

Allocations and Welfare in General Equilibrium Tax Model with Money

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Abstract

Impacts of fiscal and monetary policies are assessed in the an open economy two sector multi-household general equilibrium tax model with money. There is alarming gap in the distribution of income among households which none of the stabilisation policies could possibly address. Impacts of fiscal expansions are positive for all categories of households but the gains are much higher for household in the upper income group than for the households in the bottom income group despite claim of the government to help out to those with the least means. Model results show that demand, output, employment are sensitive to the consumer preference and confidences and sectoral shift in the production technology and impacts vary across households. Simulation results show that monetary policy is less effective in promoting output, employment and other economic policies than the fiscal policy. Relative price based optimal allocation mechanisms and redistribution provide several hints on how the complex mechanism of reallocation and redistribution functions occur in a real economy. Business cycles occur with technological shocks in economies with cash in advance and money in utility functions.

JEL Classification: F41, O11, O33, O41

Keywords: general equilibrium with money, open economy

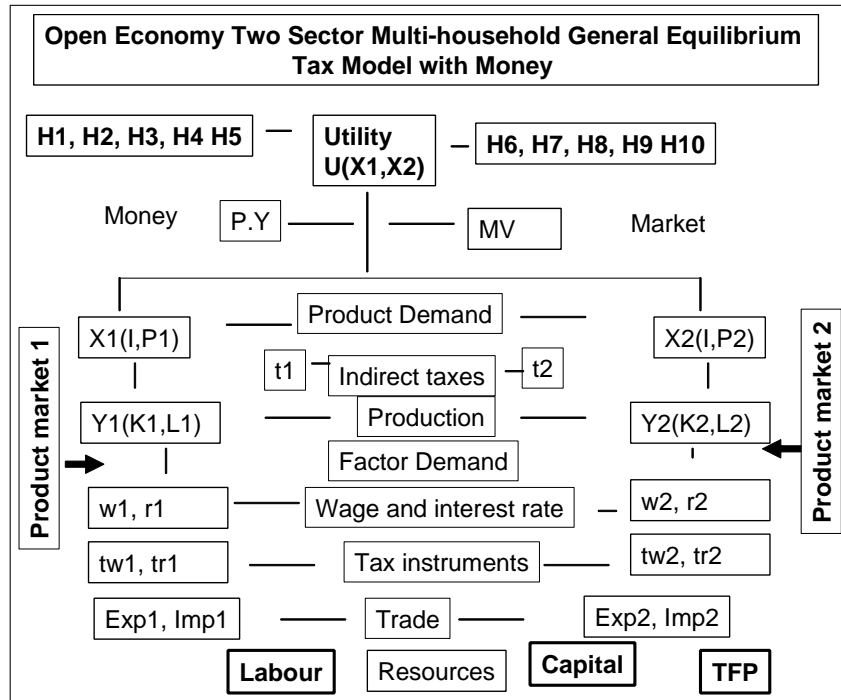
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1 Introduction

Economic crises of 2008/09 that originated from the burst of housing market bubble and the credit crises in the US has spread around the globe. Recession has hit the US, UK, EU, Japan and many other advanced countries hard this year. It has slowed down growth and other economic activities in many developing countries too. Output, employment, investment, capital accumulation, exports, imports are predicted to shrink causing alarming loss of income, deterioration in living standards of households and loss of business or profit prospects of small, medium and large scale firms. Governments have attempted to stimulate the aggregate demand by expanding the public expenditure and cutting down the taxes despite risk of accumulation of public debt. Central banks have reduced the basic interest rate to a record low rate since the beginning of central banking (on January 2009 Federal fund rate is zero, BOE bank rate is 0.5 percent, ECB 1.5 (<http://www.ecb.int/euro/intro/html/map.en.html>)) in order to expand the liquidity in the system. Credit levels of banks have been expanded under the quantitative easing.

The main aim of this paper is to show how an open economy two sector multi-household general equilibrium model with money could be applied to analyse the situation and to estimate the impacts of the economic crises and to assess the impact of policy measures adopted by the governments. For simplicity first focus remains on the steady state. The general structure of the model is as given in Figure 1. Various studies have been made earlier for evaluation of impacts of financial sector in the economy (Altig et al (1995), Arestis and Demetriades (1997) Bacchetta (1992) Bank of England (1999) Bolnick (1987), Boycko et al. (1996) ,Brunnermeier. (2009.) Cecchetti (2009). Champ et. al (1996) Chiang (1959) Cripps (1997) Dasgupta and Maskin (2000) Diamond et. al (1983) Giovannini and de Melo (1993) De Fraja (1991) Mayer et. al. (2009). Friedman (2005) King (1994) Klein (1971) Krugman (1979) Milde and Riley (1988) Prescott and Townsend (1984) Rasmusen (1987) Radelet et.al (1998) Riley (2001) Rogerson (1985),Sargent.(1987) , Smith (1958) Spencer (1984) Sproul (1947) Stiglitz, and Weiss (1981) Roubini and Sala-i-Martin (1992) Williamson (2008) Walsh (1998)) but these studies have ignored the specification of the household sector. The household sector is integrated with two sector open economy model in the next section including the specifications of preferences and technologies of the economy, monetary, government and the external sectors. Then comes the parameterisation of model and a brief discussions on results of the steady state. Cash in advance and money in utility functions with stochastic technology are added for dynamic specification required to see the business cycle implications of the credit crunch. Conclusion and list of references are at the end.



Structure of General Equilibrium Model with Money

Results of all economic activities is the welfare of the households from the consumption of goods and services that consumers purchase in product markets 1 and 2. Producers demand capital and labour factors to supply those goods and pay remunerations to them. Households pay direct and indirect taxes from these income. Exports and imports link the economy to the rest of the world economy. This economy is subject of cyclical fluctuations. Fiscal and monetary policies aim to contain these fluctuations. Specification of the household sector is given in the next section. For simplicity economy has ten households, two goods X_1 and X_2 , manufacturing and services, two types of labour L_1 and L_2 , skilled and unskilled labour and the two types of capital stock, K_1 and K_2 durable and non durable ones. Economy is linked to the rest of the world by the flows of exports EX_1 and EX_2 and imports IMP_1 and IMP_2 . Government receives revenue from direct taxes of labour and capital income and indirect taxes on consumption of X_1 and X_2 . Monetary side of the economy are first characterised by the classical quantity theory of money where the total money supply consists of both currency and demand deposits. Prices are proportional to money supply and also influenced by the volume of transaction and the velocity of money. The exchange rate is determined by the value of the ratio of exports to imports. Dynamics of monetary sector are studied under the cash in advance and money in utility function framework.

2 Households

Model categorises households in ten deciles who form a representative household sector of the entire economy. Efficient allocations of resources maximise welfare of each of these households and the aggregate household sector. Utility of individual household h given by u^h . It shares η^h of aggregate utility U

$$u^h = \eta^h . U \quad (1)$$

demand for X_1 by household h

$$x_1^h = \eta^h . X_1 \quad (2)$$

demand for X_2 by household h

$$x_2^h = \eta^h . X_2 \quad (3)$$

Total labour supply of type L_1 of household h

$$L_1^h = \theta^h . L_1 \quad (4)$$

Total labour supply of type L_2 of household h

$$L_2^h = \theta^h . L_2 \quad (5)$$

Total capital supply of type K_1 of household h

$$K_1^h = \eta^h . K_1 \quad (6)$$

Total capital supply of type K_2 of household h

$$K_2^h = \eta^h . L_2 \quad (7)$$

Income of household h

$$I^h = \eta^h . I \quad (8)$$

Aggregate utility of the economy is composite of household utilities

$$U = \sum_{h=1}^H u^h \quad (9)$$

Aggregate demand for good X_1

$$X_1 = \sum_{h=1}^H x_1^h \quad (10)$$

Aggregate demand for good X_2

$$X_2 = \sum_{h=1}^H x_2^h \quad (11)$$

Aggregate of labour demand of type L_1

$$L_1 = \sum_{h=1}^H L_1^h \quad (12)$$

Aggregate of labour demand of type L_2

$$L_2 = \sum_{h=2}^H L_2^h \quad (13)$$

Aggregate of capital demand of type K_1

$$K_1 = \sum_{h=1}^H K_1^h \quad (14)$$

Aggregate of capital demand of type K_2

$$K_2 = \sum_{h=2}^H K_2^h \quad (15)$$

Aggregate income is the total of the household income

$$I = \sum_{h=1}^H I^h \quad (16)$$

3 Economy

The representative household of the economy receives utility from consuming both goods X_1 and X_2 . A Cobb-Douglals utility function is assumed first as:

$$U = X_1^{\alpha_1} X_2^{1-\alpha_1} \quad (17)$$

Demand for X_1 is derived from the standard optimality conditions as:

$$X_1 = \frac{\alpha_1 \cdot I}{P_1 (1 + t_1)} \quad (18)$$

Demand for both goods X_2 is derived from the standard optimality conditions as

$$X_2 = \frac{(1 - \alpha_1) \cdot I}{P_2 (1 + t_2)} \quad (19)$$

Technology of production of good 1 Y_1 is

$$Y_1 = L_1^\beta K_1^{1-\beta} \quad (20)$$

Technology of production of good 2 Y_2 is

$$Y_2 = L_2^\gamma K_2^{1-\gamma} \quad (21)$$

Optimal conditions for firm 1 is given by

$$\frac{\beta}{(1-\beta)} \frac{K_1}{L_1} = \frac{w_1(1-tw_1)}{r_1(1-tr_1)} \quad (22)$$

Optimal conditions for firm 2 is given by

$$\frac{\gamma}{(1-\gamma)} \frac{K_2}{L_2} = \frac{w_2(1-tw_2)}{r_2(1-tr_2)} \quad (23)$$

Resource of firm 1 is

$$C_1 = w_1L_1 + r_1K_1 \quad (24)$$

Resource of firm 2 is

$$C_2 = w_2L_2 + r_2K_2 \quad (25)$$

Households receive income from labour and capital income and from transfers

and net borrowing as:

$$I = w_1L_1 + r_1K_1 + w_2L_2 + r_2K_2 + TR + B \quad (26)$$

Market clearing conditions in goods market are

$$X_1 = Y_1 - G_1 - (EX_1 - IMP_1) \quad (27)$$

$$X_2 = Y_2 - G_2 - (EX_2 - IMP_2) \quad (28)$$

Labour market clearing implies

$$L_1 + L_2 = \bar{L} \quad (29)$$

Capital market clearing implies

$$K_1 + K_2 = \bar{K} \quad (30)$$

Aggregate volume of output

$$P_1Y_1 + P_2Y_2 = P.Y \quad (31)$$

3.1 Monetary Sector

Quantity theory of money implies

$$P.Y = MS.V \quad (32)$$

Initial reserve of the banking system

$$R = C + D0 \quad (33)$$

Currency in circulation

$$C = \phi.R \quad (34)$$

Initial deposit

$$D0 = (1 - \phi).R \quad (35)$$

Total deposit

$$TD = \frac{D0}{rr} \quad (36)$$

Aggregate money supply in the economy:

$$MS = C + TD \quad (37)$$

3.2 Government Sector

Government collects revenue from direct taxes on capital and labour income and indirect tax on commodities as:

$$RV = t_1.P_1.X_1 + t_2.P_2.X_2 + t_{r1}.r_1.K_1 + t_{w1}.w_1.L_1 + t_{r2}.r_2.K_2 + t_{w2}.w_2.L_2 + TR + B \quad (38)$$

Aggregate government expenditure

$$G = G_1 + G_2 \quad (39)$$

Government expenditure on sector 1 goods

$$G_1 = g_1.RV \quad (40)$$

Government expenditure on sector 2 goods

$$G_2 = g_2.RV \quad (41)$$

Budget deficit

$$B = G - RV \quad (42)$$

3.3 External sector

Exports from sector 1

$$EX_1 = e_1.Y_1 \quad (43)$$

Exports from sector 2

$$EX_2 = e_2.Y_2 \quad (44)$$

Imports by sector 1

$$IMP_1 = m_1.Y_1 \quad (45)$$

Imports by sector 2

$$IMP_2 = m_2.Y_2 \quad (46)$$

Exchange rate

$$ER = \frac{P_1.EX_1 + P_2.EX_2}{PM_1.IMP_1 + PM_2.IMP_2} \quad (47)$$

4 Parameterisation of the Model

The open economy model presented in equations (1) to (47) above has 112 variables. In theory it is possible to solve all 112 endogenous variables in terms of the parameters. In practice it is difficult to produce analytical solutions for so many variables simultaneously. Therefore this model is solved using non-linear programming numerical technique. Model variables are uniquely determined in terms of parameters of preferences (α^h), share of labour (θ^h), and share of capital income (η^h), of household as given in Table 1. Income distribution structure taken as they were found in the income distribution tables of the UK. Model is calibrated to the UK economy in the next section.

Table 1: Preference and share parameters of households

	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9	h_{10}
α^h	0.44	0.48	0.49	0.41	0.47	0.57	0.47	0.55	0.48	0.47
θ^h	0.02	0.03	0.04	0.05	0.06	0.08	0.11	0.15	0.20	0.25
η^h	0.02	0.02	0.03	0.04	0.04	0.05	0.06	0.08	0.09	0.57

Model includes six tax policy instruments $t_1, t_2, t_{w1}, t_{w2}, t_{r1}, t_{r2}$, two export shares e_1 and e_2 two propensities to import m_1 and m_2 , preference and technology parameter for the aggregate economy α, β, γ and endowments \bar{L} and \bar{K} , aggregate resources available for firms C_1 and C_2 velocity of money v , reserve requirement ration rr , Transfer, share of public spending in sectors 1 and 2,

g_1 and g_2 reserves and share of currency in initial reserves ϕ as shown in Table 2.

Table 2: Policy and Technology Parameters

t_1	t_2	t_{w1}	t_{w2}	t_{r1}	t_{r2}	e_1	e_2	m_1	m_2	R	$g1$
0.25	0.15	0.40	0.10	0.05	0.3	0.25	0.30	0.15	0.50	100	0.25
α	β	γ	\bar{L}	\bar{K}	C_1	C_2	v	ϕ	rr	TR	$g2$
0.4	0.6	0.5	200	500	1000	800	1	0.03	0.05	0	0.1

Plausible values of these parameters given in Table 1 and Table 2 are used for simulation in the following section though these should be obtained from the econometric estimation or statistical inferences. This model is solved using the non-linear algorithm in GAMS.

5 Numerical Example

The optimal allocation of resources in the economy are given by the equilibrium prices of goods P_1 and P_2 wage rate w_1 and w_2 rental rate r_1 and r_2 and the exchange rate ER . Unemployment level is exogenous to the model in the current form so that equilibrium can be computed for various levels of unemployment level set at the policy level. This is a macroeconomic model with detailed specification of households and firms. The gap between the poorest and the richest household is enormous as shown in the level of utility of these two categories of households in the following chart made for four different solutions of the model.

Figure 1: Level of Welfare of the Household in the Lowest Income Decile

	Util_a	Util_b	Util_gm	Util_R	Util_f	Util_m	Util_fm
h1.sc1	0.792	0.726	0.385	0.449	0.556	0.556	0.556
h1.sc2	0.792	0.792	0.556	0.556	0.556	0.556	0.556
h1.sc3	0.784	0.743	0.541	0.556	2.197	0.556	0.540
h1.sc4	0.777	0.695	0.526	0.556	2.197	0.556	0.523
h1.sc5	0.770	0.647	0.512	0.556	2.197	0.556	0.506
h1.sc6	0.762	0.601	0.499	0.556	2.197	0.556	0.489
h1.sc7	0.755	0.556	0.487	0.556	2.197	0.556	0.473
h1.sc8	0.748	0.512	0.476	0.556	2.197	0.556	0.456
h1.sc9	0.740	0.469	0.466	0.556	2.197	0.556	0.439
h1.sc10	0.733	0.426	0.457	0.556	2.197	0.556	0.422

Figure 2: Level Welfare of the Household in the Highest Income Decile

	Util_a	Util_b	Util_gm	Util_R	Util_f	Util_m	Util_fm
h10.sc1	22.564	20.680	10.964	12.785	15.856	15.856	15.856
h10.sc2	22.564	22.564	15.856	15.856	15.856	15.856	15.856
h10.sc3	22.353	21.163	15.414	15.856	62.606	15.856	15.380
h10.sc4	22.142	19.794	14.996	15.856	62.606	15.856	14.903
h10.sc5	21.932	18.454	14.603	15.856	62.606	15.856	14.426
h10.sc6	21.722	17.142	14.234	15.856	62.606	15.856	13.949
h10.sc7	21.513	15.856	13.891	15.856	62.606	15.856	13.471
h10.sc8	21.304	14.597	13.574	15.856	62.606	15.856	12.993
h10.sc9	21.096	13.362	13.283	15.856	62.606	15.856	12.515
h10.sc10	20.888	12.151	13.020	15.856	62.606	15.856	12.036

Figure 3: Baseline Model and Sensitivity of aggregate Preference (Alpha)

	sc1	sc2	sc3	sc4	sc5	sc6	sc7	sc8	sc9	sc10
X1	116.26	116.26	122.02	127.77	133.52	139.26	145.01	150.75	156.49	162.22
X2	190.35	190.35	184.06	177.78	171.49	165.20	158.91	152.62	146.32	140.02
L1	88.89	88.89	92.99	97.06	101.10	105.11	109.09	113.04	116.97	120.86
L2	111.11	111.11	107.01	102.94	98.90	94.89	90.91	86.96	83.03	79.14
K1	173.91	173.91	183.41	192.98	202.64	212.39	222.22	232.14	242.15	252.25
K2	326.09	326.09	316.59	307.02	297.36	287.61	277.78	267.86	257.85	247.75
P1	6.19	6.19	6.20	6.20	6.20	6.20	6.21	6.21	6.21	6.21
P2	5.67	5.67	5.67	5.67	5.67	5.67	5.66	5.66	5.66	5.66
R1	2.30	2.30	2.18	2.07	1.97	1.88	1.80	1.72	1.65	1.59
R2	1.23	1.23	1.26	1.30	1.35	1.39	1.44	1.49	1.55	1.62
W1	6.75	6.75	6.45	6.18	5.94	5.71	5.50	5.31	5.13	4.96
W2	3.60	3.60	3.74	3.89	4.04	4.22	4.40	4.60	4.82	5.06
UTIL	156.28	156.28	154.87	153.73	152.84	152.20	151.80	151.64	151.73	152.05
ALPHA	0.40	0.42	0.44	0.46	0.48	0.50	0.52	0.54	0.56	0.58

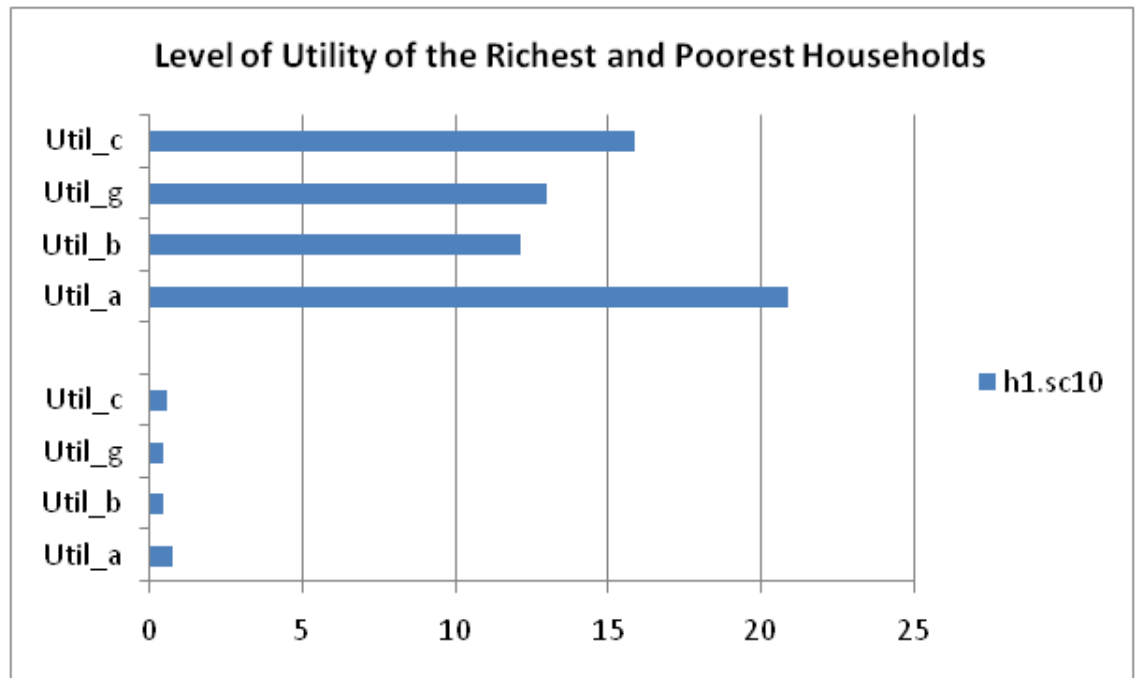


Figure 4: Sensitivity to technical progress in sector 1 (Beta)

	sc1	sc2	sc3	sc4	sc5	sc6	sc7	sc8	sc9	sc10
X1	167.96	116.26	114.53	112.89	111.36	109.92	108.59	107.37	106.25	105.25
X2	133.72	190.35	190.62	190.94	191.31	191.74	192.21	192.75	193.33	193.98
L1	124.73	88.89	90.51	92.09	93.62	95.11	96.55	97.96	99.33	100.66
L2	75.27	111.11	109.49	107.91	106.38	104.90	103.45	102.04	100.67	99.34
K1	262.44	173.91	168.14	162.16	155.96	149.53	142.86	135.92	128.71	121.21
K2	237.56	326.09	331.86	337.84	344.04	350.47	357.14	364.08	371.29	378.79
P1	6.22	6.19	6.29	6.38	6.47	6.55	6.63	6.71	6.78	6.84
P2	5.65	5.67	5.67	5.66	5.65	5.63	5.62	5.60	5.59	5.57
R1	1.52	2.30	2.26	2.22	2.18	2.14	2.10	2.06	2.02	1.98
R2	1.68	1.23	1.21	1.18	1.16	1.14	1.12	1.10	1.08	1.06
W1	4.81	6.75	6.85	6.95	7.05	7.15	7.25	7.35	7.45	7.55
W2	5.31	3.60	3.65	3.71	3.76	3.81	3.87	3.92	3.97	4.03
UTIL	152.62	156.28	155.48	154.74	154.08	153.48	152.96	152.53	152.17	151.90
BETA	0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74	0.76	0.78

Model is rich enough to assess the impacts of fiscal and monetary policies. Fiscal policy can operate by changing the tax instruments or setting the level of public spending or its allocation in sector 1 and 2 according to spending objectives, change in the borrowing requirement resulting from the fiscal operation.

In the current situation tax rates are being cut, and the level of spending raised and borrowing being raised to stimulate the economy. Household adjust to the tax, spending and borrowing plan of the public sector. Higher budget deficit means less income left for the households who internalise the public deficit in spirit of the Ricardian equivalence.

Monetary policy operates through a number of channels. Most important is the credit market channel that is controlled by the rate of deposit creation and credit expansion in the economy. In the current economic crises the rate of deposit creation has rather been slow rendering economy towards credit crunch. This is equivalent to loss in confidence and increase in the reserve requirement in the banking system. Thus parameter rr and ϕ are crucial for the determination. The consequences of the monetary policy in the economy are given by the classical quantity theory of money contained in the model where the aggregate prices are proportional to the total supply of money.

Model so far discussed is able to comparative static analysis. Dynamic analysis requires intertemporal optimisation and process accumulation of assets and capital. Two further modifications are suggested in order to make the above model for dynamic economy. Extension 1 proposes a cash in advanced economy (CA) with money and assets in line of theoretical structured contained in Sargent (1987) and Williamson (2008). The role of money in the CA economy is

Figure 5: Sensitivity of Technical Progress in Sector 2 (Gamma)

	sc1	sc2	sc3	sc4	sc5	sc6	sc7	sc8	sc9	sc10
X1	104.38	116.26	115.96	115.72	115.55	115.45	115.42	115.45	115.56	115.74
X2	194.68	190.35	186.68	183.12	179.68	176.35	173.12	170.00	166.99	164.07
L1	101.96	88.89	86.96	85.11	83.33	81.63	80.00	78.43	76.92	75.47
L2	98.04	111.11	113.04	114.89	116.67	118.37	120.00	121.57	123.08	124.53
K1	113.40	173.91	178.57	183.49	188.68	194.18	200.00	206.19	212.77	219.78
K2	386.60	326.09	321.43	316.51	311.32	305.83	300.00	293.81	287.23	280.22
P1	6.90	6.19	6.21	6.22	6.23	6.24	6.24	6.24	6.23	6.22
P2	5.55	5.67	5.79	5.90	6.01	6.12	6.24	6.35	6.47	6.58
R1	1.94	2.30	2.24	2.18	2.12	2.06	2.00	1.94	1.88	1.82
R2	1.04	1.23	1.20	1.16	1.13	1.10	1.07	1.04	1.00	0.97
W1	7.65	6.75	6.90	7.05	7.20	7.35	7.50	7.65	7.80	7.95
W2	4.08	3.60	3.68	3.76	3.84	3.92	4.00	4.08	4.16	4.24
UTIL	151.72	156.28	154.30	152.41	150.60	148.86	147.20	145.62	144.12	142.69
GAMA	0.50	0.52	0.54	0.56	0.58	0.60	0.62	0.64	0.66	0.68

Figure 6: Sensitivity of increase in financial resources for firm 1 and Firm 2.

	sc1	sc2	sc3	sc4	sc5	sc6	sc7	sc8	sc9	sc10
X1	115.99	116.26	116.26	116.26	116.26	116.26	116.26	116.26	116.26	116.26
X2	161.25	190.35	190.35	190.35	190.35	190.35	190.35	190.35	190.35	190.35
L1	74.07	88.89	88.89	88.89	88.89	88.89	88.89	88.89	88.89	88.89
L2	125.93	111.11	111.11	111.11	111.11	111.11	111.11	111.11	111.11	111.11
K1	227.27	173.91	173.91	173.91	173.91	173.91	173.91	173.91	173.91	173.91
K2	272.73	326.09	326.09	326.09	326.09	326.09	326.09	326.09	326.09	326.09
P1	6.21	12.39	13.76	15.14	16.51	17.89	19.27	20.64	22.02	23.40
P2	6.70	11.35	12.61	13.87	15.13	16.39	17.65	18.91	20.17	21.44
R1	1.76	4.60	5.06	5.52	5.98	6.44	6.90	7.36	7.82	8.28
R2	0.94	2.45	2.76	3.07	3.37	3.68	3.99	4.29	4.60	4.91
W1	8.10	13.50	14.85	16.20	17.55	18.90	20.25	21.60	22.95	24.30
W2	4.32	7.20	8.10	9.00	9.90	10.80	11.70	12.60	13.50	14.40
UTIL	141.34	156.28	156.28	156.28	156.28	156.28	156.28	156.28	156.28	156.28
C1	2000.00	2200.00	2400.00	2600.00	2800.00	3000.00	3200.00	3400.00	3600.00	3800.00
C2	1600.00	1800.00	2000.00	2200.00	2400.00	2600.00	2800.00	3000.00	3200.00	3400.00

medium of exchange and store of value. The extension 2 proposes a dynamic economy with money in the utility function adapted from Walsh(1998). Both of these models are applied with two different scenarios with low and high velocity of money, and natural growth rates of labour and output, and variation on intertemporal preferences and productivity of capital. Theoretically money is supernuetral in both of these models and it does not affect capital stock, output or consumption in the steady state. However it is argued that when economy is subject to shocks consumer and producer confidence this super neutrality does not hold, particularly when the economy is moving towards recession. This is empirically validated by computing CA and MIU models by GAMS and only sketch of model solutions will be presented leaving further elaborations for future research.

6 Extenstions:

6.1 Dynamics with Cash in Advance Constraint

Dynamic optimisation process in the cash in advance monetary economy occurs when households maximise lifetime utility from consumption $U(C_t)$ and leisure but experience disutility from labour $V(L_t)$ as:

$$\max \sum_{t=0}^{\infty} \beta^t [U(C_t) - V(L_t)] \quad (48)$$

Subject to the technology constraint:

$$Y_t = zL_t \quad (49)$$

and the Cash in advance constraint

$$P_t C_t + q_t B_{t+1} + P_t s_t X_{t+1} + P_t T_t = M_t + B_t + P_t X_t \quad (50)$$

where $P_t C_t$ is consumption expenditure P_t price of goods, C_t consumption, B_{t+1} is the amount of nominal bonds q_t is the price of nominal bonds, X_{t+1} real bonds s_t prices of real bonds, T_t lump sum tax payment, M_t money.

Budget constraint of the consumer include income from production and allocation of money for the next period

$$P_t C_t + q_t B_{t+1} + P_t s_t X_{t+1} + P_t T_t + M_{t+1} = M_t + B_t + P_t X_t + P_t z L_t \quad (51)$$

Government controls the money supply and transfer process through its budget constraint:

$$\bar{M}_{t+1} - \bar{M}_t = -P_t T_t \quad (52)$$

Assuming a constant rate of money growth α and $\bar{M}_{t+1} = \alpha \bar{M}_t$

$$\alpha \bar{M}_t = -P_t T_t \quad (53)$$

Normalising the cash in advance and budget constraints by $\frac{1}{\bar{M}_t}$ and denoting the real values in small case letters, the cash in advance constraint and budget constraints become

$$p_t C_t + q_t b_{t+1} (1 + \alpha) + p_t s_t X_{t+1} + p_t T_t = m_t + b_t + p_t X_t \quad (54)$$

and

$$p_t C_t + q_t b_{t+1} (1 + \alpha) + p_t s_t X_{t+1} + p_t T_t + m_{t+1} (1 + \alpha) = m_t + b_t + p_t X_t + p_t z L_t \quad (55)$$

The representative agent chooses $C_t, L_t, b_{t+1}, X_{t+1}, m_{t+1}$ from $t = 0, 1, 2, \dots$ to ∞ .

Bellman value function for this problem is:

$$v(m_t, b_t, X_t, p_t, q_t, s_t) \underset{C_t, L_t, b_{t+1}, X_{t+1}, m_{t+1}}{max} [U(c_t) - V(L_t)] + \beta v(m_{t+1}, b_{t+1}, X_{t+1}, p_{t+1}, q_{t+1}, s_{t+1}) \quad (56)$$

The Lagrange multiplier of this problem is:

$$\begin{aligned} & \mathcal{L}(C_t, L_t, b_{t+1}, X_{t+1}, m_{t+1}, \lambda_t, \mu_t) \quad (57) \\ = & \sum_{t=0}^{\infty} \beta^t [U(C_t) - V(L_t)] \\ & + \lambda_t [m_t + b_t + p_t X_t - p_t C_t - q_t b_{t+1} (1 + \alpha) - p_t s_t X_{t+1} - p_t T_t] \\ & + \mu_t [m_t + b_t + p_t X_t + p_t z L_t - p_t C_t - q_t b_{t+1} (1 + \alpha) - p_t s_t X_{t+1} - p_t T_t - m_{t+1} (1 + \alpha)] \end{aligned}$$

CA model stated above is solved analytically by expressing prices and quantities in terms of prices using the first order conditions for optimisations as following:

$$C_t : U'(C_t) - (\lambda_t + \mu_t) p_t = 0 \quad (58)$$

$$L_t : -V'(L_t) + \mu_t p_t z = 0 \quad (59)$$

$$b_{t+1} : -q_{t+1} (1 + \alpha) (\lambda_t + \mu_t) + \beta \frac{\partial v}{\partial b_{t+1}} = 0 \quad (60)$$

$$X_{t+1} : -p_t s_t (\lambda_t + \mu_t) + \beta \frac{\partial v}{\partial X_{t+1}} = 0 \quad (61)$$

$$m_{t+1} : -(1 + \alpha) \mu_t + \beta \frac{\partial v}{\partial m_{t+1}} = 0 \quad (62)$$

By envelop theorem on differentiating the Bellman equation:

$$\frac{\partial v}{\partial b_t} = (\lambda_t + \mu_t) \quad (63)$$

$$\frac{\partial v}{\partial X_t} = p_t (\lambda_t + \mu_t) \quad (64)$$

$$\frac{\partial v}{\partial m_t} = (\lambda_t + \mu_t) \quad (65)$$

Combining above these last three and the first two conditions the middle three FOC can be expressed as:

$$-\frac{q_t (1 + \alpha) U'(C_t)}{p_t} + \beta \frac{U'(C_{t+1})}{p_{t+1}} = 0 \quad (66)$$

$$-sU'(C_t) + \beta U'(C_{t+1}) = 0$$

$$-\frac{(1 + \alpha) V'(L_t)}{p_t z} + \beta \frac{U'(C_{t+1})}{p_{t+1}} = 0$$

Market clearing conditions:

$$m_t = 1, b_t = 0, X_t = 0 \quad (67)$$

Putting these equilibrium conditions in the budget constraint (in cash advance model all money is held for consumption) the explicit steady state solution of this model is:

$$p_t C_t = 1 + \alpha \quad (68)$$

All output is demanded for consumption

$$C_t = zL_t \quad (69)$$

Analytical solutions for prices consistent with these steady state conditions $C_t = C, L_t = L, p_t = p, q_t = q, s_t = s$ can be expressed as following:

$$q_t = \frac{\beta}{1 + \alpha} \quad (70)$$

$$s = \beta \quad (71)$$

$$(1 + \alpha) V'(L_t) - \beta z U'(zL) = 0 \quad (72)$$

$$p = \frac{1 + \alpha}{C} \quad (73)$$

$$C = zL \quad (74)$$

Lagrange multipliers

$$\begin{aligned} \lambda &= \frac{U'(C)}{p} - \mu = \frac{CU'(C)}{1+\alpha} - \frac{V'(L)}{pz} = \\ \frac{CU'(C)}{1+\alpha} - \frac{\beta}{1+\alpha} \frac{U'(C)}{p} &= \frac{CU'(C)}{1+\alpha} \left(1 - \frac{\beta}{1+\alpha}\right) = \frac{CU'(C)}{1+\alpha} (1-q) \end{aligned} \quad (75)$$

Nominal interest rate

$$R = \frac{1}{q} - 1 = \frac{1+\alpha}{\beta} - 1 \quad (76)$$

Real interest rate equals the subjective rate of time preference:

$$r = \frac{1}{s} - 1 = \frac{1}{\beta} - 1 \quad (77)$$

Inflation rate

$$i = \frac{P_{t+1}}{P_t} - 1 = \frac{p_{t+1}\bar{M}_{t+1}}{p_t\bar{M}_t} - 1 = 1 + \alpha - 1 = \alpha \quad (78)$$

Fisher equation

$$1 + r = \frac{1 + R}{1 + i} = \frac{1 + \alpha}{\beta} \div 1 + \alpha = \frac{1}{\beta} \quad (79)$$

Thus the prices $q, s, p, R, r, i, \lambda$ are all solved in terms of parameters α and β . From the equilibrium condition $Y = C = zL = \frac{1+\alpha}{zP}$ and $L = \frac{1+\alpha}{zP}$. The output, consumption and employment decline as inflation distorts the intertemporal decisions.

Impact of growth rate of money supply on employment is obtained by total differentiation

$$\frac{dL}{d\alpha} = \frac{-V''}{(1+\alpha)V'' - \beta z^2 U''} < 0 \quad (80)$$

Period t labour income is eroded when it is consumed in period $t + 1$. In line with the inflation targetting regimes the optimal rate inflation can be solved from the social planner's problem as

$$\max_{\{C_t, L_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t [U(C_t) - V(L_t)] \quad (81)$$

subject to $Y_t = C_t = zL_t$. The solution of the optimal planner is characterised by

$$zU'(zL^*) - V'(L^*) = 0 \quad (82)$$

Table 3: Parameters of CA Model

Parameters	δ	α	β	g_y	L_0	v	g_m	z	M_0
Scenario 1	0.05	0.5	0.95	0.01	100	1	0.01	(0.1, 0.1)	100
Scenario 2	0.05	0.4	0.99	0.02	100	5	0.02	(0.1, 0.1)	100

where L^* is optimal employment. The optimal growth rate of money supply is given by the Friedman rule is $\alpha = \beta - 1$ where the nominal interest rate is zero, the real interest rate is $r = \frac{1}{\beta} - 1$, cash in advance constraint does not bind $\alpha = \beta - 1$ because $\lambda = 0$. Rate of return in all assets are equal in equilibrium.

Transitional dynamics of above models is analytically less tractable. Therefore a simple version of the CA model is implemented for two different scenarios for finding the transitional dynamics of the model with slight variation in parameterisation as given in Table 3. Model is subject to idiosyncratic technological shocks and is solved for 30 years to obtained from time profiles of capital, output, prices, money, consumption, investment, labour supply and lifetime utilities of households.

Two scenarios are constructed for comparison of impacts of financial crises in these economies. First scenario is for more regular economy and second scenario for a crisis hit economy.

Financial crises occur because of shift in the investor and consumer confidences, change in perceptions and beliefs and technological shocks that hit the system. The impact of such changes can be very sudden which affect the velocity of circulation money, technological progress, discount factors or the belief in the underlying growth rate of the economy. This impacts on prices, trend of output, prices and other features of the economy. Path of model variables implied by the solutions are given in the pdf files in the appendix. Initial financial crises has adverse impact on investment and that lowers the capital formation, output and consumption as shown in the following diagramme.

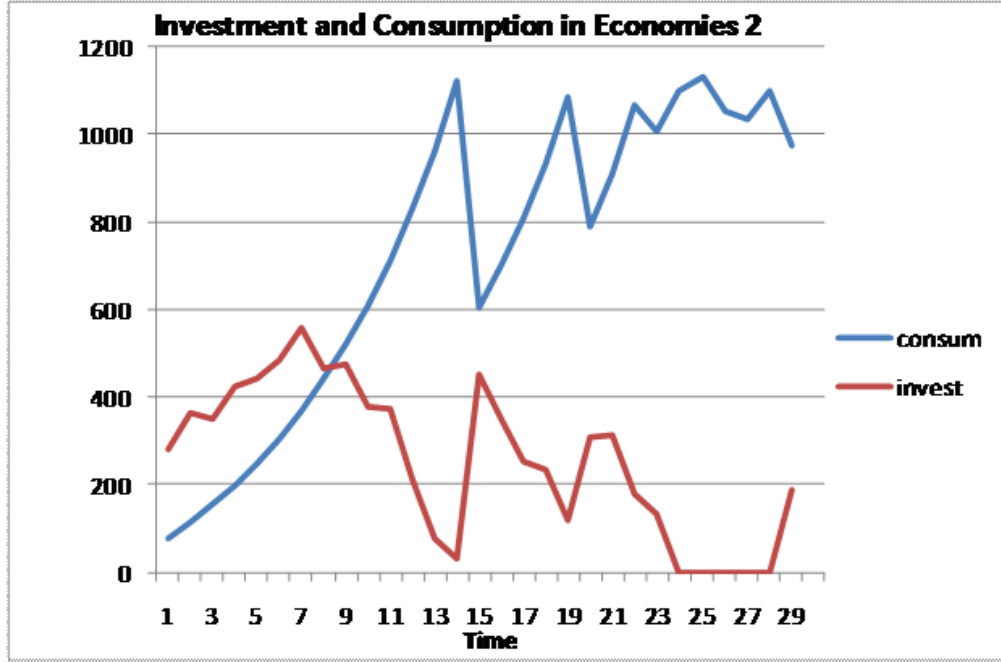
6.2 Dynamics with Money in Utility Function

Role of money was for pure exchange in the cash in advance model but there are circumstances when household prefer to store cash, particularly during the financial crises. This feature is captured by the money in the utility function model. When this desire is excessive it causes a crisis in the system like the current one. The problem of household in this is again to maximise the lifetime welfare that is obtained by consumption and money.

$$\max W = \sum_{t=0}^{\infty} [\beta^t U(c_t, m_t)] \quad (83)$$

Subject to the technology constraint:

$$Y_t = zF(K_t, L_t) \quad (84)$$



Under constant returns to scale $y_t = f(k_t)$ where $y_t = \frac{Y_t}{L_t}$ and $k_t = \frac{K_t}{L_t}$.
Economy wide budget constraint is given by

$$Y_t + \tau_t L_t + (1 - \delta) K_{t-1} + \frac{M_{t-1}}{P_{t-1}} = C_t + K_t + \frac{M_t}{P_t} \quad (85)$$

where Y_t is output, P_t price of goods, C_t consumption, K_{t+1} is capital stock, τ_t is net transfer for each individual, M_t money, L_t employment and δ is the rate of depreciation of capital

In per capita terms

$$\omega_t = f(k_{t-1}) + \tau_t + \left(\frac{1 - \delta}{1 + n} \right) k_{t-1} + \frac{m_{t-1}}{(1 + \pi_t)(1 + n)} = c_t + k_t + m_t \quad (86)$$

The dynamic program of this problem is:

$$V(\omega_t) = u(c_t, m_t) + \beta V(\omega_{t+1}) \quad (87)$$

$$V(\omega_t) = \max \left\{ u(c_t, m_t) + \beta V \left[f(\omega_t - c_t - m_t) + \tau_{t+1} + \left(\frac{1 - \delta}{1 + n} \right) (\omega_t - c_t - m_t) + \frac{m_t}{(1 + \pi_{t+1})(1 + n)} \right] \right\} \quad (88)$$

The Lagrange multiplier of this problem is:

$$\mathcal{L}(c_t, m_t, \lambda_t) = \sum_{t=0}^{\infty} [\beta^t u(c_t, m_t)] + \sum_{t=0}^{\infty} \lambda_t \left[f(\omega_t - c_t - m_t) + \tau_{t+1} + \left(\frac{1-\delta}{1+n} \right) (\omega_t - c_t - m_t) + \frac{m_t}{(1+\pi_{t+1})(1+n)} \right] \quad (89)$$

where λ_t is the lagrange multipliers.

As before solving MIU model explicitly means expressing the prices and quantities like y_t, k_t, c_t, m_t in terms of the preference and technology parameters as β, δ, α and n . In other words the optimal values of variables are determined by subjective discount factor (β), depreciation (δ) and productivity (α) of capital. This is done using the first order conditions:

$$c_t : u_c(c_t, m_t) - \beta \left[f_k(k_t) + \left(\frac{1-\delta}{1+n} \right) \right] V_\omega(\omega_{t+1}) = 0 \quad (90)$$

Here marginal utility of holding capital $\beta \left[f_k(k_t) + \left(\frac{1-\delta}{1+n} \right) \right] V_\omega(\omega_{t+1})$ should equal the marginal utility of consumption $u_c(c_t, m_t)$.

$$m : u_m(c_t, m_t) - \beta \left[f_k(k_t) + \left(\frac{1-\delta}{1+n} \right) \right] V_\omega(\omega_{t+1}) + \frac{\beta V_\omega(\omega_{t+1})}{(1+\pi_{t+1})(1+n)} = 0 \quad (91)$$

Transversality conditions

$$\lim_{t \rightarrow \infty} \beta^t \lambda_t k_t = 0; \lim_{t \rightarrow \infty} \beta^t \lambda_t m_t = 0 \quad (92)$$

By envelop theorem:

$$\lambda_t = V_\omega(\omega_t) = u_c(c_t, m_t) \quad (93)$$

This condition can be inserted into the first order condition to get:

$$u_m(c_t, m_t) + \frac{\beta u_c(c_{t+1}, m_{t+1})}{(1+\pi_{t+1})(1+n)} = u_c(c_t, m_t) \quad (94)$$

Thus marginal utility of holding money should equal to marginal utility of consumption. Left hand side gives the total marginal benefit of holding money. First term in the left hand side is the direct utility of money and the second component is real balance effect of holding money m_t at t on $t+1$. By constant returns to scale assumption $r^k k + w = f_k k + (f(k) - f_k k) = f(k)$

Consider a steady state with $n = 0$ and $V_\omega(\omega_t) = V_\omega(\omega_{t+1}) = V_\omega(\omega^{ss})$. From the first first order conditions $1 - \beta [f_k(k^{ss}) + (1-\delta)] = 0$

$$f_k(k^{ss}) + (1-\delta) = \frac{1}{\beta}$$

Assuming a Cobb-Douglas production function $f(k) = k^\alpha$ this condition converts to $\alpha k^{\alpha-1} + (1-\delta) = \frac{1}{\beta}$

Table 4: Parameters of MIU Model

Parameters	δ	α	β	g_y	L_0	v	g_m	z	M_0
Economy 1	0.05	0.5	0.95	0.01	100	1	0.02	(0.1, 0.1)	100
Economy 2	0.05	0.4	0.99	0.02	100	5	0.02	(0.1, 0.1)	100

$$k^{ss} = \left[\frac{\alpha\beta}{1 + \beta(1 - \delta)} \right]^{\frac{1}{1-\alpha}} \quad (95)$$

Consumption

$$c^{ss} = f(k^{ss}) - \delta k^{ss} = \left[\frac{\alpha\beta}{1 + \beta(1 - \delta)} \right]^{\frac{\alpha}{1-\alpha}} - \delta \left[\frac{\alpha\beta}{1 + \beta(1 - \delta)} \right]^{\frac{1}{1-\alpha}} \quad (96)$$

Steady state inflation rate equals growth rate of money supply

$$\frac{\Delta m^{ss}}{m^{ss}} = \frac{\theta^{ss} - \pi^{ss}}{(1 + \pi^{ss})} = 0$$

where $\Delta m^{ss} = 0$ implies growth rate of money supply, $\theta^{ss} = \frac{\Delta M^{ss}}{M^{ss}}$, and inflation are equal $\theta^{ss} = \pi^{ss}$.

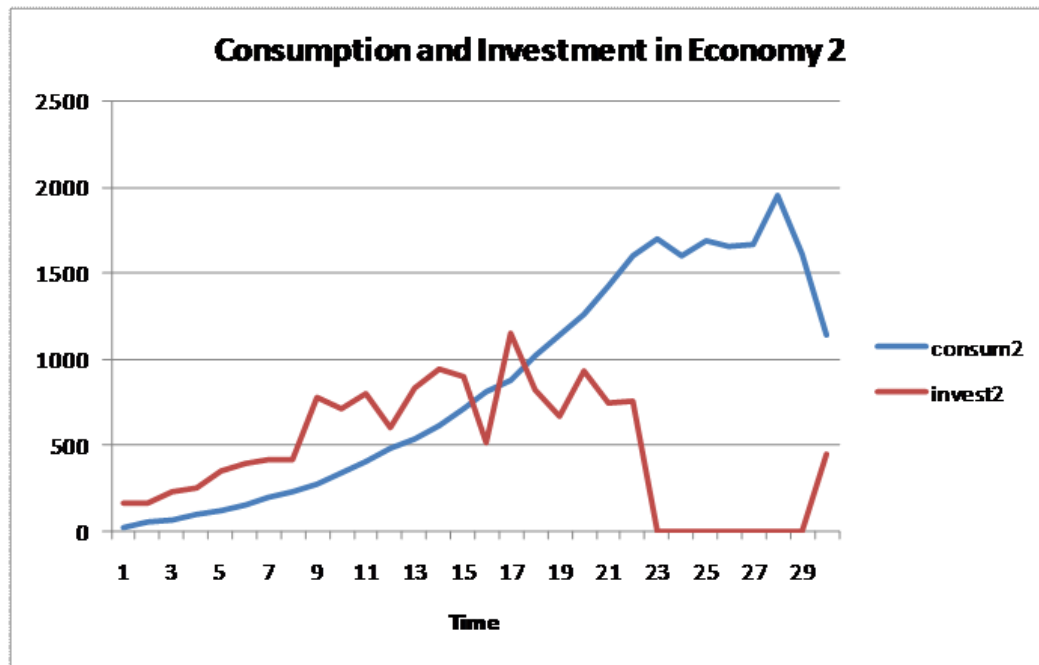
As in the CA model the transitional dynamics of the MIU model is found numerically for two scenarios representing more stable and less stable crisis prone scenarios with the set of parameters in Table 4.

Detailed time path of variables y_t, k_t, c_t, k_t, p_t are shown in series of diagram at the end. Underlying factors for such fluctuations are the moral hazard and adverse selection between lenders and borrowers or the public or the private sectors or the shocks to the technology or sudden shift in the belief. Analysis of results of all economic activities is the welfare of the households from the consumption of goods and services reveals how financial crises impacts their prospects. Producers face lower stock of capital and to combine labour and can supply less of those goods and have to pay more for factors. Direct and indirect taxes distort choices of households.

The allocations presented in the model 1 are steady state counterparts of the the dynmic CA and MIU models and depict the long run behaviour of the economy. Steady state is a condition where growth ceases to exist. This makes the comparative static conclusions efficiency, and redistribution an aproximation to full-fledged dynamic model discussed in the last two sections.

7 Conclusion

Impact of fiscal and monetary policies are assessed in the an open economy two sector multi-household general equilibrium tax model with money. There is alarming gap in the distribution of income among households which none of



the stabilisation policies could possibly address. Impacts of fiscal expansions are positive for all categories of households but the gains are much higher for household in the upper income group than for the households in the bottom income group despite claim of the government to help out to those with the least means. Model results show that demand, output, employment are sensitive to the consumer preference and confidences and sectoral shift in the production technology and impacts vary across households. Simulations result show that monetary policy is less effective in promoting output, employment and other economic policies than the fiscal policy. This is consistent to super neutrality of money found in the steady state of the cash in advance and money in the utility function models. Relative price based optimal allocation mechanisms and redistribution provide several hints on how the complex mechanism of relocation and redistribution functions occur in a real economy.

References

- [1] Altig D E, C.T. Carlstrom and K.L. Lansing (1995) Computable General Equilibrium Models and Monetary Policy Advice, *Journal of Money Credit and Banking*, vol. 27, no. 4, Nov., pp.1472-1493.
- [2] Arestis Philip and Panicos Demetriades (1997) Financial Development and Economic Growth: Assessing the Evidence *The Economic Journal*,

Vol. 107, No. 442 :May,, pp. 783-799

- [3] Bacchetta Philippe (1992) Liberalization of Capital Movements and of the Domestic Financial System *Economica*, New Series, Vol. 59, No. 236 Nov., pp. 465-474
- [4] Bank of England (1999) The Transmission Mechanism of Monetary Policy, www.bankofengland.co.uk
- [5] Bank of England (2001) Financial Stability Review, [www. Bankofengland.co.uk](http://www.Bankofengland.co.uk).
- [6] Banerji S and P Basu (2009) Universal Banking and Equity Risk Premium, April; Paper presented in CEP Seminar in Hull Business School.
- [7] Bolnick Bruce R. (1987) Financial Liberalization with Imperfect Markets: Indonesia during the 1970s *Economic Development and Cultural Change*, Vol. 35, No. 3 :Apr., pp. 581-599
- [8] Boycko Maxim , Andrei Shleifer and Robert W. Vishny (1996) A Theory of Privatisation *The Economic Journal*, Vol. 106, No. 435 :Mar., , pp. 309-319
- [9] Brunnermeier, Markus K. (2009.) "Deciphering the Liquidity and Credit Crunch 2007-2008." *Journal of Economic Perspectives*, 23(1): 77–100.
- [10] Cecchetti, Stephen G. (2009). "Crisis and Responses: The Federal Reserve in the Early Stages of the Financial Crisis." *Journal of Economic Perspectives*, 23(1): 51–75.
- [11] Champ Bruce , Bruce D. Smith and Stephen D. Williamson (1996) Currency Elasticity and Banking Panics: Theory and Evidence *The Canadian Journal of Economics / Revue canadienne d'Economie*, Vol. 29, No. 4 :Nov., pp.826-824
- [12] Chiang Alpha C. (1959) Instalment Credit Control: A Theoretical Analysis *The Journal of Political Economy*, Vol. 67, No. 4 :Aug., pp. 363-376
- [13] Cripps Martin W. (1997) Bargaining and the Timing of Investment *International Economic Review*, Vol. 38, No. 3 :Aug., pp. 527-546
- [14] Dasgupta Partha and Eric Maskin (2000) Efficient Auctions *The Quarterly Journal of Economics*, Vol. 115, No. 2 :May, pp. 341-388
- [15] Diamond P, Douglas W. and Dybvig, Philip H. (1983) Bank Runs, Deposit Insurance, and Liquidity, *JPE*, 401-419.
- [16] Giovannini Alberto and Martha de Melo (1993) Government Revenue from Financial Repression *The American Economic Review*, Vol. 83, No. 4 Sept., pp. 953-963

- [17] De Fraja Giovanni (1991) Efficiency and Privatisation in Imperfectly Competitive Industries *The Journal of Industrial Economics*, Vol. 39, No. 3 :Mar., pp. 311-321
- [18] Fama, Eugene F. (1980) Banking in Theory of Finance, *Journal of Monetary Economics*, 6- 39-57.
- [19] Mayer, Christopher, Karen Pence, and Shane M. Sherlund. (2009). "The Rise in Mortgage Defaults." *Journal of Economic Perspectives*, 23(1): 27–50.
- [20] Friedman Milton (2005) A Natural Experiment in Monetary Policy Covering Three Episodes of Growth and Decline in the Economy and the Stock Market, *The Journal of Economic Perspectives*, Vol. 19, No. 4 :Autumn,, pp. 145-150
- [21] GAMS Development Corporation (1999), *GAMS: A User's Guide*, Washington DC 20007, USA.
- [22] Sinn H.W (2009) Risk Taking, Limited Liability and the Banking Crisis, Ifo Institute, University of Munich
- [23] King Mervyn (1994) Enrique Sentana and Sushil Wadhvani Volatility and Links between National Stock Markets *Econometrica*, Vol. 62, No. 4 :Jul., pp. 901-933
- [24] Klein Michael A.(1971) A Theory of the Banking Firm *Journal of Money, Credit and Banking*, Vol. 3, No. 2, Part 1 May, pp. 205-218
- [25] Krugman Paul (1979) A Model of Balance of Payment Crisis, *Journal of Money Credit and Banking*, 11, Aug.
- [26] Lim G.C. and P D. McNeilis (2008) *Computaitonal Macroeconomics for the Open Economy*, MIT Press.
- [27] Milde Hellmuth and John G. Riley (1988) Signaling in Credit Markets *The Quarterly Journal of Economics*, Vol. 103, No. 1 :Feb., pp. 101-129
- [28] Prescott Edward C. and Robert M. Townsend (1984) Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard *Econometrica*, Vol. 52, No. 1 :Jan., pp. 21-45
- [29] Rasmusen Eric(1987) Moral Hazard in Risk-Averse Teams *The RAND Journal of Economics*, Vol. 18, No. 3 :Autumn., pp. 428-435
- [30] Radelet Steven , Jeffrey D. Sachs, Richard N. Cooper and Barry P. Bosworth (1998) *The East Asian Financial Crisis: Diagnosis, Remedies, Prospects* *Brookings Papers on Economic Activity*, Vol. 1998, No. 1, pp. 1-90

- [31] Riley John G (2001) Silver Signals: Twenty-Five Years of Screening and Signalling, *Journal of Economic Literature*, 39:2:432-478
- [32] Rogerson William P. (1985), Repeated Moral Hazard *Econometrica*, Vol. 53, No. 1 :Jan., pp. 69-76
- [33] Roubini N. and Sala-i-Martin (1992) Financial Repression and Economic Growth, *Journal of Development Economics* 39: 5-30: July.
- [34] Sargent.T J (1987) *Dynamic Macroeconomics*, Oxford University Press.
- [35] Smith Warren L.(1958) The Discount Rate as a Credit-Control Weapon *The Journal of Political Economy*, Vol. 66, No. 2 :Apr., pp. 171-177
- [36] Spencer P. D. (1984) Precautionary and Speculative Aspects of the Behaviour of Banks in the United Kingdom Under Competition and Credit Control 1972-1980.*The Economic Journal*, Vol. 94, No. 375 :Sep., pp. 554-568
- [37] Sproul Allan (1947) Monetary Management and Credit Control *The American Economic Review*, Vol. 37, No. 3:Jun., pp. 339-350
- [38] Stiglitz, J.E. and A. Weiss (1981) Credit Rationing in Markets with Imperfect Competition, *American Economic Review*, June, 393-410.
- [39] Williamson S.D. (2008) *Macroeconomics*, 3rd Edition, Pearson International.
- [40] Walsh C.E (1998) *Monetary Theory and Policy*, MIT Press.

