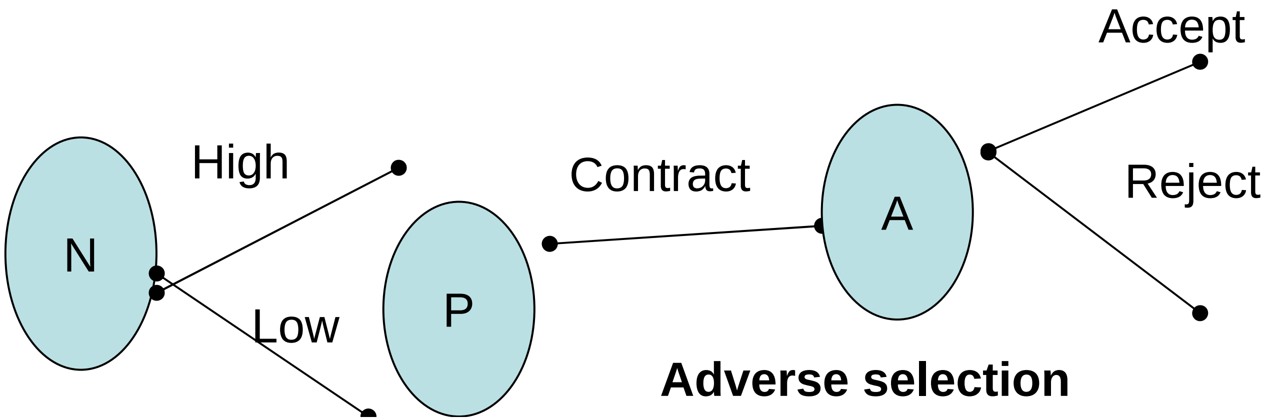
Principal Agent Model (Spence)



Moral Hazard game with Hidden action



Moral Hazard with Post contractual hidden knowledge

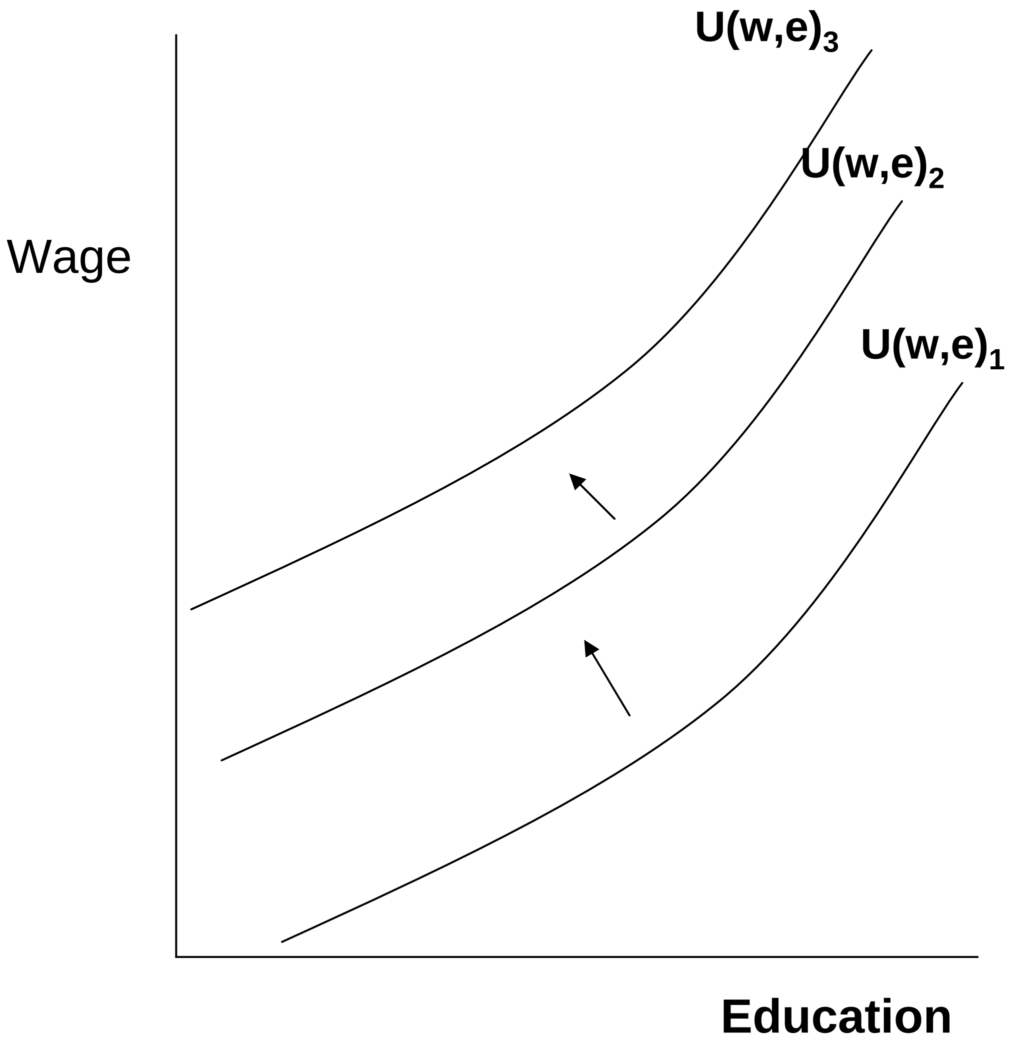


Adverse selection

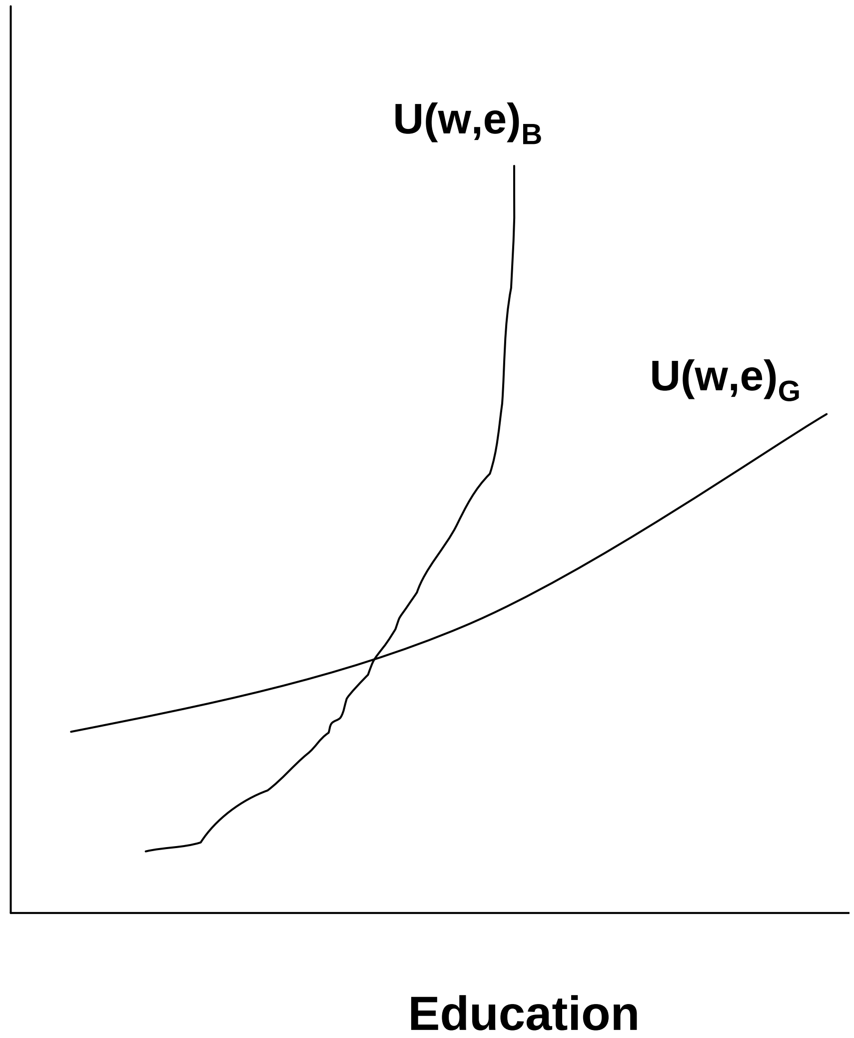
Education as Productivity Signalling Game

- Players consisting of {workers, firms and nature}.
- There are two types of workers [t = {1, 2}].
- Type 1 has less productive and type 2 more productive.
- Employer does not know which one is low or high quality worker but sees level of education
- Nature decides whether a worker is high or low productivity type.
- Level of education signals the quality of worker

Preference over wage and education



Single Crossing property



More Productive Worker Can Get Education Easily than Less Productive Worker

Preferences over Wage and Education

Workers choose level of education according to their beliefs about its impact on wage offer: $w_t(e)$.

Utility from wage and education is given by

$$u_t(w,e)$$

Utility is rising in wage received

$$\partial u_t(w,e)/\partial w > 0$$

Utility falls in work efforts

$$\partial u_t(w,e)/\partial e < 0$$

It is costly to get education.

The utility function satisfies the single-crossing property

Specification of Utility Function

$$U_t(w,e) = f(w) - k_t g(e) \quad \text{for } k_t > 0 \text{ for } t = 1,2.$$

k_t indicates the cost of education for the worker type *t*.

It is more expensive for less productive worker to produce education signal

$$k_1 > k_2.$$

More Specifically

$$U_t(w,e) = 42\sqrt{w} - k_t e^{1.5} \qquad k_1 = 2 \quad k_2 = 1$$

Level of Education Chosen by Low Productive Worker

In perfect information equilibrium, firms pay according to the marginal productivity

$$w_1(e) = e$$

$$w_2(e) = 2e$$

The type 1 workers optimisation problem

$$U_t(w,e) = 42\sqrt{w} - k_1 e^{1.5} = 42\sqrt{e} - 2e^{1.5}$$

$$\frac{\partial U_t(w,e)}{\partial e} = 42 \times \frac{1}{2\sqrt{e}} - 3e^{0..5} = 0 \qquad e^* = 7$$

Level of Education Chosen by More Productive Worker

The type 2 worker's optimisation problem

$$U_t(w,e) = 42\sqrt{w} - k_1 e^{1.5} = 42\sqrt{2e} - e^{1.5}$$

$$\frac{\partial U_t(w,e)}{\partial e} = 42 \times \frac{1}{2\sqrt{2e}} - 1.5e^{0.5} = 0$$

$$42 \times \frac{1}{\sqrt{2e}} = 1.5e^{0..5}; \quad \frac{42}{1.5\sqrt{2}} = e;$$

$$e^* = 19.8$$

Popular Principal Agent Games

Principal	Agent	Action
Plantation owner	Share cropper	Labour input
Insurance company	Policy holder	Careful behaviour
Patient	Doctor	Intervention
Owner	Renter	Maintenance
Firms	Workers	Work effort

Both principal and agents have their objective functions they like to maximize.

The principal is interested in maximising profit from the business,

Agents aim to maximise utility(payoff) choosing the best contract available from the principal with proper allowances for its efforts.

Rasmusen (2007) has interesting examples on this topic.

Incomplete Information and Adverse Selection

Principal wants to produce output employing workers with a scheme of wage contract that matches efforts put by a worker to produce.

Worker knows his type but the principal does not.

Principal knows the distribution of quality of workers *F(s)*, where s denotes either good or bad state such as probability of observing good is 0.5 and of bad 0.5.

Principal offers the agent a wage contract W(q).

Worker accepts or rejects this contract based on self-selection and participation constraints.

Objectives of Principal and Agents

Basically worker evaluates the utility from the wage and disutility from work and decides the amount of work to put in.

Output from good workers is $q(e, good) = 3e$ and from bad state is $q(e, bad) = e$

If agent rejects the contract there is no work both worker and principal get zero payoff.

If worker accepts the contract

Agent's utility: $\pi_{agent} = U(e, w, s) = w - e^3$

Principal's profit: $\pi_{principal} = V(q - w) = q - w$.

Objectives and Optimal Efforts for Agents

Good worker maximises

$$\underset{e_g}{Max} \quad 3e_g - e_g^2$$

The first part is wage income and the second part of disutility of work.

The optimal level of efforts by good agent is:

$$3 - 2e_g = 0$$

$$e_g = 1.5$$

Bad worker's Objective and Optimal Efforts

$$\underset{e_b}{Max} \quad e_b - e_b^2$$

$$1 - 2e_b = 0 \; ; \; e_g = 0.5$$

The principal does not know what levels of efforts are appropriate for good and bad workers.

Principal's Objective and Contracts

Principal maximises expected profit

$$\underset{q_g, q_b, w_g, w_b}{Max} \quad \left[0.5\left(q_g - w_g\right) + 0.5\left(q_b - w_b\right)\right]$$

by designing separate contracts for good and bad worke
$\left(q_g, w_g\right)$ and $\left(q_b, w_b\right)$.

Wage for good worker: $q(e, good) = 3e$ or $e = \dfrac{q_g}{3}$

Wage for bad worker: $q_b = e$

Incentive Compatibility Constraints for Agents

Self selection constraint for good worker

$$\pi_{agent}\left(q_g, w_g / good\right) = w_g - \left(\frac{q_g}{3}\right)^2 \geq \pi_{agent}\left(q_b, w_b / good\right) = w_b - \left(\frac{q_b}{3}\right)^2$$

Self selection constraint for bad worker

$$\pi_{agent}\left(q_b, w_b / bad\right) = w_b - \left(q_b\right)^2 \geq \pi_{agent}\left(q_g, w_g / bad\right) = w_g - \left(q_g\right)^2$$

Participation constraints for good worker

$$\pi_{agent}\left(q_g, w_g / good\right) = w_g - \left(\frac{q_g}{3}\right)^2 \geq 0$$

Participation constraint for good worker

$$\pi_{agent}\left(q_b, w_b / bad\right) = w_b - \left(q_b\right)^2 \geq 0$$

Binding Constraints

Participation constraint of bad worker

$$w_b = (q_b)^2$$

Self selection constraint for good worker

$$w_g = \left(\frac{q_g}{3}\right)^2 + w_b - \left(\frac{q_b}{3}\right)^2 = \left(\frac{q_g}{3}\right)^2 + (q_b)^2 - \left(\frac{q_b}{3}\right)^2$$

Solving the Principal Agent Game

Principal's objective function

$$\underset{q_g, q_b, w_g, w_b}{Max} \quad \lfloor 0.5(q_g - w_g) + 0.5(q_b - w_b) \rfloor$$

Including binding constraints of agents:

$$\underset{q_g, q_b}{Max} \quad \left[0.5\left(q_g - \left(\frac{q_g}{3}\right)^2 - (q_b)^2 + \left(\frac{q_b}{3}\right)^2 \right) + 0.5\left(q_b - (q_b)^2\right) \right]$$

First order conditions with respect to q_g and q_b

$$0.5\left(1 - \frac{2q_g}{9}\right) = 0 \quad \text{or} \quad q_g = 4.5$$

$$0.5\left(-2q_b + \frac{2q_b}{9}\right) + 0.5(1 - 2q_b) = 0 \quad \text{or} \quad \left(-2q_b + \frac{2q_b}{9}\right) + (1 - 2q_b) = 0$$

$$\left(-4q_b + \frac{2q_b}{9} + 1\right) = 0; \quad 34q_b = 9 \quad \text{or} \quad q_b = 0.265$$

Incentive Compatible First Best Choices of Good and Bad Worker

Now wages can be found from the constraints

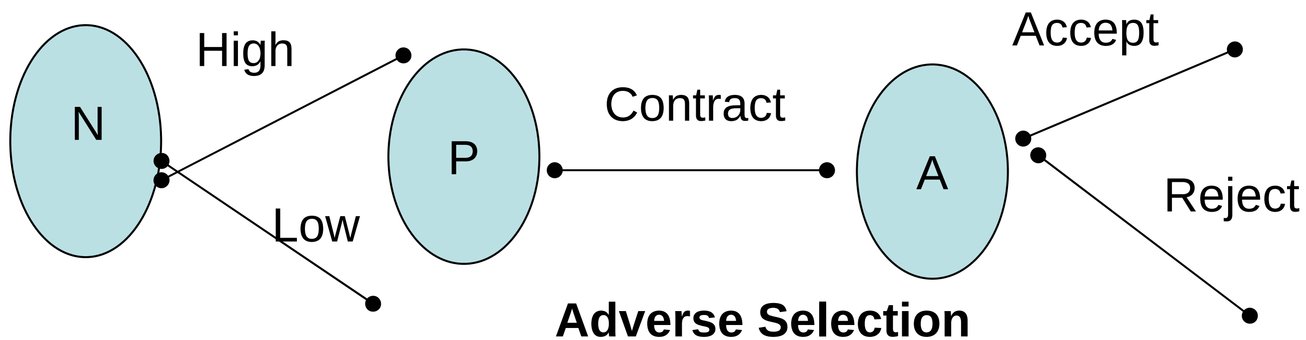
$$w_b = (q_b)^2 = (0.265)^2 = 0.07$$

$$w_g = \left(\frac{q_g}{3}\right)^2 + (q_b)^2 - \left(\frac{q_b}{3}\right)^2 = \left(\frac{4.5}{3}\right)^2 + (0.265)^2 - \left(\frac{0.265}{3}\right)^2 = 2.32$$

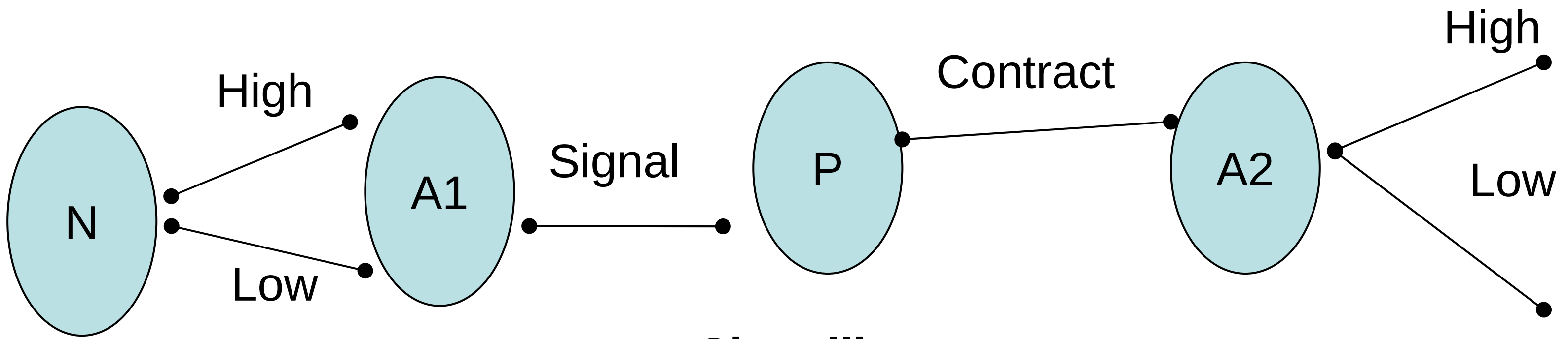
Thus in the presence of information asymmetry , the efforts by the good worker is at **the first best level** as the bad effort by him is not as attractive as the good effort.

It is not profitable for good worker to pretend as a bad worker. Good worker is not attracted by the contract for bad worker.

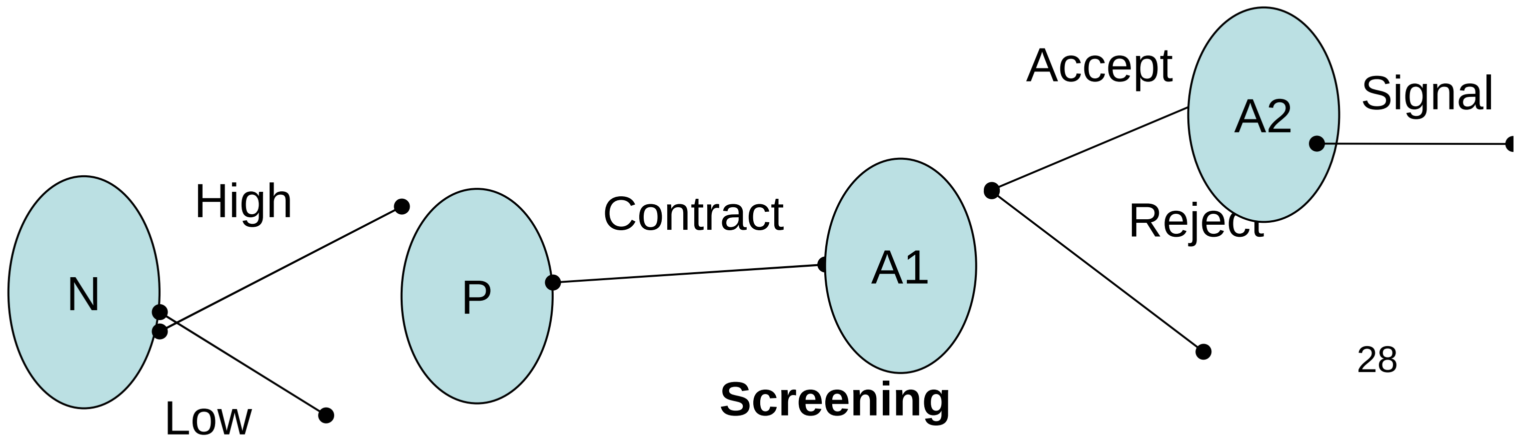
It is very costly for the bad worker to accept the contract of good worker. Bad worker’s **first best to put low effort**.



Adverse Selection



Signalling



Screening