

# Input-Output and Regional General Equilibrium and Dynamic General Equilibrium Models of Humber and Hull Sub Regions of East Yorkshire in England

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<http://www.hull.ac.uk/hubs/>



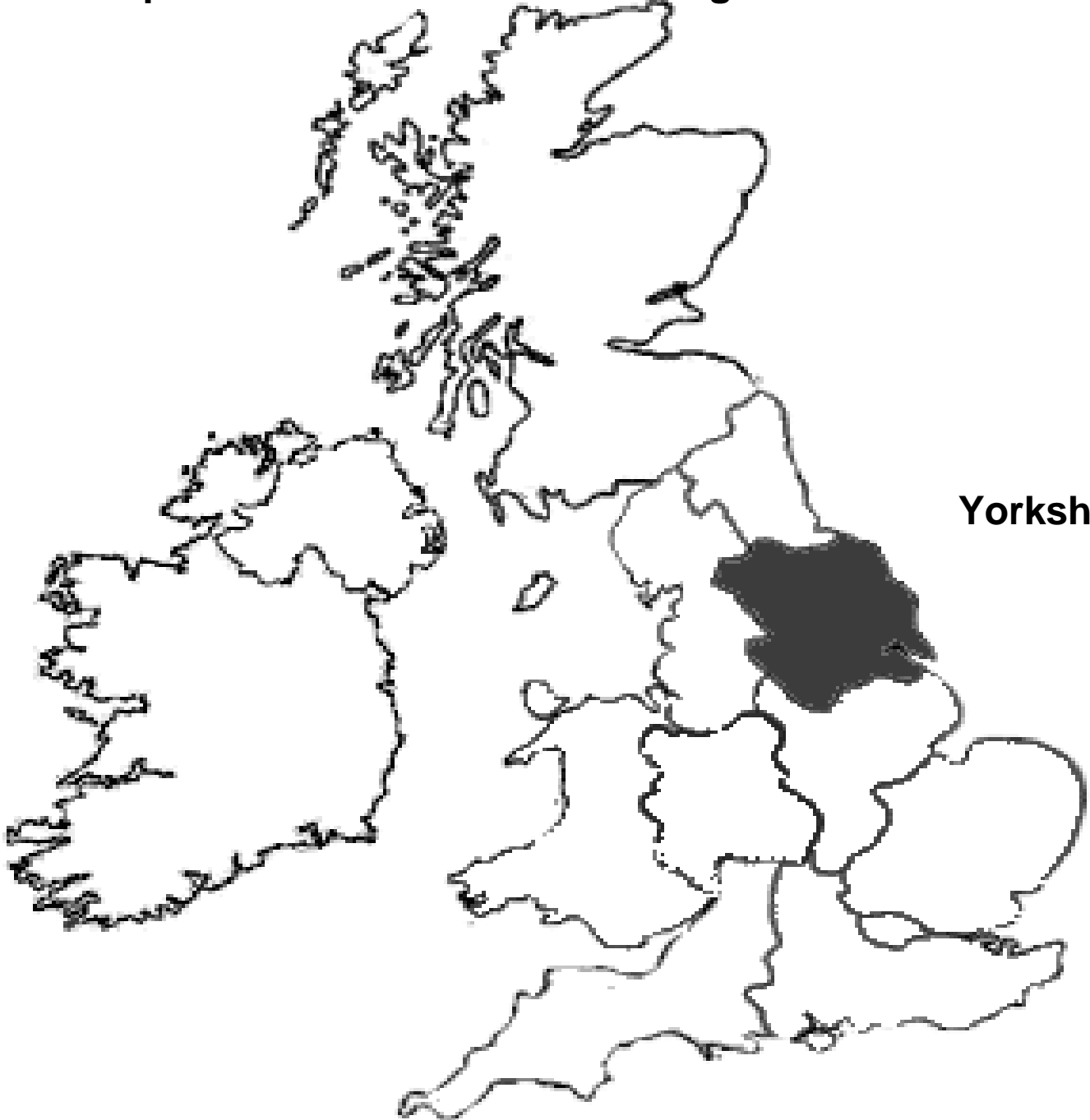
Regional and Urban Modelling  
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### Regional Differences in Economic Indicators in UK, 2004

	GVADEDED	Perhead	Population	pop-gr91-03	Labforce	EmplRate	Earning_M	Earning_F	Perc W&S
North East	34,188	13,433	2,545.1	-1.8	1,063	69.7	488.5	391.0	66
North West	101,996	14,940	6,827.2	-0.6	2,987	72.9	532.3	409.6	68
Yorkshire and the Humber	75,219	14,928	5,038.8	1.5	2,246	73.8	513.0	395.5	67
East Midlands	65,770	15,368	4,279.7	6.0	1,965	76.0	531.2	400.4	71
West Midlands	81,745	15,325	5,334.0	1.7	2,383	74.6	518.9	402.0	67
East	100,307	18,267	5,491.3	6.7	2,611	78.6	609.7	443.1	70
London	164,961	22,204	7,429.2	8.2	3,334	69.3	731.4	556.1	71
South East	158,187	19,505	8,110.2	5.9	3,892	78.6	644.4	465.1	67
South West	78,650	15,611	5,038.2	6.6	2,341	78.9	535.0	402.2	64
England	861,022	17,188	50,093.8	4.1	22,823	74.8	581.8	441.5	68
Wales	39,243	13,292	2,952.5	2.3	1,239	70.8	499.5	402.0	64
Scotland	82,050	16,157	5,078.4	-0.5	2,331	74.6	522.0	425.8	68
Northern Ireland	23,058	13,482	1,710.3	5.9	713	68.0	487.1	402.3	64

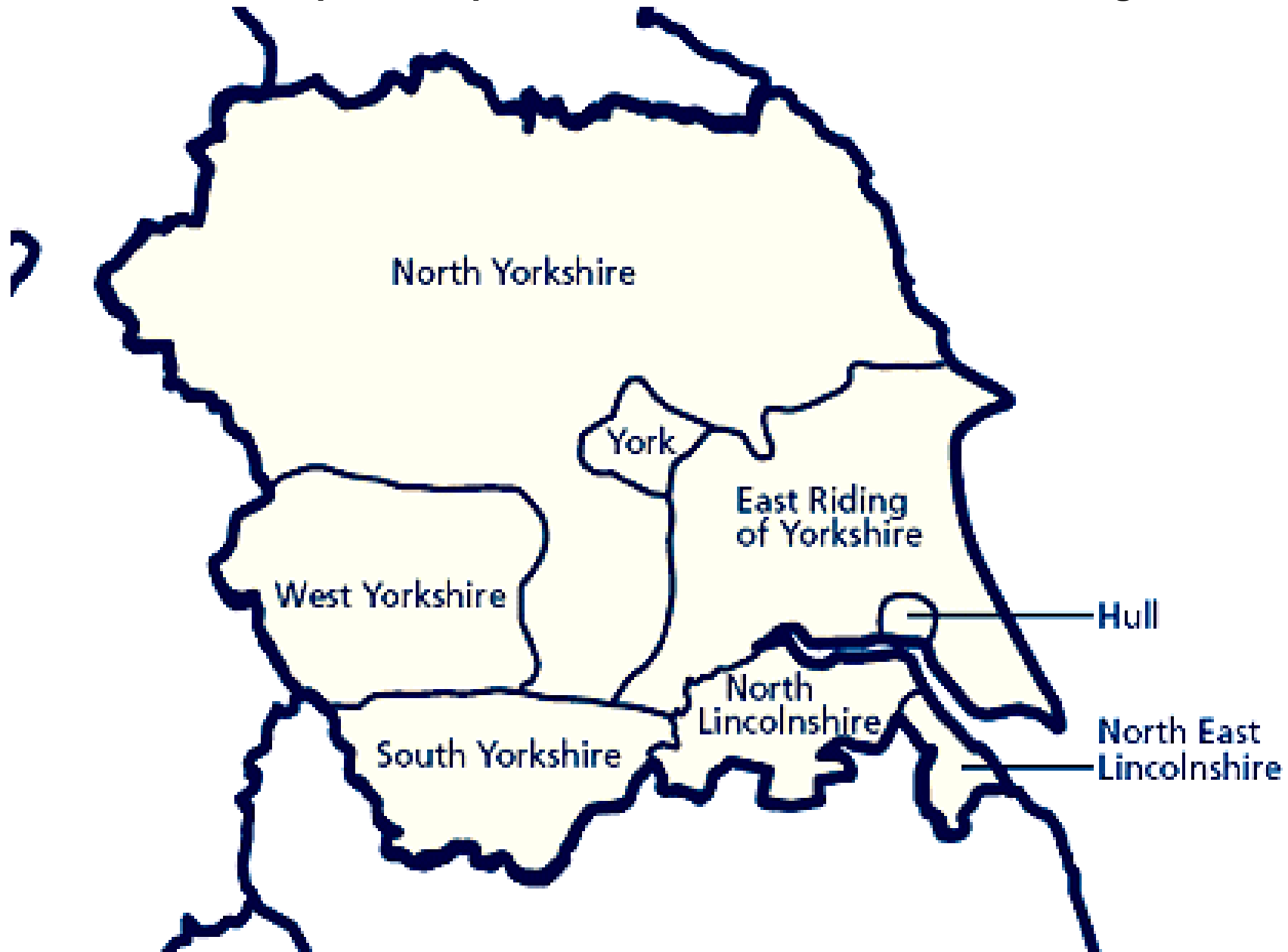
Source: Compiled from the Office of National Statistics: Regional Data Base ([www.statistics.gov.uk](http://www.statistics.gov.uk))

**Map 1: Yorkshire and Humber Region in the United Kingdom**



**Yorkshire and Humber**

**Map 2: Composition of Yorkshire and Humber Region**





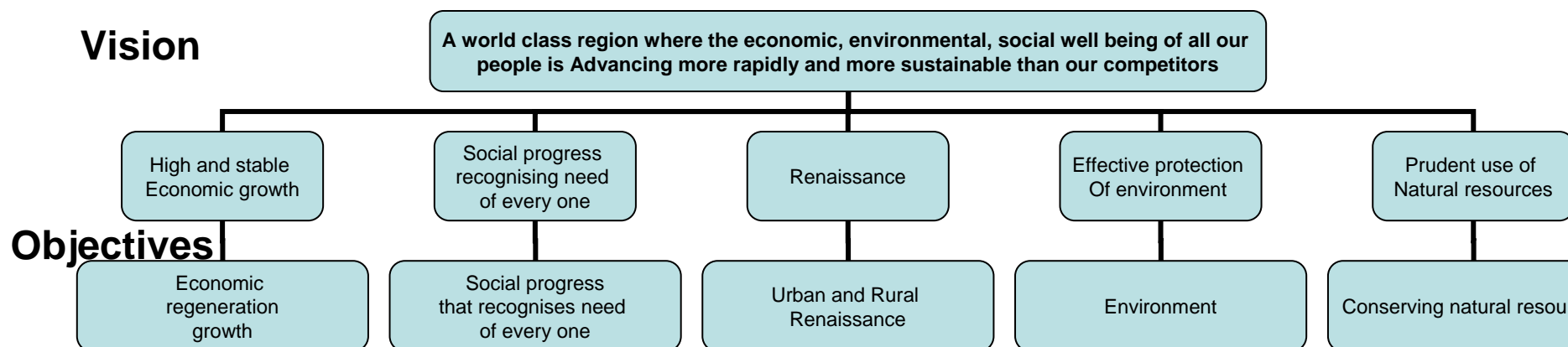
# EUROPE



[http://www.lib.utexas.edu/maps/europe/europe\\_ref\\_2003.jpg](http://www.lib.utexas.edu/maps/europe/europe_ref_2003.jpg)



# Regional Strategic Policies for Yorkshire and Humber to 2016



**Strategic Themes** Sustainable development policies

**Strategic policies** Regional Spatial Strategies

## Specific policies

Economy

Housing

Transport

Social infrastructure

Built Natural Environment

Natural resource

Source: Government Office Yorkshire and Humber 2004, RPG12, TSO

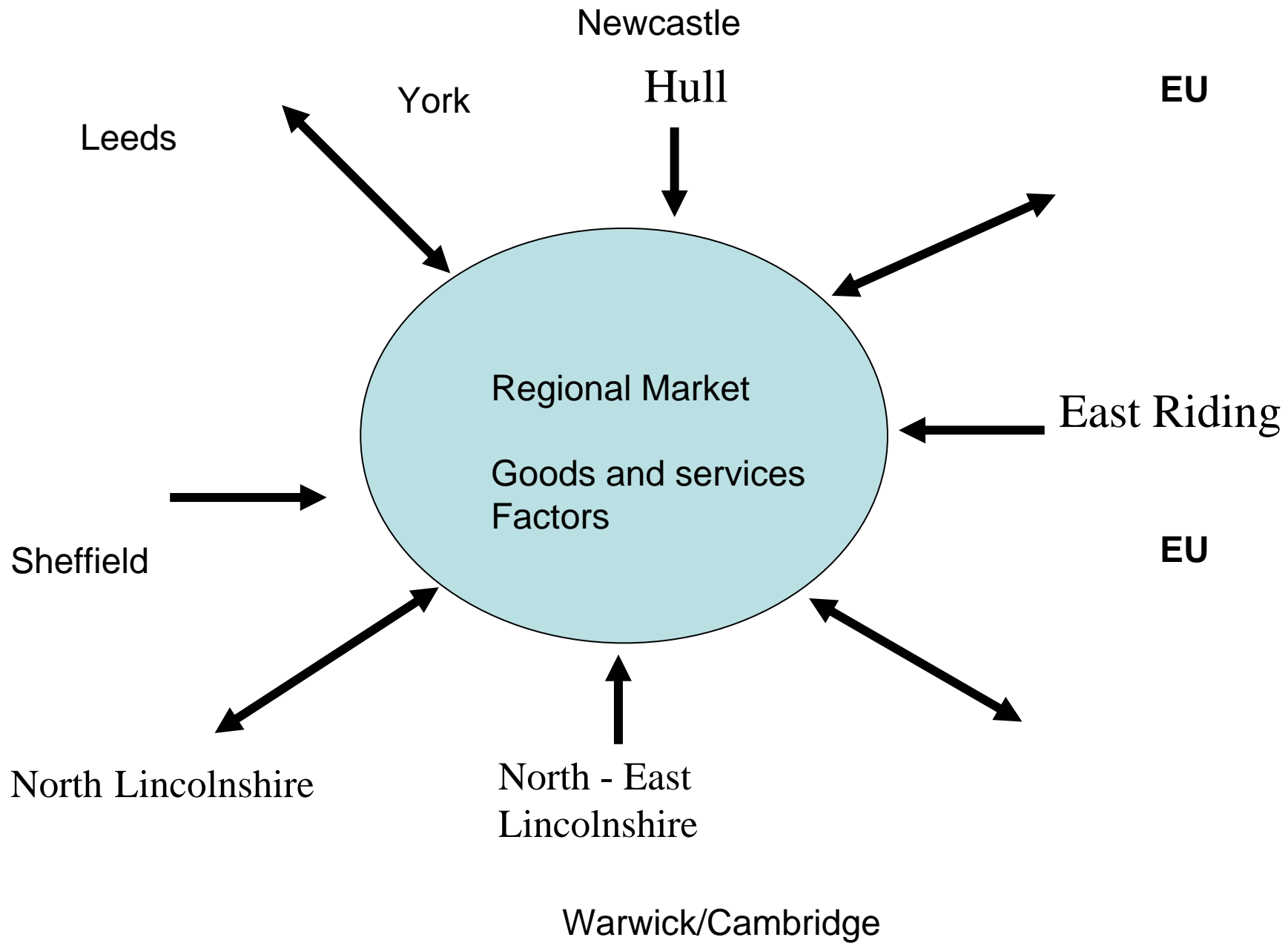








Table 1

**Economy of Hull: Sectoral Composition of Employment and Output**

	Employers	Employment	Output 2002 (ml £)	Output 2012
Agriculture and Fishing	0.3%	0.01%	0.10%	0.10%
Energy and Water	0.1%	0.60%	1.30%	1.30%
Manufacturing	11.7%	20.70%	28.20%	28.20%
Construction	7.3%	4.10%	4.30%	4.30%
Distribution, hotels and restaurants	35.3%	24.30%	17.00%	17.00%
Transport and communication	5.4%	5.40%	9.70%	9.70%
Banking, Finance and insurance	19.6%	12.60%	12.40%	12.40%
Public administration, education and health	11.8%	27.40%	23.50%	23.50%
Other services	8.6%	4.90%	3.40%	3.40%
Total	7562	120856	3125881	3,125,881,000

Source: [www.humberforum.com](http://www.humberforum.com)

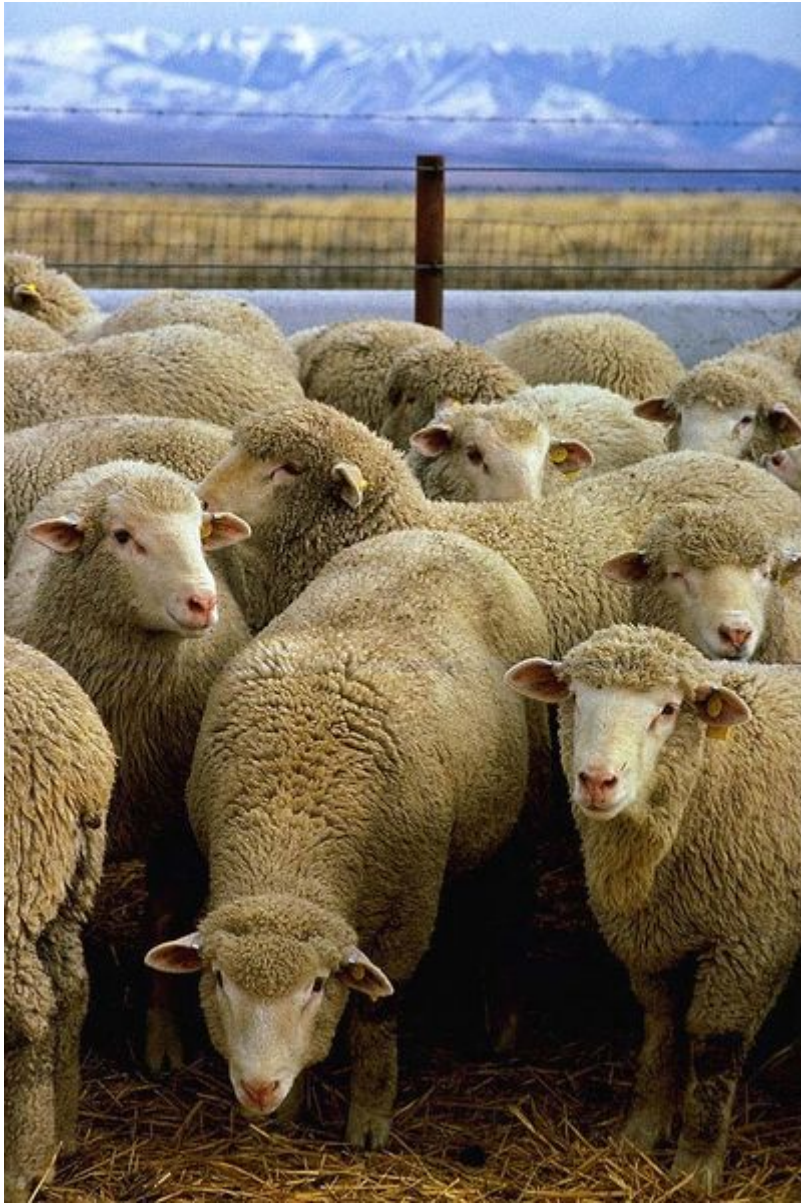
















## Material Balance Equation

$$X_1 = X_{1,1} + X_{1,2} + \dots + X_{1,9} + F_1$$

$$X_2 = X_{2,1} + X_{2,2} + \dots + X_{2,9} + F_2$$

$$X_9 = X_{9,1} + X_{9,2} + \dots + X_{9,9} + F_9$$

## Input-Output Model

$$X_1 = a_{1,1}X_1 + a_{1,2}X_2 + \dots + a_{1,9}X_9 + F_1$$

$$X_2 = a_{2,1}X_1 + a_{2,2}X_2 + \dots + a_{2,9}X_9 + F_2$$

$$X_9 = a_{9,1}X_1 + a_{9,2}X_2 + \dots + a_{9,9}X_9 + F_9$$

## Input-Output Model

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \\ X_8 \\ X_9 \end{bmatrix} = \begin{bmatrix} (1-a_{1,1}) & -a_{1,2} & -a_{1,3} & -a_{1,4} & -a_{1,5} & -a_{1,6} & -a_{1,7} & -a_{1,8} & -a_{1,9} \\ -a_{2,1} & (1-a_{2,1}) & & & & & & & \\ & & & & (1-a_{i,j}) & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & (1-a_{8,8}) & & \\ -a_{9,1} & -a_{9,1} & & -a_{9,j} & & & & & (1-a_{9,9}) \end{bmatrix}^{-1} \cdot \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \\ F_8 \\ F_9 \end{bmatrix}$$

$$X = (I - A)^{-1} F$$

$$|I - A| \neq 0$$

## Derivation of Hull Regional Input-Output Tables

- Take the share of employment and output
- Production and household surveys is the best strategy
- Get the coefficients from the national 123 sector I-O table
- Get the structure of final demand and value added from the national table
- Use for the input-output model, general equilibrium analysis

## Steps for Computing Input-Output Table for Hull

1. Take the total employment and output and sectoral composition of these employments and output from the existing data from the HF (Table 1).
2. Split the total output in labour and capital income using employment and income per capita information. Sectoral employment is obtained by multiplying the sectoral share by total employment. Then sectoral labour income is obtained by multiplying the sectoral employment by the gross value added per worker. Sectoral output is obtained by multiplying the sectoral share of output by total output. Capital income is the difference between total output and the labour income. All these numbers are presented in Table 2.
3. Analyses of inter-sectoral linkages require information on input-output coefficients which do not exist for the Hull and Humber. If the structure of production for each sector is not very different in Hull than in the national economy the Hull economy input-output coefficients can be approximated by the national input-output coefficients. Some adjustment is necessary for education and public services sector, and the mining and utility sector from the existing nine sector IO table for the national economy as given in Table 3. This needs to be refined when better data becomes available. It may not be too unrealistic if the regional coefficients do not vary significantly from the national coefficients.
4. Derive the gross output of Hull using the above Leontief coefficients assuming that final demand to be equal to total value added as in Table 4. Then decompose this final demand into consumption, investment, government consumption and export components again based on approximations from the national input-output table as in Table 5 until such information is obtained from the primary survey in Hull.
5. Derive tax or imports components as residuals in the supply side - gross output minus intermediate, labour and capitals costs altogether.

Table 2

**Economy of Hull: Capital and Labour Income**

	Capital Income	Employees	Labour Income	Total Income
Agriculture and Fishing	2,967,039.9592	12.09	158,841.04	3,125,881.00
Energy and Water	31,105,990.5520	725.14	9,530,462.45	40,636,453.00
Manufacturing	552,697,487.5440	25,017.19	328,800,954.46	881,498,442.00
Construction	69,288,056.2720	4,955.10	65,124,826.73	134,412,883.00
Distribution, hotels and restaurants	145,416,040.8560	29,368.01	385,983,729.14	531,399,770.00
Transport and communication	217,436,294.9680	6,526.22	85,774,162.03	303,210,457.00
Banking, Finance and insurance	187,469,532.5920	15,227.86	200,139,711.41	387,609,244.00
Public administration, education and health	299,357,583.2080	33,114.54	435,224,451.79	734,582,035.00
Other services	28,447,844.0080	5,921.94	77,832,109.99	106,279,954.00
<b>Total</b>	<b>1,534,185,869.96</b>	<b>120,868.09</b>	<b>1,588,569,249.04</b>	<b>3,122,755,119.00</b>

### Input-Output Coefficients for Hull

	Agric	Enrwtr	Manu	Const	Distb	Trans	Busi	Edupub	OthSect
Agric	0.086583	0.000604	0.035177	4.74E-05	0.003639	0.000487	5.68E-05	0	0.000832
Enrwtr	0.000826	0.120376	0.016645	0.004751	0.000677	0.000173	3.03E-05	0.13229	0.000321
Manu	0.203363	0.068337	0.202288	0.152745	0.102854	0.083622	0.043803	0.040461	0.056192
const	0.007105	0.00526	0.000636	0.249816	0.003891	0.001532	0.022375	0	0.000821
Distb	0.041515	0.017505	0.037544	0.016244	0.026867	0.025066	0.008611	0.008509	0.004443
Trans	0.010121	0.044796	0.027601	0.010509	0.09595	0.158739	0.065376	0.004386	0.017857
Busi	0.080511	0.049237	0.073745	0.124203	0.14469	0.125707	0.249916	0.045159	0.075562
Edupub	0.011525	0.006898	0.019606	0.003223	0.007749	0.008697	0.004566	0.294183	0.003965
OthSect	0.015615	0.001811	0.012468	0.002867	0.006459	0.013893	0.015504	0.004291	0.043622

Matrix A

$$X = (I - A)^{-1} F$$

Agric	62,648,983
Utils	242,181,122
Manu	1,452,733,597
const	218,010,053
Distb	647,427,906
Trans	587,275,170
BusiFin	1,025,074,800
Edupub	1,107,405,717
OthSect	166,690,690
Total	5,509,448,037

## Structure of final demand for Hull IO Table

	Consumption	Investment	Public spending	Exports	Total
Agric	0.772232	0.000115	0.004819	0.222834	1
Enrwtr	0.040784	0.00012	0.005654	0.953441	1
Manu	0.219586	0.097767	0.04372	0.638926	1
Const	0.921867	5.64E-05	0.074581	0.003495	1
Distb	0.062893	0.858263	0.078844	0	1
Trans	0.863897	0.020094	0.00955	0.106459	1
BusiFin	0.558103	0.022052	0.07465	0.345195	1
Edupub	0.727305	0.078453	0.078222	0.11602	1
OthSect	0.27582	6.32E-06	0.695715	0.028458	1

## Input-Output Table of Hull (preliminary version September 2006)

	Agric	Utils	Manu	const	Distb	Trans	BusiFin	Edupub	OthSect	Total IntD	Cons	Inv	Gov	Exp	FinDem	Gross Output
Agric	5,424,334	146,182	51,103,260	10,332	2,356,000	286,072	58,173	0	138,751	59,523,102	2,413,905	359	15,064	696,553	3,125,881	62,648,983
Utils	51,759	29,152,785	24,181,417	1,035,782	438,617	101,317	31,026	146,498,529	53,438	201,544,669	1,657,334	4,889	229,778	38,744,453	40,636,453	242,181,122
Manu	12,740,455	16,549,844	293,870,093	33,299,988	66,590,412	49,108,956	44,901,826	44,806,943	9,366,636	571,235,155	193,565,032	86,181,836	38,539,146	563,212,427	881,498,442	1,452,733,597
const	445,127	1,273,868	924,286	54,462,477	2,518,915	899,934	22,935,688	0	136,876	83,597,170	123,910,811	7,577	10,024,705	469,790	134,412,883	218,010,053
Distb	2,600,885	4,239,266	54,540,953	3,541,288	17,394,296	14,720,767	8,826,788	9,423,261	740,631	116,028,136	33,421,309	456,080,801	41,897,660	0	531,399,770	647,427,906
Trans	634,047	10,848,762	40,096,965	2,291,118	62,120,697	93,223,579	67,015,336	4,857,625	2,976,586	284,064,713	261,942,717	6,092,623	2,895,527	32,279,591	303,210,457	587,275,170
BusiFin	5,043,906	11,924,241	107,131,988	27,077,550	93,676,055	73,824,349	256,182,410	50,009,645	12,595,412	637,465,556	216,326,008	8,547,703	28,934,907	133,800,626	387,609,244	1,025,074,800
Edupub	722,037	1,670,647	28,482,588	702,575	5,016,943	5,107,570	4,680,990	325,779,390	660,943	372,823,682	534,264,949	57,630,395	57,460,555	85,226,136	734,582,035	1,107,405,717
OthSect	978,243	438,545	18,112,753	625,085	4,181,482	8,159,000	15,892,873	4,751,447	7,271,307	60,410,736	29,314,185	672	73,940,538	3,024,559	106,279,954	166,690,690
Total IntDem	28,640,792	76,244,139	618,444,303	123,046,194	254,293,417	245,431,543	420,525,111	586,126,840	33,940,580							2,386,692,918
Labour	158,841	9,530,462	328,800,954	65,124,827	385,983,729	85,774,162	200,139,711	435,224,452	77,832,110							1,588,569,249
Capital	2,967,040	31,105,991	552,697,488	69,288,056	145,416,041	217,436,295	187,469,533	299,357,583	28,447,844							1,534,185,870
Taxes	31,265,941	118,511,672	-46,141,443	-28,339,275	-134,049,400	30,906,536	157,066,037	-109,267,613	18,187,479							38,139,935
Imports	-383,631	6,788,858	-1,067,705	-11,109,749	-4,215,881	7,726,634	59,874,408	-104,035,545	8,282,676							-38,139,935
Gross output	62,648,983	242,181,122	1,452,733,597	218,010,053	647,427,906	587,275,170	1,025,074,800	1,107,405,717	166,690,690	2,386,692,918	1,396,816,251	614,546,854	253,937,879	857,454,135	3,122,755,119	

## Input-Output Table of East Riding (preliminary version September 2006)

	Agric	Utils	Manu	const	Distb	Trans	BusiFin	Edupub	OthSect	Cons	Inv	Gov	Exp	FinDem	Gross Output
Agric	25,267,934	123,704	42,455,598	14,280	2,277,232	245,991	50,538	0	110,998	170,886,181	25,392	1,066,452	49,310,693	221,288,717	291,834,991
Utils	241,106	24,670,047	20,089,453	1,431,577	423,953	87,122	26,954	104,707,348	42,749	2,170,601	6,403	300,939	50,743,395	53,221,337	204,941,645
Manu	59,348,301	14,005,023	244,141,580	46,024,656	64,364,117	42,228,370	39,008,879	32,025,005	7,493,089	144,545,774	64,356,770	28,779,323	420,582,039	658,263,905	1,206,902,925
const	2,073,514	1,077,989	767,878	75,273,803	2,434,701	773,845	19,925,592	0	109,498	183,340,677	11,211	14,832,735	695,109	198,879,733	301,316,553
Distb	12,115,589	3,587,407	45,311,567	4,894,493	16,812,759	12,658,261	7,668,354	6,735,117	592,487	32,415,454	442,354,481	40,636,698	0	515,406,632	625,782,665
Trans	2,953,551	9,180,580	33,311,782	3,166,605	60,043,836	80,162,155	58,220,196	3,471,905	2,381,199	217,789,452	5,065,645	2,407,455	26,838,518	252,101,070	504,992,879
BusiFin	23,495,803	10,090,685	89,003,180	37,424,486	90,544,215	63,480,924	222,560,848	35,743,548	10,076,034	171,964,767	6,794,854	23,001,323	106,362,585	308,123,530	890,543,254
Edupub	3,363,432	1,413,756	23,662,782	971,045	4,849,213	4,391,955	4,066,654	232,845,313	528,739	374,857,653	40,435,359	40,316,193	59,797,427	515,406,632	791,499,521
OthSect	4,556,908	371,111	15,047,724	863,944	4,041,684	7,015,854	13,807,081	3,396,017	5,816,875	21,632,997	496	54,565,917	2,232,035	78,431,444	133,348,642
Labour	9,647,823	9,647,823	184,380,610	48,239,113	249,771,407	55,742,975	100,766,147	377,337,062	35,375,350						1,070,908,309
Capital	211,640,894	43,573,514	473,883,295	150,640,620	265,635,225	196,358,095	207,357,383	138,069,570	43,056,094						1,730,214,691
imports	-63,650,855	82,475,457	34,059,348	-48,582,454	-131,286,684	33,477,865	157,170,426	-73,167,422	19,077,523						9,573,205
taxes	780,992	4,724,549	788,127	-19,045,613	-4,128,993	8,369,466	59,914,201	-69,663,942	8,688,007						-9,573,205
Gross output	291834991.5	204941645	1206902925	301316553.2	625782665.5	504992878.8	890543254.2	791499521.1	133348642	1,319,603,554	559,050,611	205,907,034	716,561,801	2,801,123,000	

## Input-Output Table of North East Lincolnshire (preliminary version September 2006)

	Agric	Utils	Manu	const	Distb	Trans	BusiFin	Edupub	OthSect	Total IntD	Cons	Inv	Gov	Exp	FinDem
Agric	3,662,300	80,906	28,077,835	7,676	1,399,997	178,139	31,282	0	65,900	33,504,035	6,791,111	1,009	42,381	1,959,634	8,794,135
Utils	34,946	16,134,984	13,286,077	769,476	260,638	63,091	16,684	78,823,409	25,381	109,414,685	1,004,258	2,962	139,233	23,477,124	24,623,578
Manu	8,601,863	9,159,724	161,462,031	24,738,378	39,569,776	30,580,468	24,145,357	24,108,337	4,448,724	326,814,659	103,505,463	46,084,206	20,608,124	301,167,843	471,365,636
const	300,532	705,039	507,833	40,459,874	1,496,805	560,394	12,333,360	0	65,010	56,428,849	97,284,282	5,949	7,870,550	368,839	105,529,620
Distb	1,756,017	2,346,276	29,966,619	2,630,803	10,336,149	9,166,718	4,746,488	5,070,178	351,766	66,371,013	20,021,831	273,226,065	25,099,791	0	318,347,687
Trans	428,084	6,004,387	22,030,610	1,702,059	36,913,753	58,050,932	36,036,602	2,613,641	1,413,742	165,193,809	173,216,847	4,028,915	1,914,747	21,345,770	200,506,278
BusiFin	3,405,450	6,599,625	58,861,888	20,115,763	55,664,777	45,970,905	137,758,670	26,907,647	5,982,245	361,266,969	106,013,577	4,188,921	14,179,955	65,570,863	189,953,316
Edupub	487,491	924,641	15,649,284	521,939	2,981,199	3,180,517	2,517,140	175,285,322	313,918	201,861,451	286,541,495	30,908,821	30,817,731	45,709,202	393,977,248
OthSect	660,472	242,718	9,951,751	464,372	2,484,747	5,080,663	8,546,180	2,556,512	3,453,538	33,440,953	12,613,132	289	31,814,692	1,301,389	45,729,502
Total IntDem	19,337,155	42,198,301	339,793,928	91,410,341	151,107,840	152,831,826	226,131,763	315,365,045	16,120,224	1,354,296,423	806,991,996	358,447,137	132,487,203	460,900,664	1,758,827,000
Labour	4,416,096	4,416,096	153,680,145	43,277,742	242,885,287	68,007,880	107,752,745	230,520,217	27,379,796	882,336,005					
Capital	4,378,039	20,207,482	317,685,491	62,251,878	75,462,400	132,498,398	82,200,571	163,457,031	18,349,706	876,490,995					
imp	14,342,866	63,574,560	-12,685,724	-25,129,902	-82,153,096	9,889,587	97,838,609	-58,143,850	11,900,966	19,434,015					
tax	-175,986	3,641,824	-293,545	-9,851,590	-2,583,732	2,472,397	37,296,597	-55,359,744	5,419,764	-19,434,015					
Gross output	42298170.28	134038262.9	798180294.6	161958468.5	384718700.2	365700087.2	551220284.7	595838699.5	79170455.36	3,132,557,438	806,991,996	358,447,137	132,487,203	460,900,664	1,758,827,000

## Input-Output Table of North Lincolnshire (preliminary version September 2006)

	Agric	Utils	Manu	const	Distb	Trans	BusiFin	Edupub	OthSect	Total IntD	Cons	Inv	Gov	Exp	FinDem
Agric	7,271,493	87,967	40,772,323	11,184	1,700,571	226,136	43,015	0	97,661	50,210,350	26,080,291	3,875	162,760	7,525,695	33,772,620
Utils	69,384	17,543,091	19,292,948	1,121,233	316,596	80,090	22,941	68,976,307	37,612	107,460,203	1,561,049	4,605	216,429	36,493,553	38,275,636
Manu	17,078,989	9,959,097	234,461,881	36,047,206	48,065,243	38,820,014	33,201,980	21,096,576	6,592,745	445,323,732	156,724,940	69,779,355	31,204,217	456,019,524	713,728,036
const	596,706	766,568	737,434	58,955,581	1,818,163	711,386	16,959,450	0	96,341	80,641,629	143,215,785	8,758	11,586,527	542,982	155,354,052
Distb	3,486,570	2,551,037	43,515,059	3,833,441	12,555,277	11,636,582	6,526,836	4,436,780	521,296	89,062,878	23,789,476	324,640,878	29,822,990	0	378,253,344
Trans	849,960	6,528,393	31,991,040	2,480,133	44,838,983	73,692,070	49,553,483	2,287,129	2,095,082	214,316,272	215,902,975	5,021,767	2,386,602	26,606,045	249,917,388
BusiFin	6,761,517	7,175,577	85,474,392	29,311,423	67,615,776	58,357,222	189,430,231	23,546,179	8,865,331	476,537,649	157,071,763	6,206,386	21,009,294	97,151,056	281,438,500
Edupub	967,913	1,005,335	22,724,603	760,537	3,621,251	4,037,470	3,461,288	153,387,609	465,207	190,431,212	240,717,248	25,965,825	25,889,302	38,399,302	330,971,676
OthSect	1,311,367	263,900	14,451,114	676,654	3,018,211	6,449,587	11,751,746	2,237,137	5,117,939	45,277,655	19,872,383	455	50,125,039	2,050,379	72,048,256
Total IntDem	38,393,900	45,880,965	493,420,793	133,197,392	183,550,072	194,010,558	310,950,971	275,967,717	23,889,214	1,699,261,580	984,935,909	431,631,905	172,403,159	664,788,535	2,253,759,508
Labour	2,708,849	7,223,598	196,843,045	76,750,728	211,290,241	82,168,427	97,518,573	197,745,995	31,603,241	903,852,697					
Capital	31,063,771	31,052,038	516,884,991	78,603,324	166,963,103	167,748,961	183,919,927	133,225,681	40,445,015	1,349,906,811					
Taxes	11,963,238	58,242,838	-47,009,274	-37,754,856	-91,606,162	16,244,571	119,885,636	-43,817,306	14,695,865	844,552					
Imports	-146,788	3,336,400	-1,087,786	-14,800,907	-2,881,032	4,061,143	45,701,041	-41,719,199	6,692,576	-844,552					
Gross output	83,982,970	145,735,839	1,159,051,768	235,995,681	467,316,222	464,233,660	757,976,149	521,402,888	117,325,911	3,953,021,088	984,935,909	431,631,905	172,403,159	664,788,535	2,253,759,508

## Short-Run Impact Analysis with Input-Output Model

$$\Delta X = (I - A)^{-1} \Delta F$$

$$\Delta X = (I - A)^{-1} \Delta C_{man} \quad (0.1)$$

$$\Delta L = \sum_i l_i (I - A)^{-1} \Delta C_{man} \quad (0.1)$$

$$\Delta K = \sum_i k_i (I - A)^{-1} \Delta C_{man} \quad (0.1)$$

$$M_{s,i} = IOM_{s,i} X_i$$

## Production Functions Across Regions of the Humber Economy

$$Y_{i,r} = \min \left( \frac{INT_{j,i,r}}{a_{i,j,r}}, \left( K_{i,r}^{\beta_r} L_{i,r}^{1-\beta_r} \right) \right) \quad (1)$$

Here  $Y_{i,r}$  is output of the sector  $i$  good in region  $r$ ,

$K_{i,r}$  is capital services originating in region  $r$  but used to produce the good  $i$  in region  $r$ ,

$L_{i,r}$  are labour services originating in region  $r$  but used to produce the sector  $i$  good in region  $r$ ,

$INT_{j,i,r}$  is an intermediate input originating in sector  $j$  of region  $r$  but used to produce the sector  $i$  good in region  $r$ ,

$a_{j,i,r}$  is a coefficient that gives the amount of the sector  $j$  intermediate input of region  $r$  used to produce the sector  $i$  good in region  $r$ , and  $\beta_r$  is the share of capital income in sectoral output in region  $r$ .

Land and natural resources are additional inputs in case of agriculture sector.

## Production, Trade and Transport Across Regions of Humber Economy

$$Y_{i,r} = \left( \delta_{i,r} YD_{i,r}^{\eta_{i,r}} + (1 - \delta_{i,r}) X_{i,r}^{\eta_{i,r}} \right)^{\frac{1}{\eta_{i,r}}} \quad (2)$$

where  $YD_{i,r}$  is domestic sales of output of good  $i$  in region  $r$ ,  $X_{i,r}$  is exports of good  $i$  from a region  $r$ ,  $\delta_{i,r}$  is the share of domestic sales of gross output,  $Y_{i,r}$ , and  $\eta_{i,r}$  is the

Absorption of region,  $r$  is given by a CES aggregation of imports and domestic supplies

$$A_{i,r} = \left( \mu_{i,r} YD_{i,r}^{\sigma_{i,r}} + (1 - \mu_{i,r}) M_{i,r}^{\sigma_{i,r}} \right)^{\frac{1}{\sigma_{i,r}}} \quad (3)$$

Transportation services are proportional to trade:

$$T_{i,r,s} = \tau_{i,r,s} M_{i,r,s} \quad (4)$$

Here  $T_{i,r,s}$  transportation services,  $\tau_{i,r,s}$  is transport cost per unit of traded goods

$M_{i,r,s}$  amount of good  $i$  traded from region  $r$  to  $s$ .

Household consumption is a Cobb-Douglas aggregation of sector  $i$  commodities over all  $r$  regions.

$$U_r = \prod_{i,r} C_{i,r}^\gamma \quad (5)$$

Households receive factor income from all regions and transfers from their own government. The income of the representative household in each region is

$$I_r = \sum_i w_r L_{i,r} + \sum_r r_r K_{i,r} + RV_r \quad (7)$$

where  $I_r$  is income,  $w_r$  is wage rate and  $r_r$  is the interest rate and  $RV_r$  is the transfer received by a representative household in region  $r$ .

Government consumption demand reflects a Cobb-Douglas aggregate of all sector  $i$  commodities over all  $r$  regions.

$$G_r = \prod_{i,r} GD_{i,r}^\gamma \quad (8)$$

$GD_{i,r}^g$  is the government consumption of good  $i$  in region  $r$ . The government in each region collects taxes from factors income, intermediate inputs, imports and domestic sales.

## Market Clearing Conditions in Humber Market

$$G_r = \tau_k r_r \bar{K}_r + \tau_w w_r \bar{L}_r + \tau_{i,r} P_{i,r} Y_{i,r} + \tau_{N,r} P_{i,r} INT_{j,i,r} \quad (9)$$

Here  $G_r$  is total government revenue,  $\tau_{k,r}$  is tax rate on capital income,  $\tau_{w,rr}$  is tax rate on labour income,  $\tau_{w,r}$  is tax rate in wage income,  $\tau_{i,r}$  is tax rate on intermediate income,  $\tau_{N,r}$  is tax rate on intermediate input.

the market clearing condition for the goods market is given by

$$Y_{i,r} = \sum_r C_{i,r} + \sum_{rr,j} a_{i,j,r} INT_{i,j,r} \quad (10)$$

The global capital market clearing condition implies

$$\sum_r \bar{K}_r = \sum_{i,r} K_{r,ri} \quad (11)$$

and labour market clears at the regional level:

$$LS_r = \sum_i LS_{i,r} \quad (12)$$

When there are  $r.n$  different markets in the economy, relative prices that clear  $rn-1$  markets also clear the  $rn$ th market as well.

# Sectoral and Regional Comparison of Labour, Capital, Domestic Output and Supply: Results from the Regional General Equilibrium Model of Humber Regions

	LABOUR	CAPITAL	DOMOUT	SUPPLY	DOMOUTR	SUPPLYR	LABOURR	CAPITALR
AGRIC .HULL	158841	2967040	62336060	61952430	1.000	1.000	1.000	1.000
AGRIC .ER	9647823	211640900	306175200	242524300	4.912	3.915	60.739	71.331
AGRIC .NL	2708849	31063770	76604060	76457270	1.229	1.234	17.054	10.470
AGRIC .NEL	4416096	4378039	25995670	40338540	0.417	0.651	27.802	1.476
UTILS .HULL	9530462	31105990	196647800	203436700	1.000	1.000	1.000	1.000
UTILS .ER	9647823	43573510	71722790	154198200	0.365	0.758	1.012	1.401
UTILS .NL	7223598	31052040	105905900	109242300	0.539	0.537	0.758	0.998
UTILS .NEL	4416096	20207480	46986580	110561100	0.239	0.543	0.463	0.650
MANU .HULL	328801000	552697500	890588900	889521200	1.000	1.000	1.000	1.000
MANU .ER	184380600	473883300	752261500	786320900	0.845	0.884	0.561	0.857
MANU .NL	196843000	516885000	704120000	703032200	0.791	0.790	0.599	0.935
MANU .NEL	153680100	317685500	509698200	497012500	0.572	0.559	0.467	0.575
CONST .HULL	65124830	69288060	228650000	217540300	1.000	1.000	1.000	1.000
CONST .ER	48239110	150640600	349203900	300621400	1.527	1.382	0.741	2.174
CONST .NL	76750730	78603320	250253600	235452700	1.094	1.082	1.179	1.134
CONST .NEL	43277740	62251880	186719500	161589600	0.817	0.743	0.665	0.898
DISTB .HULL	385983700	145416000	651643800	647427900	1.000	1.000	1.000	1.000
DISTB .ER	249771400	265635200	757069300	625782700	1.162	0.967	0.647	1.827
DISTB .NL	211290200	166963100	470197300	467316200	0.722	0.722	0.547	1.148
DISTB .NEL	242885300	75462400	466871800	384718700	0.716	0.594	0.629	0.519
TRANS .HULL	85774160	217436300	547268900	554995600	1.000	1.000	1.000	1.000
TRANS .ER	55742970	196358100	444676500	478154400	0.813	0.862	0.650	0.903
TRANS .NL	82168430	167749000	433566500	437627600	0.792	0.789	0.958	0.771
TRANS .NEL	68007880	132498400	334464700	344354300	0.611	0.620	0.793	0.609
BUSIFIN.HULL	200139700	187469500	831399800	891274200	1.000	1.000	1.000	1.000
BUSIFIN.ER	100766100	207357400	627010200	784180700	0.754	0.880	0.503	1.106
BUSIFIN.NL	97518570	183919900	615124100	660825100	0.740	0.741	0.487	0.981
BUSIFIN.NEL	107752700	82200570	387810800	485649400	0.466	0.545	0.538	0.438
EDUPUB .HULL	435224500	299357600	1126215000	1022180000	1.000	1.000	1.000	1.000
EDUPUB .ER	377337100	138069600	804869500	731702100	0.715	0.716	0.867	0.461
EDUPUB .NL	197746000	133225700	524722800	483003600	0.466	0.473	0.454	0.445
EDUPUB .NEL	230520200	163457000	608273300	550129500	0.540	0.538	0.530	0.546
OTHSECT.HULL	77832110	28447840	155383500	163666100	1.000	1.000	1.000	1.000
OTHSECT.ER	35375350	43056090	112039100	131116600	0.721	0.801	0.455	1.514
OTHSECT.NL	31603240	40445010	108583000	115275500	0.699	0.704	0.406	1.422
OTHSECT.NEL	27270800	48240710	65069100	77660070	0.425	0.476	0.352	0.645

## Dynamic Model of Humber Region: Consumer's Problem and Demand

$$U(C_t^h) = \frac{(C_t^h)^{\frac{1}{1-\sigma}} - 1}{1-\sigma}$$

$$C_t^h = \left( \prod_{i=1}^{11} C_{i,t}^{\alpha_i^h} \right)$$

$$U_t^h = \left( \frac{\prod_{i=1}^{11} C_{i,t}^{\alpha_i^h}}{1-\sigma} \right)^{1-\sigma}$$

$$\sum_{t=0}^{\infty} P_t C_t^h = \sum_t w_t^h L_t^h + M_o^h$$

$$\frac{C_{t+1}}{C_t} = \frac{\bar{C}_{t+1}}{\bar{C}_t} \left( \frac{P_t}{P_{t+1}} \right)^{\sigma} \left( \frac{\bar{P}_{t+1}}{\bar{P}_t} \right)^{\sigma} (1+g)(1+r)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right)^{\frac{1}{\sigma}}$$

## Producer and Investor's Problem

$$\Pi_{j,t}^y = [(\theta_j^x PX_{j,t}^{1+\eta} + (1 - \theta_j^x) PD_{j,t}^{1+\eta})]^{\frac{1}{1+\eta}} - \theta^v PV_j^v - (1 - \theta^v) \sum_j a_{i,j} P_{i,t} \leq 0$$

$$\Pi_{j,t}^I = P_{t+1}^k - \sum_i P_{i,t}^A a_{i,j}^I \leq 0$$

$$\Pi_{j,t}^k = (1 - \delta) P_{j,t+1}^k + r_{j,t}^k - P_{j,t}^k \leq 0$$

$$K_{j,t+1} = I_{j,t} + (1 - \delta) K_{j,t}$$

$$I_{j,t} = (g + \delta_j) K_{j,t}$$

# Armington Conditions for Trade

$$\mathfrak{R}_{i,,w,,t} = PE_{i,t}E_{i,t} + PXE_{i,t}XE - P_{i,t}Y_{i,t}$$

$$+ \Phi_{e,w,t} \left[ Y_{i,t} - \Psi(\gamma_i E_{i,t}^{\chi_i} + (1 - \gamma_i) XE_{i,t}^{\chi_i})^{\frac{1}{\chi_i}} \right]$$

$$Y_{i,t} = \Psi(\gamma_i E_{i,t}^{\chi_i} + (1 - \gamma_i) XE_{i,t}^{\chi_i})^{\frac{1}{\chi_i}}$$

$$PD_{i,t}Y_{i,t} = PE_{i,t}E_{i,t} + PXE_{i,t}XE_{i,t}$$

## Definition of a Competitive Equilibrium in the Economy

prices of composite commodities,  $P_{i,t}$  ;

Prices of domestic goods sold in domestic markets,  $PD_{i,t}$  ;

prices of exported commodities,  $PX_{i,t}$  ;

prices of capital goods ,  $P_{j,t}^k$  ;

prices of terminal capital ,  $PTK_{j,t}$  ;

wage rates for each categories of labor,  $w_{h,t}$  ;

prices of government services,  $PG_t$  ;

prices of provisions for tourism,  $PT_t$  ;

prices of transfer,  $PR_t$  ;

prices of consumption,  $PU_t$  ;

price of aggregate welfare,  $PW_t$  ;

price of foreign exchange,  $PFX_t$  ,

present value of foreign exchange,  $PVPFX_t$  ;

rental rate of capital for each sector,  $r_1^k : \mathbf{R}_+ \rightarrow \mathbf{R}$ , and

sequence of gross output,  $Y_{i,t}$  ; total supply of commodities,  $A_{i,t}$  ; sectoral capital

stock,  $K_{i,t}$  ; sectoral investment,  $I_{i,t}$  ;

exports,  $X_{i,t}$  ; government services,  $GOV_t$  ;

level of household utility from consumption,  $U_t$  ; and total welfare,  $W$  such that given these prices and commodities

- i) households solve intertemporal utility maximization problems
- ii) investors solve intertemporal profit maximization problem,
- iii) markets for goods and services, labor , capital clear
- iv) government constraint is satisfied
- v) and balance of payments condition is fulfilled

# Calibration of the Dynamic Model

$$P_1^I = 1 = P_2^k = (1-r)P_1^k \Rightarrow P_1^k = \frac{1}{1-r}$$

$$\frac{P_{t+1}^k}{P_t^k} = (1-r) \quad P_1^k = r_1^k + (1-\delta)(1-r)P_1^k$$

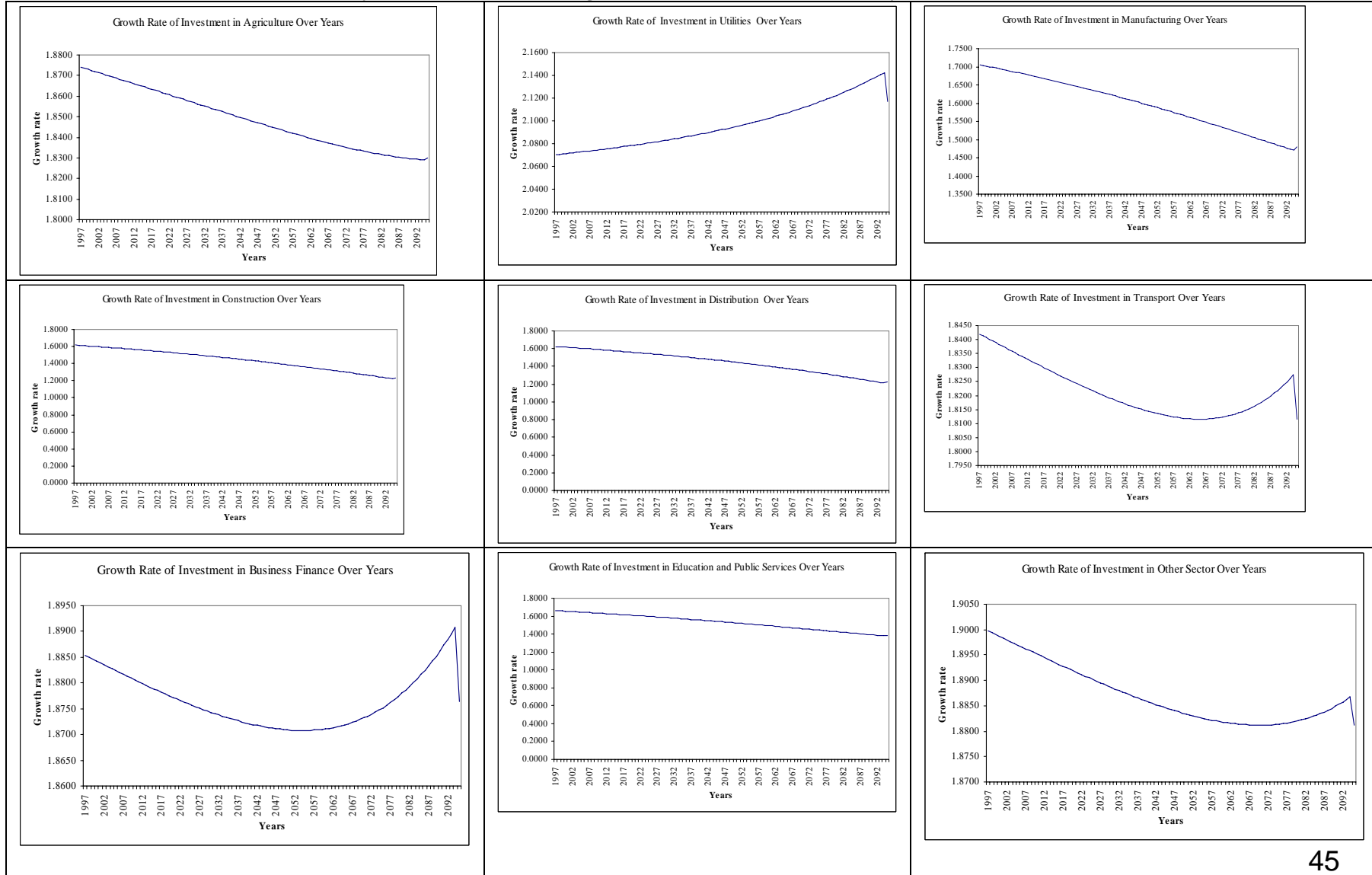
$$\frac{1}{1-r} = r_1^k + (1-\delta) \quad V_1 = r_1^k K_1$$

$$K_1 = \frac{V_1}{\frac{r}{1-r} + \delta} \quad \frac{I_1}{V_1} = \frac{\delta + g}{\frac{r}{1-r} + \delta} \quad \frac{I_1}{V_1} = 1$$

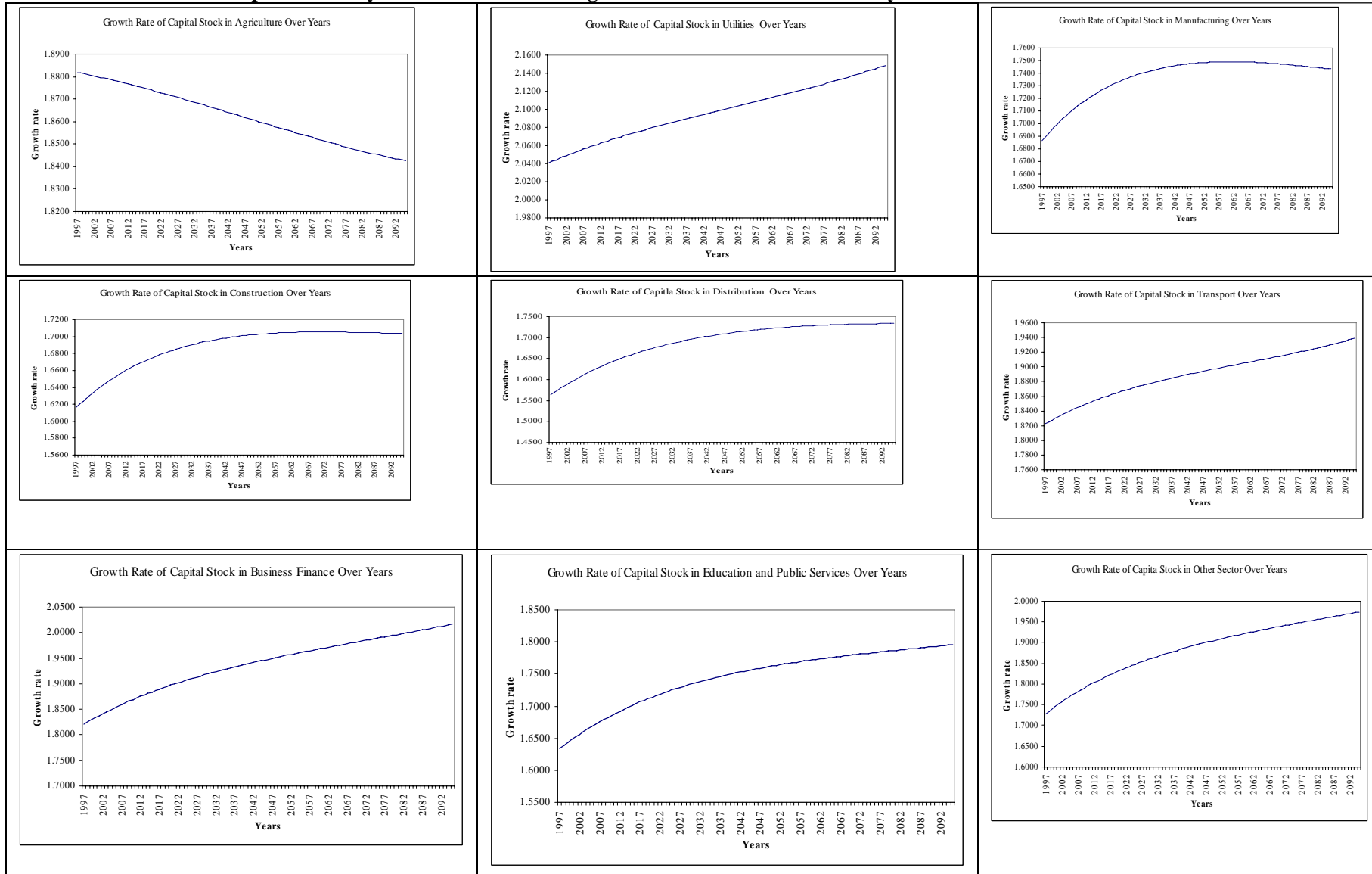
$$g = \frac{r}{1-r} \quad r = \frac{g}{1+g} \quad \frac{I_1}{V_1} \neq 1$$

$$\delta_j = g \frac{V_j}{I_j - V_j} - \frac{r}{1-r} \frac{I_j}{I_j - V_j}$$

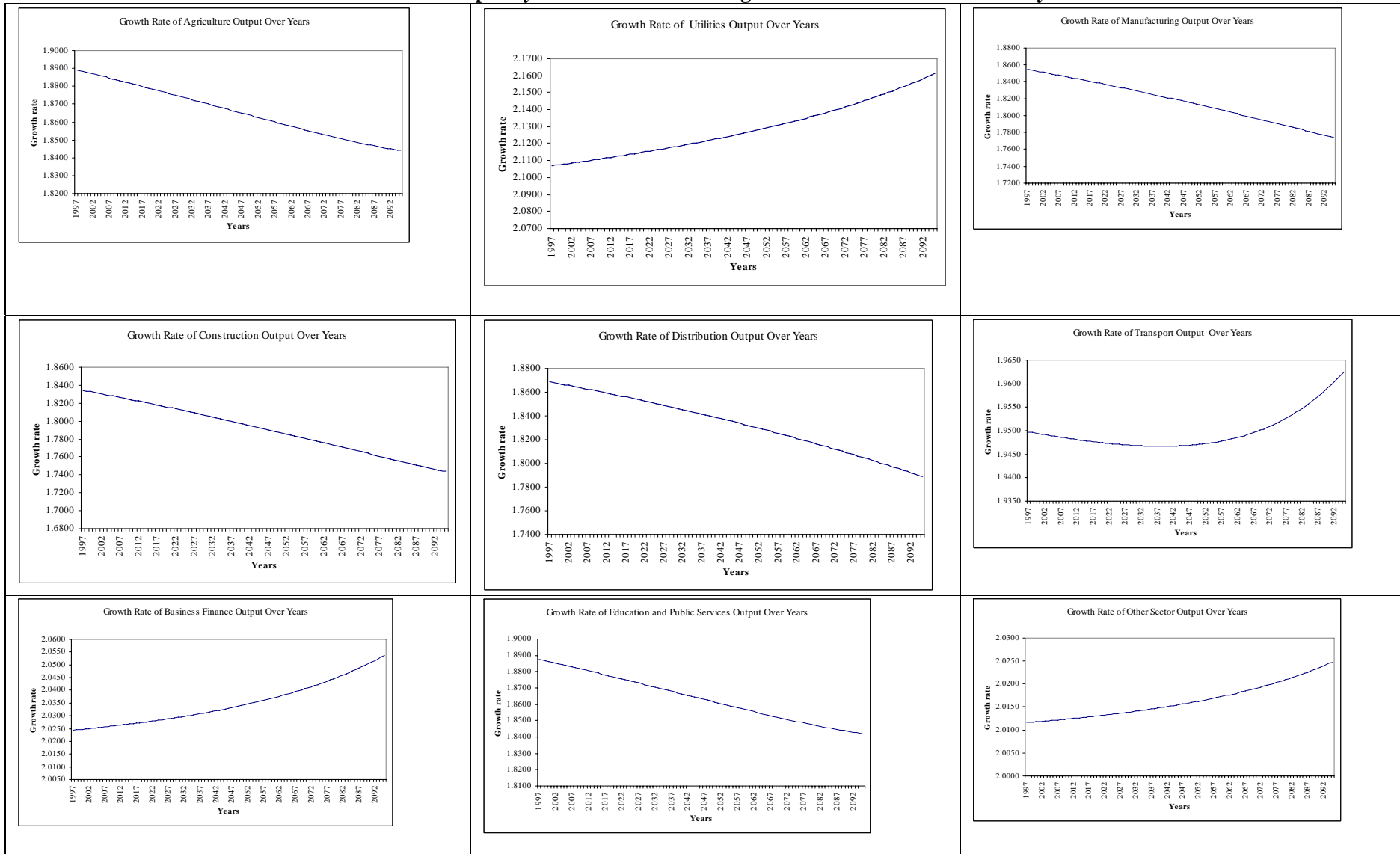
**Figure 1**  
**Growth Rates of Investment by Sectors in Humber Region GE Model Over 21<sup>st</sup> Century**



**Figure 2**  
**Growth Rates of Capital Stock by Sectors in Humber Region GE Model Over 21<sup>st</sup> Century**



**Figure 3**  
**Growth Rates of Output by Sectors in Humber Region GE Model Over 21<sup>st</sup> Century**



## Conclusions of this study

- **First input output table for Humber Region: Hull, East Riding, North Lincolnshire and North East Lincolnshire**
- **First regional multisectoral general equilibrium model for Humber regions.**
- First dynamic model for Hull
- **Regional comparison is made for output, employment, capital and aggregate supply**
  - **dynamic growth paths are evaluated based on current preferences and technology for coming hundred years**
- Useful for policy analysis.
- Programmes and worksheets containing model are available.

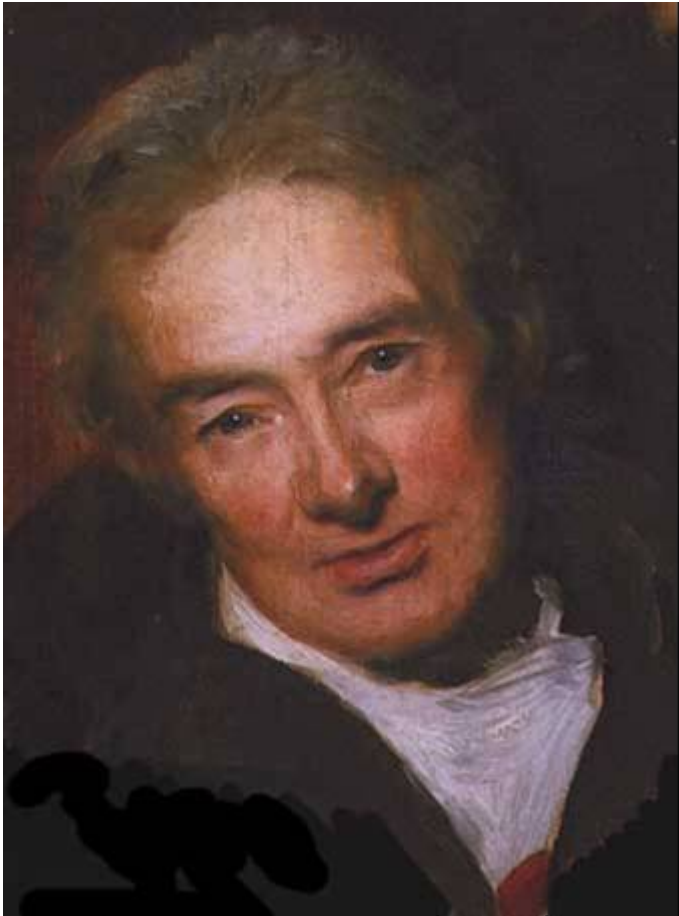
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