

# Economic Modelling

Ramsey Growth Model

Panel data growth regressions

## Determinants of Growth in Ramsey Model

- Preferences of households for current consumption relative to future consumption,  $\beta$
- Productivity of capital,  $\alpha$
- Rate of depreciation of capital
- Technology of production,  $A$

Analytical solution and numerical examples

# One Sector Ramsey (1928) Model of Accumulation and Growth

Preference: 
$$U_t = \sum_{t=0}^{\infty} \beta^t \ln(C_t) \quad 0 < \beta < 1$$

Technology: 
$$Y_t = AK_t^\alpha \quad 0 < \alpha < 1$$

Market clearing: 
$$C_t + I_t = Y_t \quad K_0 = K_0 \quad C_t = Y_t - I_t$$

Accumulation 
$$K_{t+1} = K_t(1 - \delta) + I_t$$

State and control: 
$$C_t = AK_t^\alpha - \{K_{t+1} - K_t(1 - \delta)\}$$

Solve the infinite horizon problem taking slice for two periods:

$$U = +\beta^t \ln[AK_t^\alpha - \{K_{t+1} - K_t(1 - \delta)\}] + \beta^{t+1} \ln[AK_{t+1}^\alpha - \{K_{t+2} - K_{t+1}(1 - \delta)\}] + \dots$$

Ramsey (1928) Cass (1965) Koopman(1965) Uzawa (1965)

## Optimality Conditions and Steady State in Ramsey Model (Euler Equation)

$$\frac{\partial U}{\partial K_{t+1}} = -\frac{\phi\beta^t}{C_t} + \frac{\beta^{t+1}}{C_{t+1}} \left( \alpha AK_{t+1}^{\alpha-1} + (1-\delta) \right) = 0$$

$$\frac{\phi\beta^t}{C_t} = \frac{\beta^{t+1}}{C_{t+1}} \left( \alpha AK_{t+1}^{\alpha-1} + (1-\delta) \right)$$

$$\frac{C_{t+1}}{C_t} = \frac{\beta^{t+1}}{\beta^t} \left( \alpha AK_{t+1}^{\alpha-1} + (1-\delta) \right)$$

$$\dots = K_{t-1} = K_t = K_{t+1} = \dots = \bar{K} \quad \dots = C_{t-1} = C_t = C_{t+1} = \dots = \bar{C}$$

$$\frac{C_{t+1}}{C_t} = \frac{\bar{C}}{\bar{C}} = \beta \left( \alpha A \bar{K}^{\alpha-1} + (1-\delta) \right)$$

## Steady State in an Optimal Growth Model

$$\frac{C_{t+1}}{C_t} = \frac{\bar{C}}{\bar{C}} = \beta(\alpha A \bar{K}^{\alpha-1} + (1-\delta)) \quad (\alpha A \bar{K}^{\alpha-1} + (1-\delta)) = \left(\frac{1}{\beta}\right) \quad 0 < \delta < 1$$

$$(\bar{K}^{\alpha-1}) = \frac{1}{\alpha A} \left( \frac{1}{\beta} - (1-\delta) \right) \quad (\bar{K}^{\alpha-1}) = \frac{1}{\alpha A} \left( \frac{1-\beta(1-\delta)}{\beta} \right)$$

$$\bar{K} = \left( \frac{1-\beta(1-\delta)}{\alpha A \beta} \right)^{\frac{1}{\alpha-1}} \quad \bar{K} = \left( \frac{\alpha A \beta}{1-\beta(1-\delta)} \right)^{\frac{1}{1-\alpha}}$$

$$\bar{Y} = A \bar{K}^{\alpha}$$

$$\bar{Y} = A^{\frac{2-\alpha}{1-\alpha}} \left( \frac{\alpha \beta}{1-\beta(1-\delta)} \right)^{\frac{\alpha}{1-\alpha}}$$

$$\bar{I} = \bar{K} - (1-\delta)\bar{K}$$

$$\bar{I} = \delta \bar{K}$$

$$\bar{I} = \delta \bar{K} = \delta \left( \frac{\alpha A \beta}{1-\beta(1-\delta)} \right)^{\frac{1}{1-\alpha}}$$

$$\bar{C} = \bar{Y} - \bar{I}$$

$$\bar{C} = \left( \frac{\alpha A \beta}{1-\beta(1-\delta)} \right)^{\frac{\alpha}{1-\alpha}} - \delta \left( \frac{\alpha A \beta}{1-\beta(1-\delta)} \right)^{\frac{1}{1-\alpha}}$$

# Growth Regressions: Determinants of Growth

- Dependent variable:
  - Growth rate of per capita real GDP
- Explanatory variables
  - Capital per worker
  - Growth rate of population
  - Technology knowledge
  - Human capital
  - Economic policy (inflation, trade and budget deficit, regimes)
  - Country and time specific factors

# Determinants of Growth: Panel Regression Model

$$y_{i,t} = \alpha_i + x_{i,t}\beta_{i,t} + \lambda_t + e_{i,t}$$

$$\begin{bmatrix} y_{i,1} \\ y_{i,2} \\ \cdot \\ \cdot \\ y_{i,T} \end{bmatrix} = \begin{bmatrix} x_{i,1}\beta \\ x_{i,2}\beta \\ \cdot \\ \cdot \\ x_{i,T}\beta \end{bmatrix} + \begin{bmatrix} \alpha_i \\ \alpha_i \\ \cdot \\ \cdot \\ \alpha_i \end{bmatrix} + \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \cdot \\ \cdot \\ \lambda_T \end{bmatrix} + \begin{bmatrix} e_{1i} \\ e_{2i} \\ \cdot \\ \cdot \\ e_{mi} \end{bmatrix}$$

$$y_i = x_i\beta + \lambda_t + \iota_i\alpha_i + e_i$$

Baltagi (1995), Doornik and Hendry (2001)

## Estimator of a Panel Data Model

$$y_i = x_i \beta + D_i \delta + e_i$$

$$y = W\beta + e$$

$$\hat{\beta} = (W'W)^{-1}W'y$$

# Dynamic Panel Data Model

$$y_{i,t} = \sum_{s=1}^p a_k y_{i,t-s} + \beta^t(L)x_{i,t} + \lambda_t + \alpha_i + e_{i,t}$$

$$y_i = W_i \delta + \iota_i \alpha_i + e_i$$

GMM Estimator ( Hansen (1982), Baltagi (1995), Doornik and Hendry (2001)):

$$\hat{\delta} = \left[ \left( \sum_i W_i^* Z_i \right) A_N \left( \sum_i Z_i' W_i \right) \right]^{-1} \left( \sum_i W_i^* Z_i \right) A_N \left( \sum_i Z_i' y_i^* \right)$$

$$A_N = \left( \frac{1}{N} \sum_i Z_i' H_i Z_i \right)^{-1}$$

## Growth Panel Regression

DPD( 1) Modelling rgdpwok by OLS (using Advanced\_Penn.xls)

	Coefficient	Std.Error	t-value	t-prob		
ki	931.226	260.5	3.57	0	I0	Canada
openk	480.134	64.62	7.43	0	I1	France
kc	1007.88	345	2.92	0.004		German
kg	-593.585	386	-1.54	0.125	I2	y
Constant	-59755.5	2.81E+04	-2.13	0.034	I3	Ireland
I1	15282	2610	5.86	0	I4	Italy
I2	6925.64	923.7	7.5	0	I5	Japan
I3	-21775.3	2974	-7.32	0		Netherla
I4	6660.39	1849	3.6	0	I6	nds
I5	8304.64	3861	2.15	0.032	I7	Spain
I6	-963.007	2715	-0.355	0.723	I8	UK
I7	-22.6079	1448	-0.0156	0.988	I9	USA
I8	10505.5	1487	7.07	0		
I9	22697.9	2142	10.6	0		

### Steps for estimating growth regression

1. Formulate, specify growth, regression model
2. Formulate a growth panel data (ONS, Navidata, Mimas, Penn World Table, WBIndicators or IMF, OECD, EUROSTAT, FAO)
3. Read Excel Data file in GiveWin/PcGive software
4. Choose Panel data package
5. Choose dependent, independent variables and years
6. Run the regression (pooled, fixed effect, within and between group effects)
7. Report the coefficients and write a paper

## Diagnostic Tests on Reliability of Coefficients in the Panel Data Model

sigma            5550.889    sigma^2            3.081237e+007  
R^2              0.8246781  
RSS              15591057230    TSS              88928202576  
no. of observations    520    no. of parameters    14  
Using robust standard errors

Transformation used:    none

constant:            yes    time dummies:            0  
group dummies:            0    time\*group:            0    individual:            9  
number of individuals    10 (derived from year)    longest time series    54  
[1950 - 2003]    shortest time series    34 (unbalanced panel)

Wald (joint):    Chi^2(4) =    129.3 [0.000] \*\*

Warning: invertgen: singular matrix

../dpd/dpd.ox (1201): TestWald

Wald (dummy):    Chi^2(10) =    106.8 [0.000] \*\*

AR(1) test:        N(0,1) =    2.393 [0.017] \*

AR(2) test:        N(0,1) =    2.316 [0.021] \*