

**Dynamic Multi-household General Economic  
Models for Policy Simulations:  
France, Germany, Spain and UK  
and  
General Equilibrium with Unemployment:  
Theory and Application**

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Modelling Employment and Inequality  
Loughborough University**

# European Economy

- Aims
  - macroeconomic stability
  - higher economic growth
  - greater efficiency in allocation of resources
  - more equal distributions for peace and prosperity
- Miles Stones
  - Treaty of Rome (1957)
  - Treaty of Maastricht (1992)
  - Stability and Growth Pact (1997)
  - Euro (1999)
  - Lisbon Agenda (2000)
  - Economic Accord to Contain Crises (2008)

## Selected CGE Literature for EU

- Allen, Gasiorek, Smith, Flam, Sørensen (1998)
- Baldwin, Francois, Portes, Rodrik, (1997)
- Broer, Westerhout, Bovenberg (1994)
- Francois (1996)
- Haaland (1992)
- Harrison., Rutherford, Tarr (1997)
- Haaland, Norman, Wergeland, Rutherford (1987)
- Kohler, Casella, Sapir (1996)
- Nordhaus and Yang (1996)
- Piazolo (2001)
- Saito (2004)
- Wren-Lewis, Darby, Ireland, Ricchi (1996)
- Wright (1988)
- Krusell, Ohanian Ríos-Rull, and Violante (2000)

### Dynamic models

- Bhattarai (2008,2007, 1999)
- Daniel and Blanchard (1988)

### Comparative static models:

- Bhattarai and Whalley (2006, 2003)
- Maureen and Oates (1992)
- Abrego and Whalley(2000)
- Touhami, (1998)
- Armington (1969).
- Edwards and Whalley (2007).
- Haskel, and Slaughter (2001).
- Winchester, Greenaway and Reed (2006).
- Wright (1988)
- Research Centres of member countries
- European Union Studies
- Ecomod Conference series

# Main Aspects of the Model

- **Demand Side of the Economy**
- **Supply side of the economy**
- **Trade and Aggregate Supply**
- **Public sector**
- **Markets and the relative prices**
- **Pareto optimal allocations**
- **Growth, investment and accumulation**

Figure 1

# Macro Aggregaates of Germany (Euro)

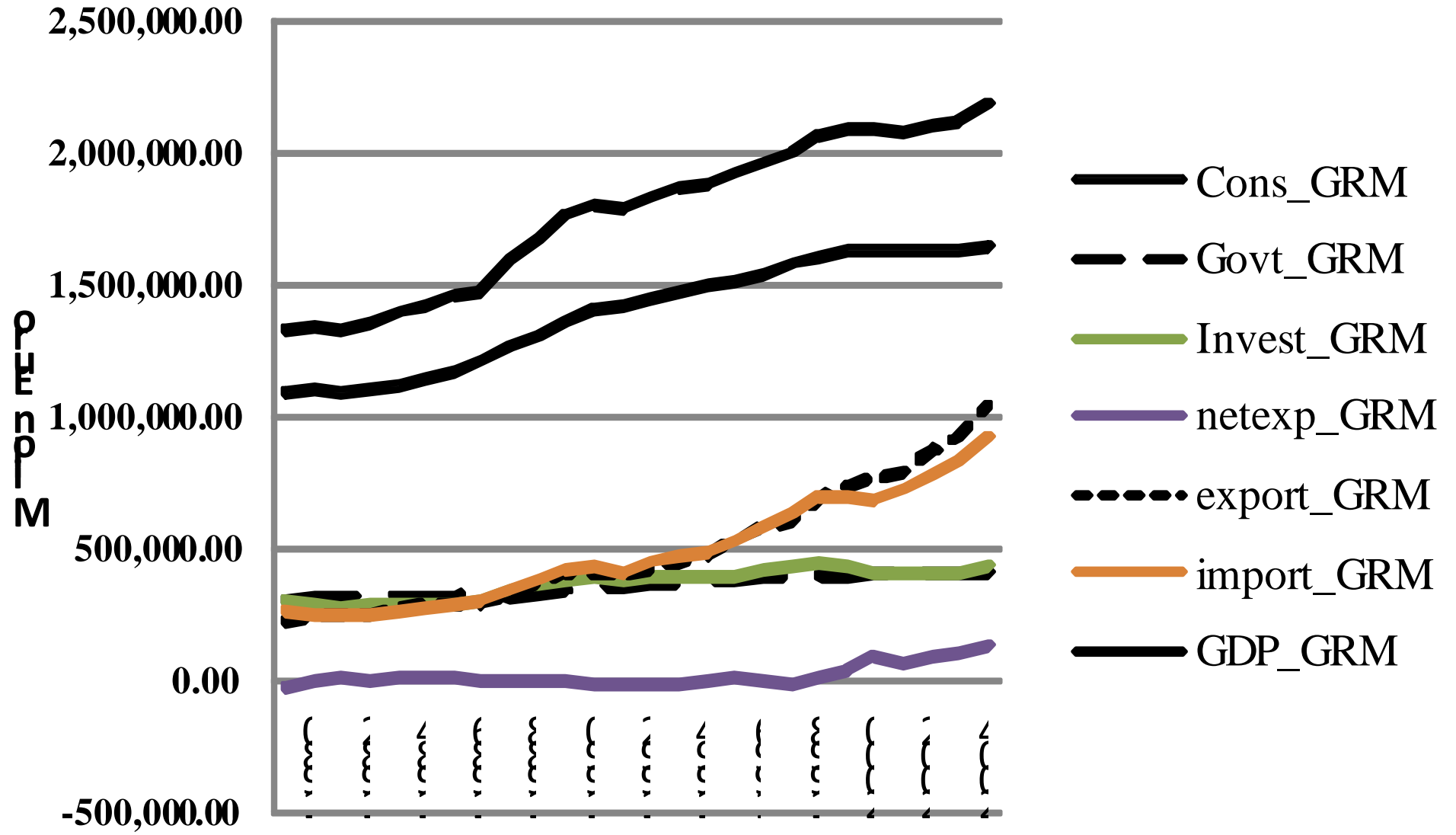


Table 1 (a)  
Input-Output Transaction Table for Germany

	Primary	Manu	MatManu	Machman	Utility	Constru	TRHTRST	TRNCMM	FINREST	BUSSRV	PRSRV
Primary	2718	35960	31325	799	4974	2283	1717	379	891	394	2289
Manu	4375	43348	7290	6716	108	8584	14610	528	635	610	8478
MatManu	6261	20204	155823	91274	3238	46371	18009	9428	3133	7827	16544
Machman	2271	3545	13458	176105	4018	13668	9071	7065	1449	3512	12942
Utility	1641	3455	11378	4565	3089	509	5025	2327	1816	1488	5449
Constru	1073	1140	3631	3080	1845	5560	3073	2333	18184	2259	7887
TRHTRST	3111	12414	16180	21302	1431	9776	23608	7294	1336	4077	11411
TRNCMM	1331	7203	15180	16333	1413	3693	12045	64052	4672	4622	12161
FINREST	1906	6505	14552	14897	3049	22752	40787	9386	79174	21705	22605
BUSSRV	5722	18510	41951	44076	6834	12964	29434	13544	50591	53838	29702
PRSRV	2164	2402	8577	2544	4635	1500	9268	2316	4842	14094	34441

Data Source: OECD, DSTO/DOC(2006)8

**Table 1 (b)**  
**Value added, Final Demand and production Taxes for Germany**

	Primary	Manu	MatManu	Machman	Utility	Constru	TRHTRST	TRNCMM	FINREST	BUSSRV	PRSRV
Capital	17209	20979	47323	23148	16614	27837	51196	46146	227050	139478	96230
Wages	15870	43810	114080	152750	15780	65210	158090	61370	64650	101660	306690
Prodtax	-4983	2026	6043	4229	2513	2452	11879	4872	16750	2356	8212
Conshh	20715	145901	69456	75220	24741	4607	225554	69690	241371	14698	147103
Cons_gov	128	1269	8463	4033	334	816	11370	2966	3592	5697	342683
GFCFR	3318	9387	16058	151324	2917	169171	14325	5899	3830	30274	5478
Export	5703	43103	164654	260311	1476	2082	34403	33646	2973	19255	2826
Imports	52917	73441	149952	176171	670	3578	9782	23867	13912	19169	9834

Data Source: OECD, DSTO/DOC(2006)8

**Table 1 (c)**  
**Share of income, consumption, labour and capital income in Germany**

	h1	h2	h3	h4	h5	h6	h7	h8	h9	h10
Incshare	0.0322	0.0530	0.0686	0.0686	0.0890	0.0890	0.1155	0.1155	0.1481	0.2207
Consshare	0.0320	0.0530	0.0685	0.0685	0.0890	0.0890	0.1155	0.1155	0.1480	0.2210
Capital	22965.36	37800.12	48926.20	48926.20	63440.02	63440.02	82340.08	82340.08	105626.39	157405.42
Wages	35418.71	58297.88	75457.26	75457.26	97841.44	97841.44	126990.38	126990.38	162904.08	242761.17
Conshh	33249.76	55069.92	71175.27	71175.27	92475.90	92475.90	120010.85	120010.85	153780.14	229631.16

Data Source: OECD, DSTO/DOC(2006)8 World Bank data at [www.mimas.ac.uk](http://www.mimas.ac.uk) for income and Table 2.7 of WDI for consumption Share.

Figure 2

### Macroeconomic Trends of France (Euro)

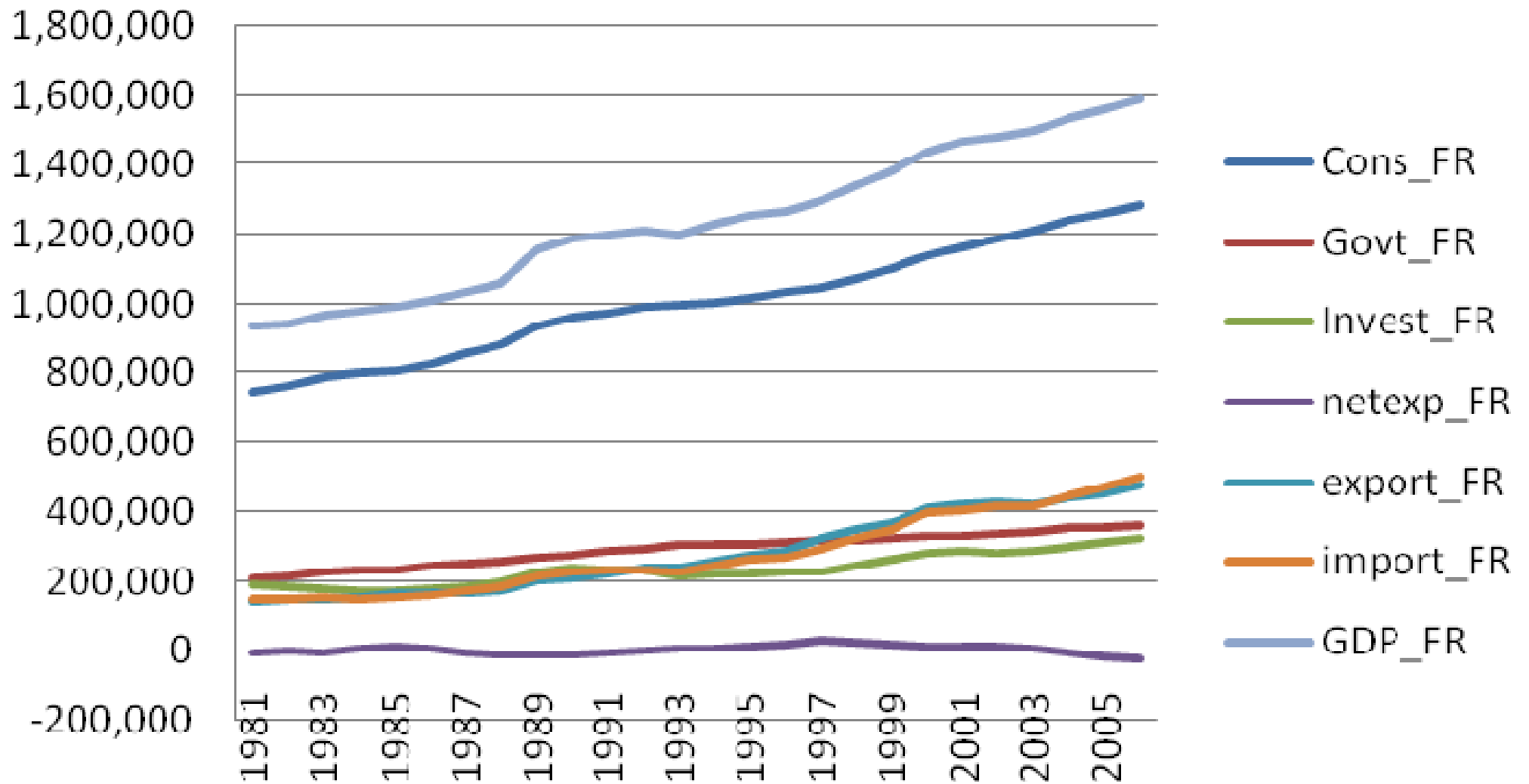


Table 2 (a)  
Input Output Table of France

	Primary	Manu	MatManu	Machman	Utility	Constru	TRHTRST	TRNCMM	FINREST	BUSSRV	PRSRV
Primary	11739	31811	23653	35	5466	2577	1747	15	16	95	2067
Manu	6417	36129	5087	3704	26	4304	10810	737	1453	1028	9501
MatManu	8550	17561	109128	45261	3933	21534	10377	7539	3593	12225	17932
Machman	2628	1935	6781	99367	853	9201	3344	7564	994	8146	10964
Utility	959	2443	5902	1727	5984	395	2710	907	711	946	5175
Constru	523	253	1682	570	2459	18955	985	390	2416	1074	5732
TRHTRST	3093	6836	11115	10439	232	4529	14725	2042	981	5066	8413
TRNCMM	1222	4405	10590	4245	504	2791	12233	42415	5257	11679	10230
FINREST	2041	6136	7885	6333	1471	10360	18571	6636	42577	14996	18179
BUSSRV	1950	17724	28227	33167	4614	17657	14928	8701	26126	57193	23378
PRSRV	607	1606	3869	3336	1107	1483	2420	1624	3718	6572	13475

Data Source: OECD, DSTO/DOC(2006)8

**Table 2 (b)**  
**Value added, Final Demand and production Taxes for France**

	Primary	Manu	MatManu	Machman	Utility	Constru	TRHTRST	TRNCMM	FINREST	BUSSRV	PRSRV
Capital	27835	20683	37599	18944	12421	19514	48867	27192	152201	56339	59525
Wages	9079	31161	59709	48630	10569	37910	106261	49965	47681	103165	233997
Prodtax	1607	4654	15216	11684	3391	6837	14024	8931	19052	11160	16163
Conshh	21454	109208	47331	47392	21746	7502	165238	41960	150087	12207	95474
Consgov	3	32	10244	856	11	43	6619	370	10509	7207	293960
GFFCFR	2138	4395	15746	81476	633	115462	9541	1834	10102	34869	4117
Export	10068	47065	98407	149023	3048	0	16197	22272	3024	19227	3742
Imports	34633	56559	102918	143080	268	0	3061	7347	2131	17493	2379

Data Source: OECD, DSTO/DOC(2006)8

**Table 2 (c)**  
**Share of income, consumption, labour and capital income in France**

	h1	h2	h3	h4	h5	h6	h7	h8	h9	h10
Incshare	0.0275	0.0443	0.0631	0.0631	0.0860	0.0860	0.1140	0.1140	0.1512	0.2509
Consshare	0.0280	0.0440	0.0630	0.0630	0.0860	0.0860	0.1140	0.1140	0.1510	0.2510
Capital	13230.77	21313.57	30358.61	30358.61	41352.18	41352.18	54847.57	54847.57	72745.19	120712.76
Wages	20298.49	32699.03	46575.81	46575.81	63442.02	63442.02	84146.48	84146.48	111604.80	185196.06
Conshh	20148.74	31662.30	45334.66	45334.66	61885.41	61885.41	82034.15	82034.15	108659.27	180619.05

Data Source: OECD, DSTO/DOC(2006)8

Figure 3

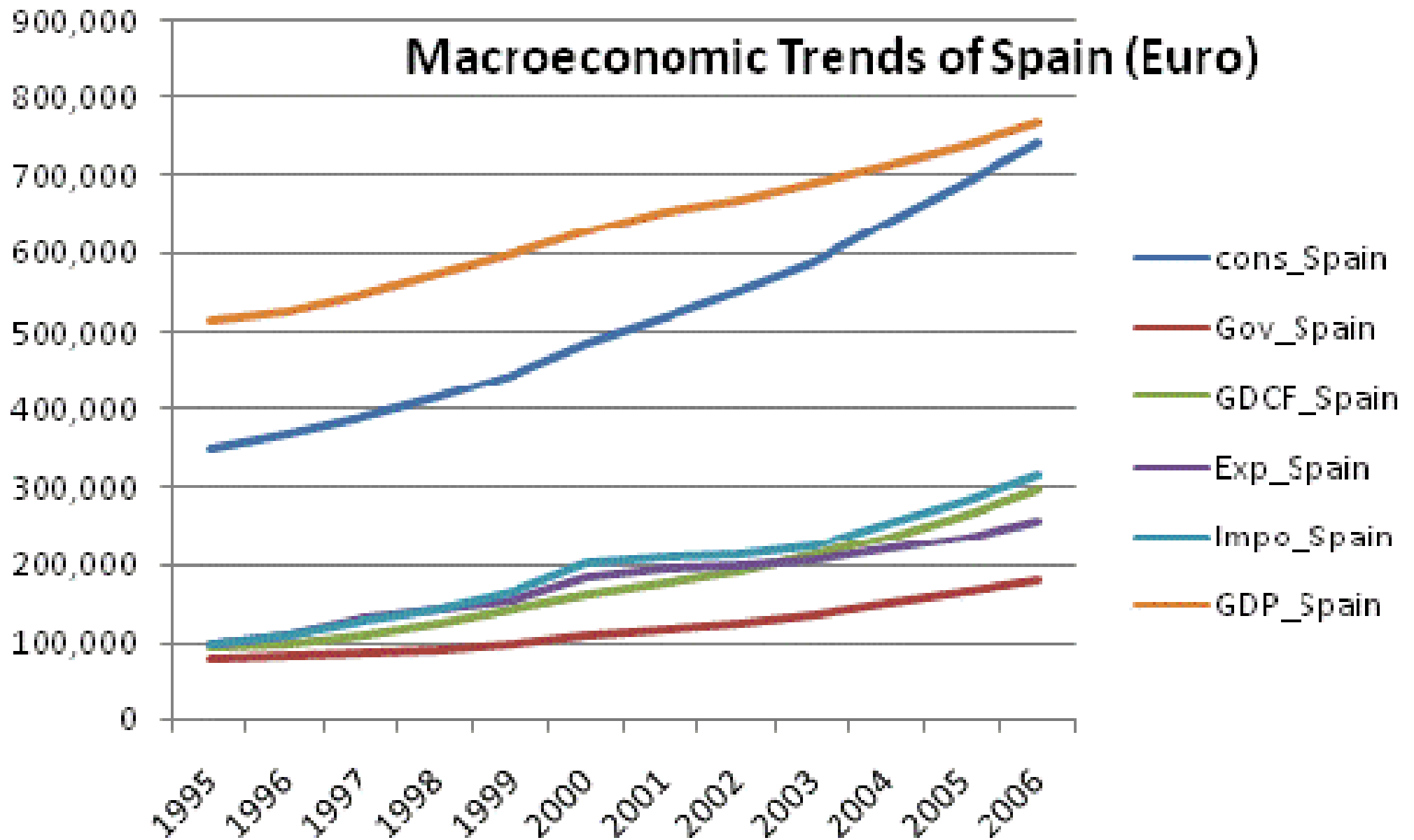


Table 3 (a)  
Input Output Table of Spain

	Primary	Manu	MatManu	Machman	Utility	Constru	TRHTRST	TRNCMM	FINREST	BUSSRV	PRSRV
Primary	3368	22010	16903	25	4684	1134	2151	16	30	18	437
Manu	5564	29048	2383	1227	35	2639	16069	510	83	1003	2042
MatManu	4439	11549	50776	23065	2520	23923	4248	5055	1016	4131	6813
Machman	784	1276	6278	34162	499	4699	4681	2803	128	1171	3756
Utility	861	1397	4770	855	2325	393	2821	585	344	724	2167
Constru	274	269	747	243	269	16079	1832	584	9413	588	2084
TRHTRST	1626	4775	6037	2146	192	5667	6770	5153	517	1514	2609
TRNCMM	1846	6612	10310	3033	349	2978	6427	13620	1860	2722	3875
FINREST	1126	2178	3154	1571	792	2842	11556	2766	10262	4234	5781
BUSSRV	404	4487	7330	3936	777	5223	5884	3752	2684	7925	6356
PRSRV	446	1071	1662	526	164	581	1754	763	504	1715	6129

Data

Source: OECD, DSTO/DOC(2006)8

**Table 3 (b)**  
**Value added, Final Demand and production Taxes for Spain**

	Primary	Manu	MatManu	Machman	Utility	Constru	TRHTRST	TRNCMM	FINREST	BUSSRV	PRSRV
Capital	17626	9132	16915	6365	8814	14598	63417	23003	51897	12055	18119
Wages	5190	19533	29216	18782	2978	30908	42831	21519	17757	24634	86529
Prdntax	-1270	-2699	904	138	-56	1261	2481	2133	5115	519	4319
Conshh	6642	51587	18682	20947	6603	3272	124237	20328	46279	8274	35697
Consgov	8	43	4291	150	6	7	2397	710	1	83	99780
GFFCFR	760	3474	5163	35543	401	76977	2541	1468	8163	10317	880
Export	8307	20049	39515	52322	208	294	7828	11139	1973	8631	1394
Imports	24209	25116	47801	73124	118	9	1087	5013	1305	13112	2050

Data Source: OECD, DSTO/DOC(2006)8

**Table 3 (c)**  
**Share of income, consumption, labour and capital income in Spain**

	h1	h2	h3	h4	h5	h6	h7	h8	h9	h10
Incshare	0.0257	0.0440	0.0605	0.0605	0.0822	0.0822	0.1126	0.1126	0.1539	0.2661
Consshare	0.0260	0.0440	0.0605	0.0605	0.0820	0.0820	0.1125	0.1125	0.1540	0.2660
intr	6217.9	10645.4	14625.3	14625.3	19875.4	19875.4	27230.4	27230.4	37234.7	64380.4
wage	7706.8	13194.6	18127.6	18127.6	24634.9	24634.9	33751.2	33751.2	46151.1	79797.3
Conshh	8906.3	15072.2	20724.2	20724.2	28089.0	28089.0	38536.8	38536.8	52752.6	91118.1
Leisure	5780.1	9895.9	13595.7	13595.7	18476.2	18476.2	25313.4	25313.4	34613.3	59848.0

Data Source: OECD, DSTO/DOC(2006)8

Figure 4

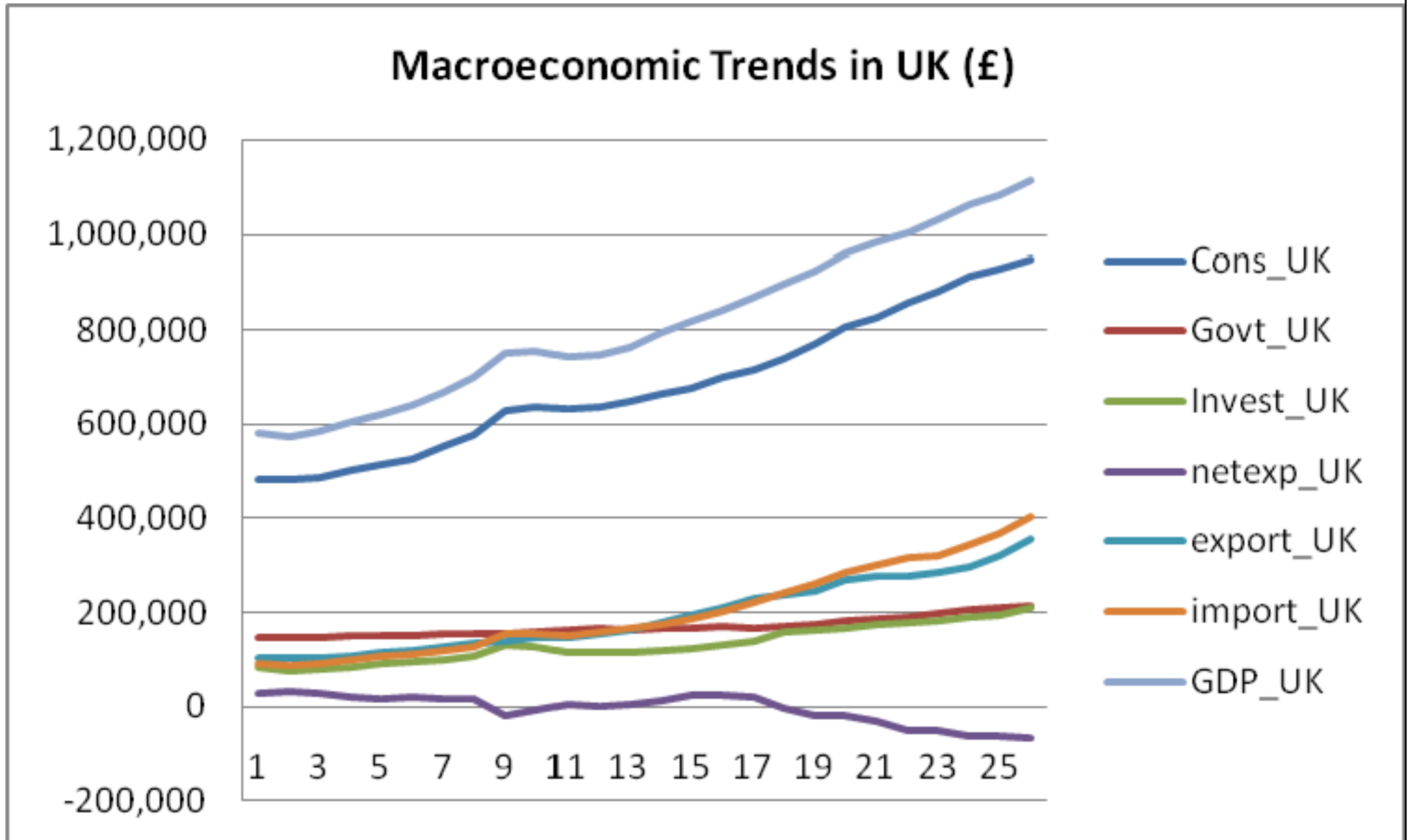


Table 4 (a)  
Input Output Table of UK

	Primary	Manu	MatManu	Machman	Utility	Constru	TRHTRST	TRNCMM	FINREST	BUSSRV	PRSRV
Primary	4894	8984	13549	51	8679	2319	1826	243	152	240	601
Manu	1509	14435	3260	1479	80	2550	14753	886	707	1133	4388
MatManu	2966	10251	42359	22719	1210	12526	9282	6568	5537	5387	14439
Machman	1015	2178	4502	38747	985	3303	6360	8106	1463	2863	15696
Utility	704	1485	3938	1641	16801	393	2027	941	904	1206	2739
Constru	825	280	622	412	569	27619	1640	1564	9889	841	4585
TRHTRST	2659	9933	13218	13115	703	4892	14015	5126	3613	4120	10920
TRNCMM	1362	3545	6805	3192	365	1727	29331	30061	18767	8462	9603
FINREST	2779	3760	7285	5699	1380	5869	21927	6678	20526	8164	11586
BUSSRV	2028	4858	7318	6501	1374	7813	22711	14639	28627	48562	24121
PRSRV	614	952	2861	1357	314	667	2987	3040	4988	7112	0

Data Source: OECD, DSTO/DOC(2006)8

**Table 4 (b)**  
**Value added, Final Demand and production Taxes for UK**

	Primary	Manu	MatManu	Machman	Utility	Constru	TRHTRST	TRNCMM	FINREST	BUSSRV	PRSRV
Capital	26537	9910	13109	9826	9345	19347	37196	20480	79837	34277	30218
Wages	6207	23942	50056	36519	4850	23637	82230	45321	37008	70597	151951
Prdtax	1090	2485	4619	3345	2555	4204	13705	7442	6267	5416	7826
Conshh	11347	77290	43828	47161	16043	4092	135849	36316	108326	11082	81348
Consgov	24	0	0	0	0	0	18	2	0	145	176902
GFCFR	1887	4068	8368	47980	577	63768	12516	1764	2903	11027	3243
Export	16694	11335	41235	77902	210	224	32351	12647	14541	24662	6608
Imports	16280	40870	53157	113677	397	63	3059	12855	3138	17087	4321

Data Source: OECD, DSTO/DOC(2006)8

**Table 4 (c)**  
**Share of income, consumption, labour and capital income in UK**

	h1	h2	h3	h4	h5	h6	h7	h8	h9	h10
Incshare	0.0206	0.0408	0.0571	0.0571	0.0798	0.0798	0.1125	0.1125	0.1553	0.2849
Consshare	0.0210	0.0400	0.0570	0.0570	0.0800	0.0800	0.1125	0.1125	0.1550	0.2850
Capital	5975.69	11835.35	16549.18	16549.18	23148.54	23148.54	32619.72	32619.72	45049.73	82644.36
Wages	10965.75	21718.57	30368.74	30368.74	42478.98	42478.98	59859.16	59859.16	82668.99	151657.40
Conshh	12026.31	22907.26	32642.85	32642.85	45814.52	45814.52	64426.67	64426.67	88765.63	163214.23

Data Source: OECD, DSTO/DOC(2006)8

# Structure of the Multi-Household Dynamic General Equilibrium Model

- Consumption
  - 10 categories of representative household
  - Investor
  - Trader
  - Government
- Labour-leisure choice and labour supply

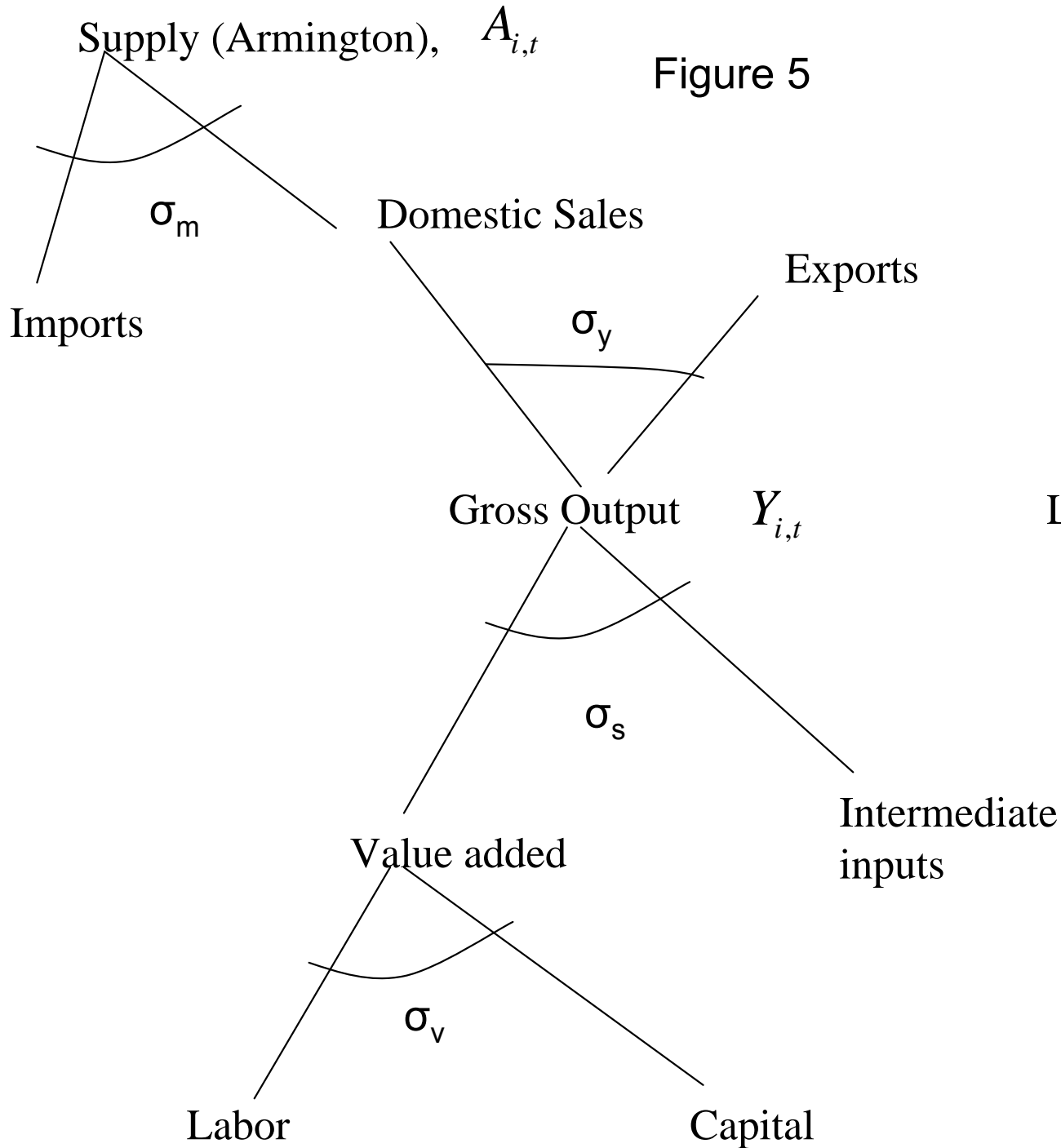
- Production
  - Capital
  - Labour
  - Technology
  - Input-output structure

Nested functions

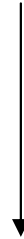
Price based substitutions

Separate models with  
nine, eleven, 16, 21  
and 123 sectors

Figure 5



Dynamic Analysis



Law of capital accumulation

and

Investment by origin  
and destination

and

Steady state and  
Transitional dynamics

Figure 6

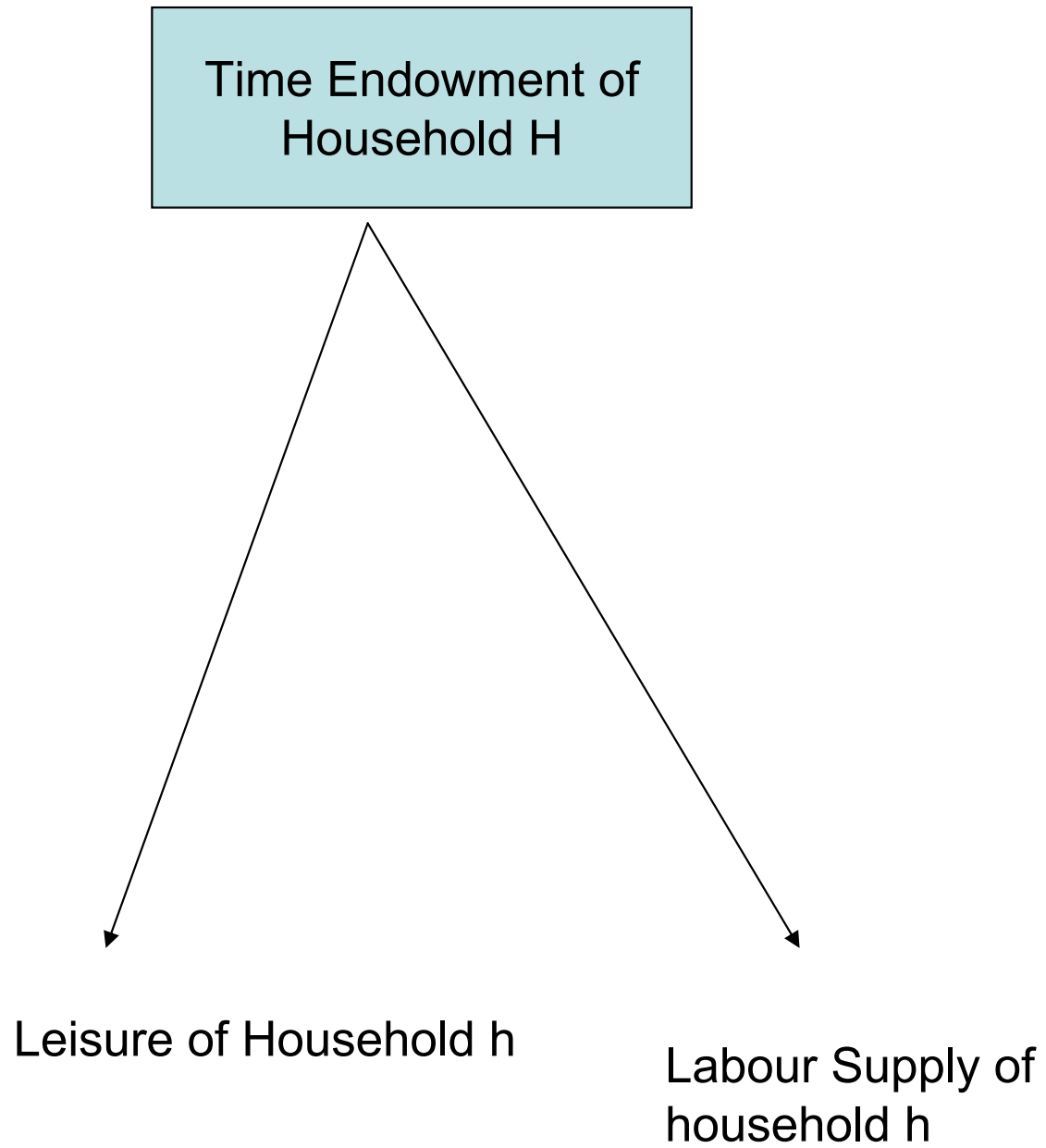


Figure 7

**Lifetime and Instantaneous Utility Function of a Household in the BHI CGE Model**

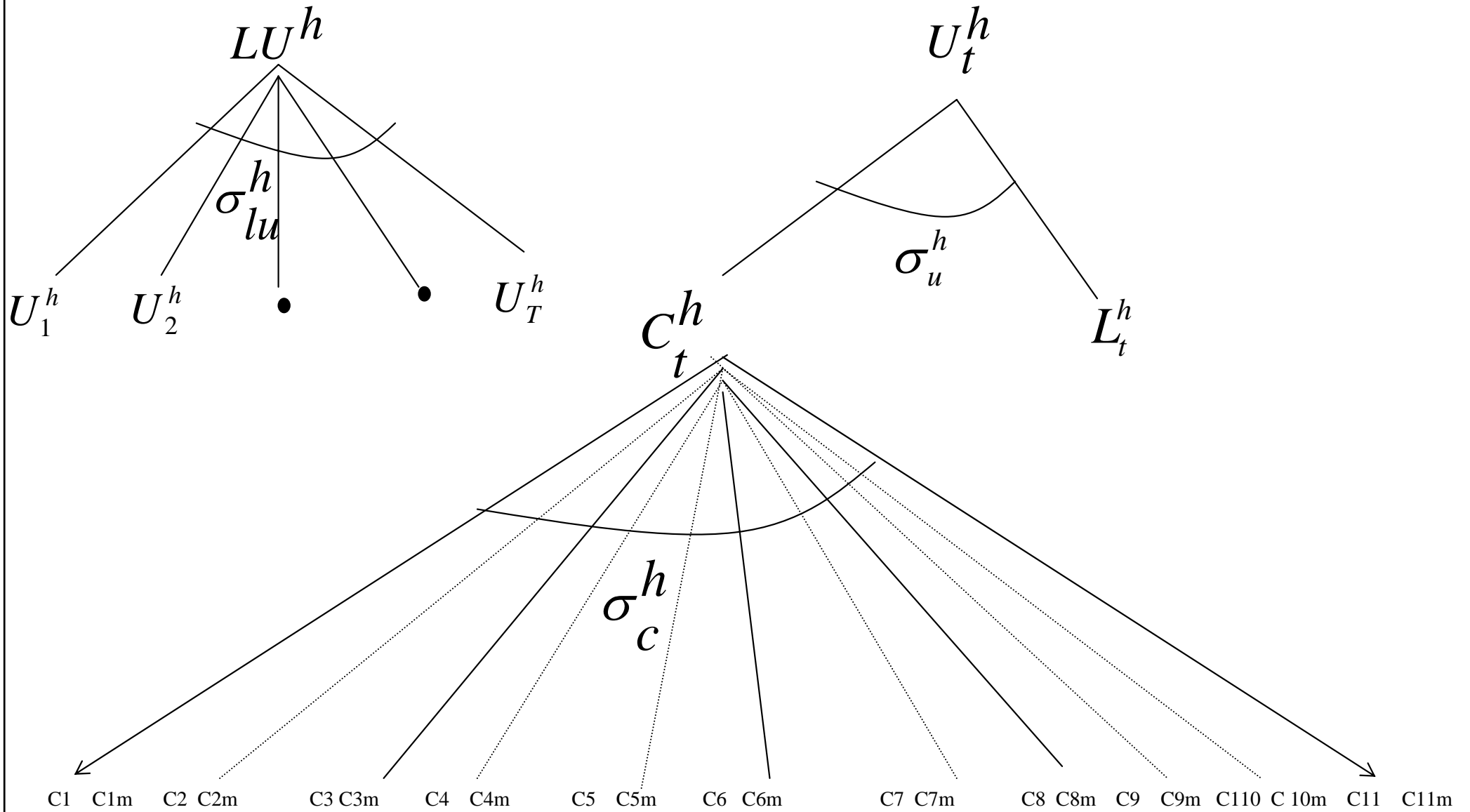


Figure 8  
Investment by Origin by Sectors

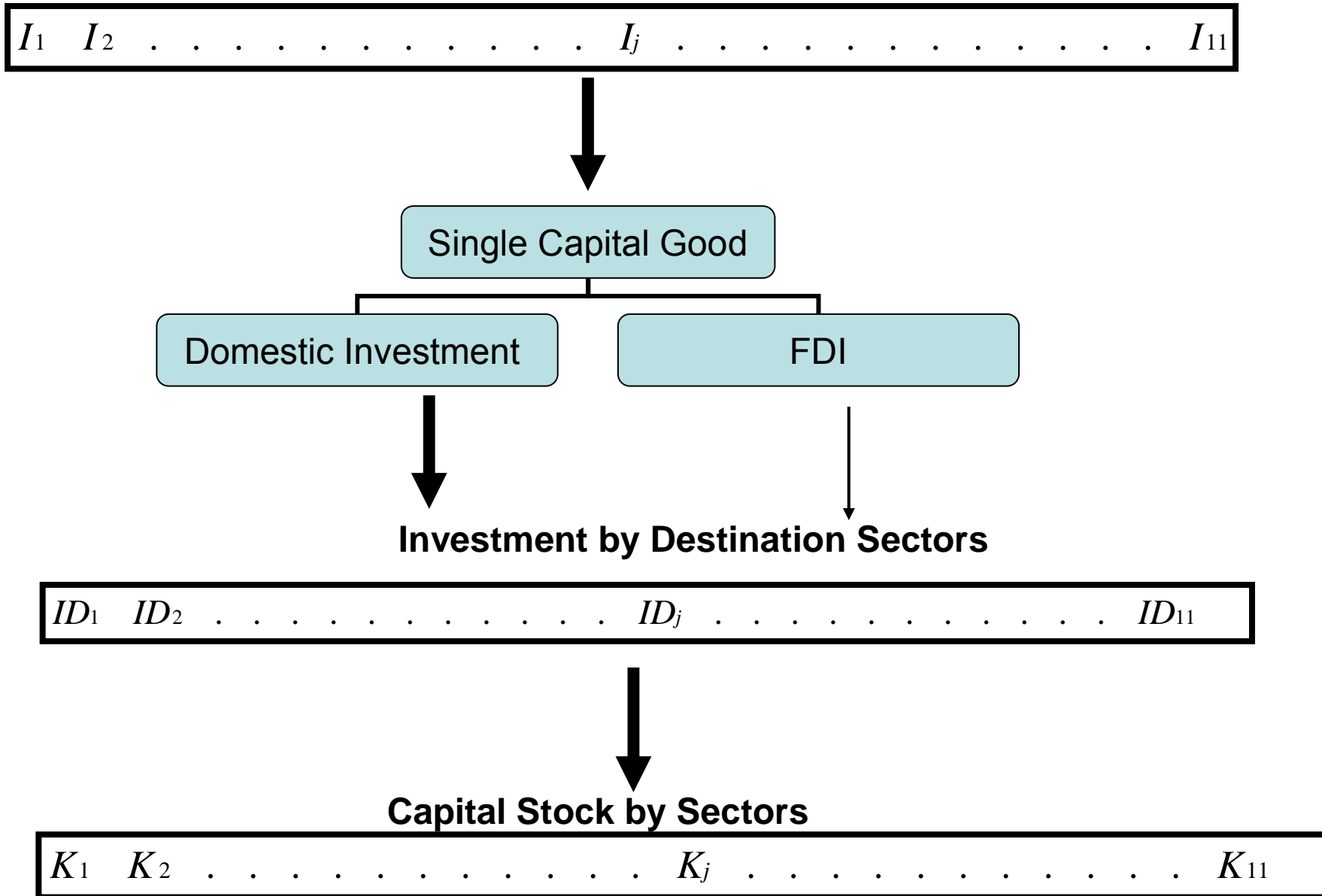


Table 5

**Elasticities and Growth and Interest Rates in the Benchmark**

Elasticity of substitution	1.5
Steady State growth rate of output	0.03
Benchmark interest rate	0.05
Rate of depreciation	0.02
Intertemporal substitution elasticity	0.95
Elasticity of substitution on capital and labour	1.5
Armington elasticities	1.2
Household income tax rates (vary by household)	
Ealsticity of substitution in public consumption	1.5

## Structure of Taxes on Inputs

	UK		Germany		France		Spain	
	captax	labtax	captax	labtax	captax	labtax	captax	labtax
Primary	0.01	0.13	0.01	0.13	0.01	0.13	-0.02	-0.18
Manu	0.06	0.08	0.06	0.11	0.06	0.11	-0.07	-0.10
MatManu	0.09	0.07	0.10	0.19	0.10	0.19	0.01	0.02
Machman	0.09	0.07	0.15	0.18	0.15	0.18	0.01	0.01
Utility	0.07	0.40	0.07	0.24	0.07	0.24	0.00	-0.01
Constru	0.05	0.13	0.09	0.14	0.09	0.14	0.02	0.03
TRHTRST	0.09	0.13	0.07	0.10	0.07	0.10	0.01	0.04
TRNCMM	0.09	0.12	0.08	0.13	0.08	0.13	0.02	0.07
<b>FINREST</b>	<b>0.02</b>	<b>0.13</b>	<b>0.03</b>	<b>0.30</b>	<b>0.03</b>	<b>0.30</b>	<b>0.02</b>	<b>0.22</b>
BUSSRV	0.04	0.06	0.05	0.08	0.05	0.08	0.01	0.02
PRSRV	0.06	0.04	0.07	0.05	0.07	0.05	0.06	0.04

This model contains much more on data series and input-output tables. It is not presented here for space reasons. Counterfactual scenarios here eliminate taxes on real estate sector.

Figure 9

### Income Distribution Among German Households Over Time (Euro) (Results of a Dynamic General Equilibrium Model, 2008)

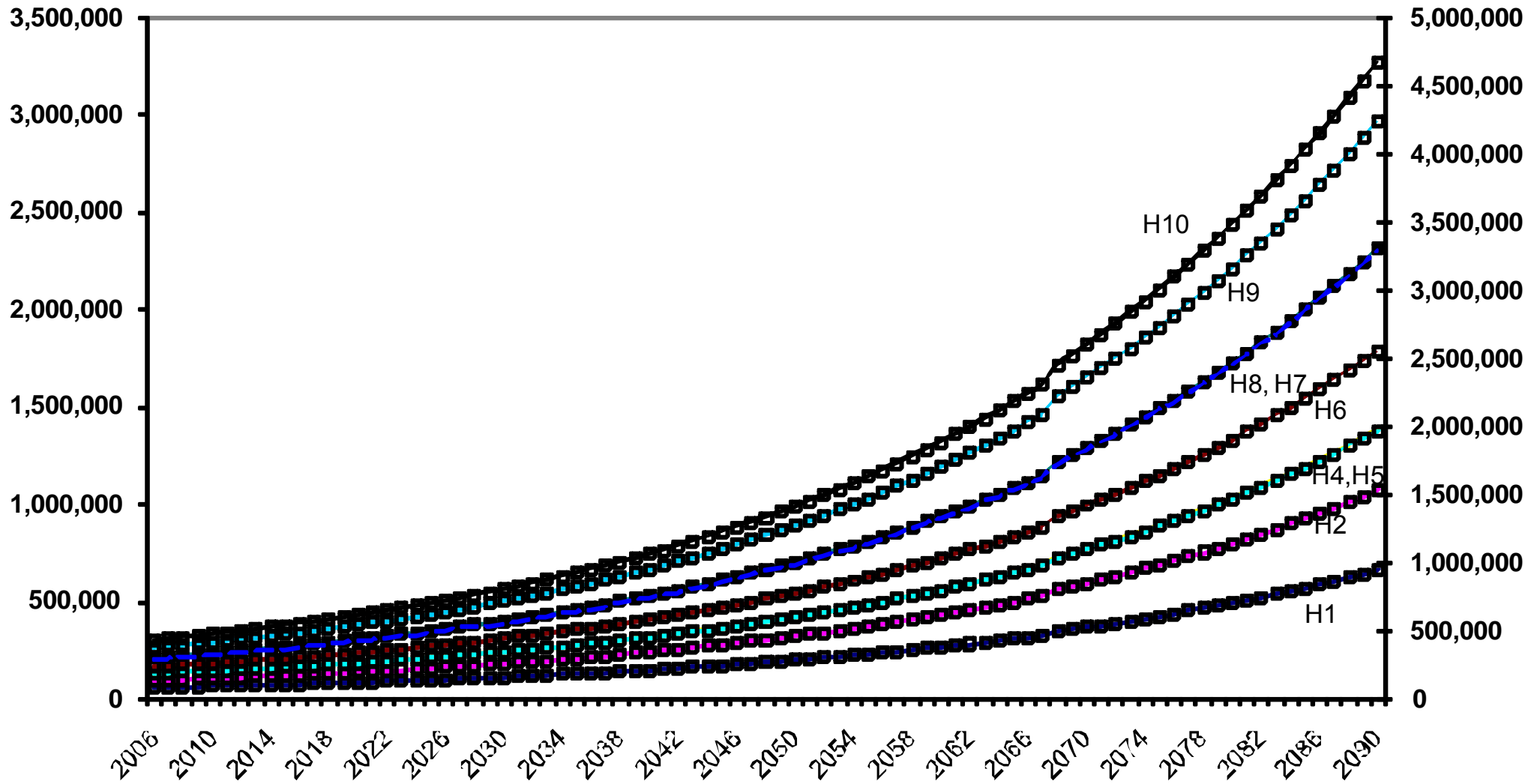


Figure 10

### Income Distribution Among French Households Over Time (Euro) (Results of a Dynamic General Equilibrium Model, 2008)

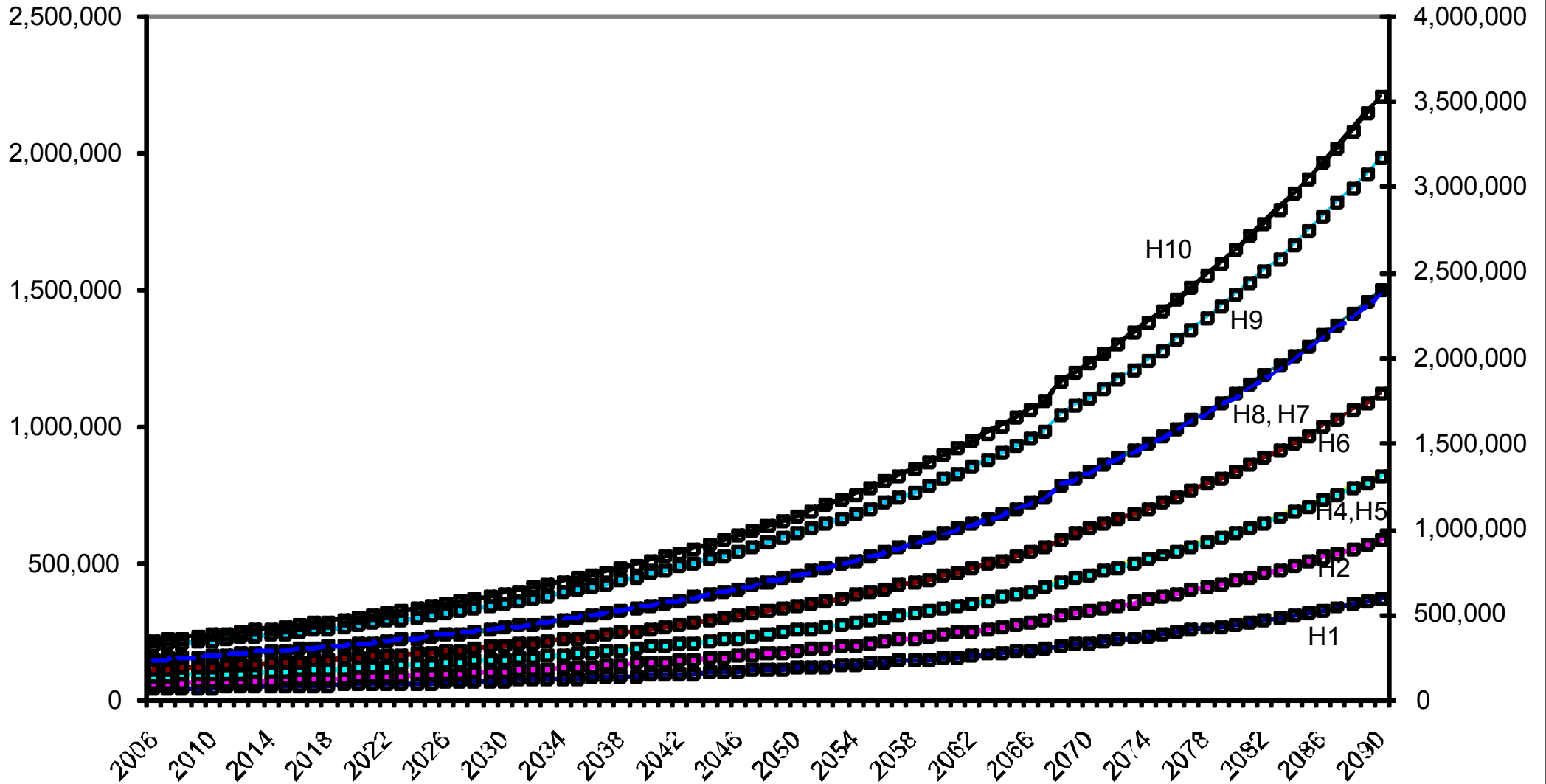


Figure 11

# Income Distribution Among Spanish Households Over Time (Euro)

(Results of a Dynamic General Equilibrium Model, 2008)

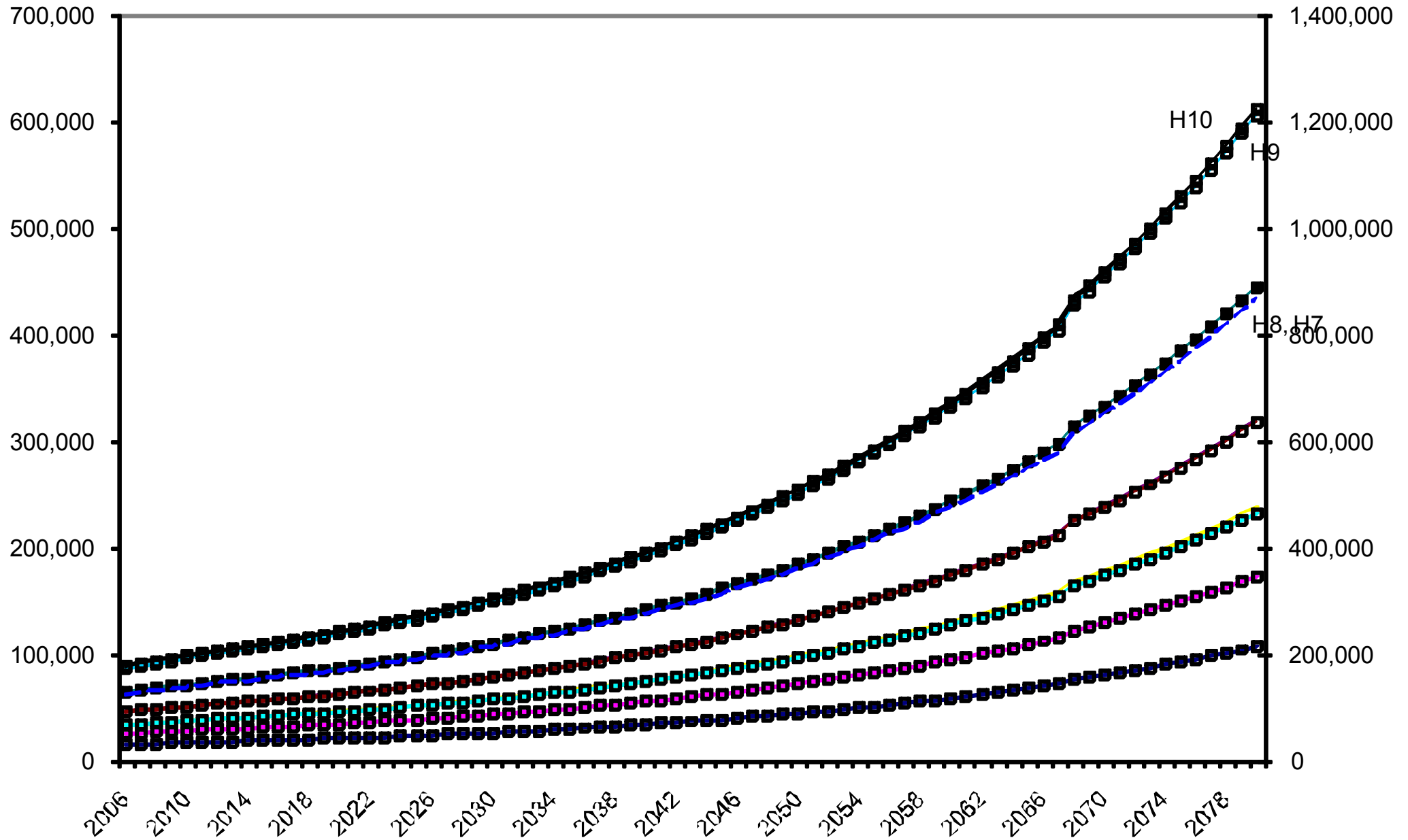
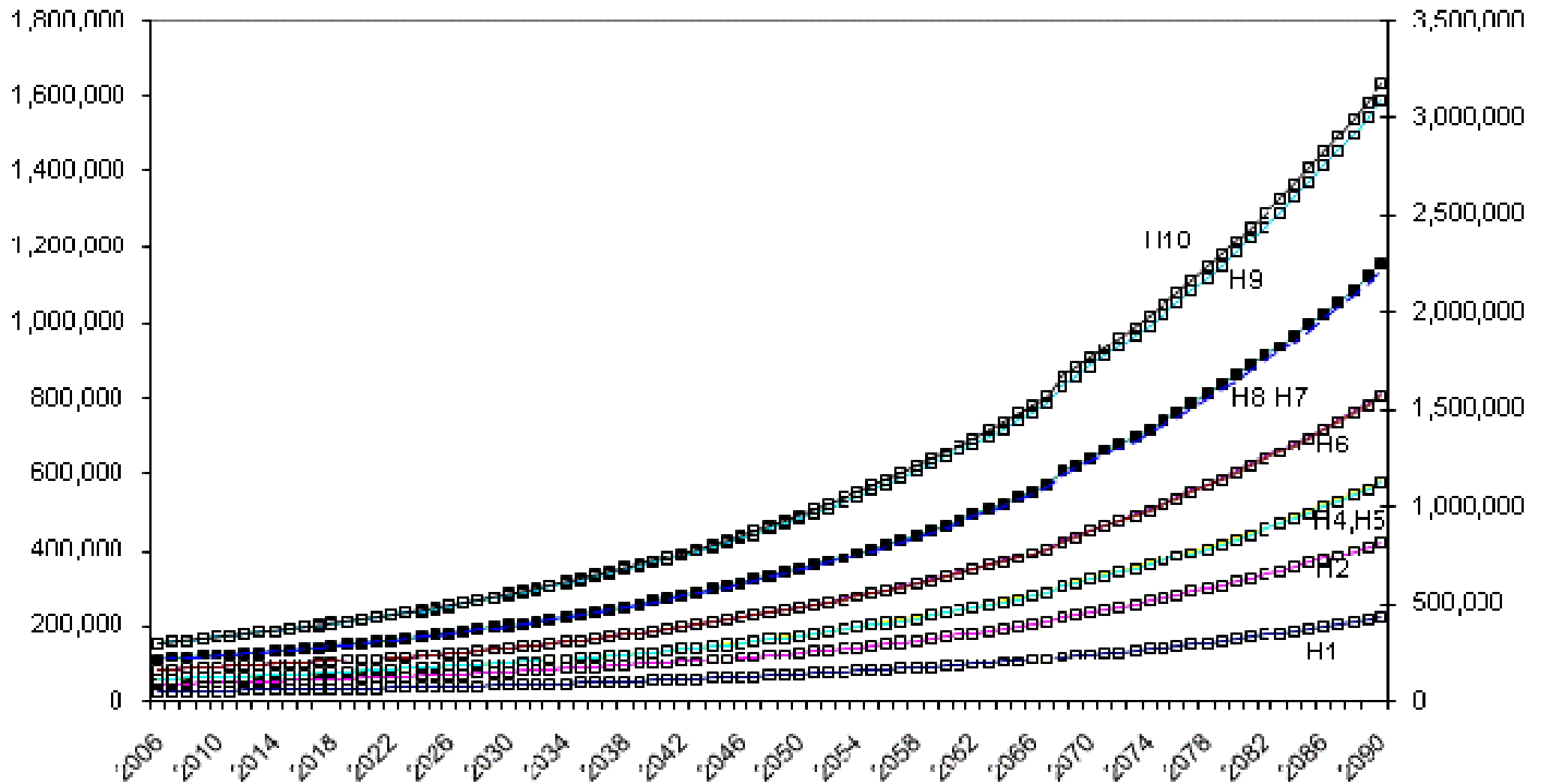


Figure 12

Income Distribution Among UK Households Over Time (£)  
(Results of a Dynamic General Equilibrium Model, 2008)



# Conclusion of the Study

- The dynamic model presented here shows evolution of the economy for 84 years from 2006 to 2090, most parts of the 21st century.
- It shows how the rate of investment and accumulation of capital, level of production among all sectors, consumption and welfare of households, relative prices of goods and services, revenue and expenditure of the public sectors, exports, imports and net trade balance evolve over time.
- Inequality in the distribution of income among households will not decrease if the current policies continue, they will widen over time as shown in from the solutions of dynamic general equilibrium model as presented in Figures.

## Structure of the Dynamic MH- GE Model of EU -1

Preference:

$$\text{Max } U_0^{i,h} = \sum_{t=0}^{\infty} \beta^{i,t} U_t^{i,h} (C_t^{i,h}, l_t^{i,h}) \quad (1)$$

Subject to

$$\sum_{t=0}^{\infty} R_{i,t}^{-1} [P_{i,t} (1 + t^{i,vc}) C_t^{i,h} + w_{i,t} (1 - t_{i,l}) l_t^h] = \sum_{t=0}^{\infty} [(1 - t_l) w_{i,t} L_t^{i,h} + (1 - t_{i,k}) r_{i,t} K_t^{i,h} + TR_t^{i,h}] \quad (2)$$

Technology:

$$\Pi_{j,g,t}^y = [((1 - \delta_{i,g}^e) PD_{i,,g,t}^{\frac{\sigma_y-1}{\sigma_y}} + \delta_{i,g}^e PE_{i,g,t}^{\frac{\sigma_y}{\sigma_y}})]^{\frac{1}{\sigma_y-1}} - \theta_j^v PY_{j,,g,t}^v - \theta_j^d \sum_i a_{i,j}^d P_{i,g,t} \quad (3)$$

## Structure of the Dynamic MH- GE Model of EU -2

Trade:

$$A_{i,g,t} = \Phi \left( \delta_{i,g}^d D_{i,g,t}^{\frac{\sigma_m-1}{\sigma_m}} + \delta_{i,g}^m M_{i,g,t}^{\frac{\sigma_m-1}{\sigma_m}} \right)^{\frac{\sigma_m}{\sigma_m-1}} \quad (4)$$

$$\sum_g PE_{i,g,t} E_{i,g,t} = \sum_g PM_{i,g,t} M_{i,g,t} \quad (5)$$

Public Sector:

$$G_{i,t} = \sum_{i=1}^N g_{i,g,t} \quad (6)$$

$$RV_{i,t} = \sum_{h=1}^H t_w^h w_{i,t} LS_t^{i,h} + \sum_{h=1}^H t_k^i r_t K_{i,,g,t} + \sum_{h=1}^H t_{i,t}^h p_{i,t} C_{i,g,t}^h \quad (7)$$

$$\sum_{t=1}^{\infty} G_{i,t} = \sum_t \left( RV_{i,t} + \sum_{h=1}^H R_t^{i,h} \right) \quad (8)$$

## Structure of the Dynamic MH- GE Model of EU - 3

Capital accumulation

$$K_{i,g,t} = (1 - \delta_{i,g})K_{i,g,t-1} + I_{i,g,t}; K_{i,g,t} = K_{i,g,0} \quad K_{i,g,T} = (g_{i,g} + \delta_{i,g})K_{i,g,T-1} \quad (9)$$

Labour supply

$$LS_t^{i,h} + L_t^{i,h} = \bar{L}_t^{i,h}; \bar{L}_t^{i,h} = \bar{L}_0 e^{n_{i,t}} \quad (10)$$

Labour force:

$$E_{i,t} + U_{i,t} = \sum_{h=1}^H \bar{L}_t^{i,h}$$

## Structure of the Dynamic MH- GE Model of EU -4

### Calibration Process

$$P_{i,t}^k = R_{i,t}^t + (1 - \delta_i)P_{i,t+1}^k \quad (11)$$

$$R_{i,t}^t = (r_i + \delta_i)P_{i,t} = (r_i + \delta_i)P_{i,t+1}^k \quad (12)$$

$$\frac{P_{i,t+1}^k}{P_{i,t}^k} = \frac{1}{1 + r_i} \approx 1 - \delta_i. \quad (13)$$

$$\bar{V}_{i,g} = (r_i + \delta_i)P_{i,t+1}^k K_{i,g}, \text{ or } K_{i,g} = \frac{\bar{V}_{i,g}}{(r_i + \delta_i)} \quad \text{Since } P_{i,t} = P_{i,t+1}^k = 1 \quad (14)$$

$$I_{i,g} = \frac{(g_i + \delta_i)}{(r_i + \delta_i)} \bar{V}_{i,g} \quad (15)$$

# **GENERAL EQUILIBRIUM WITH UNEMPLOYMENT: THEORY AND APPLICATION**

Integration of Beveridge Curve to  
General Equilibrium Modelling

# Main Features of the Model

- A general equilibrium model with equilibrium unemployment rate
- to evaluate the cost of unemployment in an economy by comparing responses to policy changes in an economy with equilibrium unemployment rate to the economy with full employment.
- Model can assess the cost and benefits of unemployment and transfer programmes.
- A lower rate of frictional unemployment is not necessarily growth retarding in the long run when economy goes through the dynamic adjustment process.

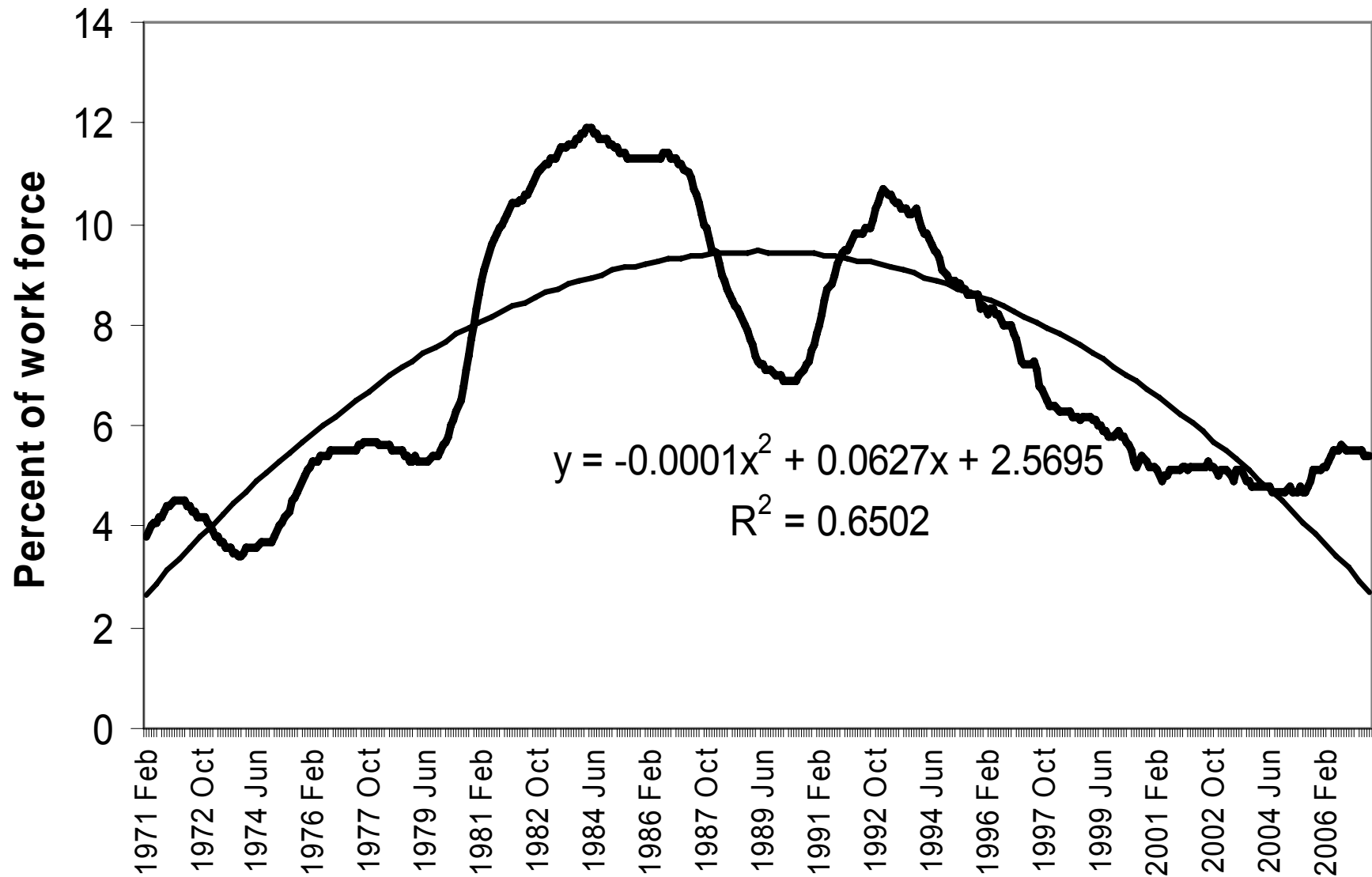
## Impacts uniform capital and labour income taxes across sectors and households

- Unemployment of some workers may raise labour supply of others as the existing workers supply more labour in response to higher wage rates.
- Labour income, consumption and saving by households and production and accumulation of capital rise.
- Inequality in income and consumption can be reduced by usual tax more productive worker and transfers to job trainings and other activities transitionally for unemployed ones.
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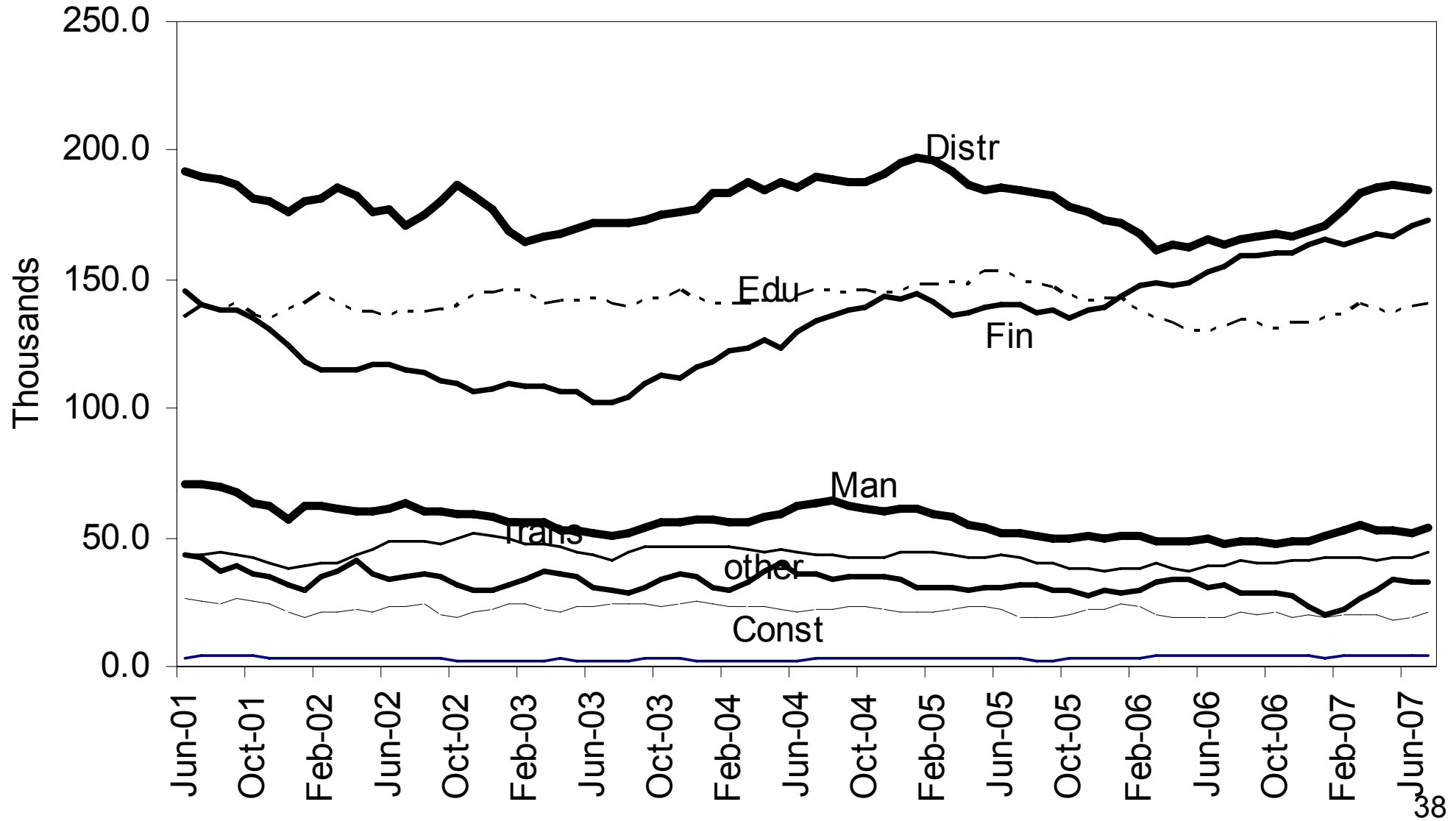
## Literature Equilibrium and Disequilibrium in Labour Markets

- Do markets clear and are full employment assumptions valid in tradition of Shoven and Whalley (1973) and Whalley (1975)?
- Job matching and search models developed by Phelps (1968), Mortensen and Pissarides (1984), Shapiro and Stiglitz (1984), Dixon (1988), Lindbeck and Snower (1988), Lockwood, Miller and Zhang (1989), Lockwood and Manning (1989), Blanchard et. al (1989), Layard and Nickell (1990), Manning (1990) Blanchflower and Oswald (1994), Ball and Mankiw (2002), Hall (2003), Gilles (2004), Lonquivists and Sargent (2007)
- Recent Studies (Hutton-Ruocco (1999), Rutherford et al (2000), Bohringer, Beoters and Feil (2005))

# Monthly Unemployment Rate in the UK, 2007



# Vacancies by Industry in UK

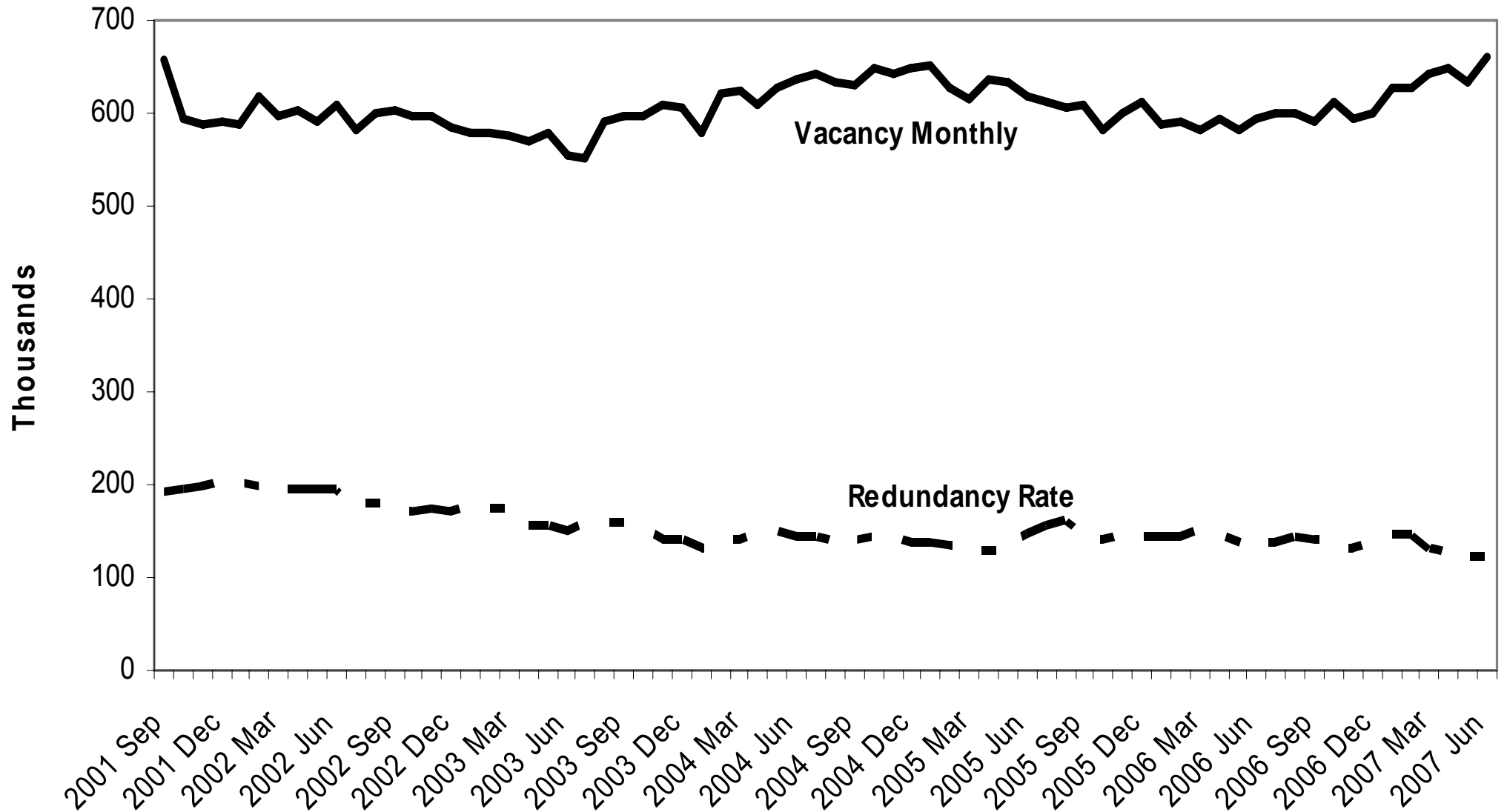


**Evidence of Beveridge Curve in UK, 2001 to 2007**  
(Source: [www.statistics.gov.uk](http://www.statistics.gov.uk))

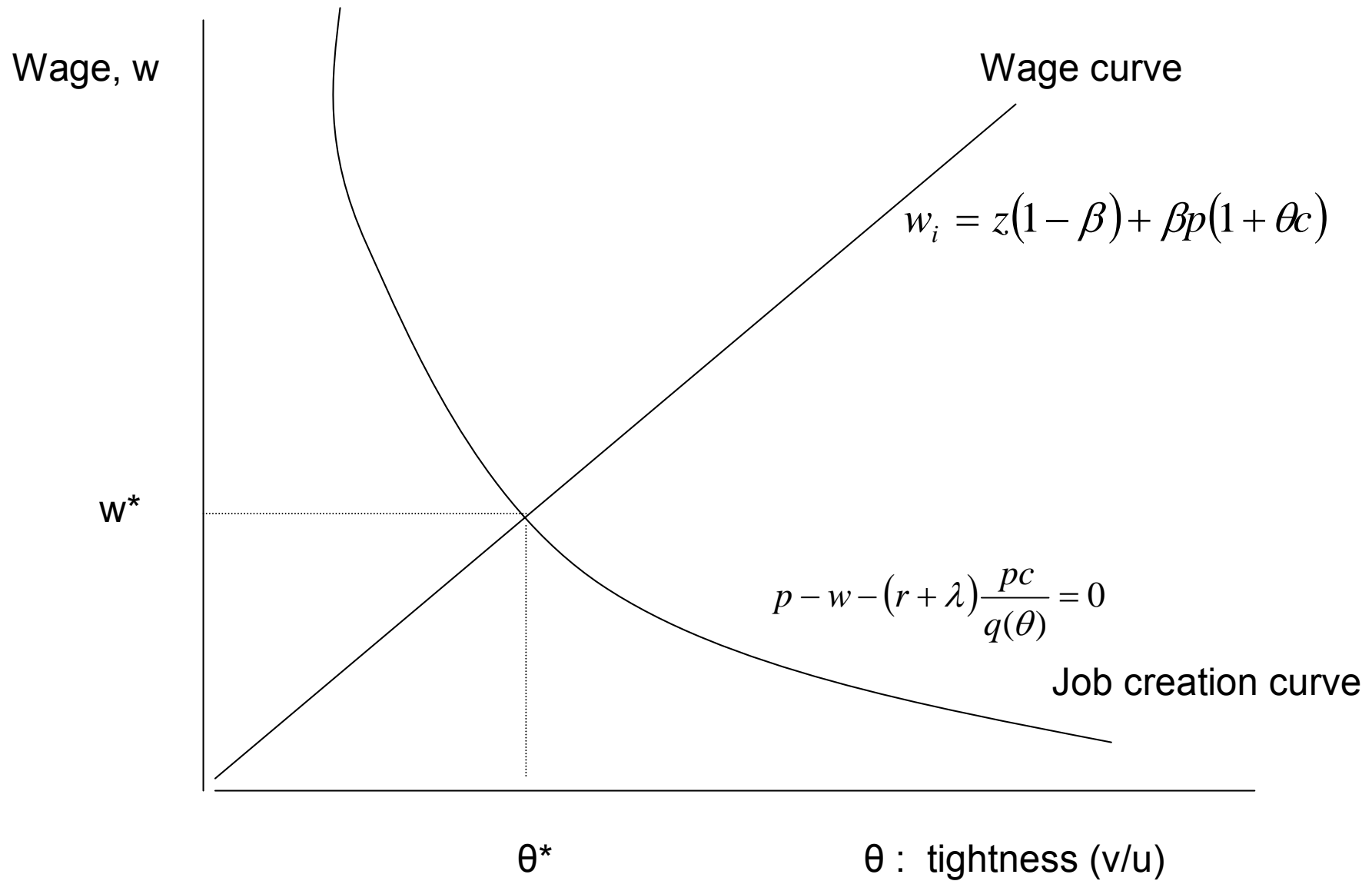


# Monthly Vacancy and Redundancy of Jobs in UK

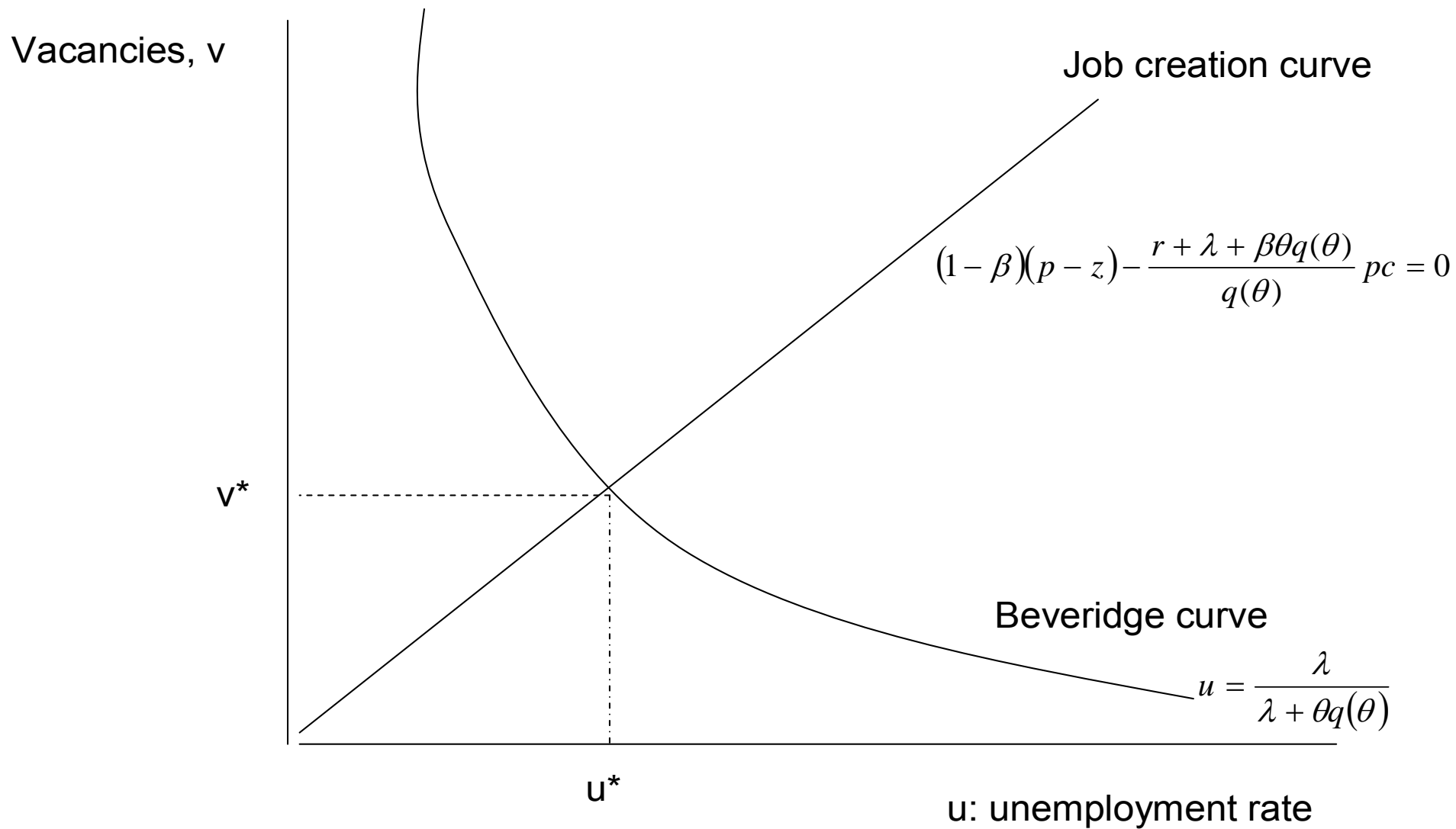
(Source <http://www.statistics.gov.uk>)



**Diagram 1: Equilibrium wage and market tightness**



**Diagram 2: Beveridge and job creation curve**



## What is Equilibrium Unemployment ?

Standard Equilibrium unemployment and job creation Model  
(Pissarides (2000) or Shimer (2005):

$$M = M(V, U) = V^\gamma U^{(1-\gamma)} \quad (1)$$

$$(W_i - U)^\beta (J_i - V)^{1-\beta} \quad (2)$$

Nash Bargaining and symmetric solution of this satisfies joint profit maximisation condition as:

$$(W_i - U) = \beta(J_i + W_i - V - U) \quad (3)$$

A job search model is often explained using three simple equations

1) Dynamics of unemployment depends on the rate of job destruction-

$\lambda(1-u)$ , and the rate of job creation -  $\theta q(\theta)u$ .

$\dot{u} = \lambda(1-u) - \theta q(\theta)u$  and in equilibrium

Equilibrium Unemployment:

$$u = \frac{\lambda}{\lambda + \theta q(\theta)} \quad (4)^3$$

# Job Creation Curve 1

Return from vacancy

$$rV = -pc + q(\theta)(J - V)$$

In equilibrium  $V = 0$

$$J = \frac{pc}{q(\theta)}$$

Returns from an occupied job

$$rJ = p - w - \lambda J$$

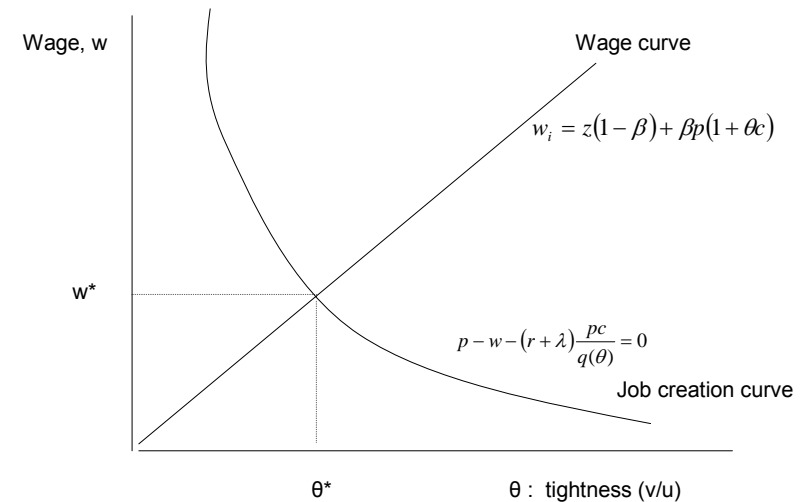
Combining these two gives the job creation curve:

$$p - w - (r + \lambda) \frac{pc}{q(\theta)} = 0$$

Probability of filling the job:

$$q(\theta)$$

Diagram 1: Equilibrium wage and market tightness



**Value of remaining unemployed:**

$$rU = z + \theta q(\theta)(W - U)$$

Return for employed

$$rW = w + \lambda(U - W)$$

$$W = \frac{w}{(r + \lambda)} + \frac{\lambda}{(r + \lambda)}U$$

$$rU + \theta q(\theta)U = z + \theta q(\theta) \left[ \frac{w}{(r + \lambda)} + \frac{\lambda}{(r + \lambda)}U \right] = z + \theta q(\theta) \frac{w}{(r + \lambda)} + \theta q(\theta) \frac{\lambda}{(r + \lambda)}U$$

**Deriving the return from remaining unemployed:**

$$rU - \theta q(\theta) \frac{\lambda}{(r + \lambda)} U + \theta q(\theta) U = z + \theta q(\theta) \frac{w}{(r + \lambda)}$$

$$rU(r + \lambda) - \theta q(\theta) \lambda U + \theta q(\theta) U(r + \lambda) = z(r + \lambda) + \theta q(\theta) w$$

$$rU[(r + \lambda) + \theta q(\theta)] = z(r + \lambda) + \theta q(\theta) w$$

$$rU = \frac{z(r + \lambda) + \theta q(\theta) w}{r + \lambda + \theta q(\theta)}$$

## Deriving the return from Working

$$(r + \lambda)W = w + \lambda U$$

$$(r + \lambda)W = w + \frac{\lambda}{r} \left[ \frac{z(r + \lambda) + \theta q(\theta)w}{r + \lambda + \theta q(\theta)} \right]$$

$$(r + \lambda)W = \frac{wr(r + \lambda + \theta q(\theta)) + \lambda(z(r + \lambda) + \theta q(\theta)w)}{r(r + \lambda + \theta q(\theta))}$$

$$rW = \frac{wr + \theta q(\theta)w + \lambda z}{r + \lambda + \theta q(\theta)}$$

## Nash-product of the bargaining game

$$(W_i - U)^\beta (J_i - U)^{1-\beta}$$

$$rJ_i = p - w_i - \lambda J_i \quad J_i = \frac{p - w_i}{r + \lambda}$$

$$(W_i - U) = \beta (J_i + W_i - V - U)$$

$$rW_i = w_i - \lambda(W_i - U)$$

$$W_i = \frac{w_i}{r + \lambda} + \frac{\lambda}{r + \lambda} U$$

$$V = 0$$

$$W_i(1 - \beta) = \beta J_i + (1 - \beta)U$$

$$\left[ \frac{w_i}{r + \lambda} + \frac{\lambda}{r + \lambda} U \right] (1 - \beta) = \beta \left[ \frac{p - w_i}{r + \lambda} \right] + (1 - \beta)U$$

## Derivation of Wage Curve

$$w_i(1 - \beta) + (1 - \beta)\lambda U = \beta(p - w_i) + (1 - \beta)(r + \lambda)U$$

$$w_i(1 - \beta) = \beta(p - w_i) + (1 - \beta)rU$$

$$w_i = \beta p + (1 - \beta)rU$$

$$w_i = \beta p + (1 - \beta)(z + \theta q(\theta)(W_i - U))$$

$$(W_i - U) = \beta(J_i + W_i - V - U)$$

$$(W_i - U) = \frac{\beta}{1 - \beta} J_i$$

$$(W_i - U) = \frac{\beta}{1 - \beta} \frac{pc}{q(\theta)} \quad w_i = \beta p + (1 - \beta) \left( z + \theta q(\theta) \frac{\beta}{1 - \beta} \frac{pc}{q(\theta)} \right)$$

$$w_i = z(1 - \beta) + \beta p(1 + \theta c)$$

## Derivation of Beveridge Curve

Balance between job destruction and job creation

$$\dot{u} = \lambda(1 - u) - \theta q(\theta)u$$

$$\theta = \frac{V}{U}$$

$$mL\delta t = u\theta q(\theta)L\delta t$$

the transition probability from unemployed to employed

$$\theta q(\theta)$$

**Beveridge curve**

$$u = \frac{\lambda}{\lambda + \theta q(\theta)}$$

## Job Creation Curve 2

$$p - w - (r + \lambda) \frac{pc}{q(\theta)} = 0$$

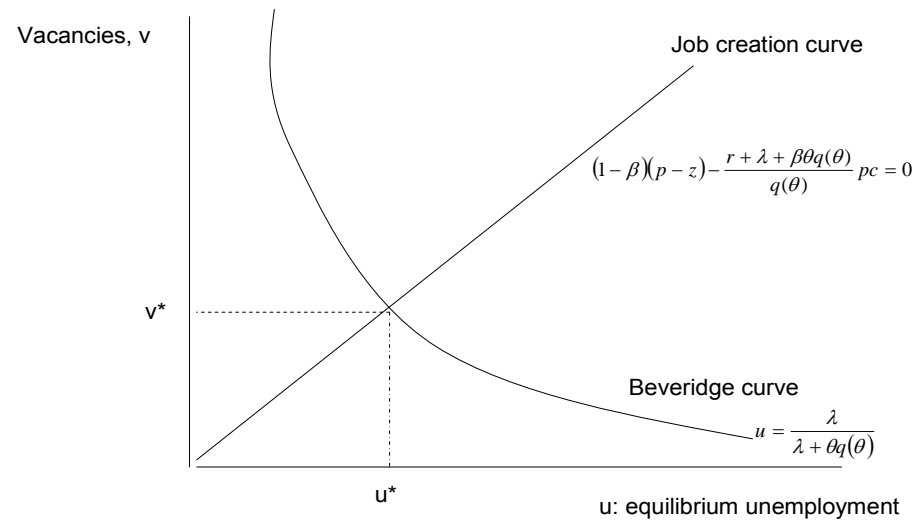
**Substitution wage curve**

$$p - z(1 - \beta) - \beta p(1 + \theta c) - (r + \lambda) \frac{pc}{q(\theta)} = 0$$

**Job Creation Curve**

$$(1 - \beta)(p - z) - \frac{r + \lambda + \beta\theta q(\theta)}{q(\theta)} pc = 0$$

Diagram 2: Beveridge and job creation curve



# Unemployment in General Equilibrium

- How these features of equilibrium unemployment affect the economy wide general equilibrium?
- What are the impacts of vacancies, redundancies and unemployment in the growth of output, employment, relative prices, allocation of factors between private and public sectors and welfare of various categories of households in the economy?

$$\text{Max} \sum_{t=0}^{\infty} \beta^t U(C_t^h, L_t^h) \quad (7)$$

subject to

$$\sum_{t=0}^{\infty} P_t (1+t_c) C_t^h = \sum_{t=0}^{\infty} (1-t_w) (w_t L S_t^h + r(1-t_k) K_t^h + R_t^h)$$

for employed (8)

$$\sum_{t=0}^{\infty} P_t (1+t) C_t^h = \sum_{t=0}^{\infty} (r(1-t_k) K_t^h + R_t^h)$$

for unemployed (9)

## Production Technology and Supply of Goods

$$\Pi_{j,t}^y = \left[ \left( (1 - \delta_i^e) P D_{i,t}^{\frac{\sigma_y - 1}{\sigma_y}} + \delta_i^e P E_{i,t}^{\frac{\sigma_y - 1}{\sigma_y}} \right) \right]^{\frac{1}{\sigma_y - 1}} - \theta_j^v P Y_{j,t}^v - \theta_j^d \sum_i a_{i,j}^d P_{i,t} - \theta_j^m \sum_i a_{i,j}^m P_{i,t} \quad (10)$$

Trade takes place between the EU and the ROW and given by the standard Armington functions.

$$A_{i,t} = \Phi \left( \delta_i^d D_{i,t}^{\frac{\sigma_m - 1}{\sigma_m}} + \delta_i^m M_{i,t}^{\frac{\sigma_m - 1}{\sigma_m}} \right)^{\frac{\sigma_m}{\sigma_m - 1}} \quad (11)$$

The value of exports balances to the value of imports.

$$\sum_i P E_{i,t} E_{i,t} = \sum_i P M_{i,t} M_{i,t} \quad (12) \quad 54$$

## Market Clearing and Boundary Conditions

$$K_{i,t} = (1 - \delta_i)K_{i,t-1} + I_{i,t}$$

$$K_{i,t} = K_{i,0}$$

$$K_{i,T} = (g_i + \delta_i)K_{i,T-1}$$

$$LS_t^h + L_t^h = \bar{L}_t^h$$

$$\bar{L}_t^h = \bar{L}_0^h e^{nt}$$

$$E_t + U_t = \sum_{h=1}^H \bar{L}_t^h$$

## Public Sector in the Model

Tax transfer system

$$G_t = \sum_{i=1}^N g_{i,t} \quad (15)$$

$$RV_t = \sum_{h=1}^H t_w^h w_t LS_t^h + \sum_{h=1}^H t_k^i r_t K_{i,t} + \sum_{h=1}^H t_{i,t}^h p_{i,t} C_{i,t}^h \quad (16)$$

Revenue is balanced over the model horizon:

$$\sum_{t=1}^{\infty} G_t = \sum_t \left( RV_t + \sum_{h=1}^H R_t^h \right) \quad (17)$$

Table1 : Utility under unemployment and full employment

	1995	2000	2010	2020	2030	2040	2050	2060	2070
PERSWELF- H1	0.9939	0.9972	0.9995	1.0005	1.0009	1.0009	1.0008	1.0007	1.0007
PERSWELF- H2	0.9939	0.9974	0.9996	1.0006	1.0010	1.0010	1.0010	1.0009	1.0008
PERSWELF- H3	0.9939	0.9974	0.9996	1.0006	1.0010	1.0010	1.0010	1.0009	1.0008
PERSWELF- H4	0.9939	0.9974	0.9996	1.0006	1.0010	1.0010	1.0010	1.0009	1.0008
PERSWELF- H5	0.9939	0.9974	0.9996	1.0006	1.0010	1.0010	1.0010	1.0009	1.0008
PERSWELF- H6	0.9939	0.9974	0.9996	1.0006	1.0010	1.0010	1.0009	1.0009	1.0008
PERSWELF- H7	0.9939	0.9973	0.9996	1.0006	1.0010	1.0010	1.0009	1.0009	1.0008
PERSWELF- H8	0.9939	0.9974	0.9996	1.0006	1.0010	1.0010	1.0010	1.0009	1.0008
PERSWELF- H9	0.9939	0.9973	0.9996	1.0006	1.0010	1.0010	1.0009	1.0009	1.0008
PERSWELF- H10	0.9939	0.9971	0.9994	1.0005	1.0008	1.0008	1.0007	1.0006	1.0006

Table 2 : Labour supply under unemployment and full employment

	1995	2000	2010	2020	2030	2040	2050	2060	2070
LSUPPLY- H1	1.0087	1.0045	1.0028	1.0019	1.0015	1.0013	1.0012	1.0012	1.0012
LSUPPLY- H2	1.0098	1.0048	1.0028	1.0017	1.0012	1.0010	1.0009	1.0009	1.0009
LSUPPLY- H3	1.0096	1.0048	1.0028	1.0017	1.0013	1.0011	1.0010	1.0010	1.0009
LSUPPLY- H4	1.0096	1.0048	1.0028	1.0017	1.0013	1.0011	1.0010	1.0009	1.0009
LSUPPLY- H5	1.0095	1.0048	1.0028	1.0018	1.0013	1.0011	1.0010	1.0010	1.0010
LSUPPLY- H6	1.0095	1.0047	1.0028	1.0018	1.0013	1.0011	1.0010	1.0010	1.0010
LSUPPLY- H7	1.0094	1.0047	1.0028	1.0018	1.0013	1.0011	1.0010	1.0010	1.0010
LSUPPLY- H8	1.0095	1.0047	1.0028	1.0018	1.0013	1.0011	1.0010	1.0010	1.0010
LSUPPLY- H9	1.0094	1.0047	1.0028	1.0018	1.0013	1.0011	1.0010	1.0010	1.0010
LSUPPLY- H10	1.0085	1.0045	1.0029	1.0020	1.0016	1.0014	1.0014	1.0013	1.0013

Table 3 : Consumption under unemployment and full employment

		1995	2000	2010	2020	2030	2040	2050	2060	2070
PERSCONS-	H1	0.996299244	0.998687	1.001271	1.00231	1.00255	1.00246	1.002285	1.002129	1.002019
PERSCONS-	H2	0.996989357	0.999274	1.001888	1.002928	1.003157	1.003054	1.002869	1.002704	1.00259
PERSCONS-	H3	0.996884951	0.999187	1.001796	1.002836	1.003067	1.002966	1.002783	1.00262	1.002506
PERSCONS-	H4	0.996906914	0.999205	1.001816	1.002855	1.003086	1.002985	1.002801	1.002637	1.002524
PERSCONS-	H5	0.996863077	0.999169	1.001777	1.002816	1.003048	1.002947	1.002765	1.002602	1.002488
PERSCONS-	H6	0.996826926	0.999138	1.001745	1.002784	1.003017	1.002917	1.002735	1.002572	1.002459
PERSCONS-	H7	0.996783182	0.999101	1.001706	1.002745	1.002979	1.002879	1.002698	1.002536	1.002423
PERSCONS-	H8	0.996851556	0.999159	1.001767	1.002806	1.003038	1.002938	1.002755	1.002592	1.002479
PERSCONS-	H9	0.996806259	0.999121	1.001727	1.002766	1.002999	1.002899	1.002717	1.002555	1.002442
PERSCONS-	H1	0.996040295	0.998461	1.001035	1.002073	1.002317	1.00223	1.002059	1.001905	1.001797

Table 4 : Output under unemployment and full employment

		1995	2000	2010	2020	2030	2040	2050	2060	2070
YB- AGRIC		0.968546242	0.966887	0.967177	0.967277	0.967205	0.96708	0.966957	0.966841	0.966705
YB- MIN		0.976989834	0.972979	0.983125	0.989095	0.992242	0.993733	0.994359	0.994533	0.994406
YB- MANU		0.968478411	0.95863	0.953069	0.950062	0.948277	0.94724	0.946636	0.946253	0.945926
YB- UTILS		0.990352354	0.990635	0.996252	0.999202	1.000631	1.001235	1.001447	1.001482	1.001428
YB- CONST		0.988618742	0.985697	0.982126	0.980133	0.979012	0.978392	0.978051	0.977858	0.97773
YB- DISTB		0.955651642	0.954547	0.953532	0.952452	0.951538	0.950857	0.950385	0.950065	0.949826
YB- TRANS		0.972655088	0.969151	0.967693	0.966669	0.965906	0.965373	0.965012	0.964752	0.964512
YB- BUSI		0.997706518	0.996777	0.997057	0.997031	0.996821	0.996571	0.99635	0.996164	0.995988
YB-OTHSEC		1.107553454	1.10361	1.099735	1.097308	1.095888	1.095092	1.094657	1.094418	1.094282

Table 5: Employment under unemployment and full employment

	1995	2000	2010	2020	2030	2040	2050	2060	2070
EMPLYMENT- AGI	0.925354791	0.938238	0.939336	0.94032	0.941029	0.941509	0.941812	0.941986	0.942055
EMPLYMENT- MI	0.945335863	0.951614	0.962096	0.966864	0.969898	0.971722	0.972777	0.973363	0.973671
EMPLYMENT- MA	0.959766853	0.956562	0.95755	0.958417	0.959072	0.959531	0.959831	0.960016	0.960122
EMPLYMENT- UTI	0.932260427	0.950536	0.954522	0.957216	0.958986	0.960085	0.960742	0.961115	0.961307
EMPLYMENT- CO	0.957620025	0.954475	0.951218	0.949461	0.948502	0.947991	0.94772	0.94757	0.947472
EMPLYMENT- DIS	0.919147105	0.931442	0.93178	0.931887	0.931896	0.931874	0.931844	0.931813	0.931772
EMPLYMENT- TR/	0.942127625	0.945378	0.945587	0.945874	0.946096	0.946251	0.946347	0.946388	0.946366
EMPLYMENT- BU	0.964748361	0.963073	0.963094	0.963512	0.963958	0.964323	0.96458	0.964736	0.964803
EMPLYMENT-OTF	1.101058056	1.08853	1.085081	1.082981	1.081793	1.081154	1.080819	1.080644	1.08055

Table 6: Investment under unemployment and full employment

	2000	2010	2020	2030	2040	2050	2060	2070
INVST- AGRIC	1.022535	1.007629	1.001058	0.998235	0.996969	0.99637	0.996035	0.995762
INVST- MIN	#DIV/0!	1.043781	1.036518	1.03288	1.031147	1.030304	1.029855	1.029625
INVST- MANU	1.04255	1.026502	1.020065	1.017268	1.016011	1.015431	1.015143	1.014978
INVST- UTILS	1.048531	1.02946	1.022358	1.019133	1.017655	1.016955	1.016583	1.01634
INVST- CONST	1.025175	1.010397	1.005309	1.003118	1.002132	1.001675	1.001434	1.001231
INVST- DISTB	1.028724	0.998832	0.990331	0.987164	0.98584	0.985249	0.984948	0.984731
INVST- TRANS	1.032614	1.012809	1.005598	1.002679	1.001403	1.000803	1.000454	1.000134
INVST- BUSI	1.041295	1.02996	1.024609	1.022111	1.020946	1.020384	1.020065	1.019816
INVST-OTHSEC	1.172972	1.152316	1.146368	1.143903	1.142814	1.142323	1.142088	1.141933

Table 7: Capital stock under unemployment and full employment

	1995	2000	2010	2020	2030	2040	2050	2060	2070
KSTCK- AGRIC	1	0.980878	0.989187	0.993718	0.995706	0.996375	0.996475	0.996367	0.996185
KSTCK- MIN	1	1	1.013154	1.021769	1.026252	1.028349	1.029237	1.029555	1.029618
KSTCK- MANU	1	1.000035	1.008367	1.012842	1.014797	1.015447	1.015539	1.015437	1.01529
KSTCK- UTILS	1	0.993735	1.005179	1.011573	1.014706	1.016034	1.016503	1.016601	1.016543
KSTCK- CONST	1	0.997854	1.0017	1.003378	1.003613	1.003235	1.002725	1.002273	1.001913
KSTCK- DISTB	1	0.973774	0.98123	0.984805	0.986042	0.986178	0.985929	0.985606	0.985311
KSTCK- TRANS	1	0.988343	0.995769	0.999587	1.001067	1.001394	1.001273	1.001023	1.000743
KSTCK- BUSI	1	1.006843	1.014206	1.018226	1.019967	1.020519	1.020564	1.02043	1.02024
KSTCK-OTHSEI	1	1.138001	1.142666	1.14448	1.144649	1.144158	1.143549	1.14303	1.142638

Table 8: Macro Aggregates under unemployment and full employment

	1995	2000	2010	2020	2030	2040	2050	2060	2070
LSUPPLYT	1.009236328	1.004677	1.002802	1.001827	1.001367	1.001169	1.00109	1.001061	1.00105
EMPLYMNTT	1.00923633	1.004677	1.002802	1.001827	1.001367	1.001169	1.00109	1.001061	1.00105
WAGEBILL	0.993646748	0.984544	0.980614	0.978594	0.977685	0.977333	0.977223	0.977203	0.977206
EXPORTT	1.017023295	1.02037	1.02587	1.028604	1.030051	1.030794	1.031172	1.031373	1.031517
IMPORTT	1.03515503	1.043036	1.056287	1.063303	1.06714	1.069153	1.070191	1.070755	1.071171
TRADEDEF	1.000180531	1.000315	1.000429	1.000483	1.000516	1.000538	1.000551	1.00056	1.000567
CAPIFLOW	0.984807601	0.983924	0.983896	0.984089	0.984527	0.984984	0.985356	0.985631	0.985837

# Conclusion

- A general equilibrium model with unemployment is developed to evaluate the cost of equilibrium unemployment in an economy by comparing responses to policy changes in equilibrium unemployment economy to the full employment equilibrium economy.
- Model can assess the cost and benefits of unemployment and transfer programmes.
- Analysis of impacts of uniform capital and labour income taxes in the multi-household multi-sectoral general equilibrium model with unemployment shows that such unemployment is not necessarily growth retarding in the long run when economy runs through the dynamic adjustment process.
- Lower labour supply raises wage rates and labour income, consumption and saving by households and production and accumulation of capital by firms.
- In base as usual scenario the inequality of income and consumption persists if no measures are taken to reduce such inequality.

Thank You! Comments Welcome!