Basics of Game Theory

Elements of Games
Players and Strategies
Solutions:
Dominant Strategy
Nash Equilibrium
Mixed Strategy
Dynamic Game: Backward Induction
GAMES with Perfect and Imperfect Information

http://cepa.newschool.edu/het/schools/game.htm

Elements of a Game

- **Rational Players**
- **Strategic Choices**
- **Payoff matrix**

Players like to
Maximise their
Own pay-off.

A

B

$$\begin{array}{ccc} & \textbf{Strategy 1} & \textbf{Strategy 2} \\ \textbf{Strategy 1} & \left(\prod^{R}_{1,1},\prod^{C}_{1,1}\right) & \left(\prod^{R}_{1,2},\prod^{C}_{1,2}\right) \\ \textbf{Strategy 2} & \left(\prod^{R}_{2,1},\prod^{C}_{2,1}\right) & \left(\prod^{R}_{2,2},\prod^{C}_{2,2}\right) \end{array}$$

$\prod^{R}_{1,1}$ payoff to row player when both row and column play strategy 1.

Types of Games

- **Cooperative Games:**
  two or many players; oligopoly/competition
- **Non-cooperative Games: two or many players**
  between opposing political parties, countries
- **Single period of multiple period: static and dynamic**
- **Full information or incomplete information**
- **Firms and consumers; government and public;**

Among individuals, clubs, parties; nations and regions

Competition and Collusion

$$\begin{array}{cccc} & & \textbf{A} & \\ & & \textbf{Strategy 1} & \textbf{Strategy 2} \\ \textbf{B} & \textbf{Strategy 1} & (10,-10) & (-10,10) \\ & \textbf{Strategy 2} & (-10,10) & (10,-10) \end{array}$$

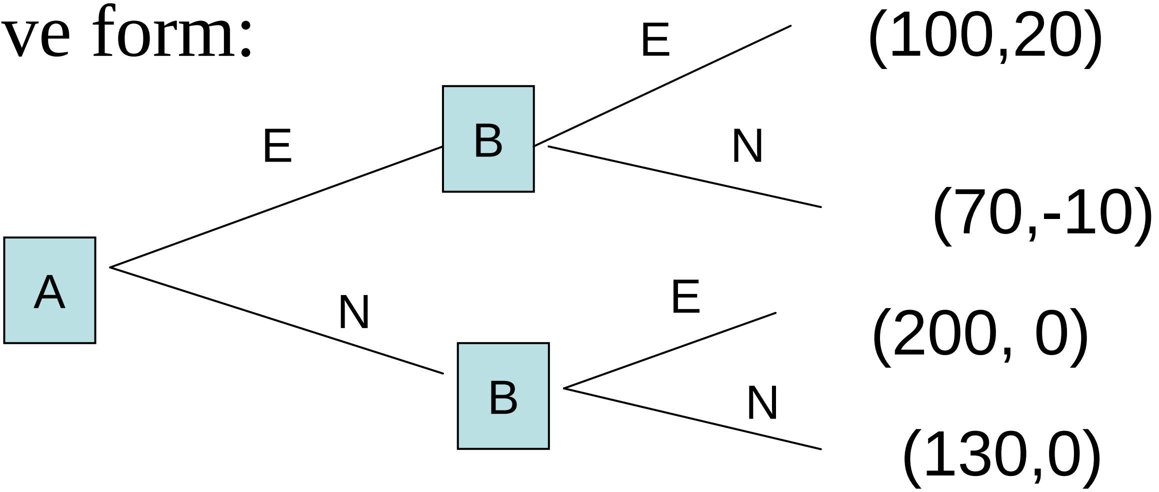
Example of a **zero sum game**: one’s gain = loss of another Sports; Market shares.

Normal and Extensive Form Representation of a Game

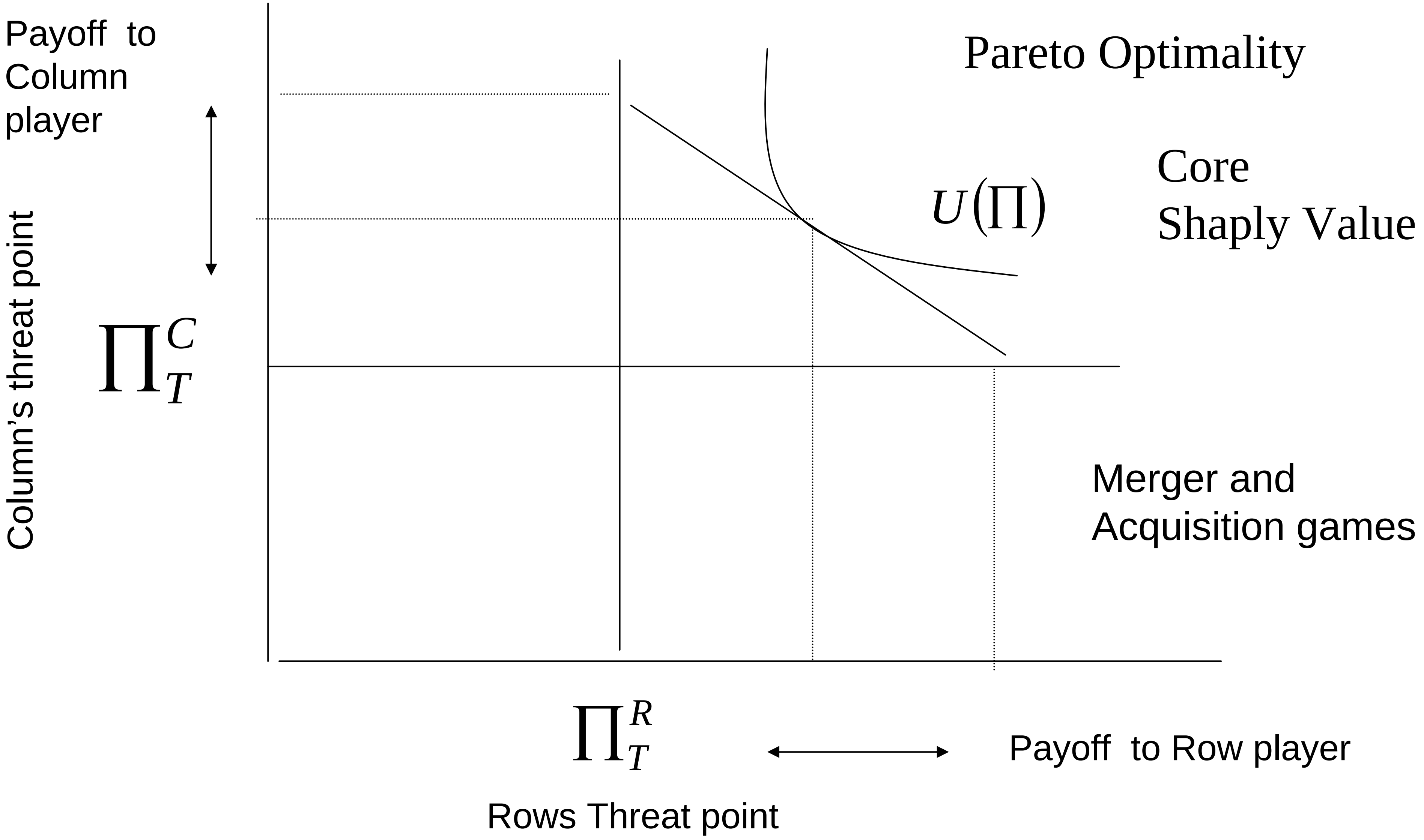
Normal form:

$$\begin{array}{cccc} & & \multicolumn{2}{c}{\text{A}} \\ & & \text{Enter} & \text{Don't Enter} \\ \text{B} & \text{Enter} & (100,20) & (200,0) \\ & \text{Don't Enter} & (70,-10) & (130,0) \end{array}$$

Extensive form:



Gains from Co-operative Solutions and Room for a Bargain



Payoff to Column player

Column's threat point

$\prod_T^C$

Pareto Optimality

$U(\Pi)$

Core

Shaply Value

Merger and Acquisition games

$\prod_T^R$

Payoff to Row player

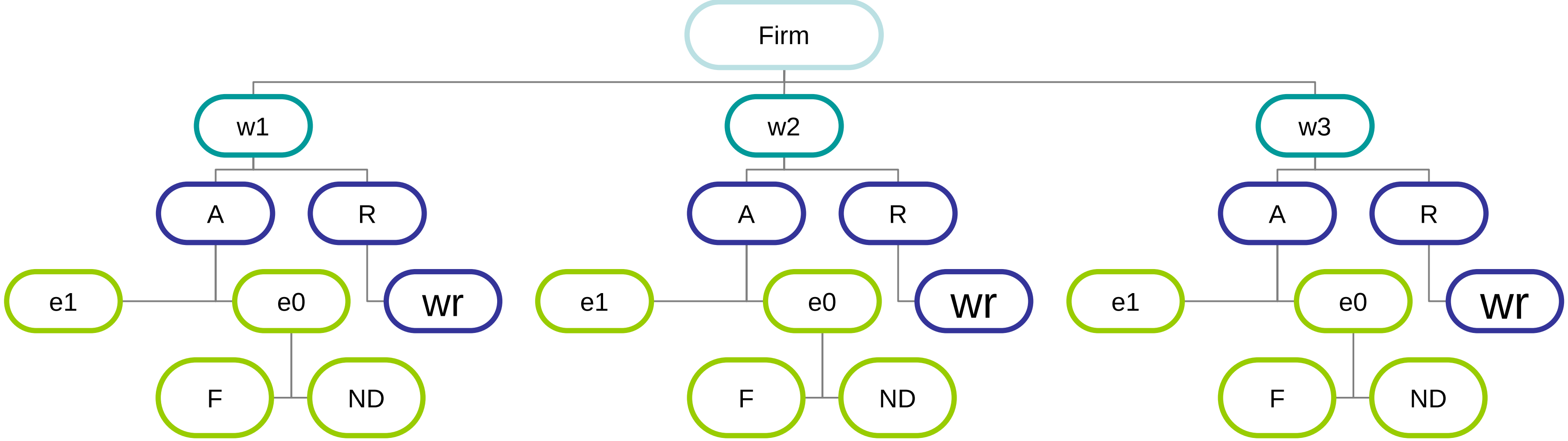
Rows Threat point

With many players there can be several coalitions.

A Dynamic GAME

Efficiency Wage Rate: Principal Agent Problem

Workers try to maximize expected Utility



First stage: Firm offers three different wage rates w1, w2 and W2

Second stage: Workers accept or reject the offer. If they reject they get wr

Third stage: either they can put hard effort (e1) or Shirk (e0), Gets efficiency wage in effort e1 but can be caught and punished in shirking.

If caught is fired (F) and gets doles (wr)

If not detected (ND) gets the efficiency wage rate as in e1

Question: How do workers and firms play this wage bargaining game?

There is an issue of Reputation and credibility over time.

Games that can be solved by a Dominant Strategy

$$\begin{array}{ccc} & & \text{A} \\ & & \begin{array}{cc} \text{Adv} & \text{DntAdv} \end{array} \\ \text{B} & \begin{array}{c} \text{Adv} \\ \text{DntAdv} \end{array} & \begin{bmatrix} (10,5) & (15,0) \\ (6,8) & (10,2) \end{bmatrix} \end{array}$$

Adv better for B

Adv better for A

Both advertise
B get 10
A get 5

$$\begin{array}{ccc} & & \text{A} \\ & & \begin{array}{cc} \text{Adv} & \text{DntAdv} \end{array} \\ \text{B} & \begin{array}{c} \text{Adv} \\ \text{DntAdv} \end{array} & \begin{bmatrix} (10,5) & (15,0) \\ (6,8) & (20,2) \end{bmatrix} \end{array}$$

Firm B does not have a dominant Strategy

Adv is dominant St. for Firm A

Game with A Nash Equilibrium

Prisoners dilemma

$$\begin{array}{ccc} & & \begin{array}{cc} & A \\ \text{Confess} & \text{Do not confess} \end{array} \\ B & \begin{array}{c} \text{Confess} \\ \text{Do not confess} \end{array} & \begin{bmatrix} (-5,-5) & (-1,-10) \\ (-10,-1) & (-2,-2) \end{bmatrix} \end{array}$$

$$\begin{bmatrix} (-\underline{5},-\underline{5}) & (-\underline{1},-10) \\ (-10,-\underline{1}) & (-2,-2) \end{bmatrix}$$

Nash Solution: (-5,-5)

Cooperation was better:
(-2,-2)

Cooperation is better but each think that other player will cheat and therefore they
Don’t cooperate, therefore stay longer in jail.

Solution by Random Mixed Strategy

All games do not have equilibrium in pure strategy.
Example: Game of Matching Pennies

$$\begin{array}{ccc} & & \text{A} \\ & & \begin{array}{cc} \text{S1} & \text{S2} \end{array} \\ \text{B} & \begin{array}{c} \text{S 1} \\ \text{S2} \end{array} & \begin{bmatrix} (1,-1) & (-1,1) \\ (-1,1) & (1,-1) \end{bmatrix} \end{array}$$

There is no solution in pure strategy.

At least on Nash Equilibrium in the Mix strategy.

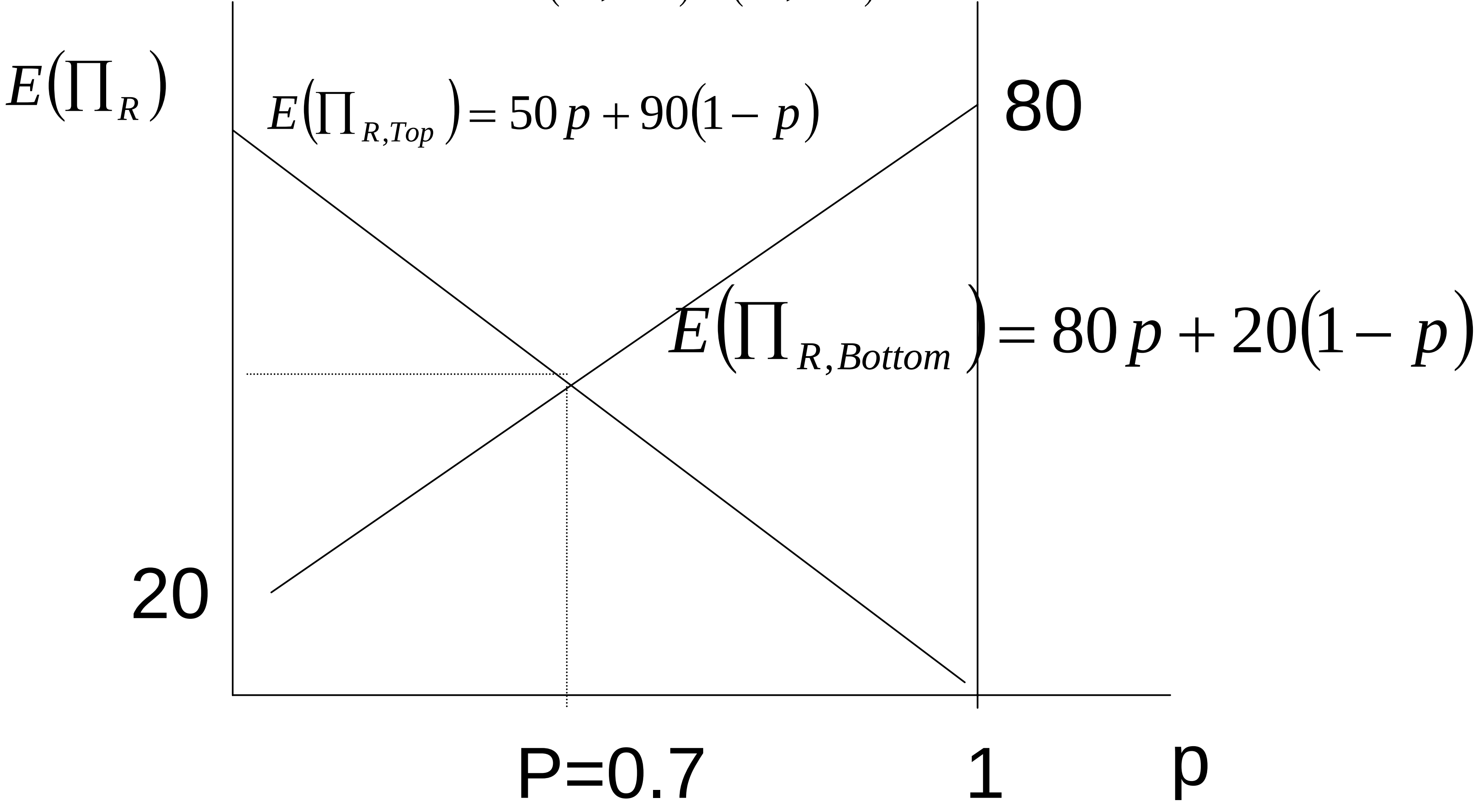
Flip the coin to randomise the chosen strategies.

If played s1 half of the time optimal payoff is zero to both players.

Probability of playing S1 or S2 is 0.5.

Finding the Mixed Strategy in a Competitive Game

	left	*right*
top	$(50,-50)$	$(80,-80)$
Bottom	$(90,-90)$	$(20,-20)$



$$50p + 90(1-p) = 80p + 20(1-p) \qquad 100p = 70$$

Does subsidy to the Airbus by EU countries deter Boeing from Producing a New Aircraft?

$$\begin{array}{rl} & \qquad\text{Airbus} \\ & \begin{array}{cc} \quad\text{S1} & \qquad\qquad \text{S2} \end{array} \\ \text{Boeing}\begin{array}{r} \text{S 1} \\ \text{S2} \end{array} & \begin{bmatrix} (-10,-10) & (100,0) \\ (0,100) & (0,0) \end{bmatrix} \end{array}$$

GAME 1

$$\begin{array}{rl} & \qquad\text{Airbus} \\ & \begin{array}{cc} \quad\text{S1} & \qquad\qquad \text{S2} \end{array} \\ \text{Boeing}\begin{array}{r} \text{S 1} \\ \text{S2} \end{array} & \begin{bmatrix} (-10,10) & (100,0) \\ (0,120) & (0,0) \end{bmatrix} \end{array}$$

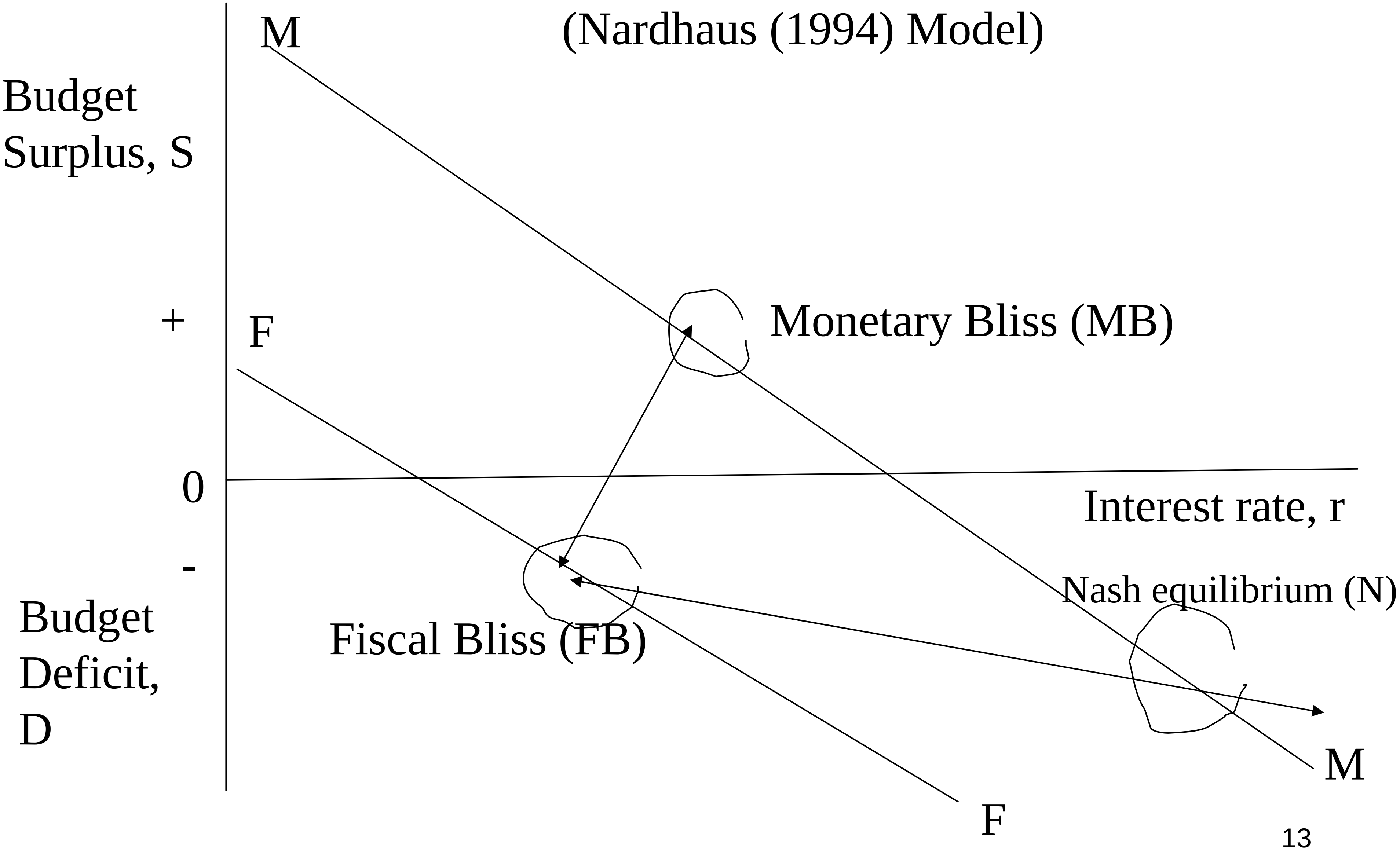
GAME 2

Entry Deterrence by an Incumbent to a Potential Entrant

$$\begin{array}{llcc}
 & & \multicolumn{2}{c}{\text{Potential Entrant}} \\
 & & \text{Enter} & \text{Don't} \\
 & \text{Enter} & (-10,-10) & (100,0) \\
\text{Incumbent} & & & \\
 & \text{Don't} & (0,100) & (0,0)
\end{array}$$

$$\begin{array}{llcc}
 & & \multicolumn{2}{c}{\text{Potential Entrant}} \\
 & & \text{Enter} & \text{Don't} \\
 & \text{Enter} & (-10,10) & (100,0) \\
\text{Incumbent} & & & \\
 & \text{Don't} & (0,120) & (0,0)
\end{array}$$

Fiscal and Monetary Policy Game in a Diagram



(Nardhaus (1994) Model)

Budget Surplus, S

+

0

-

Budget Deficit, D

M

F

Monetary Bliss (MB)

Fiscal Bliss (FB)

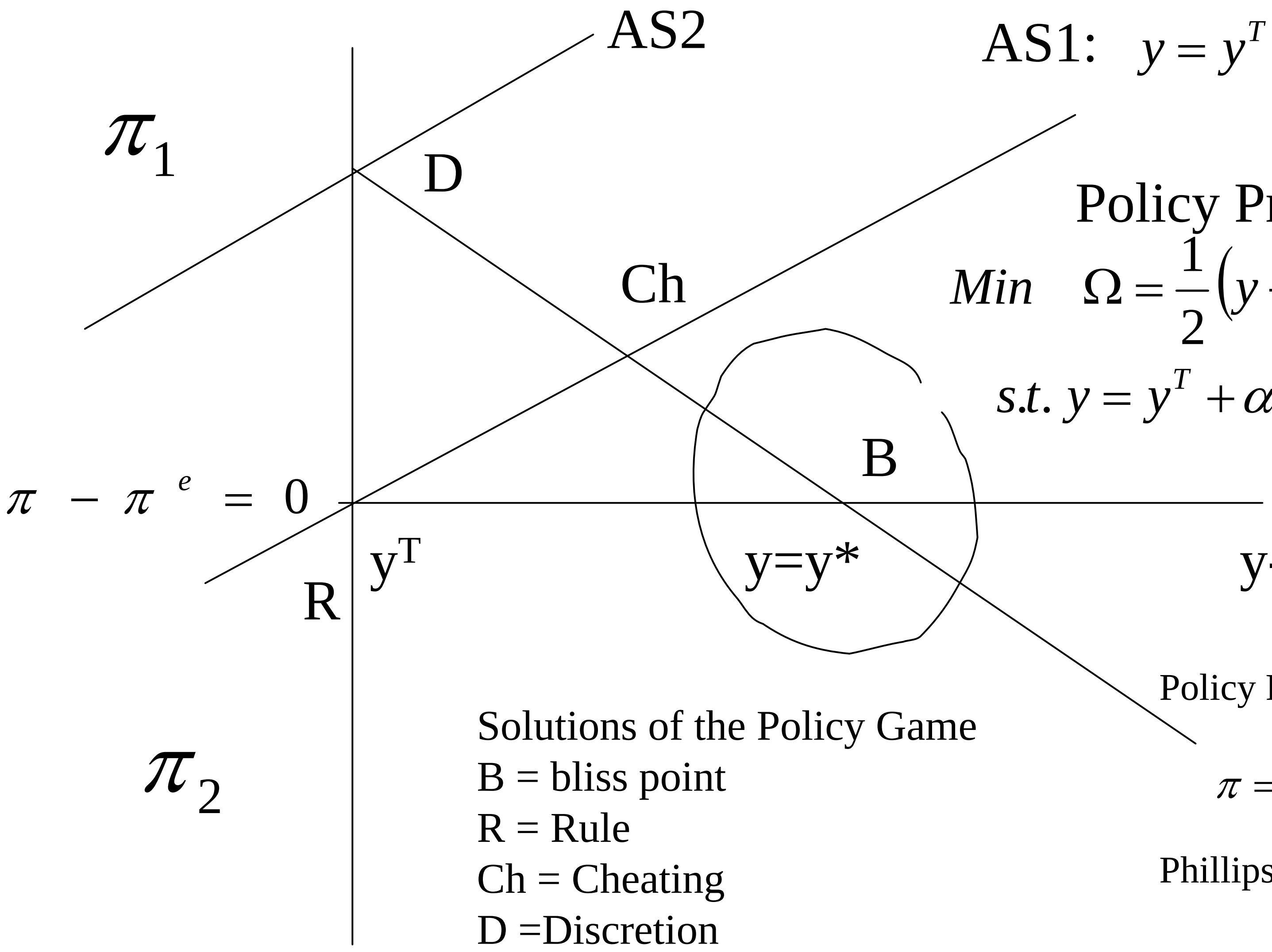
Interest rate, r

Nash equilibrium (N)

M

F

Policy Reaction Function and Lucas Supply Curve



AS1: $y = y^T + \alpha\left(\pi - \pi^e\right) + \varepsilon$

Policy Problem

$$Min \quad \Omega = \frac{1}{2}\left(y - y^*\right)^2 + \frac{\beta}{2}\pi^2$$

$$s.t.\ y = y^T + \alpha\left(\pi - \pi^e\right) + \varepsilon$$

Solutions of the Policy Game
B = bliss point
R = Rule
Ch = Cheating
D =Discretion

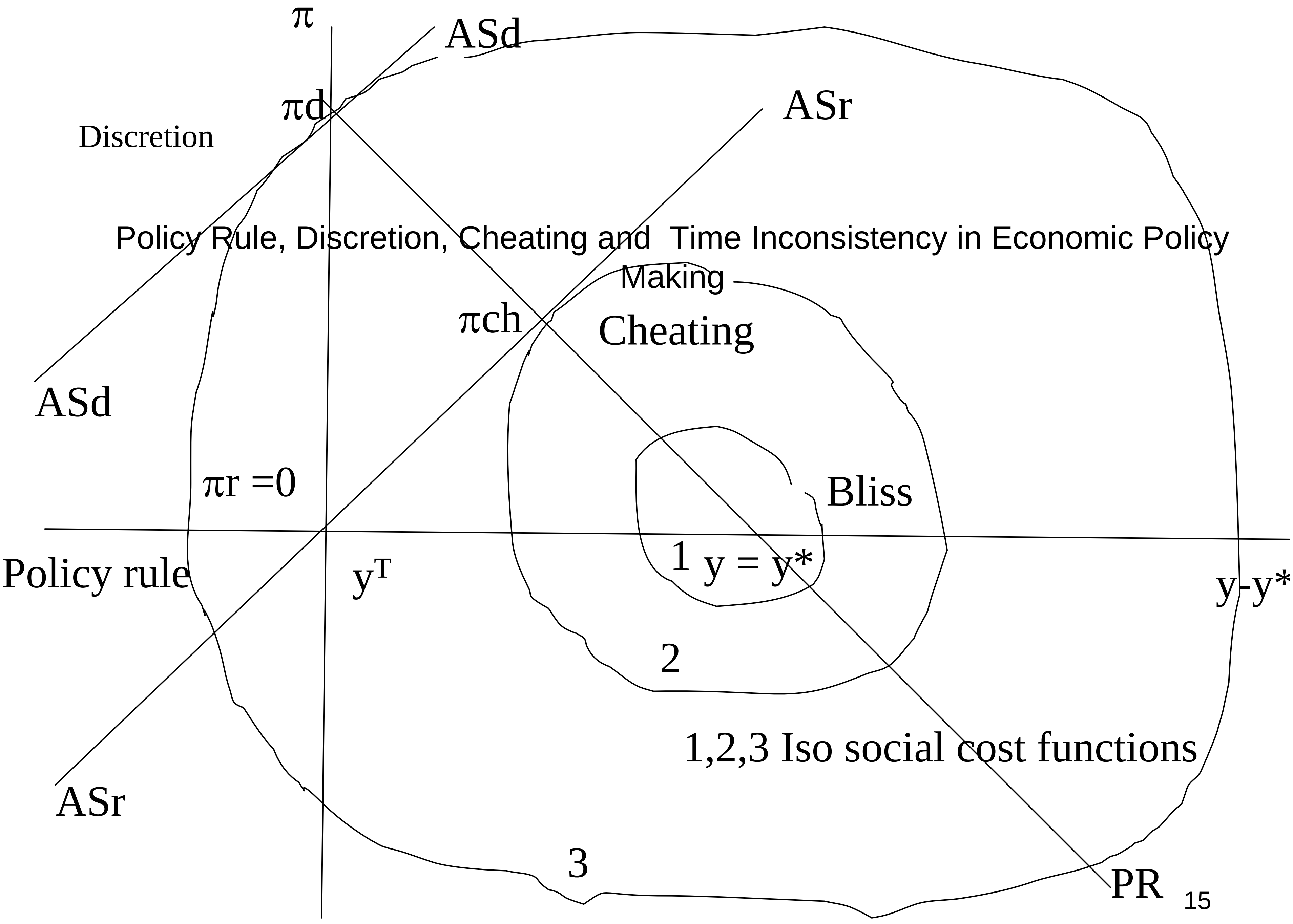
Policy Reaction Function

$$\pi = \frac{-\alpha\left(y - y^*\right)}{\beta}$$

Phillips’ Curve

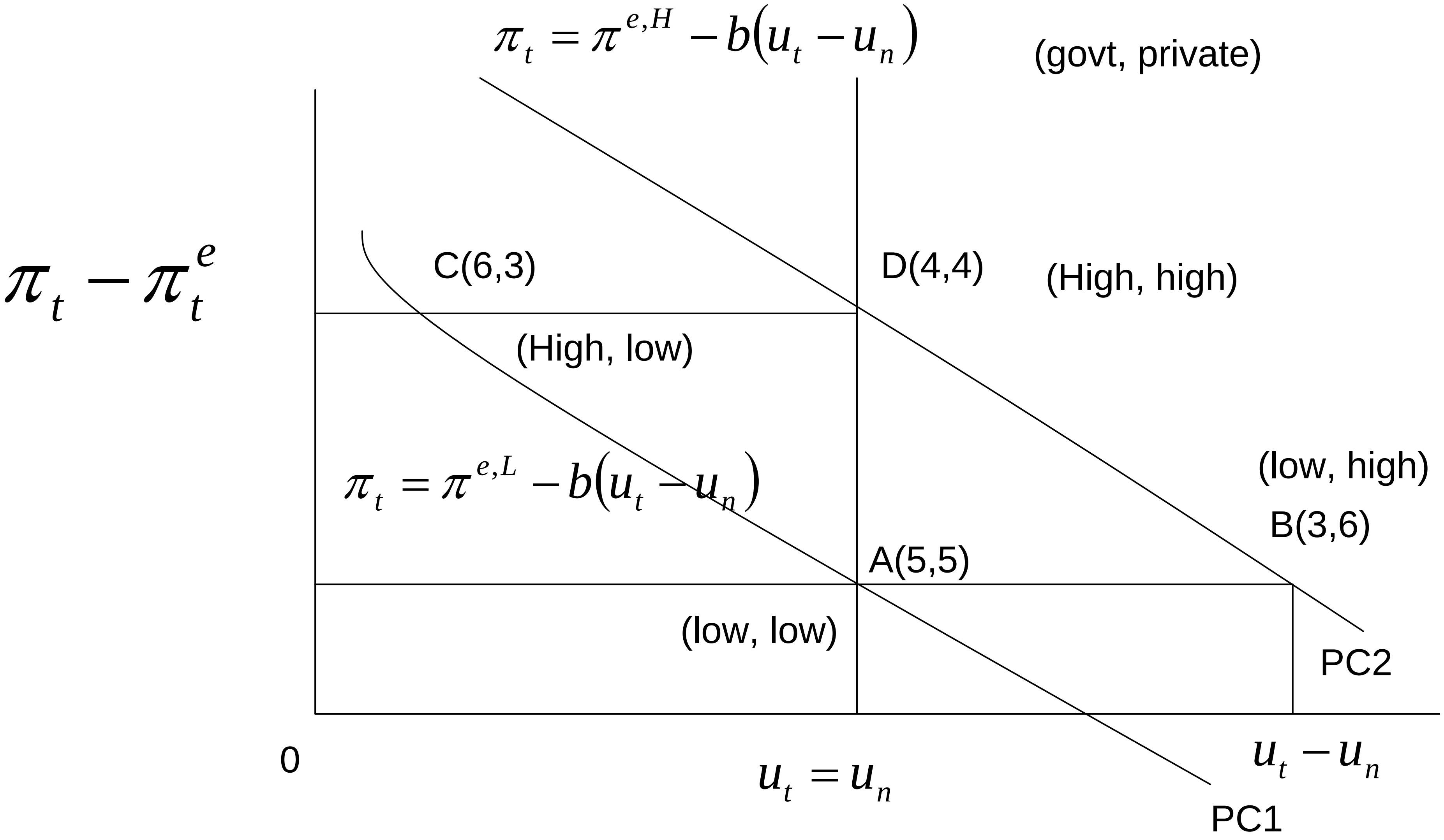
Higher rate of inflation or deflation or deviation of output from the trend are undesirable

Policy Rule, Discretion, Cheating and Time Inconsistency in Economic Policy Making



Kydland and Prescott (1977)

Inflation-Unemployment Game Between Private and Public Sectors in a diagram



First element of choice is by the government government.

Pay-Off Matrix for Inflation-unemployment Policy Game

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad \Pr ivate \quad Sector$$

$$Government \quad Sector \quad \begin{bmatrix} & H & L \\ H & 4,4 & 6,3 \\ L & 3,6 & 5,5 \end{bmatrix}$$

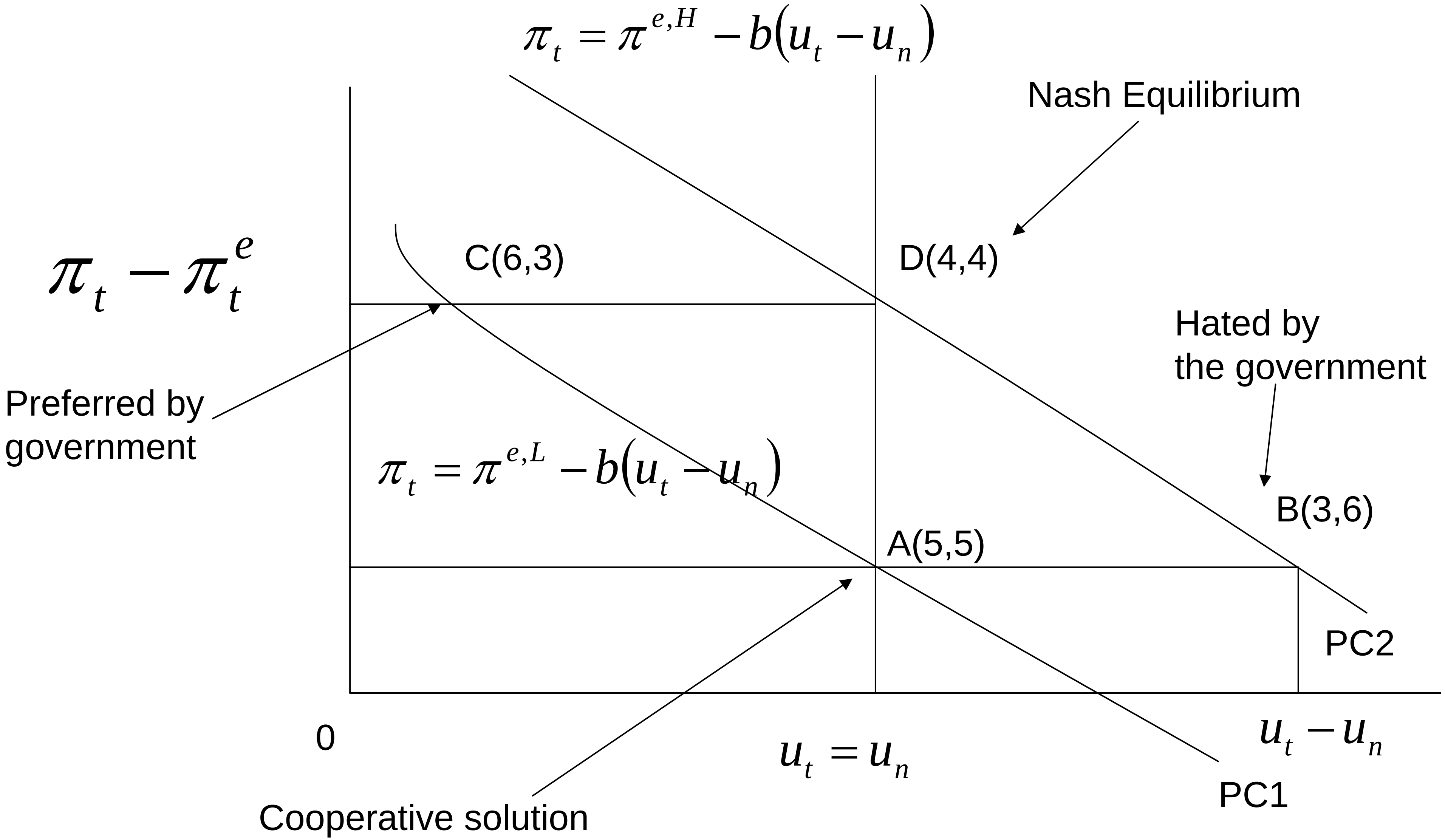
Tasks

Find a Nash Equilibrium.

Solve the game using Backward induction if the government moves first.

Find the discount factor if the game is played infinite number of times.

Inflation-Unemployment Game Between Private and Public Sectors



First element represents payoff to the government.

Process of Finding a Nash Equilibrium Unemployment Inflation Game

Private sector

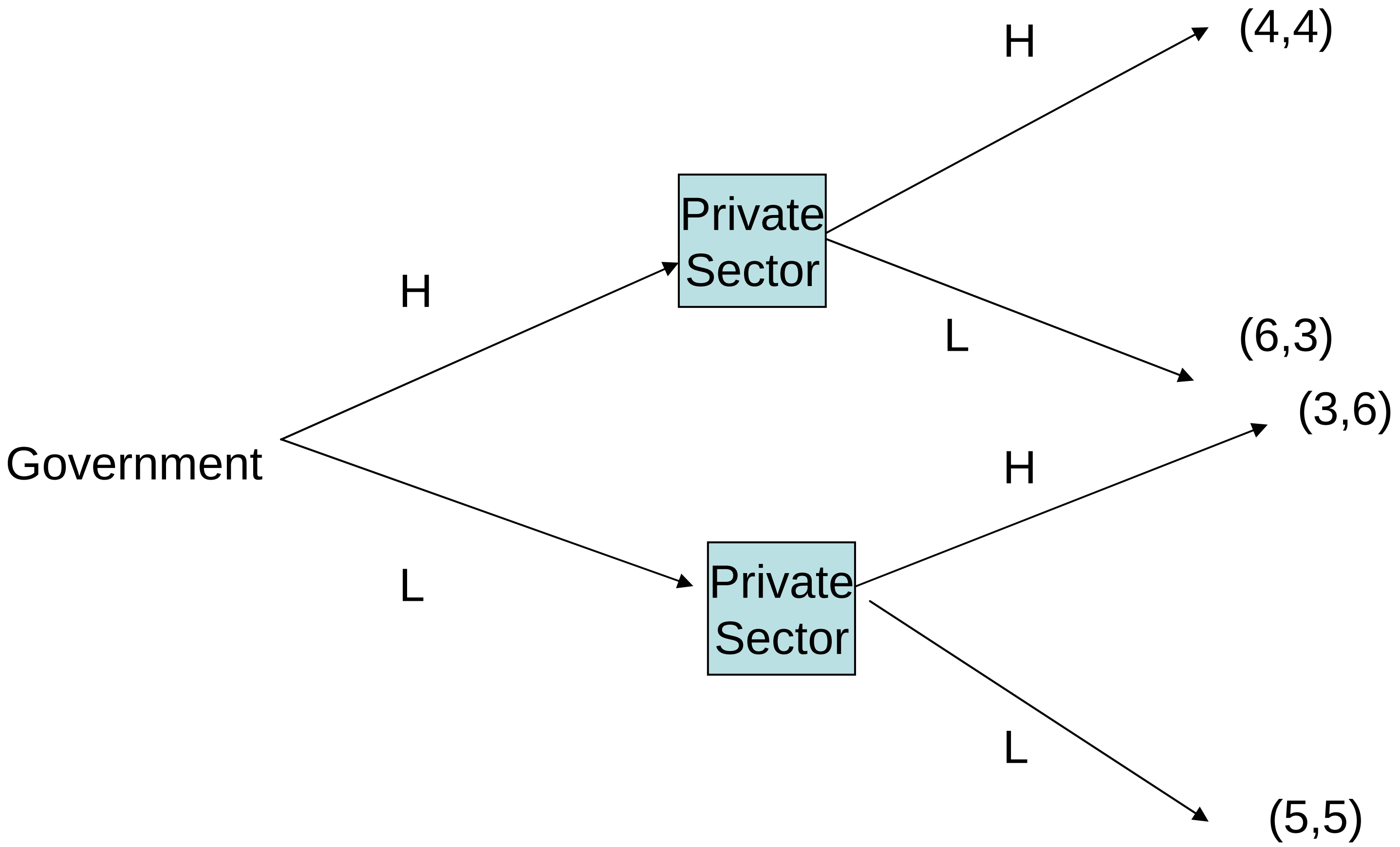
		H	L		
Government	H	$\underline{4}$ 4	$\underline{6}$ 3	H	Government Sector’ choice
	L	3 6	5 5	L	

Private sector

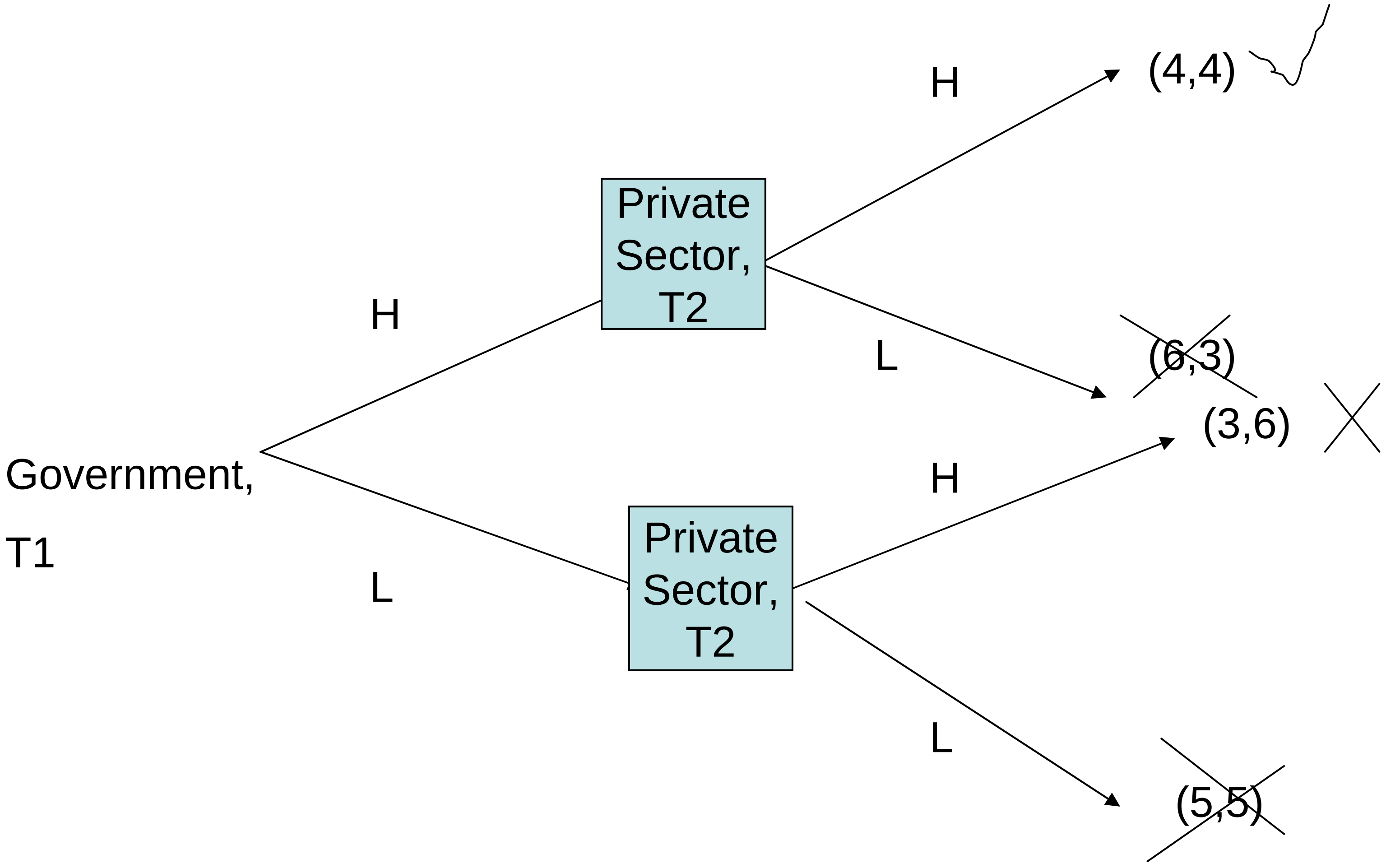
		H	L	
Government	H	4 $\underline{4}$	6 3	Private Sector’ choice
	L	3 $\underline{6}$	5 5	

	H	L	
H	$\underline{4}$ $\underline{4}$	$\underline{6}$ 3	Outcome of the Game
H	3 $\underline{6}$	5 5	

Extensive Form of Inflation-Unemployment Game



Solution by Backward Induction
Dynamic Inflation-Unemployment Game



Government, T1

H

L

Private Sector, T2

Private Sector, T2

H

L

H

L

(4,4)

~~(6,3)~~

(3,6)

~~(5,5)~~

Credibility Problem, Cheating and Discount Factor of the Game

Both gain by playing (C,C)

But this solution is not credible.
There is incentive to deviate. Trigger Strategy

Game returns to Nash path in absence of credibility.

If the game is played infinite number of times the optimal discount value ff the game is calculated as

$$PV(C,C) = 5 + 5\delta + 5\delta^2 + 5\delta^3 + ... + 5\delta^n = \frac{5}{1-\delta}$$

$$\underset{Lim \quad n \to \infty}{PV(C,C)} = 5 + 5\delta + 5\delta^2 + 5\delta^3 + ... + 5\delta^n = \frac{5}{1-\delta}$$

$$PV(cheat) = 6 + 4\delta + 4\delta^2 + 4\delta^3 + ... + 4\delta^n$$

Solution for the Discount Factor of the Game

$$\underset{Lim\quad n\to\infty}{PV(L,L)} = 5 + 5\delta + 5\delta^{2} + 5\delta^{3} + \ldots + 5\delta^{n} = \frac{5}{1-\delta}$$

$$PV(cheat) = 6 + 4\delta + 4\delta^{2} + 4\delta^{3} + \ldots + 4\delta^{n}$$

$$\delta PV(cheat) = 6\delta + 4\delta^{2} + 4\delta^{3} + \ldots + 4\delta^{n+1}$$

$$(1-\delta)PV(cheat) = 6 - 6\delta + 4\delta$$

$$\underset{\lim n\to\infty}{\delta^{n+1}} \approx 0$$

$$PV(cheat) = 6 + 4\frac{\delta}{(1-\delta)}$$

$$\frac{5}{1-\delta} = 6 + 4\frac{\delta}{(1-\delta)}$$

$$5 = 6(1-\delta) + 4\delta$$

$$6 - 5 = 2\delta$$

$$\delta = \frac{1}{2}$$

Cooperation or non-Cooperation?

$$................................ \qquad Advanced \quad Countries$$

$$Developing \quad Countries \begin{bmatrix} & NC & C \\ NC & 4,4 & 6,3 \\ C & 3,6 & 5,5 \end{bmatrix}$$

Nash Solution is non-cooperation (NC,NC) =(4,4)

$$................................ \qquad Advanced \quad Countries$$

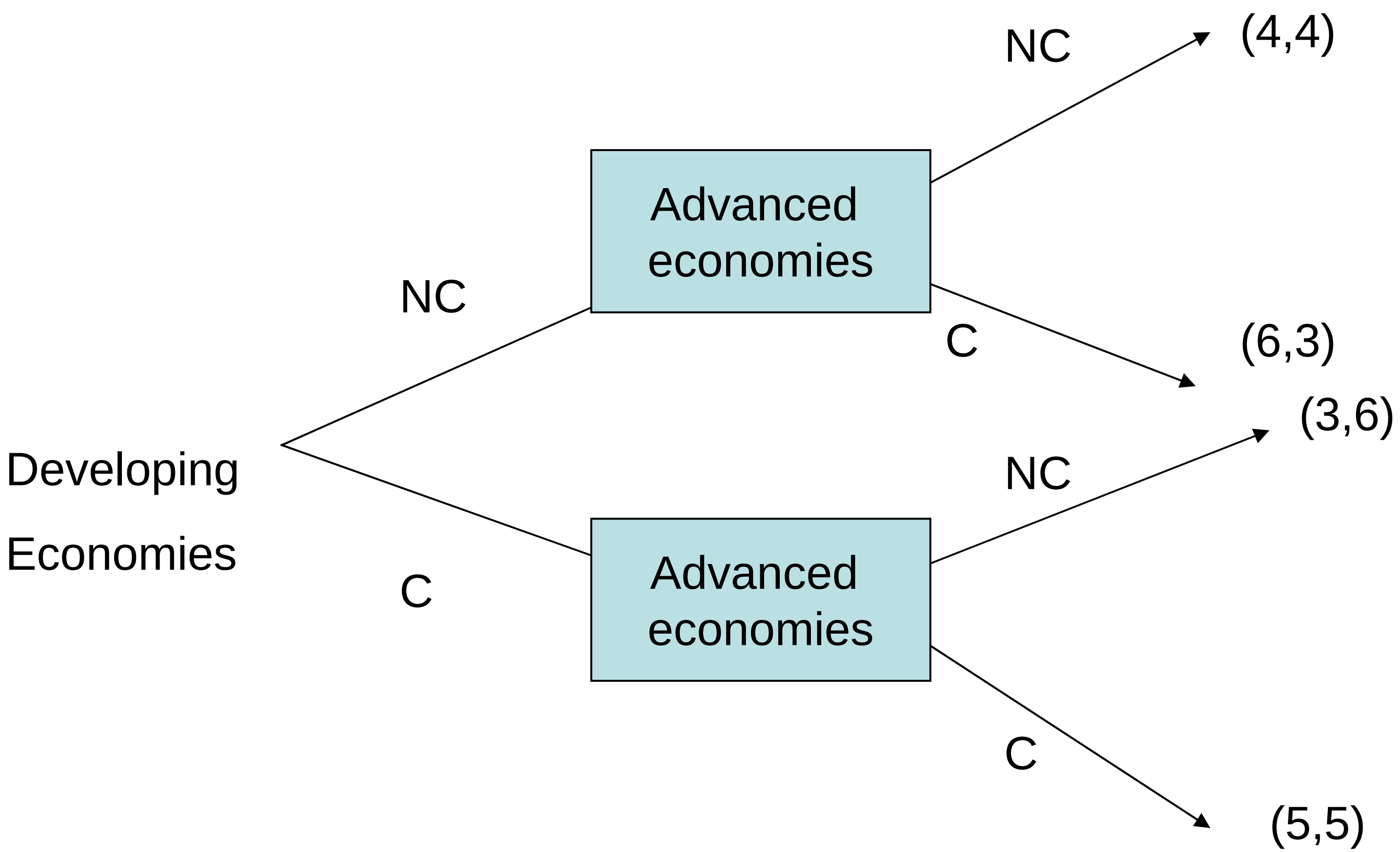
$$Developing \quad Countries \begin{bmatrix} & NC & C \\ NC & \underline{4},\underline{4} & \underline{6},3 \\ C & 3,\underline{6} & 5,5 \end{bmatrix}$$

Cooperative Solution (C,C) =(5,5)

Cooperative solution Pareto dominated Non-cooperative solution.

Pareto efficiency: at least one party gains without hurting the other.

Extensive Form of International Cooperation Game



NC

(4,4)

Advanced economies

NC

C

(6,3)

(3,6)

Developing Economies

NC

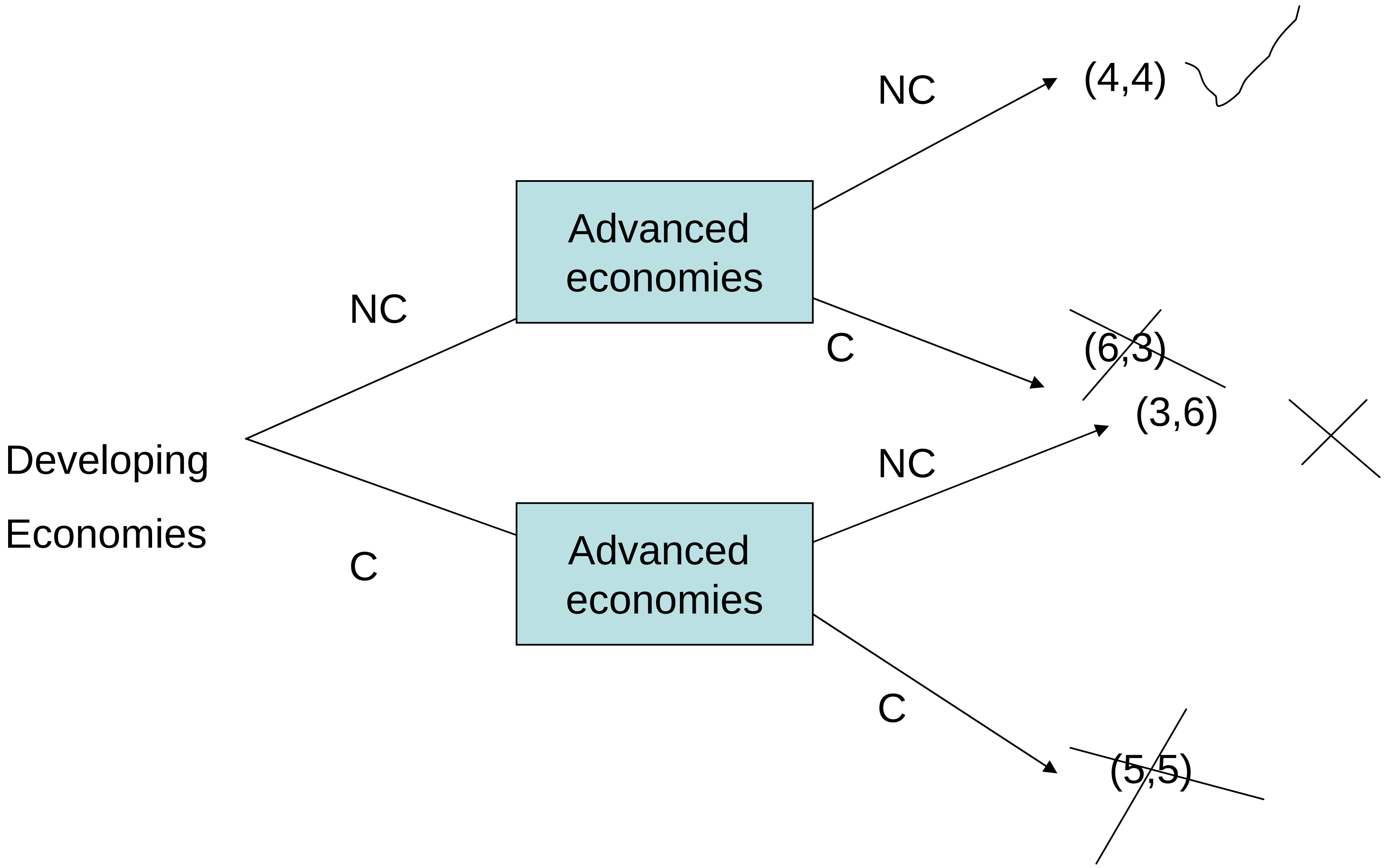
C

Advanced economies

C

(5,5)

Dynamics of International Policy Cooperation Game: Solution by Backward Induction



Developing

Economies

NC

C

Advanced economies

Advanced economies

NC

C

NC

C

(4,4)

~~(6,3)~~

(3,6)

~~(5,5)~~

Solution for the Discount Factor of the Game

$$\underset{Lim\quad n\to\infty}{PV(C,C)} = 5 + 5\delta + 5\delta^2 + 5\delta^3 + ... + 5\delta^n = \frac{5}{1-\delta}$$

$$PV(cheat) = 6 + 4\delta + 4\delta^2 + 4\delta^3 + ... + 4\delta^n$$

$$\delta PV(cheat) = 6\delta + 4\delta^2 + 4\delta^3 + ... + 4\delta^{n+1}$$

$$(1-\delta)PV(cheat) = 6 - 6\delta + 4\delta$$

$$\underset{\lim n\to\infty}{\delta^{n+1}} \approx 0$$

$$PV(cheat) = 6 + 4\frac{\delta}{(1-\delta)}$$

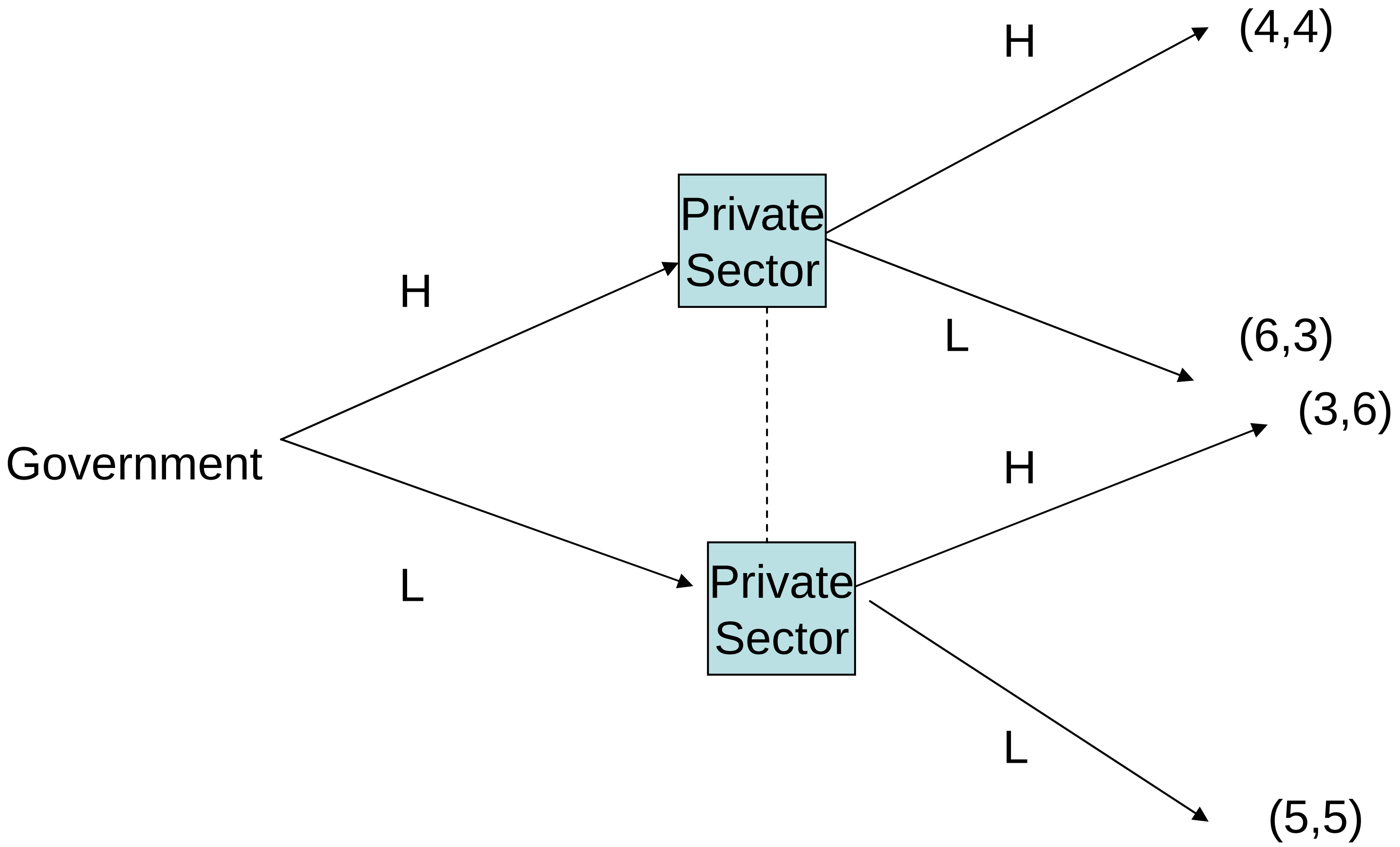
$$\frac{5}{1-\delta} = 6 + 4\frac{\delta}{(1-\delta)}$$

$$5 = 6(1-\delta) + 4\delta$$

$$6 - 5 = 2\delta$$

$$\delta = \frac{1}{2}$$

GAMES with Incomplete Information



Some Popular games

- Advertising
- Always the low price
- Bankrupty
- Blackjack
- Chicken
- Competition
- Coordination
- Devide a dollar
- Free riding
- Hawks versus Dove
- Lemongs
- Liar’s poker
- Majority rule
- Matching pennies
- Money back guarantee
- Pick the largest number
- Poker
- Principal agent
- Prisoner’s dilemma
- Roulette
- Solitaire
- Take it or leave it
- Tic-tac-toe
- This offer is good for limited time only
- Tragedy of commons

References and Readings

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- Pindyck and Rubinfeld (2005) Microeconomics, Chapter 13 .
- Varian H (2003) Microeconomics, Chapter 28-29.
- Romp Graham (1997) Game Theory, Oxford University Press.