General Equilibrium Gains from Global Trade Over Time

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Global Economy with Two Countries

Country A

Technologies: $X_{1,A} = 5L_{1,A}$; $X_{2,A} = 2L_{2,A}$

Resources $L_{1,A} + L_{2,A} = 200$

Preferences $U_A = X_{1,A}^{\alpha} X_{2,A}^{(1-\alpha)}$ α is 0.4

Production possibility frontier of A:

$$\frac{1}{5} X_{1,A} + \frac{1}{2} X_{2,A} = 200$$

Country B

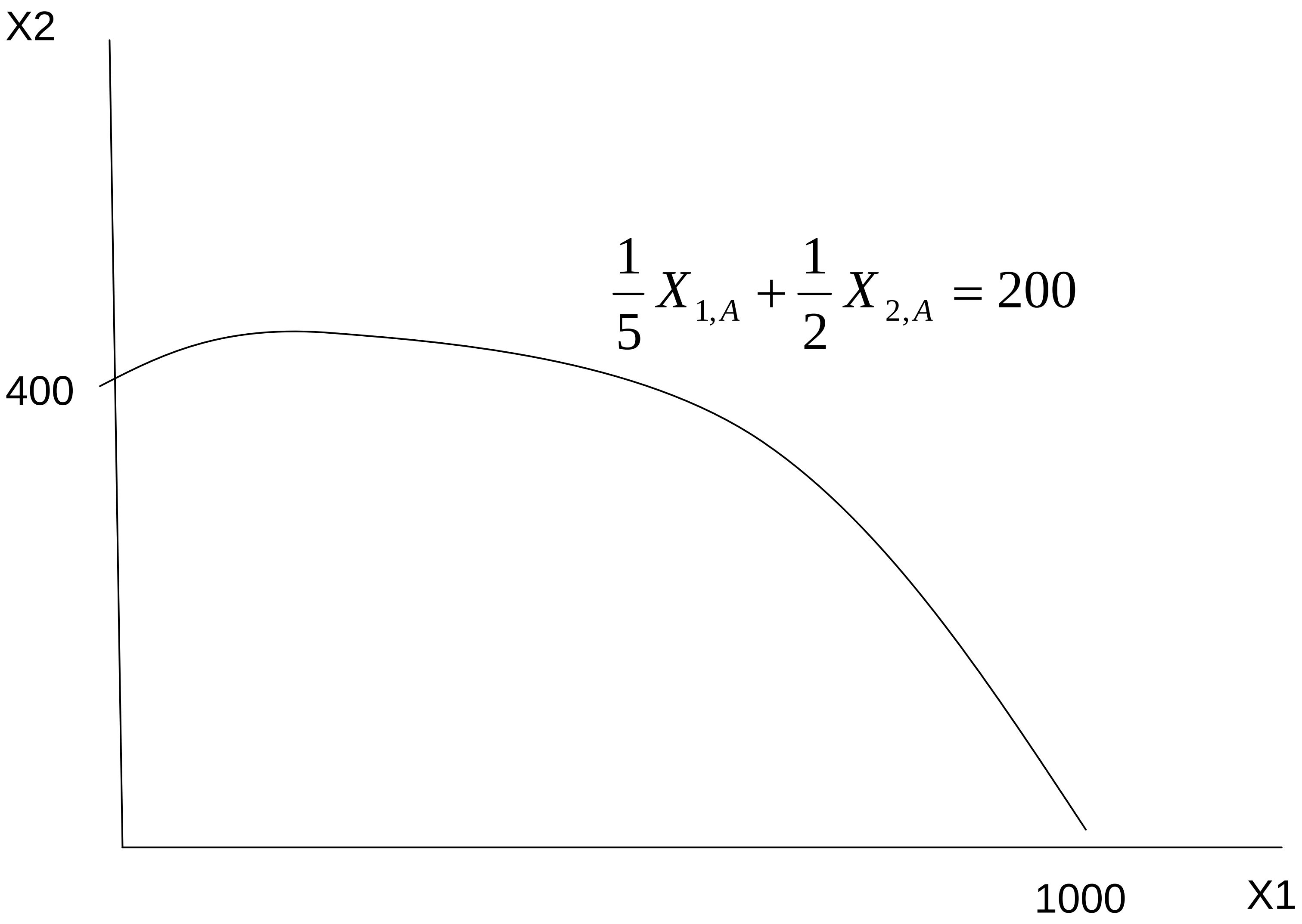
Technologies: $X_{1,B} = 2L_{1,B}$ and $X_{2,B} = 5L_{2,B}$

Resource: $L_{1,B} + L_{2,B} = 400$

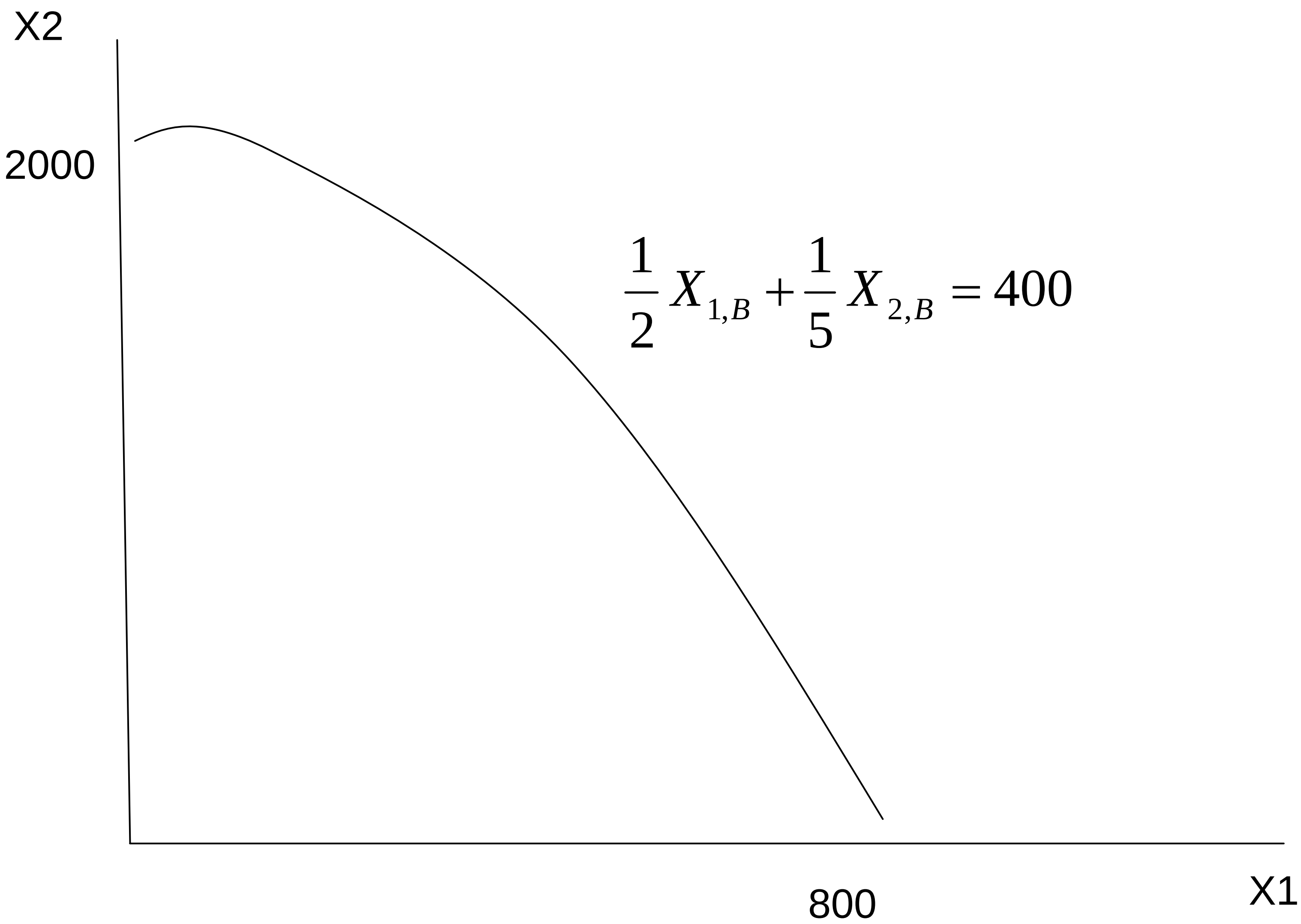
Preferences: $U_B = X_{1,B}^{\beta} X_{2,B}^{(1-\beta)}$ β 0.6.

Production possibility frontier: $\frac{1}{2} X_{1,B} + \frac{1}{5} X_{2,B} = 400$

Production Possibility Frontier of Country A



Production Possibility Frontier of Country B



Autarchy (no trade) Solution for Country A

$$L_A = X_{1,A}^{\alpha} X_{2,A}^{(1-\alpha)} + \lambda\left[200 - \frac{1}{5}X_{1,A} - \frac{1}{2}X_{2,A}\right]$$

$$\frac{\partial L_A}{\partial X_{1,A}} = \alpha X_{1,A}^{\alpha-1} X_{2,A}^{(1-\alpha)} - \frac{1}{5}\lambda = 0\ ;$$

$$\frac{\partial L_A}{\partial X_{2,A}} = (1-\alpha) X_{1,A}^{\alpha} X_{2,A}^{(-\alpha)} - \frac{1}{2}\lambda = 0$$

$$\frac{\partial L_A}{\partial \lambda} = 200 - \frac{1}{5}X_{1,A} - \frac{1}{2}X_{2,A} = 0$$

From first two FOC, $\frac{\partial L_A}{\partial X_{1,A}} = \frac{\alpha}{(1-\alpha)}\frac{X_{2,A}}{X_{1,A}} = \frac{2}{5}$ or

$X_{2,A} = \frac{2}{5}\frac{(1-\alpha)}{\alpha}X_{1,A}$ substituting this into the 3rd first order condition

$$200 = \frac{1}{5}X_{1,A} + \frac{1}{2}X_{2,A} = \frac{1}{5}X_{1,A} + \frac{1}{2}\left(\frac{2}{5}\frac{(1-\alpha)}{\alpha}X_{1,A}\right) = \left[\frac{1}{5} + \frac{1}{5}\left(\frac{0.6}{0.4}\right)\right]X_{1,A}$$

$$200 = \frac{1}{5}\left[1 + \left(\frac{3}{2}\right)\right]X_{1,A} \qquad X_{1,A} = 400$$

This implies $X_{2,A} = \frac{2}{5}\frac{(1-\alpha)}{\alpha}X_{1,A} = \frac{2}{5}\frac{0.6}{0.4}(400) = \frac{2}{5}\frac{3}{2}(400) = 240$

This solution exhausts the total resource available in the country which can be checked putting these values in the production possibility frontier:

$\frac{1}{5}X_{1,A} + \frac{1}{2}X_{2,A} = 200$ and $\frac{1}{5}(400) + \frac{1}{2}(240) = 80 + 120 = 200$

Autarchy (no trade) Solution for Country B

$$L_B = X_{1,B}^{\beta} X_{2,B}^{(1-\beta)} + \lambda\left[400 - \frac{1}{2}X_{1,B} - \frac{1}{5}X_{2,B}\right]$$

$$\frac{\partial L_B}{\partial X_{1,B}} = \beta X_{1,B}^{\beta-1} X_{2,B}^{(1-\beta)} - \frac{1}{2}\lambda = 0$$

$$\frac{\partial L_B}{\partial X_{2,B}} = (1-\beta) X_{1,B}^{\beta} X_{2,B}^{(-\beta)} - \frac{1}{5}\lambda = 0$$

$$\frac{\partial L_B}{\partial \lambda} = 400 - \frac{1}{2}X_{1,B} - \frac{1}{5}X_{2,B}$$

From first two FOC, $\frac{\partial L_B}{\partial X_{1,B}} = \frac{\beta}{(1-\beta)}\frac{X_{2,B}}{X_{1,B}} = \frac{5}{2}$ or

$X_{2,B} = \frac{5}{2}\frac{(1-\beta)}{\beta}X_{1,B}$ substituting this into the 3rd first order condition

$$400 = \frac{1}{2}X_{1,B} + \frac{1}{5}X_{2,B} = \frac{1}{2}X_{1,B} + \frac{1}{5}\left(\frac{5}{2}\frac{(1-\beta)}{\beta}X_{1,B}\right) = \left[\frac{1}{2} + \frac{1}{2}\left(\frac{0.4}{0.6}\right)\right]X_{1,B}$$

$$400 = \left[\frac{1}{2} + \frac{1}{3}\right]X_{1,\mathbf{B}} \qquad X_{1,A} = \frac{400 \times 6}{5} = 480$$

This implies $X_{2,B} = \frac{5}{2}\frac{(1-\beta)}{\beta}X_{1,B} = \frac{5}{2}\frac{4}{6}(480) = \frac{5}{3}(480) = 5 \times 160 = 800$

This satisfies the resource balance equation: $\frac{1}{2}X_{1,B} + \frac{1}{5}X_{2,B} = 400$

$$\frac{1}{2}(480) + \frac{1}{5}(800) = 240 + 160 = 400$$

Comparative Advantages of Countries A and B

Country A is more efficient in producing X_1:

$X_{1,A} = 5L_{1,A}$ implies one unit of labour can produce five units of X_1 and 1000 units of X_1 if it only produces X_1

$X_{2,A} = 2L_{2,A}$ implies it can produce 400 units of X_2 if it produces only X_2

Country B is more efficient in producing X_2 than X_1;
Given its technologies $X_{1,B} = 2L_{1,B}$ and $X_{2,B} = 5L_{2,B}$

In complete specialisation B can produce 2000 units of X_2 but only 800 units of X_1.

Global Market Clearing

A complete specialisation is desirable to reap all comparative advantage.

A competitive equilibrium implies global market clearing for both goods

$$X_{1,A} + X_{1,B} = 1000 \qquad X_{2,A} + X_{2,B} = 2000$$

Competitive price can be obtained using one of these two market clearing conditions;

Let the price of good 1 be numeraire $P_1 = 1$.

$$\frac{\alpha X_1}{P_1} + \frac{\beta X_2}{P_1} = 1000\,;\ 0.4(1000) + 0.6(2000P_2) = 1000 \text{ from this}$$

$$400 + 1200P_2 = 1000\,;\ P_2 = \frac{600}{1200} = 0.5\,;$$

This is the relative price of good two that clears the global market.

Now it is possible to determine the demand for X_1 and X_2 by A and B.

$$X_{1,A} + X_{1,B} = 400 + 600 = 1000$$

Similarly market clearing in good two implies

$$X_{2,A} + X_{2,B} = 2000 \qquad X_{2,A} = \frac{(1-\alpha)I_A}{P_2} = \frac{0.6(1000)}{0.5} = 1200$$

$$X_{2,B} = \frac{(1-\beta)I_A}{P_2} = \frac{0.4(2000 \times 0.5)}{0.5} = 800$$

$$X_{2,A} + X_{2,B} = 1200 + 800 = 2000$$

This satisfies the global market equilibrium.

Gains from complete specialisation is obvious;

If country A produced only good X_2 it could produce only 400 but if it specialises in X_1 it can consume 400 of X_1 and 1200 X_2.

The consumption of X_2 alone is three times its production capacity.

Similarly if country B only produced X_1 is can produce only 800 of X_1 but by specialising in X_2 it can consume 600 of X_1 and 800 of X_2.

Moral Hazard, Adverse Selection and Cooperative Bargaining in Global Trade

- **Moral hazard** arises when countries break trade agreements quietly for their benefit and harming another trading partner without any compensation.

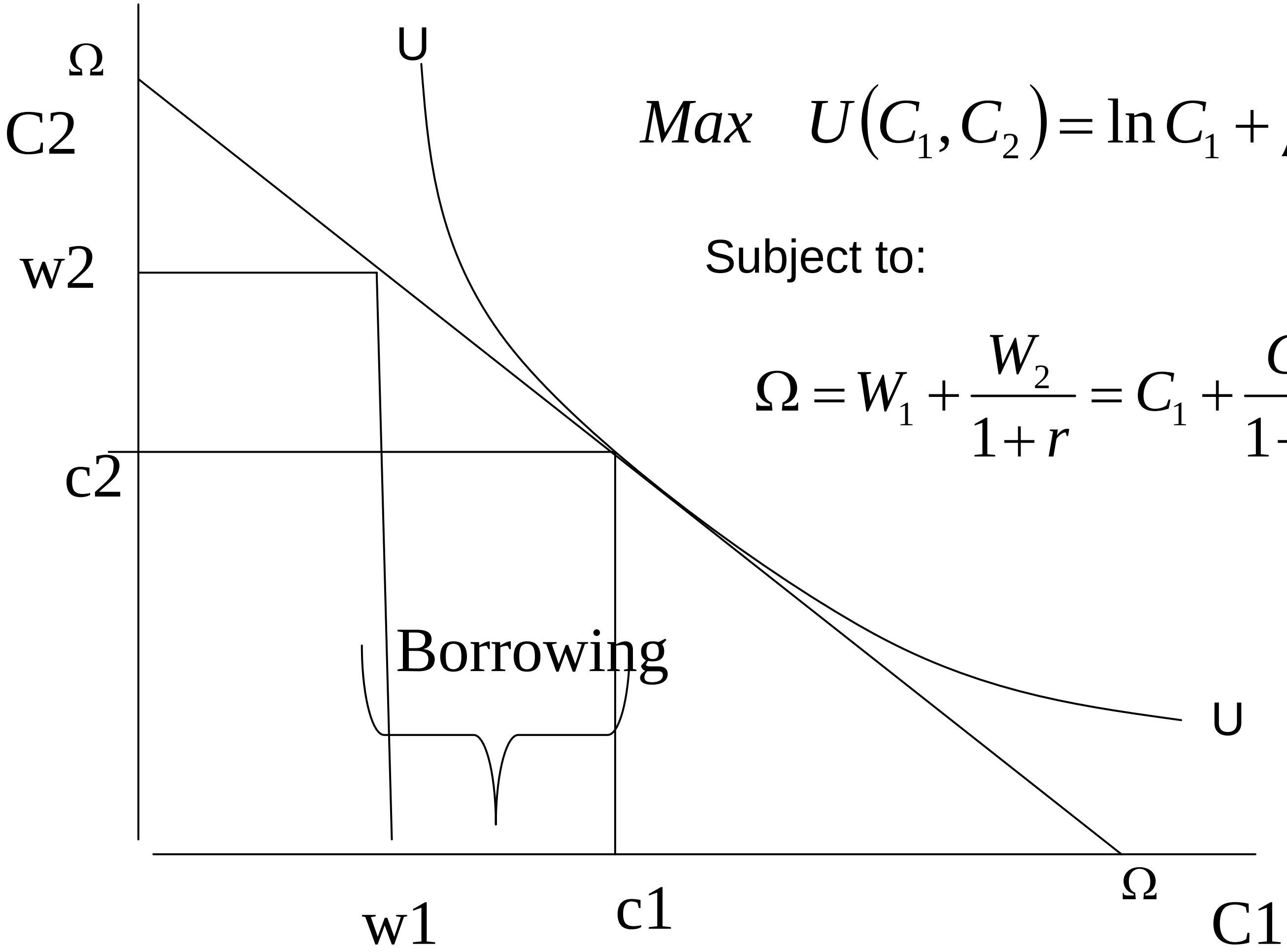
- Anti-dumping duties and violation of copy rights are examples of moral hazard.

- Lack of information about the structure of demand in foreign country or about the technology of the foreign country would give rise to problem of **adverse selection.**

- In such situation countries will can use **signalling, screening** and optimise the expected utilities assigning probabilities various contingencies.

- **Cooperative Bargaining** a better than retaliation.

Two Period Model of Consumption and Saving



$$Max \quad U(C_1, C_2) = \ln C_1 + \beta \ln C_2$$

Subject to:

$$\Omega = W_1 + \frac{W_2}{1+r} = C_1 + \frac{C_2}{1+r}$$

Consumption-Saving Problem in a Two Period Model

$$Max \quad U\left(C_1^i, C_2^i\right) = \ln C_1^i + \beta \ln C_2^i \qquad (1)$$

Subject to

$C_1^i + b^i \leq W_1^i$: budget constraint in period 1

$C_2^i \leq b^i(1+r) + W_2^i$: budget constraint in period 2 (2)

The inter temporal budget constraint is:

(4)

$$C_1^i + \frac{C_2^i}{1+r} = W_1^i + \frac{W_2^i}{1+r} \qquad (3)$$

$$L^i = \ln C_1^i + \beta \ln C_2^i + \lambda\left[C_1^i + \frac{C_2^i}{1+r} - W_1^i - \frac{W_2^i}{1+r}\right] \qquad (4)$$

Demand for period 1 and 2 consumptions

$$C_1^i = \frac{1}{1+\beta}\left[W_1^i + \frac{W_2^i}{1+r}\right] \qquad (5)$$

$$C_2^i = (1+r)\beta C_1^i \Rightarrow C_2^i = (1+r)\frac{\beta}{1+\beta}\left[W_1^i + \frac{W_2^i}{1+r}\right] \qquad (6)$$

Market clearing conditions in period 1 and 2

$$C_1^A + C_1^B = W_1^A + W_1^B$$

$$\frac{1}{1+\beta}\left[W_1^A + \frac{W_2^A}{1+r}\right] + \frac{1}{1+\beta}\left[W_1^B + \frac{W_2^B}{1+r}\right] = W_1^A + W_1^B \qquad (7)$$

Market clearing condition for period 2:

$$C_2^A + C_2^B = W_2^A + W_2^B$$

$$\frac{\beta(1+r)}{1+\beta}\left[W_1^A + \frac{W_2^A}{1+r}\right] + \frac{\beta(1+r)}{1+\beta}\left[W_1^B + \frac{W_2^B}{1+r}\right] = W_2^A + W_2^B \qquad (8)$$

The gross interest rate that clears the market

$$\frac{1}{1+\beta}\left[W_1^A+\frac{W_2^A}{1+r}\right]+\frac{1}{1+\beta}\left[W_1^B+\frac{W_2^B}{1+r}\right]=W_1^A+W_1^B$$

or

$$\frac{1}{1+\beta}\left[W_1^A+W_1^B\right]+\frac{1}{1+\beta}\left[\frac{W_2^A}{1+r}+\frac{W_2^B}{1+r}\right]=W_1^A+W_1^B$$

$$\frac{1}{1+\beta}+\frac{1}{1+\beta}\frac{1}{1+r}\left(\frac{W_2^A+W_2^B}{W_1^A+W_1^B}\right)=1 \qquad (9)$$

$$\frac{1}{1+\beta}\frac{1}{1+r}\left(\frac{W_2^A+W_2^B}{W_1^A+W_1^B}\right)=1-\frac{1}{1+\beta}=\frac{\beta}{1+\beta} \quad \text{or } 1+r=\frac{1}{\beta}\left(\frac{W_2^A+W_2^B}{W_1^A+W_1^B}\right)$$

Numerical Example

If endowments and preferences were $\{W_1^A, W_2^A, W_1^B, W_2^B\}=\{100,0,0,200\}$ and $\beta=0.9$

respectively the interest rate that clears these two markets is given by:

$$\frac{1}{1+0.9}+\frac{1}{1+0.9}\left(\frac{200}{100}\right)=1+r=2.222 \Rightarrow r=1.222 \qquad (10)$$

Same result can be obtained using the period 2 budget constraint

$$1+r=\frac{1}{\beta}\left(\frac{W_2^A+W_2^B}{W_1^A+W_1^B}\right)=\frac{1}{0.9}\left(\frac{200}{100}\right)\Rightarrow 2.222 \qquad (10)$$

Consumption Saving Decisions of Agent A

$$C_1^A = \frac{1}{1+\beta}\left[W_1^A + \frac{W_2^A}{1+r}\right] = \frac{1}{1.9}(100) = 52.63$$

$$C_2^A = (1+r)\beta\frac{1}{1+\beta}\left[W_1^A + \frac{W_2^A}{1+r}\right] = 2.222\left(\frac{0.9}{1.9}\right)100 = 105.25$$

Amount of savings in period 1 and period 2 respectively are

$$S_1^A = W_1^A - C_1^A = W_1^A - \frac{1}{1+\beta}\left[W_1^A + \frac{W_2^A}{1+r}\right] = 100 - 52.63 = 47.37$$

Consumption Saving Decisions of Agent B

$$C_1^B = \frac{1}{1+\beta}\left[W_1^B + \frac{W_2^B}{1+r}\right] = \frac{1}{1.9}\left(\frac{200}{2.222}\right) = 47.37$$

$$C_2^B = (1+r)\frac{\beta}{1+\beta}\left[W_1^B + \frac{W_2^B}{1+r}\right] = 2.222\left(\frac{0.9}{1.9}\right)\frac{200}{2.222} = 94.73$$

$$S_1^B = W_1^B - C_1^B = -47.37;\ \ S_2^B = W_2^B - C_2^B = 200 - 94.73 = 105.3$$

Scenarios of Equilibrium Allocation in Two Period Intertemporal Model

Supply of savings in Two period Model with Various parameters

Parameters of Consumption Saving Model							
	I	II	III	IV	V	VI	VII
w1a	100	100	100	100	100	500	1000
w2a	0	0	0	50	200	200	200
w1b	0	0	0	100	100	400	2000
w2b	200	200	200	200	200	200	100
Beta	0.9	0.95	1	0.9	0.9	0.9	0.9
Equilibrium Interest Rate and Consumption							
1+r	2.222222	2.105263	2	1.388889	2.222222	0.493827	0.111111
c1a	52.63158	51.28205	50	71.57895	100	476.3158	1473.684
c2a	105.2632	102.5641	100	89.47368	200	211.6959	147.3684
Ua	8.154133	8.336304	8.517193	8.315351	9.373656	10.98572	11.78916
Uanb	4.60517	4.60517	4.60517	8.125991	9.373656	10.98309	11.67624
c1b	47.36842	48.71795	50	128.4211	100	423.6842	1526.316
c2b	94.73684	97.4359	100	160.5263	200	188.3041	152.6316
Ub	7.953948	8.236282	8.517193	9.425926	9.373656	10.76324	11.85584
Ubnb	4.768486	5.033401	5.298317	9.373656	9.373656	10.75995	11.74556
Equilibrium Savings (Borrowing or Lending)							
s1a	47.36842	48.71795	50	28.42105	0	23.68421	-473.684
s2a	-105.263	-102.564	-100	-39.4737	0	-11.6959	52.63158
s1b	-47.3684	-48.7179	-50	-28.4211	0	-23.6842	473.6842
s2b	105.2632	102.5641	100	39.47368	0	11.69591	-52.6316

Steady State and Transitions Dynamics in an Overlapping Generation model

The transition dynamics in the neoclassical model is given by the law of motion of the capital stock. Consider an economy inhibited by *N* number of individuals.

At period *0* each of them is endowed by k_0 capital stock and aggregate capital stock is K_0. The level of technical know how is denoted by *A*.

Production technology is standard Cobb-Douglas production function; $Y_t = AK_t^{\beta} L_t^{(1-\beta)}$. This implies per capita output to be $y_t = A_t k_t^{\beta}$. Let the labour force L_t be fixed to *N* in each period.

The remuneration to capital is according to its marginal productivity; $r_t = \frac{\partial y_t}{\partial k_t} = \beta A_t k_t^{\beta-1}$. Labour is paid the residual amount $w_t = \frac{\partial y_t}{\partial L_t} = (1-\beta) A_t k_t^{\beta}$.

There are two types of people living in this economy, young and old. Young work and earn labour income and consume a α fraction of income $c_{yt} = \alpha w_t$ and save $(1-\alpha)$ for their old age.

Consumption Saving and Capital Accumulation of Young and Old

There are two types of people living in this economy, young and old. Young work and earn labour income and consume a α fraction of income $c_{yt} = \alpha w_t$ and save $(1-\alpha)$ for their old age.

The life time budget constraint is given by $C_{Y,t} + \frac{C_{O,t}}{(1+r_{t+1})} = W_t$. The old people earn interest in their asset and consume all of their income, $c_{ot} = a_t(1+r_t)$.

The capital stock of period t is result of the saving of old people; $a_{t+1} = (1-\alpha)w_t$. Next periods capital stock equals the assets saved today as given by the equation of accumulation;

$K_{t+1} = (1-\alpha)w_t = (1-\alpha)(1-\beta)A_t k_t^{\beta}$. Aggregate saving equals total output minus the consumption of young and old, this also it the market clearing condition in this model $S_t = Y_t - Nc_{yt} - Nc_{ot}$.

Saving equals investment in each period $S_t = I_t$ and investment adds to the capital stock $I_t = K_{t+1} - K_t$.

Numerical Results of Steady State and Transitional Dynamics in an Overlapping Generation Model

Growth, Capital Accumulation and Consumption in an Overlapping Generation Model

Time	K	K	Y	w	r	cy	co	S	I
0.0	3.000	300.000	1390.389	9.733	1.390	4.866	7.171	186.636	186.636
1.0	4.866	486.636	1607.538	11.253	0.991	5.626	9.689	76.002	76.002
2.0	5.626	562.638	1679.070	11.753	0.895	5.877	10.664	25.036	25.036
3.0	5.877	587.675	1701.144	11.908	0.868	5.954	10.980	7.726	7.726
4.0	5.954	595.400	1707.822	11.955	0.861	5.977	11.077	2.338	2.338
5.0	5.977	597.738	1709.831	11.969	0.858	5.984	11.107	0.703	0.703
6.0	5.984	598.441	1710.434	11.973	0.857	5.987	11.116	0.211	0.211
..	..	..	..	..	..	..	..	..	..
40.0	5.987	598.742	1710.693	11.975	0.857	5.987	11.120	0.000	0.000
41	5.987	598.742	1710.693	11.975	0.857	5.987	11.120	0.000	0.000

α	0.5	Mpc	k = capital labour ratio	cy = consumption of young
N	100	Population	K = capital stock	co = consumption of old
β	0.3	productivity of capital	Y = aggregate output	S=aggregate saving
A	10	technological index	w=wage rate	I= net investment
K0	300	Initial capital stock	r= interest rate	a = asset position

For details on this type of model see Auerbach and Kotlikoff (1998) Macroeconomics: An Integrated Approach, MIT Pres p.63-71.