Keynesian Models for Analysis of Macroeconomic Policy

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Vast literature in macroeconomic modelling

- Keynes (1936), Hicks (1937), Samuelson (1939),
- Phillips (1958), Friedman (1968), Phelps (1968), Tobin (1969),
- Sargent and Wallace (1975), Lucas (1976), Fisher (1977), Kydland and Prescott (1977) ,Wallis (1980), King and Plosser (1984) Mankiw (1989), Prescott (1986), Taylor (1987)
- Blanchard and Kiyotaki (1987), Manning (1995), Rankin (1992)
- Barro and Gordon (1983), Sargent (1986) Goodhart (1989), Nickell (1990), Mankiw and Romer(1993), Lockwood Miller and Zhang (1998)
- Wallis (1989), MPC (1999), Pagan and Wickens (1989), Hendry (1995), Holly Weale (2000)
- Taylor (1993), Sargent and Ljungqvists (2000), Minford and Peel (2002), Blake and Weal (2003), Garratt, Lee, Pesaran and Shin (2003)
- Solow (1956), Lucas (1988), Romer (1990), Mankiw, Romer and Weil (1992),
- Harrod (1939), Domar (1947) and Solow (1956), Parente and Prescott (1993)
- Fullerton, Shoven and Whalley (1983), Auerbach and Kotlikoff (1987), Perroni (1995), Rutherford (1995), Bank of England, NIESR) Kehoe, Srinivasan and Whalley (2005), Bhattarai (1997, 1999)

Multiplier-accelerator model of Samuelson (1939)

$$Y_t = C_t + I_t + G_0$$

$$C_t = \gamma Y_{t-1} \qquad 0 < \gamma < 1$$

$$I_t = \alpha\left(\gamma C_t - \gamma C_{t-1}\right) \qquad \alpha > 0$$

$$Y_t = \gamma\left(1+\alpha\right)Y_{t-1} - \alpha\gamma Y_{t-2} + G_0$$

$$\bar{Y} = \frac{G_0}{1-\gamma\left(1+\alpha\right)+\alpha\gamma} = \frac{G_0}{1-\gamma}$$

Transitional Dynamics

$$Y_t - \gamma(1+\alpha)Y_{t-1} + \alpha\gamma Y_{t-2} = 0$$

$$Ab^t - \gamma(1+\alpha)Ab^{t-1} + \alpha\gamma Ab^{t-2} = 0$$

$$b_1, b_2 = \frac{\gamma(1+\alpha) \pm \sqrt{\gamma^2(1+\alpha)^2 - 4\alpha\gamma}}{2}$$

$$Y_t = A_1 b_1^t + A_2 b_2^t$$

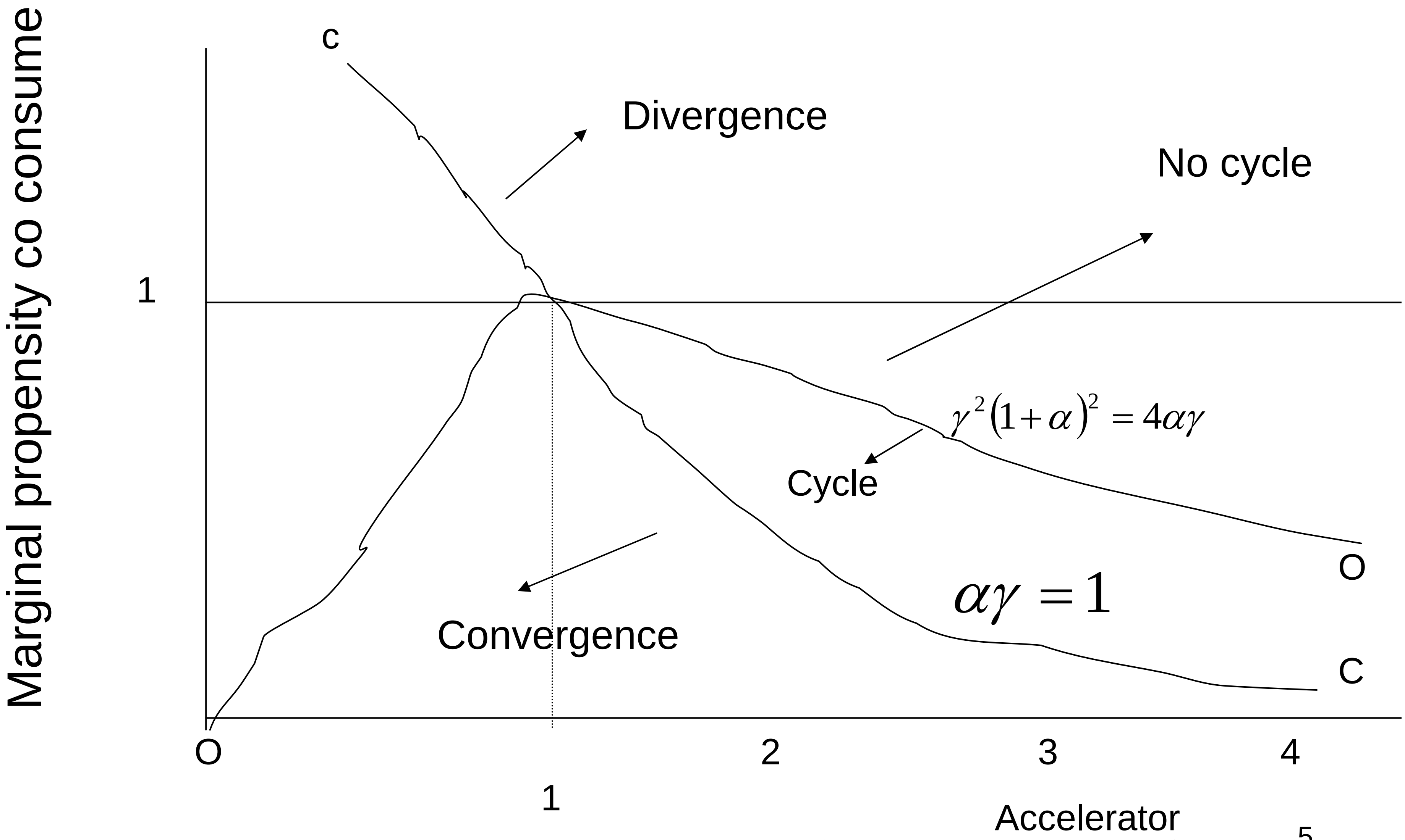
Three cases:

$$\gamma^2(1+\alpha)^2 > 4\alpha\gamma$$

$$\gamma^2(1+\alpha)^2 = 4\alpha\gamma$$

$$\gamma^2(1+\alpha)^2 < 4\alpha\gamma$$

Convergence or divergence , cycle or no cycle



$$Y_t = A_1 R^t \left(\cos\theta\cdot t + i\sin\theta\cdot t\right) + A_2 R^t \left(\cos\theta\cdot t - i\sin\theta\cdot t\right)$$

Application of the Multiplier Accelerator Model

$$C_t = \beta_0 + \beta_1 (Y_t - T_t)$$

$$I_t = \mu_0 + \mu_1 R_t + \phi \Delta Y_{t-1}$$

$$T_t = t_0 + t_1 Y_t$$

$$M_t = m_0 + m_1 Y_t + m_2 \lambda_t$$

$$C_t + T_t + S_t = Y_t = C_t + I_t + G_t + X_t - M_t$$

$$(T_t - G_t) + (S_t - I_t) = (X_t - M_t)$$

$$Y_t = \frac{\beta_0 - \beta_1 c_0 + \mu_0 - m_0 + G_t + X_t}{1 - \beta_1 + \beta_1 t_1 + m_1} + \frac{\mu_1 R_t}{1 - \beta_1 + \beta_1 t_1 + m_1} + \frac{\phi \Delta Y_{t-1}}{1 - \beta_1 + \beta_1 t_1 + m_1}$$

Parametric Specification of the Keynesian Model

	Parameters	Base Case	Tax cut	Spending	MPC	T &G	High X	High I	MMM
G	200	200	200	400	200	400	200	200	200
X	100	100	100	100	100	100	300	100	100
r	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
C0	300	300	300	300	300	300	300	300	300
b	0.8	0.8	0.8	0.8	0.9	0.8	0.8	0.8	0.8
I0	50	50	50	50	50	50	50	200	50
d	10	10	10	10	10	10	10	10	10
t0	30	30	30	30	30	30	30	30	30
t	0.3	0.3	0.2	0.2	0.3	0.2	0.3	0.3	0.3
m0	20	20	20	20	20	20	20	20	20
m1	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.4

Solutions of the Basic Keynesian Model

	Y	T	C	I	G	X	M	S	T-G	X-M	S-I	Bal
Base case	876.8	293.0	767.0	49.0	200.0	100.0	239.2	-183.2	93.0	-139.2	-232.2	-139.2
Tax cut	991.8	228.4	910.8	49.0	200.0	100.0	268.0	-147.3	28.4	-168.0	-196.3	-168.0
Spending	1166.7	380.0	929.3	49.0	400.0	100.0	311.7	-142.7	-20.0	-211.7	-191.7	-211.7
MPC	971.0	321.3	884.7	49.0	200.0	100.0	262.7	-235.0	121.3	-162.7	-284.0	-162.7
T&G	1319.7	293.9	1120.6	49.0	400.0	100.0	349.9	-94.9	-106.1	-249.9	-143.9	-249.9
High X	1166.7	380.0	929.3	49.0	200.0	300.0	311.7	-142.7	180.0	-11.7	-191.7	-11.7
High I	1094.2	358.3	888.8	199.0	200.0	100.0	293.6	-152.8	158.3	-193.6	-351.8	-193.6
MMM	720.2	246.1	679.3	49.0	200.0	100.0	308.1	-205.2	46.1	-208.1	-254.2	-208.1

Keynesian IS-LM Model

$$\left(\frac{\overline{MM}}{P}\right)_t = b_0 + b_1 Y_t - b_2 R_t$$

$$R_t = \frac{b_0}{b_2} - \frac{1}{b_2}\left(\frac{\overline{MM}}{P}\right)_t + \frac{b_1}{b_2} Y_t$$

$$Y_t = \frac{b_2\left(\beta_0 - \beta_1 t_0 + \mu_0 - m_0 + G_t + X_t\right)}{\left(1 - \beta_1 + \beta_1 t_1 + m_1\right) b_2 - \mu_1 b_1} + \frac{b_2 \phi \Delta Y_{t-1}}{\left(1 - \beta_1 + \beta_1 t_1 + m_1\right) b_2 - \mu_1 b_1} + \frac{b_2 \mu_1}{\left(1 - \beta_1 + \beta_1 t_1 + m_1\right) b_2 - \mu_1 b_1}\left[\frac{b_0}{b_2} - \frac{1}{b_2}\left(\frac{\overline{MM}}{P}\right)_t\right]$$

$$R_t = \frac{b_0}{b_2} - \frac{1}{b_2}\left(\frac{\overline{MM}}{P}\right)_t + \frac{b_1}{b_2}\left[\frac{b_2\left(\beta_0 - \beta_1 t_0 + \mu_0 - m_0 + G_t + X_t\right)}{\left(1 - \beta_1 + \beta_1 t_1 + m_1\right) b_2 - \mu_1 b_1} + \frac{b_2 \phi \Delta Y_{t-1}}{\left(1 - \beta_1 + \beta_1 t_1 + m_1\right) b_2 - \mu_1 b_1} + \frac{b_2 \mu_1}{\left(1 - \beta_1 + \beta_1 t_1 + m_1\right) b_2 - \mu_1 b_1}\left[\frac{b_0}{b_2} - \frac{1}{b_2}\left(\frac{\overline{MM}}{P}\right)_t\right]\right]$$

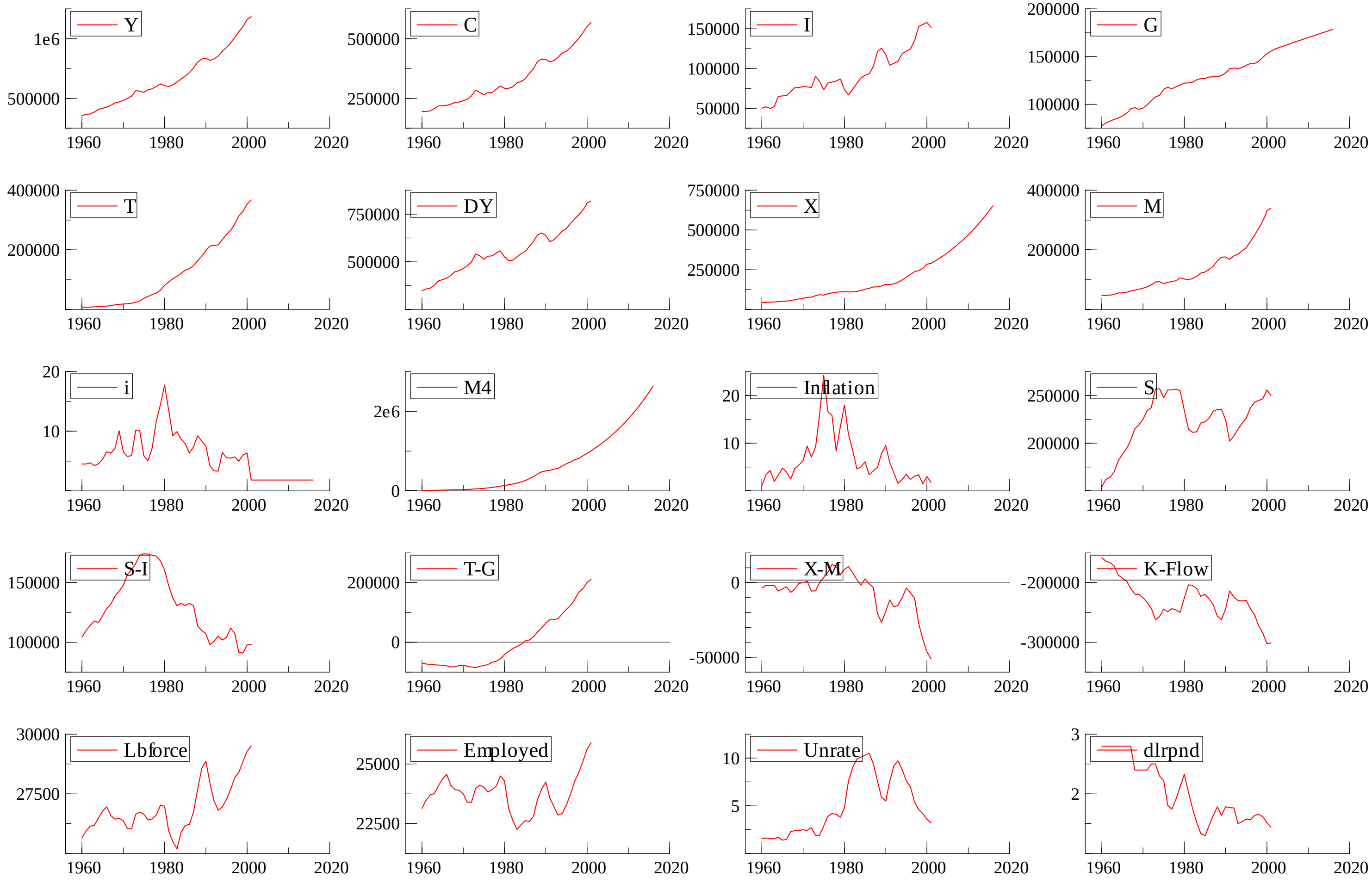
Parameters of the IS-LM Model

beta0	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	22114.16
beta1	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.6	0.9	0.459078
mu0	500	500	500	500	500	500	1000	500	500	500	105457
m0	100	100	100	100	100	100	100	100	100	100	-65167
t0	200	500	200	500	200	200	200	200	200	200	-201384
t1	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.476403
m1	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.3	1.387408
mu1	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1720.051
phi	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
G	20000	20000	25000	25000	20000	20000	20000	20000	20000	20000	155880
X	8000	8000	8000	8000	8000	8000	8000	10000	10000	8000	289225
y0	500	500	500	500	500	500	500	500	500	500	500
b0	800	800	800	800	800	800	800	800	800	800	-78809
b1	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.333992
b2	300000	300000	300000	300000	300000	600000	300000	300000	300000	300000	-1829.75
M4	10000	10000	10000	10000	15000	10000	10000	10000	10000	10000	10000
P	1	1	1	1	1	1	1	1	1	1	1

Solution of the IS-LM Model

	R	Y	C	I	G	T	M	X	S	X-M	S-I	T-G
Base case	0.0251	66901	51968	525	20000	20270	13480	8000	-5337	-5480	-5862	270
More Tax	0.024	65640	50453	524	20000	20692	13228	8000	-5505	-5228	-6029	692
More Spending	0.0324	75660	57486	532	25000	22898	15232	8000	-4724	-7232	-5256	-2102
Tax and Spend	0.032	75187	56918	532	25000	23056	15137	8000	-4787	-7137	-5319	-1944
More money supply	0.0084	66872	51949	508	20000	20262	13474	8000	-5339	-5474	-5847	262
More Sensitive Asset demand	0.0126	66977	52015	513	20000	20293	13495	8000	-5332	-5495	-5844	293
More investment	0.026	67777	52519	1026	20000	20533	13655	8000	-5276	-5655	-6301	533
More Exports	0.028	70405	54175	528	20000	21321	14181	10000	-5092	-4181	-5620	1321
Low MPC	0.01	48985	30454	510	20000	14896	9897	8000	3636	-1897	3126	-5104
High MPM	0.017	56928	45685	517	20000	17278	17178	8000	-6035	-9178	-6552	-2722

Macroeconomic Time Series of the UK,1960-2000

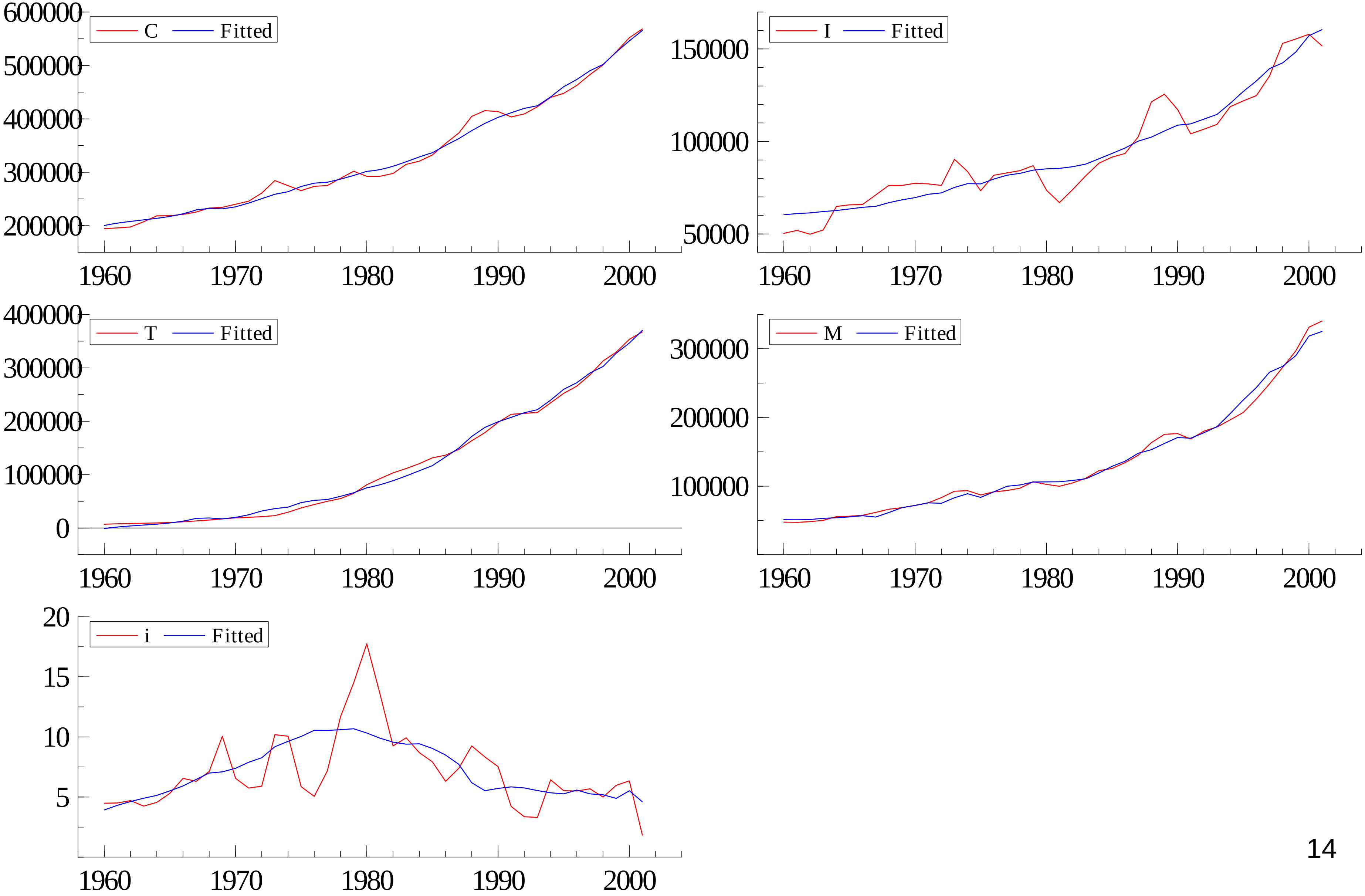


3SLS Estimation of Reduced form of a Keynesian Model

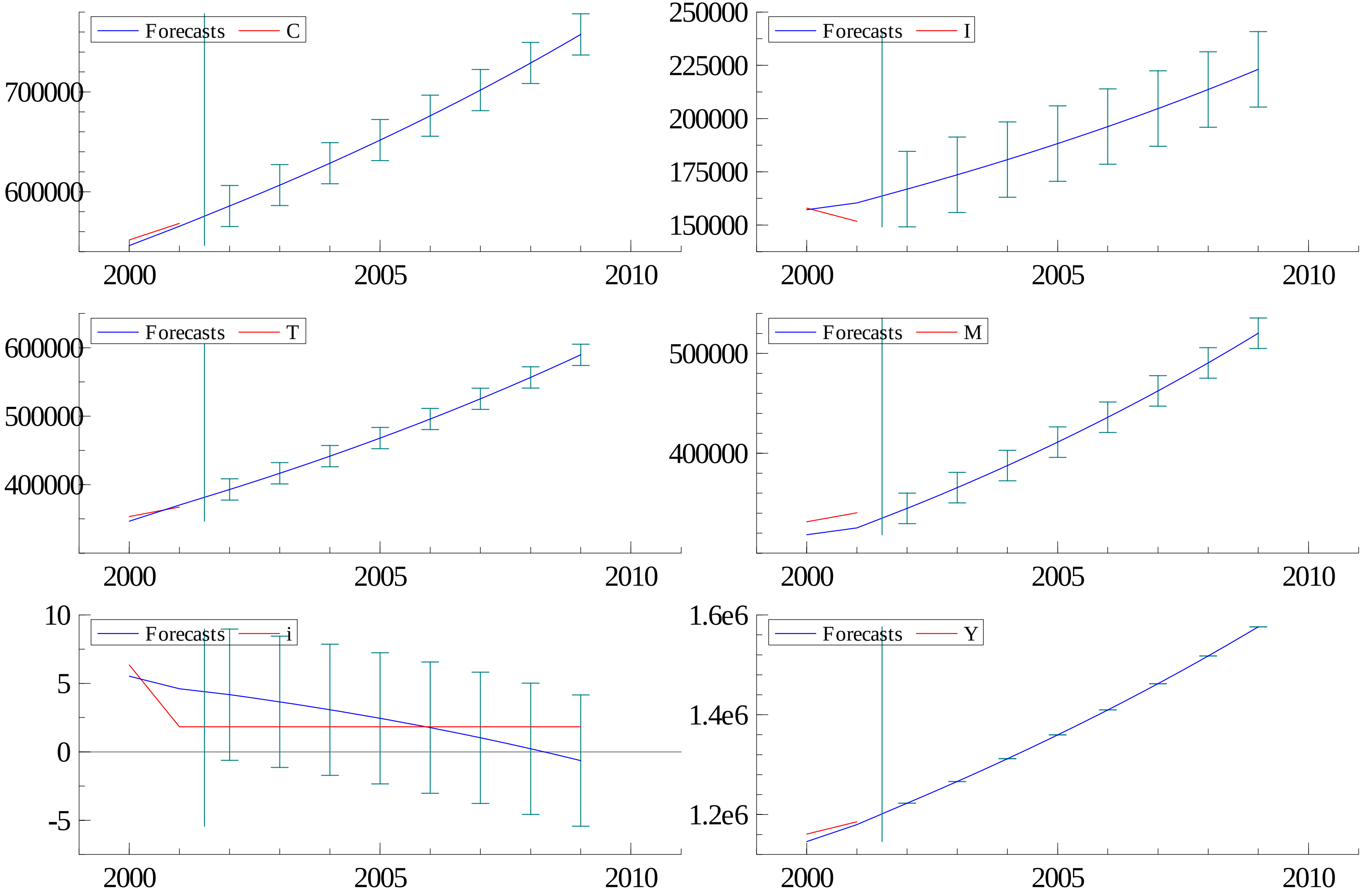
Consumption function	C =	1.407*G + 0.1767*X + 0.2128*M4 + 8.059e+004
	(SE)	(0.212) (0.154) (0.0266) (1.63e+004)
Investment function:	I =	0.0684*G + 0.2681*X + 0.02907*M4 + 4.292e+004
	(SE)	(0.182) (0.132) (0.0229) (1.4e+004)
Tax Reveneu:	T =	+ 0.9521*G - 0.08909*X + 0.3204*M4 - 7.533e+004
	(SE)	(0.159) (0.116) (0.02) (1.23e+004)
Import function:	M =	- 0.4738*G + 1.003*X + 0.06508*M4 + 4.34e+004
	(SE)	(0.157) (0.114) (0.0198) (1.21e+004)
Interest rate:	i =	0.0001148*G + 6.273e-005*X - 2.384e-005*M4 - 7.408
	(SE)	(4.93e-005) (3.58e-005) (6.2e-006) (3.79)

log-likelihood	-1798.42246	-T/2log|Omega|	-1500.44537
no. of observations	42	no. of parameters	20

Fit of the Simultaneous Equation Model



Ex-Ante Forecast of the Model Economy



Neo-classical Synthesis of the Keynesian Model: Comparative Static Analysis

$$Y = F(K, N)$$

$$F_N > 0 \qquad F_K > 0 \qquad F_{NN} < 0 \qquad F_{KK} < 0$$

$$W(N) = \begin{cases} 0 \; for & N \le \overline{N} \\ + \; for & N > \overline{N} \end{cases} \qquad W = W_0 + W(N)$$

$$\frac{W}{P} = F_N(N, K)$$

$$C = C(Y^d) \qquad Y^d = (1 - \tau)Y$$

$$\frac{M}{P} = M(Y, r) \qquad M_y > 0 \qquad M_r < 0$$

Reduced form of the Neoclassical synthesis to the Keynesian Model

$$F(N,K) = c(F(N,K)\cdot(1-\tau)) + I(r) + G + NX$$

$$\frac{W}{P} = F_N(N,K)$$

$$\frac{M}{P} = M(Y,r)$$

$$F_N dN + F_K dK = c(1-\tau)F_N dN + c(1-\tau)F_K dK - cd\tau F(N,K) + I_r dr + dG$$

$$\frac{dW}{P} - \frac{W}{P^2}dP = F_{NN}dN + F_{NK}dK$$

$$\frac{dM}{P} - \frac{M}{P^2}dP = M_y F_N dN + M_y F_K dK + M_r dr$$

$$\begin{bmatrix} (1-c(1-\tau))F_N & 0 & -I_r \\ M_y F_N & \frac{M}{P^2} & M_r \\ F_{NN} & \frac{W}{P^2} & 0 \end{bmatrix} \begin{bmatrix} dN \\ dP \\ dr \end{bmatrix} = \begin{bmatrix} c(1-\tau)F_K dK - F_K dK - cd\tau F(N,K) + dG \\ \frac{dM}{P} - M_y F_K dK \\ \frac{dW}{P} - F_{NK} dK \end{bmatrix}$$

$$\Delta = \begin{vmatrix} (1-c(1-\tau))F_N & 0 & -I_r \\ M_y F_N & \frac{M}{P^2} & M_r \\ F_{NN} & \frac{W}{P^2} & 0 \end{vmatrix} = -M_r \frac{W}{P^2}\left[(1-c(1-\tau))F_N\right] - I_r\left(M_y F_N \frac{W}{P^2} - F_{NN}\frac{M}{P^2}\right)$$

$$dN = \frac{1}{\Delta}\begin{bmatrix} c(1-\tau)F_K dK - F_K dK - cd\tau F(N,K) + dG & 0 & -I_r \\ \frac{dM}{P} - M_y F_K dK & \frac{M}{P^2} & M_r \\ \frac{dW}{P} - F_{NK} dK & \frac{W}{P^2} & 0 \end{bmatrix}$$

Employment, output, price and interest Effects in the Keynesian Model

$$dN = \frac{1}{\Delta}\left[-I_r \frac{W}{P^2}\left(\frac{dM}{P} - M_y F_K dK\right) + I_r\left(\frac{dW}{P} - F_{NK} dK\right)\frac{M}{P^2} - M_r \frac{W}{P^2}\left(c(1-\tau)F_K dK - F_K dK - cd\tau F(N,K) + dG\right)\right]$$

$$dy = \frac{F_N}{\Delta}\left[-I_r \frac{W}{P^2}\left(\frac{dM}{P} - M_y F_K dK\right) + I_r\left(\frac{dW}{P} - F_{NK} dK\right)\frac{M}{P^2} - M_r \frac{W}{P^2}\left(c(1-\tau)F_K dK - F_K dK - cd\tau F(N,K) + dG\right)\right]$$

$$\frac{dy}{d\tau} = -cF(N,K)\left(-M_r \frac{W}{P^2}\right)$$

$$dp = \frac{1}{\Delta}\begin{bmatrix} (1-c(1-\tau))F_N & c(1-\tau)F_K dK - F_K dK - cd\tau F(N,K) + dG & -I_r \\ M_y F_N & \frac{dM}{P} - M_y F_K dK & M_r \\ F_{NN} & \frac{dW}{P} - F_{NK} dK & 0 \end{bmatrix}$$

$$dr = \frac{1}{\Delta}\begin{bmatrix} (1-c(1-\tau))F_N & 0 & c(1-\tau)F_K dK - F_K dK - cd\tau F(N,K) + dG \\ M_y F_N & \frac{M}{P^2} & \frac{dM}{P} - M_y F_K dK \\ F_{NN} & \frac{W}{P^2} & \frac{dW}{P} - F_{NK} dK \end{bmatrix}$$

Phillips Curve: Short -Run Dynamics: Unemployment Inflation Trade-off; policy menu

$$\pi_t = \bar{\pi} + \left\{ \begin{array}{c} a(y - \bar{y}) \\ or \\ -b(u - \bar{u}) \end{array} \right\} + s$$

$$W_t = (1 + \gamma) P_t^e$$

$$P_t = (1 + \theta)(1 + \gamma) P_t^e$$

$$(1 + \pi_t) = \frac{P_t}{P_{t-1}}$$

$$(1 + \pi_t) = (1 + \theta)(1 + \gamma)(1 + \pi_t^e)$$

$$\pi_t = \pi_t^e + \theta + \gamma$$

$$\theta + \gamma = a(y_t - \bar{y}) = -b(u_t - \bar{u})$$

Noeclassical model for the long run

Preference: $\int_0^{\infty} e^{-\rho t} \frac{C_t^{1-\sigma}}{1-\sigma} dt$

Technology: $Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$ assume $A_t = 1$ $N_t = 1$

Capital accumulation: $\dot{K}_t = Y_t - N_t C_t - \delta K_t$

Current value Hamiltonian of this problem

$$H(c, K, \theta) = \frac{C_t^{1-\sigma}}{1-\sigma} + \theta\left[K_t^{\alpha} - C_t - \delta K_{t-1}\right]$$

C is control, K is state variable, θ is co-state variable.

Optimality and Boundary Conditions

First order conditions

$$\frac{\partial H}{\partial C_t} = 0 \;\rightarrow\; C_t^{-\sigma} = \theta_t \qquad (2)$$

$$\dot{\theta}_t = \rho\theta_t - \frac{\partial H_t}{\partial K_t} \;\rightarrow\; \dot{\theta}_t = \rho\theta_t - \theta_t \left\lfloor \alpha K_t^{\alpha-1} - \delta \right\rfloor \qquad (3)$$

$$\dot{K}_t = K_t^{\alpha} - N_t C_t - \delta K_t \qquad (4)$$

Transversality condition

$$\underset{n \to \infty}{Lim}\; e^{-\rho t}\theta_t K_t = 0 \qquad (5)$$

Characterisation of the Balanced Growth Path

Capital stock, consumption and the shadow price of capital remain constant in the balanced growth path $\frac{\dot{C}}{C} = g_c$; $\frac{\dot{K}}{K} = g_K$ and $\frac{\dot{\theta}_t}{\theta_t} = g_\theta$. From (3)

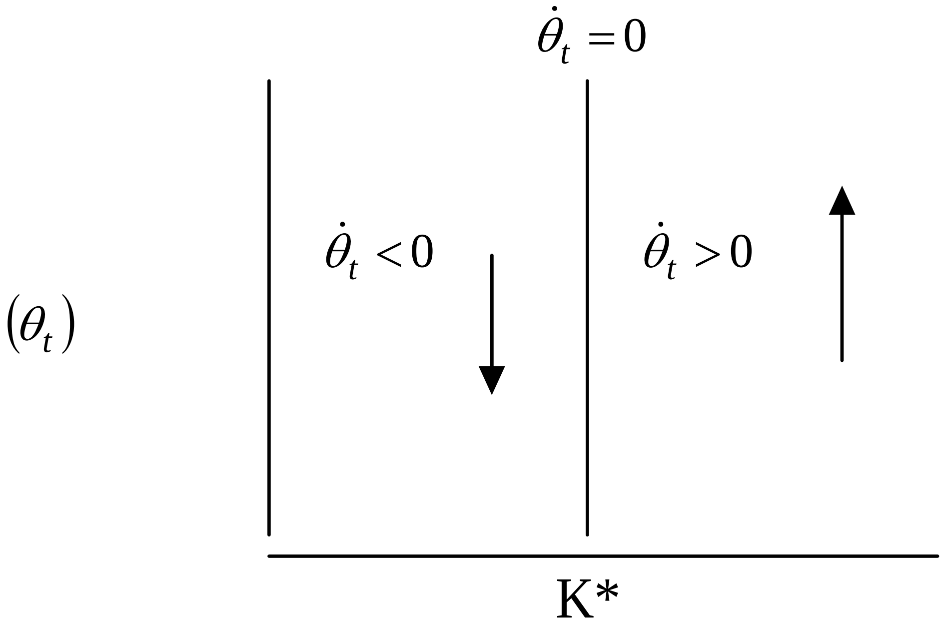
$$\frac{\dot{\theta}_t}{\theta_t} = \rho - \left[\alpha K_t^{\alpha-1} - \delta\right] \Rightarrow \alpha K_t^{\alpha-1} = \rho - \frac{\dot{\theta}_t}{\theta_t} + \delta \qquad (6)$$

Since the RHS is constant , therefore LHS also should be constant $\frac{\dot{K}}{K} = 0$. If capital stock is not growing output is not growing $\frac{\dot{Y}}{Y} = 0$ and consumption is not growing $\frac{\dot{C}}{C} = 0$.

From (2) $\frac{\dot{\theta}_t}{\theta_t} = -\sigma \frac{\dot{C}_t}{C_t} \Rightarrow \frac{\dot{\theta}_t}{\theta_t} = 0$ (7)

Transitional Dynamics-1

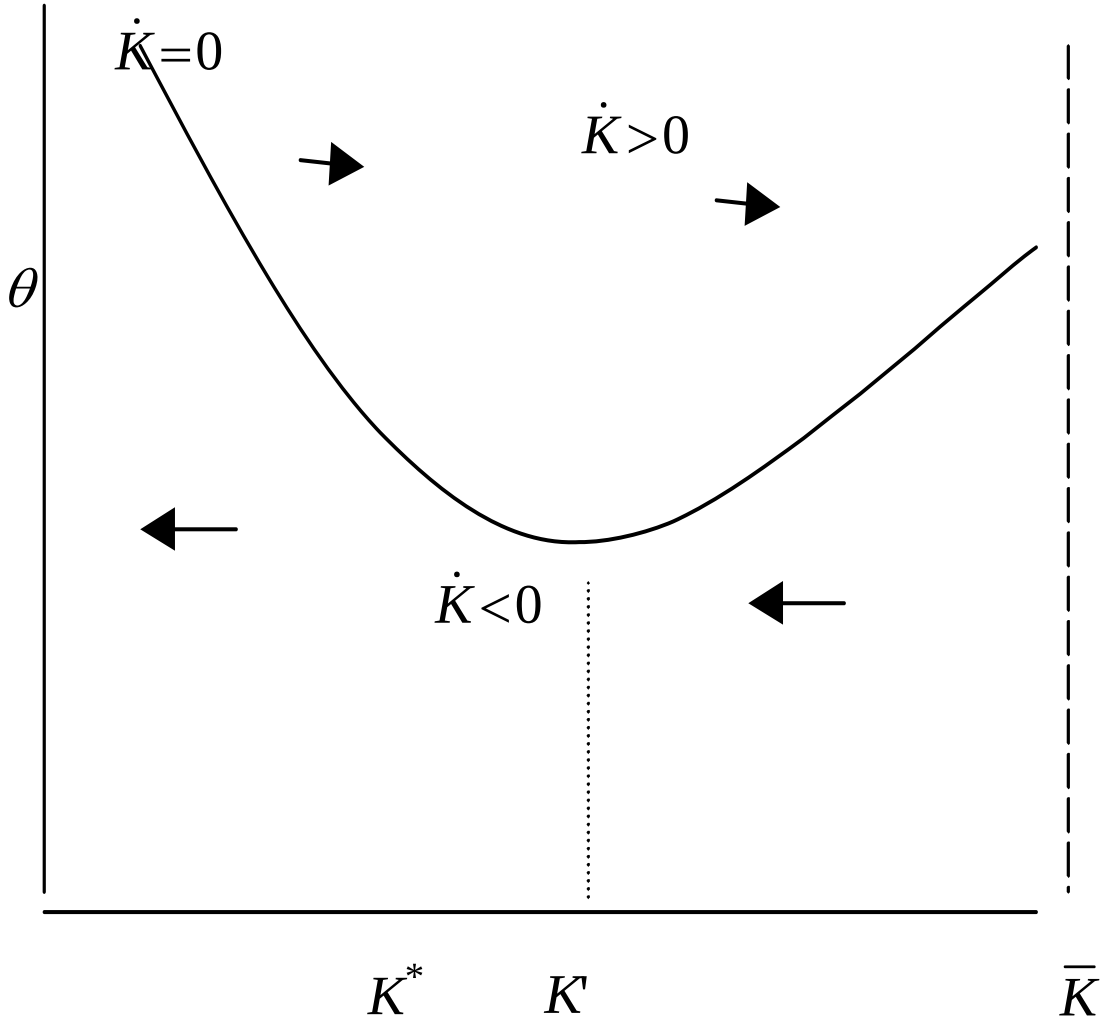
In (θ_t, K_t) space the transition dynamics of the shadowprice θ_t relative to the steady state capital stock is that



$$K^* = \left[\frac{\alpha}{\rho + \delta - \frac{\dot{\theta}}{\theta}} \right]^{\frac{1}{1-\alpha}}$$

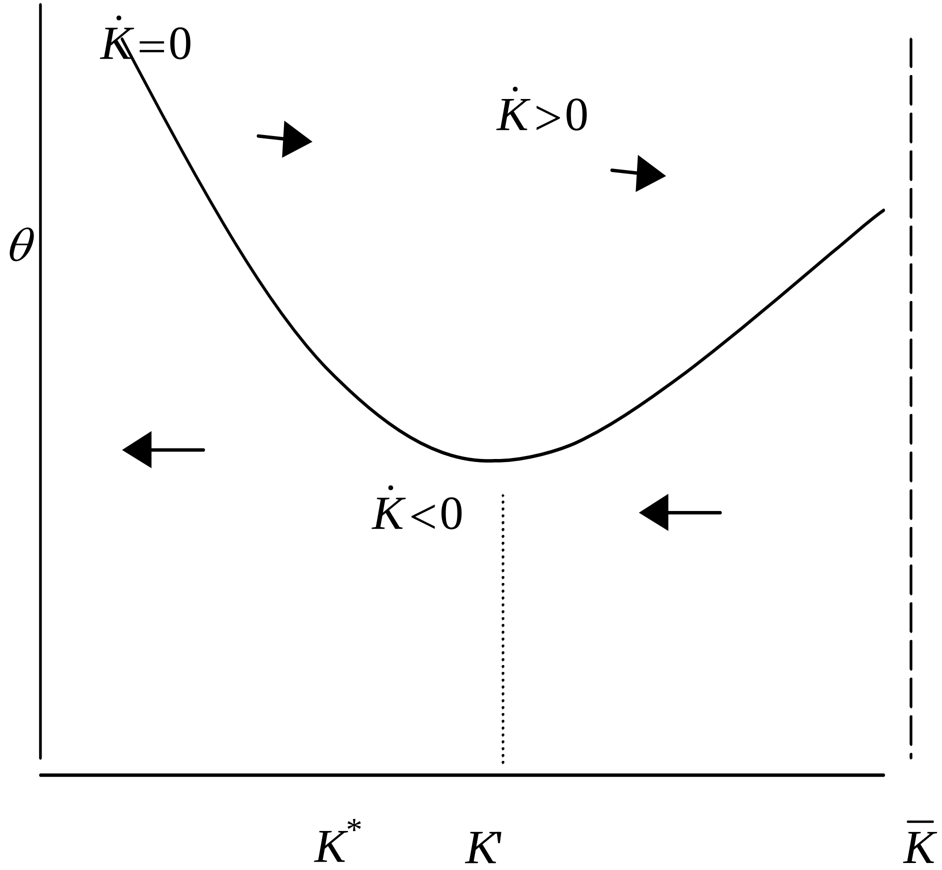
Transitional Dynamics-2

$\bar{K} > K^* > K'$.



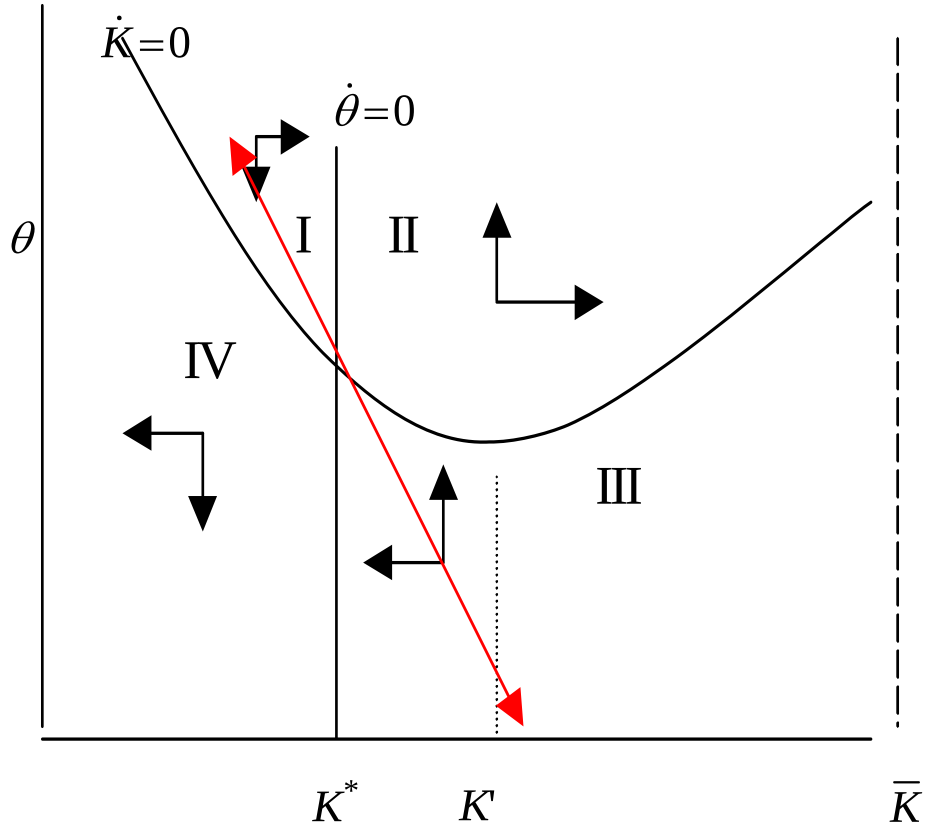
Transitional Dynamics-2

$\bar{K} > K^* > K'$.



Saddle Point Solution

Putting all these things together the convergence to the steady state can be summarised in the following diagram.



Convergence to the steady state lies in region I and III as shown by the double arrow red line.