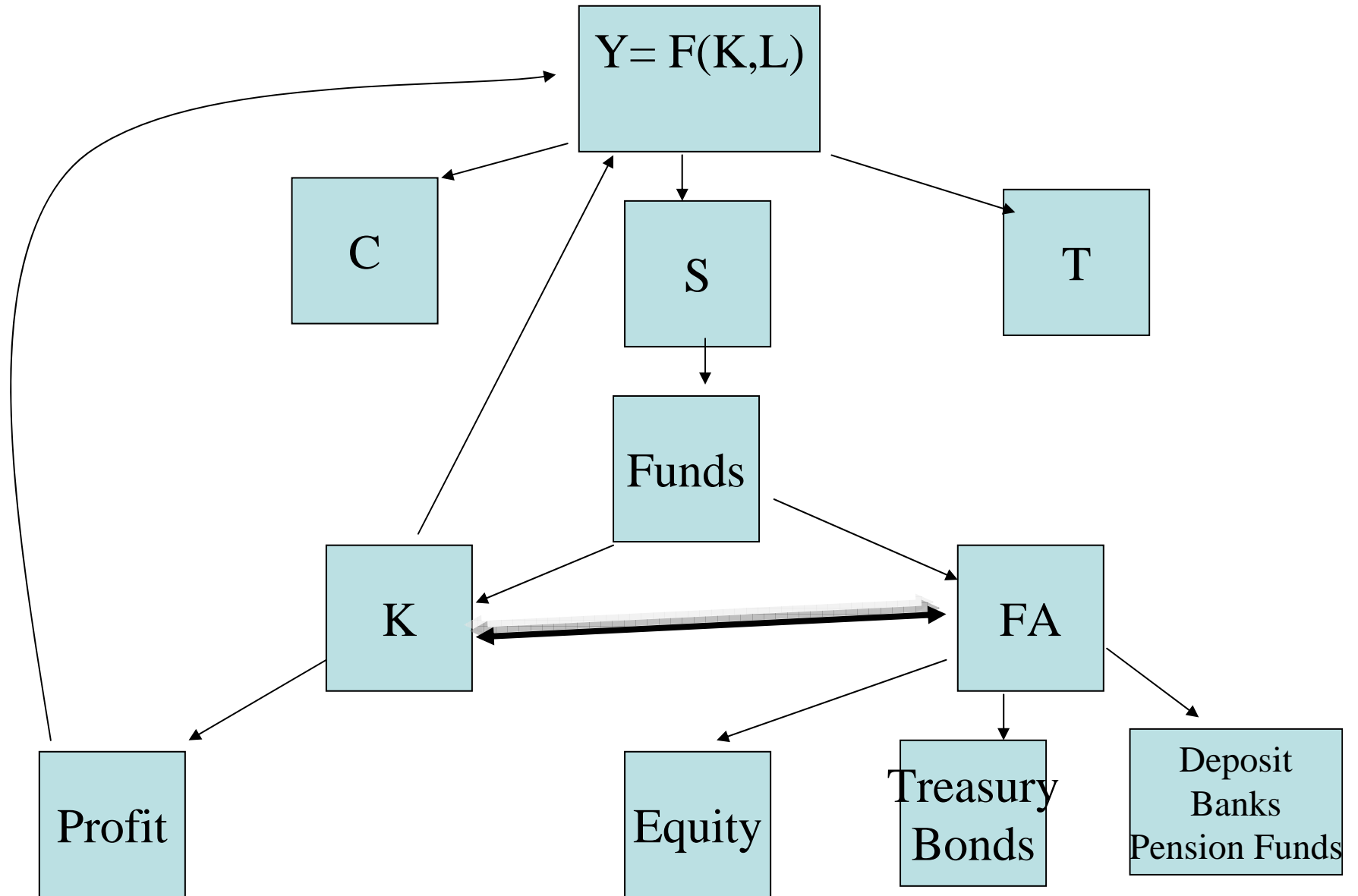


# Economic Modelling

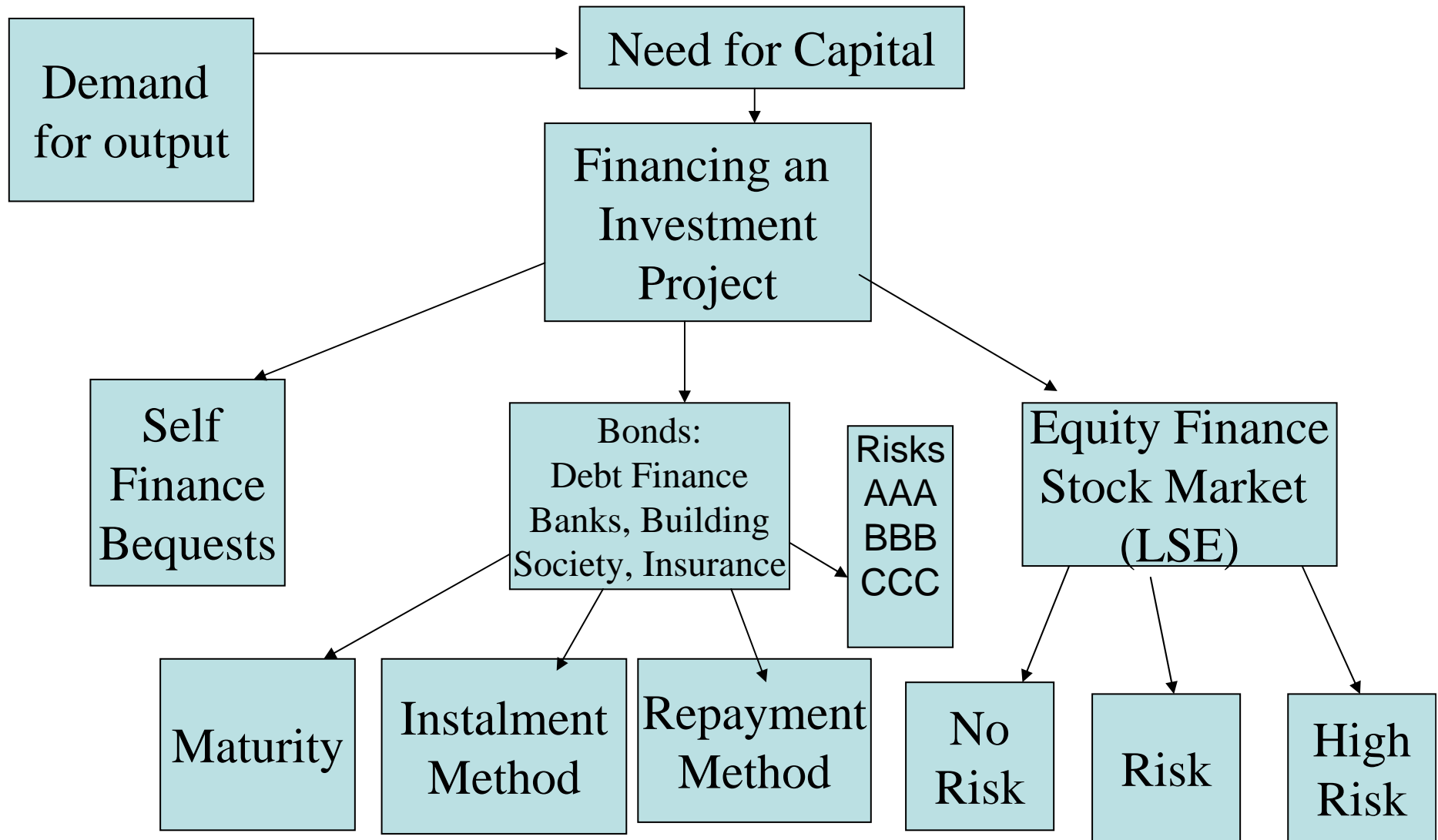
## Lecture 8

### Financial Market: Value of Bonds and Stocks

# Link Between Financial System and the Economy



# Financing of an Investment Project



# Three Sources of Financing an Investment Project

- Self-financing
  - Depends on retained earning
  - Personal savings
- Bonds
  - Various maturities and risks
- Stocks
  - Market signals and equity prices

# Investor Care about the Real Return: Fisher equation

## Gross nominal return

Time	today	Tomorrow
Amount	$P_t$	$(1+i)P_t$

## Gross real return (take account of inflation)

Time	Today	Tomorrow
Amount	$P_t$	$(1+r) = (1+i) \frac{P_t}{P_{t+1}^e}$
Approximation	$P_t$	$(1+r) = \frac{1+i}{1+\pi^e} \approx (1+i - \pi^e)$

$$\frac{P_t}{P_{t+1}^e} = \frac{1}{1+\pi_t^e} \quad \text{and} \quad \pi_t^e = \frac{P_{t+1}^e - P_t}{P_t}$$

## Fisher Approximation Rule is Good only for Small Interest Rates

If  $i=4\%$  and  $\pi^e = 2\%$   $r \approx i - \pi$

$$r = 4\% - 2\% \cong 2\%$$

$$(1+r) = \frac{1+i}{1+\pi^e} = \frac{1+0.04}{1+0.02} = 1.01961$$

$$\implies r = 1.961\% \cong 2\%$$

But the  $r \approx i - \pi$  not good for large  $r$ ,  $i$  and  $\pi$

$$r = 84\% - 54\% \cong 30\%$$

$$(1+r) = \frac{1+i}{1+\pi} = \frac{1+0.84}{1+0.54} = 1.195 \implies r = 19.5\%$$

# Key Points about Prices Financial Assets

Prices depend on

- Expected earnings from these assets
  - Expected returns from bonds
  - Expected stream of dividends from stocks
- Expected interest rates
  - Interest rates discount future earnings to present values
- Expected resale values of those assets
  - short run vs. long run
- Arbitrage implies investors reshuffle assets until the rate of return equalise in every asset

## Market Price of a bond (console)

What is the market price (value) of a console that pays 100 each year forever at the interest rate,  $r$ ?

$$PV = \frac{100}{(1+r)^1} + \frac{100}{(1+r)^2} + \frac{100}{(1+r)^3} + \dots + \frac{100}{(1+r)^n}$$

$$PV = \frac{100}{(1+r)^1} \left[ \frac{1 - \frac{1}{(1+r)^{n+1}}}{1 - \frac{1}{1+r}} \right] \quad \text{as } n \rightarrow \infty \quad \frac{1}{(1+r)^{n+1}} \rightarrow 0$$

$$PV = \frac{100}{(1+r)^1} \left[ \frac{1+r}{r} \right] = \frac{100}{r} = \frac{100}{0.1} = 1000.$$

## Market Prices of Bonds By Maturity

One Period Bond: 
$$P_{1,t} B = \frac{\text{Face value}}{\left(1+i_{1,t}\right)}$$

Two Period Bond: 
$$P_{2,t} B = \frac{\text{Face value}}{\left(1+i_{1,t}\right)\left(1+i_{1,t+1}^e\right)}$$

Derivation

$$P_{2t} = \frac{P_{1,t+1}^e}{1+i_{1,t}}$$

$$P_{1,t+1}^e = \frac{1}{1+i_{1,t+1}^e}$$

$$P_{2t} B = \frac{P_{1,t+1}^e (\text{Face value})}{1+i_{1,t}} = \frac{\text{Face value}}{\left(1+i_{1,t}\right)\left(1+i_{1,t+1}^e\right)}$$

## Yield to maturity on Bonds

Face Value      100                      Maturity              2 years

Market Price                       $P_{2t}^B = 90$

$$\left(1 + i_{2t}\right) = \sqrt{100/90} = 1.054$$

Yield to maturity 5.4%

Note

$$P_{2t}^B = \frac{\text{Face value}}{\left(1 + i_{2t}\right)^2} = \frac{100}{\left(1 + i_{2t}\right)^2}$$

# Arbitrage Between Short and Long Run Yields

Long Term

Short term

$$(1 + i_{2t})^2 = (1 + i_{1t})(1 + i_{1,t+1}^e)$$

Approximations:

$$(1 + 2i_{2t}) = (1 + i_{1t} + i_{2t}^e) \Rightarrow i_{2t} = \frac{1}{2}(i_{1t} + i_{2t}^e)$$

$$i_{nt} = \frac{1}{n}(i_{1t} + i_{2t}^e + i_{3t}^e + \dots + i_{nt}^e)$$

Long-term interest rate (yield) is average of expected short run interest rates.

# Market Price of Stocks

Arbitrage between bonds and stock:  $(1+i_{1,t}) = \frac{D_{t+1}^e + P_{S,t+1}^e}{P_{S,t}}$

By Rearranging:  $P_{S,t} = \frac{D_{t+1}^e}{(1+i_{1,t})} + \frac{P_{S,t+1}^e}{(1+i_{1,t})}$

Stock Price depends on expected stream of dividends and resale:  
(by iterating forward the above equation)

$$P_{S,t} = \frac{D_{t+1}^e}{(1+i_{1,t})} + \frac{D_{t+2}^e}{(1+i_{1,t})(1+i_{1,t}^e)} + \dots + \frac{D_{t+n}^e}{(1+i_{1,t})(1+i_{1,t}^e) \dots (1+i_{n,t}^e)}$$

$$+ \frac{P_{S,t+n}^e}{(1+i_{1,t})(1+i_{1,t}^e) \dots (1+i_{n,t}^e)}$$

## Price of a Stock: An example

$$P_{Share} = \left( \frac{D}{r - g + x} \right)$$

$P_{Share}$  = Price of a Stock

D = expected Dividend

r = interest rate

g = growth rate of dividend

x = risk premium

D = 1000 and r = 5% and g = 3% has a risk x = 0;

$$P_{Share} = \left( \frac{1000}{0.05 - 0.03 + 0} \right) = \frac{1000}{0.02} = 50,000$$

If r = 8%

$$P_{Share} = \left( \frac{1000}{0.08 - 0.03 + 0} \right) = \frac{1000}{0.05} = 20,000$$

## Price of Stock with Risks: An example

D =1000 and r =5% and g=3% has a risk x =8%

$$P_{Share} = \left( \frac{D}{r - g + x} \right) = 1000 \left( \frac{1}{0.05 - 0.03 + 0.08} \right) = \frac{1000}{0.1} = 10000$$

when r=8%

$$P_{Share} = \left( \frac{D}{r - g + x} \right) = 1000 \left( \frac{1}{0.08 - 0.03 + 0.08} \right) = \frac{1000}{0.13} = 7692.31$$

## Observations From the above Analysis of Stock Markets

- Lower the market interest rate, higher is the value of stock. Because future earnings are discounted at lower rate.
- Higher the growth rate of dividend higher the value of stock. As dividend grows earning from the share rises and hence price rise.
- Higher the risk premium lower is the value of the share. A decrease in the risk premium will increase the market value of a stock.
- Arbitrage implies same rate of risk adjusted returns in both stocks and bonds.
- (in the short run) Higher the resale value of the stock higher is its price.

## Difference in Life Time Income Due to University Degree

1. Tuition fee is rising. A person of 20 years of age is wondering whether to start working and earn £15000 salary per year that grows by 3 percent per year without going to the university or to go to the university for 3 years and start working after graduation earning £30000 a year with annual growth of 4 percent. For each university year, the tuition is £3000 and subsistence cost is £5000. Retirement age is 65 and the real interest rate is zero. What difference is made in lifetime income by going to the university? [hint:

$$LI = \left[ 1 + (1+g) + (1+g)^2 + \dots + (1+g)^n \right] Y_0 = \left[ \frac{1 - (1+g)^{n+1}}{1 - (1+g)} \right] Y_0 \quad [10]$$

**Answer:** The person choosing not to go to college works for 45 year (65-20 =45).

Applying above formula

$$LI_{no\_uni} = \left[ \frac{1 - (1+g)^{n+1}}{1 - (1+g)} \right] Y_0 = \left[ \frac{1 - (1 + 0.03)^{45+1}}{1 - (1 + 0.03)} \right] 15000 = \left[ \frac{1 - (1.03)^{46}}{1 - (1.03)} \right] 15000$$

$$LI_{no\_uni} = \left[ \frac{1 - 3.89504}{-0.03} \right] 15000 = 15000(96.50) = 1,447,521.80$$

University education costs £8000 per year (£3000 in tuition plus £5000 other expenses) for three years, which means total cost of £24000. University graduate starts working at 23 and thus works 42 years (65-23). Given this the life time income with university graduate is

$$LI_{uni} = \left[ \frac{1 - (1+g)^{n+1}}{1 - (1+g)} \right] Y_0 - Cost = \left[ \frac{1 - (1 + 0.04)^{42+1}}{1 - (1 + 0.04)} \right] 30000 - 24000 = \left[ \frac{1 - (1.04)^{43}}{1 - (1.04)} \right] 30000 - 24000$$

$$LI_{uni} = \left[ \frac{1 - 5.4005}{-0.04} \right] 30000 - 24000 = 30000 (110.01) - 24000 = 3,300,371.5 - 24000$$

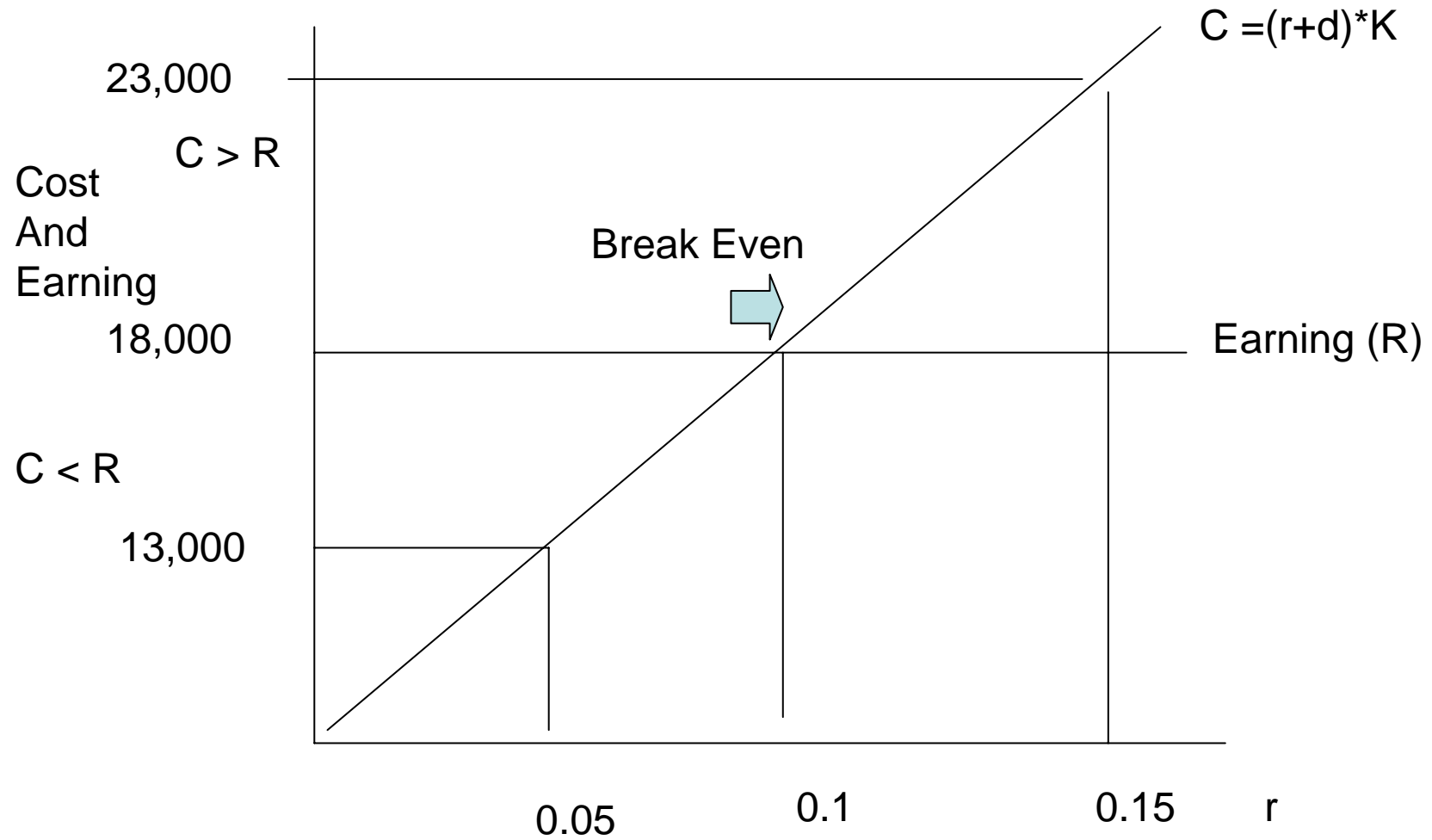
$$LI_{uni} = 3,300,371.5 - 24000 = 3,276,371$$

Benefit of going to university = Net income from after university - income without uni  
 = 3,276,371 - 1,447,521.80 = 1,828,849.20

Thus university education makes a difference of 1, 828,849.20 in the life time income of an individual.

# Analysis of Earnings and Cost from an Investment Project

$K = 100000$ ;  $d = 0.08$ ;  $R$  (Earning) = 18000



# Two Period Model of Consumption and Saving

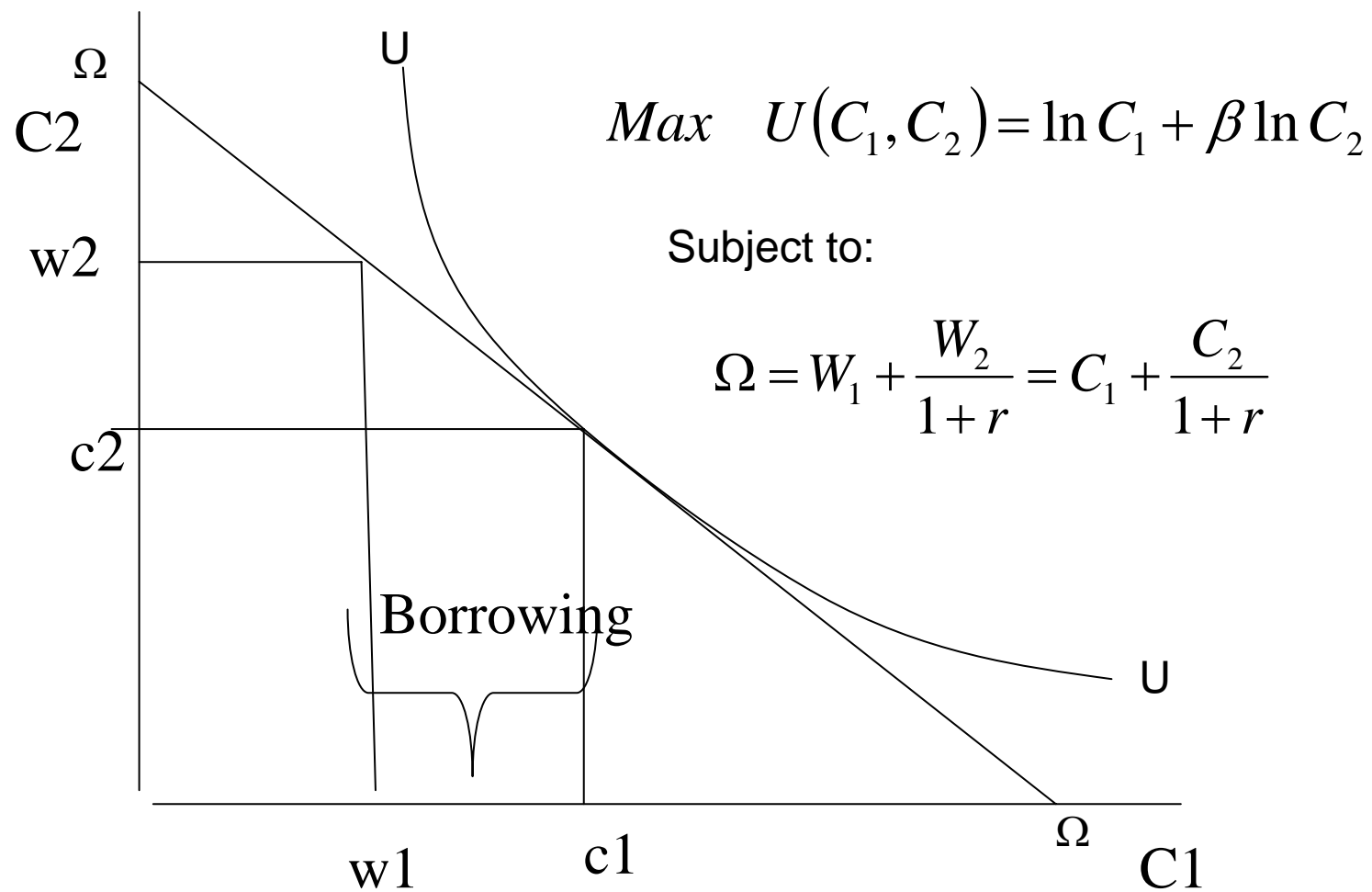
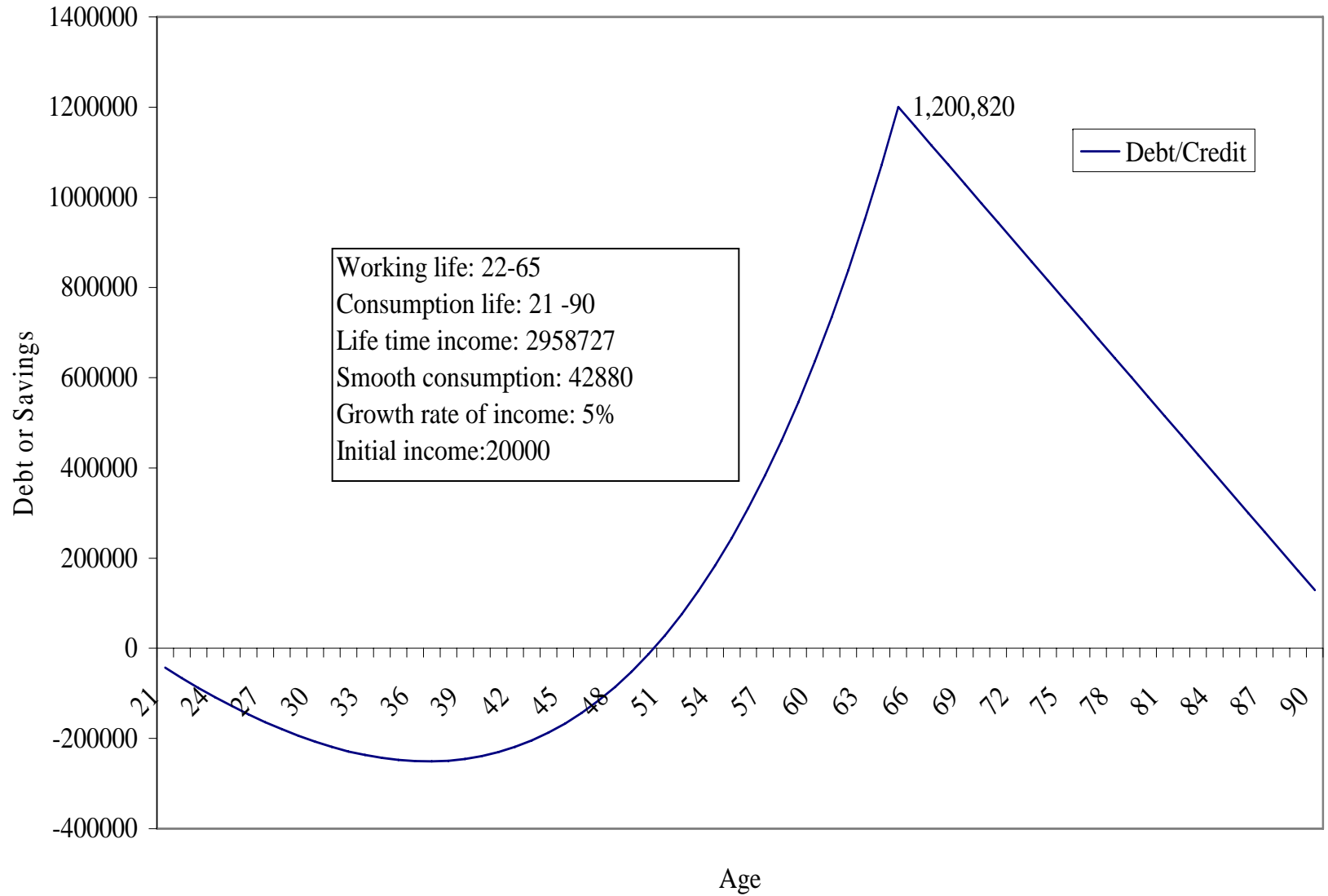


Figure 1  
Debt and Savings in Life Cycle Model



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