

Why Fuel Poverty? Dynamics of Income and Expenditure of Households in Yorkshire

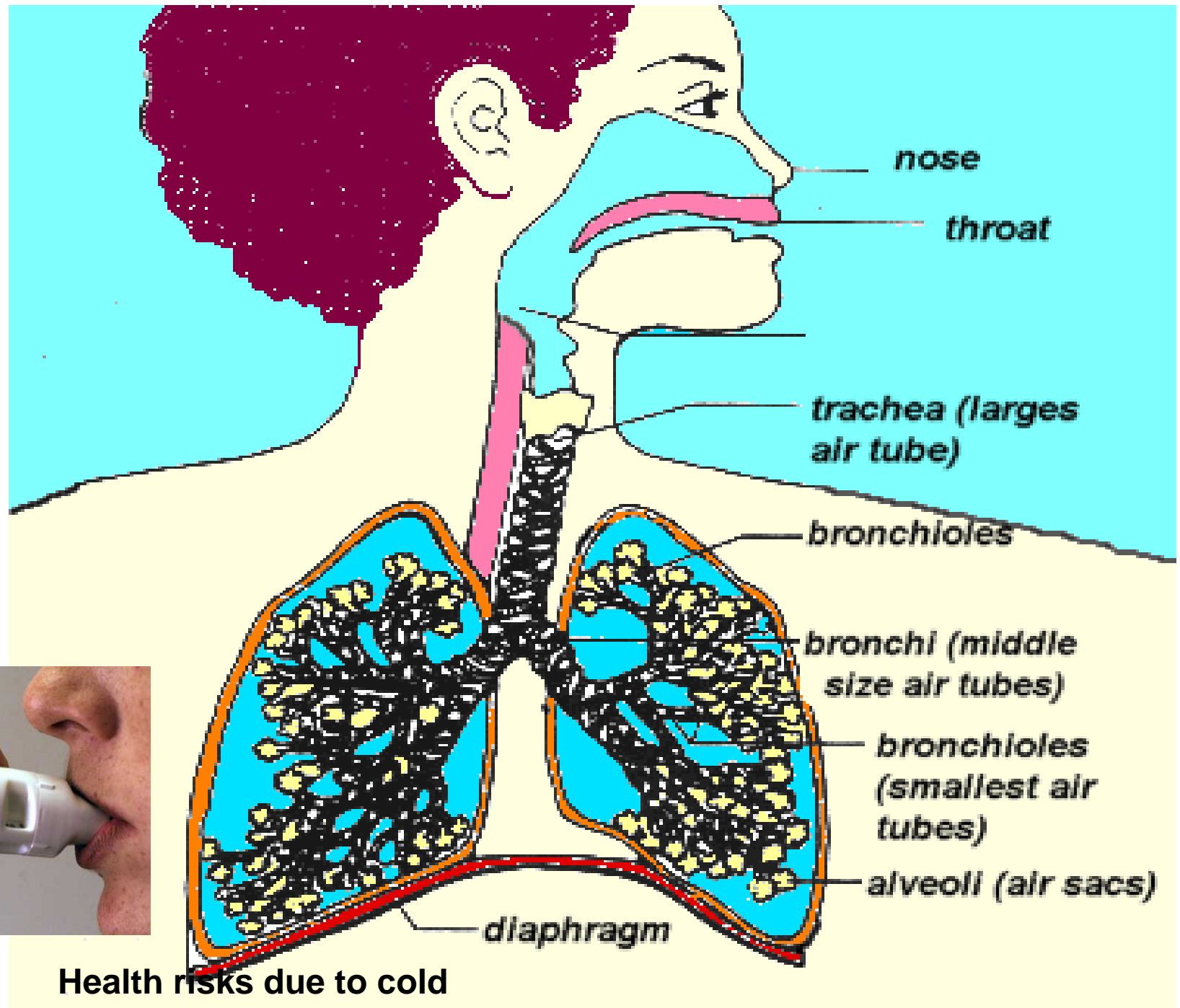
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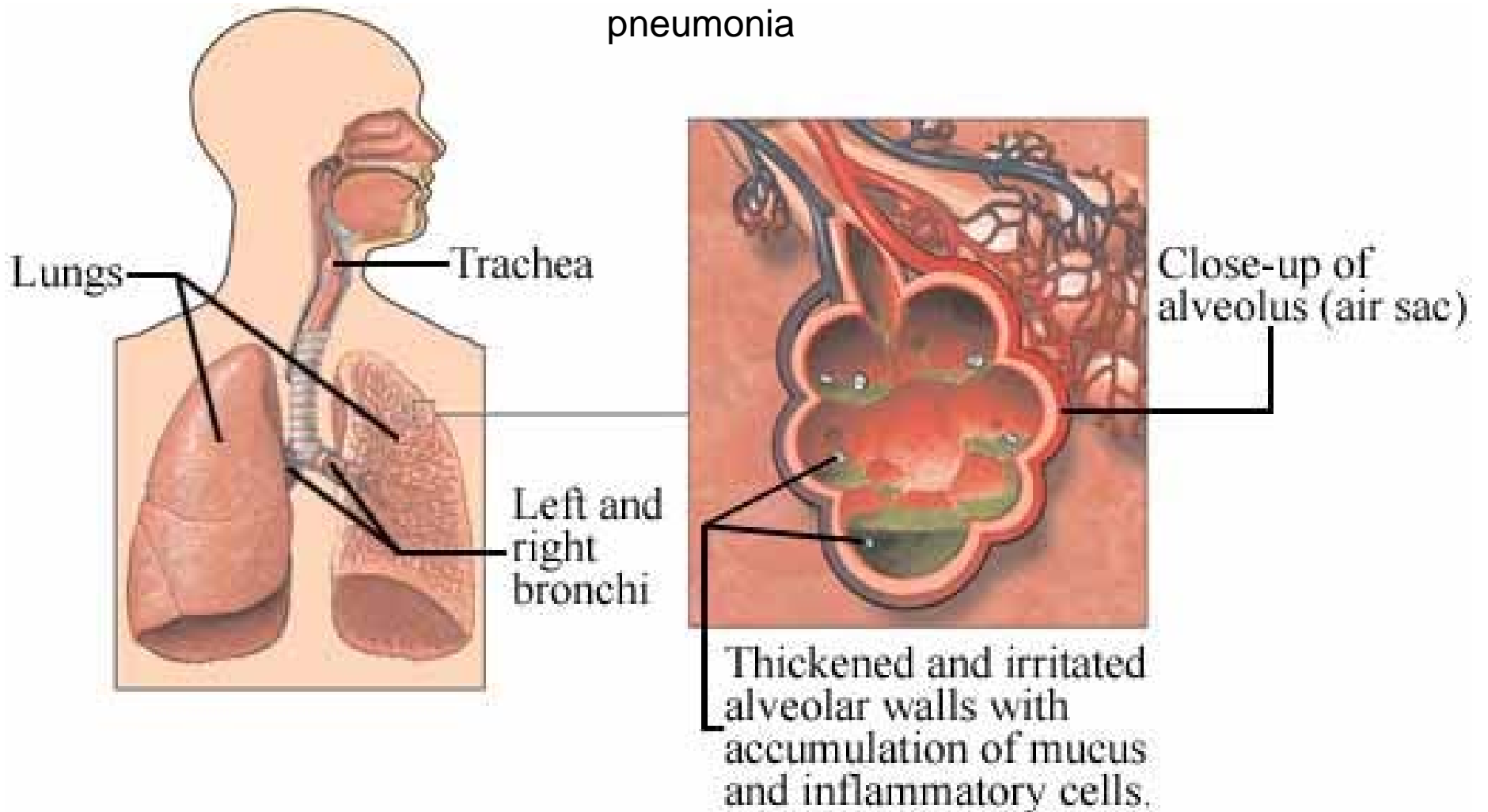
Illnesses and Diseases Due to Coldness

- Asthma
- Chronic obstructive pulmonary disease, blood pressure
- Heart attack
- Arthritis
- Loss of strengths of fingers and mental retardation
- Isolation of elderly and created obstacles in smooth education process of younger pupils
- 163,000 households in the Yorkshire and Humber (7.7% of the households)
- Excess death of more than 3200 individuals annually in Yorkshire alone.



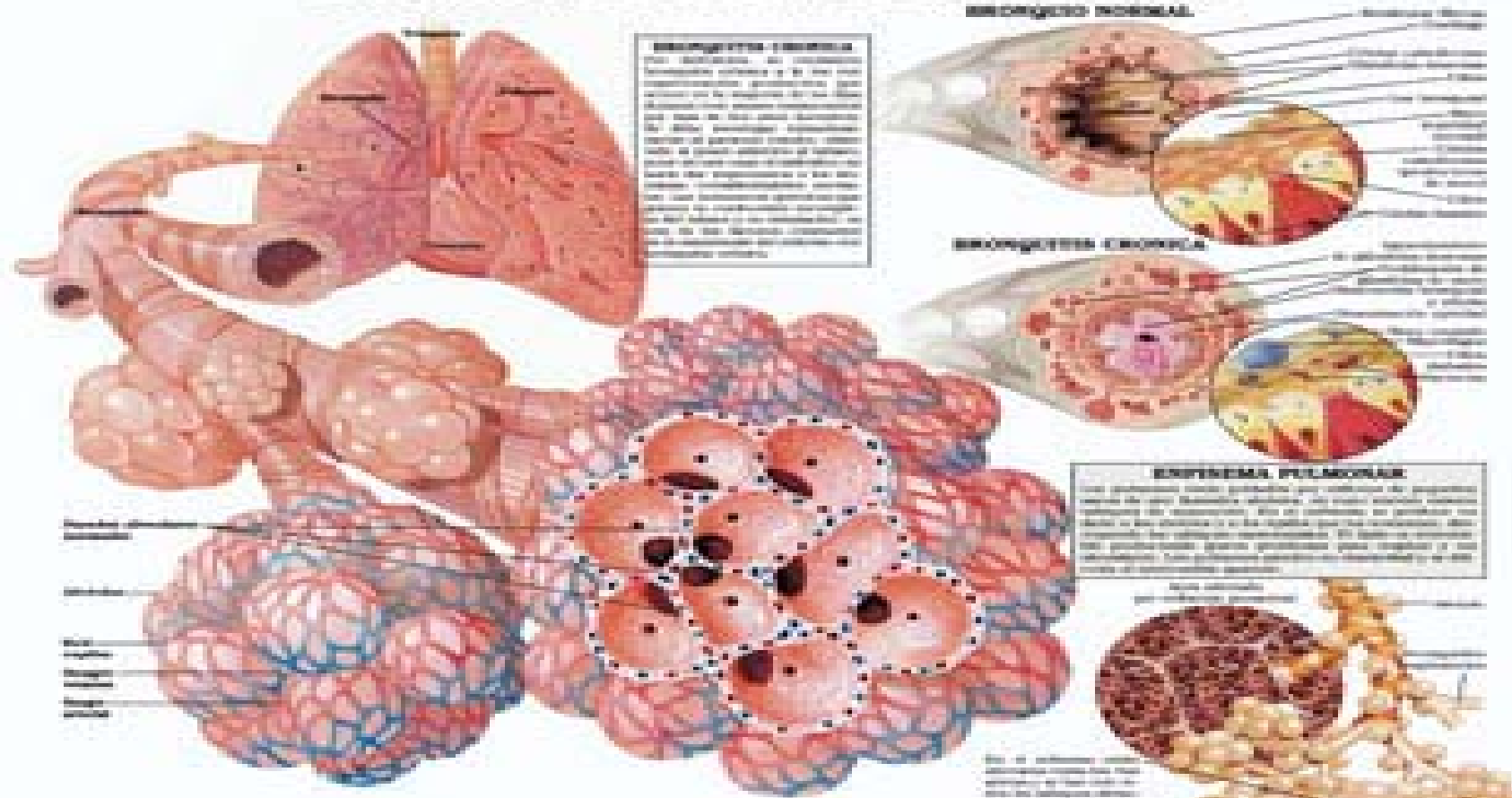
Health risks due to cold

pneumonia

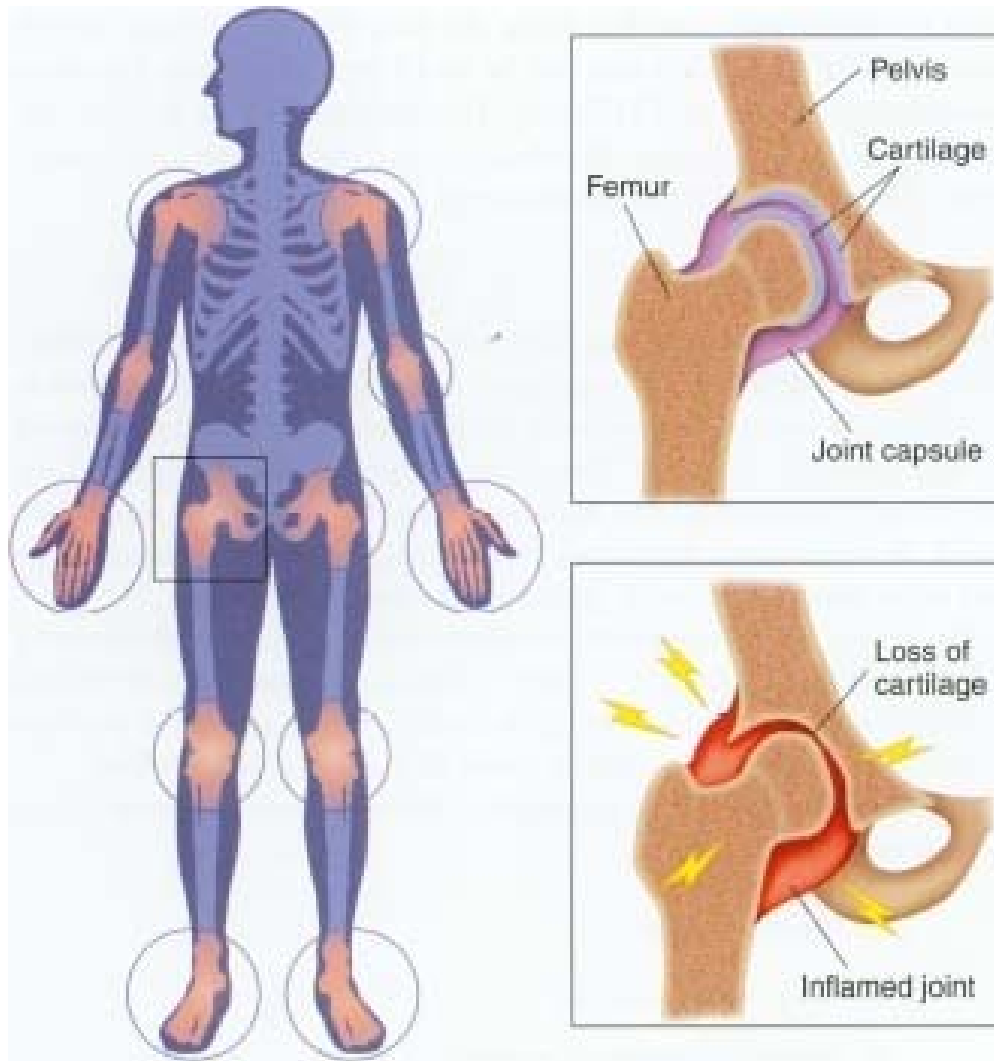


Health risks due to cold

CHRONIC OBSTRUCTIVE PULMONARY DISEASE (COPD)



Health risks due to cold

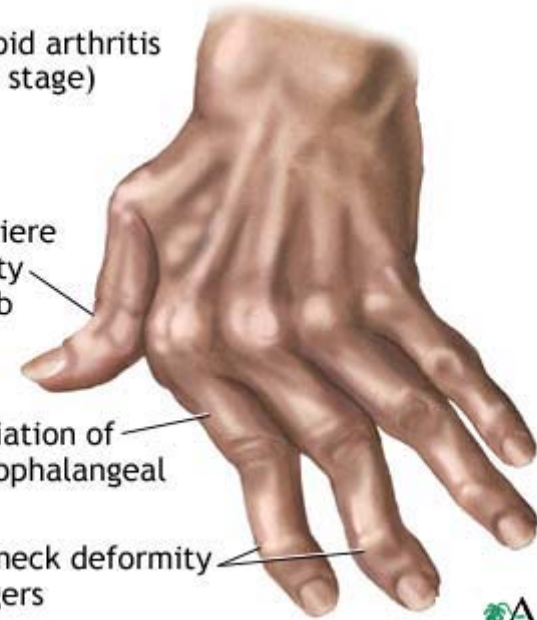


Rheumatoid arthritis
(late stage)

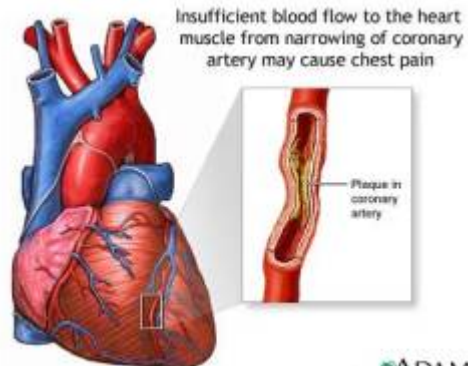
Boutonniere
deformity
of thumb

Ulnar deviation of
metacarpophalangeal
joints

Swan-neck deformity
of fingers



ADAM.



Insufficient blood flow to the heart
muscle from narrowing of coronary
artery may cause chest pain

Plaque in
coronary
artery

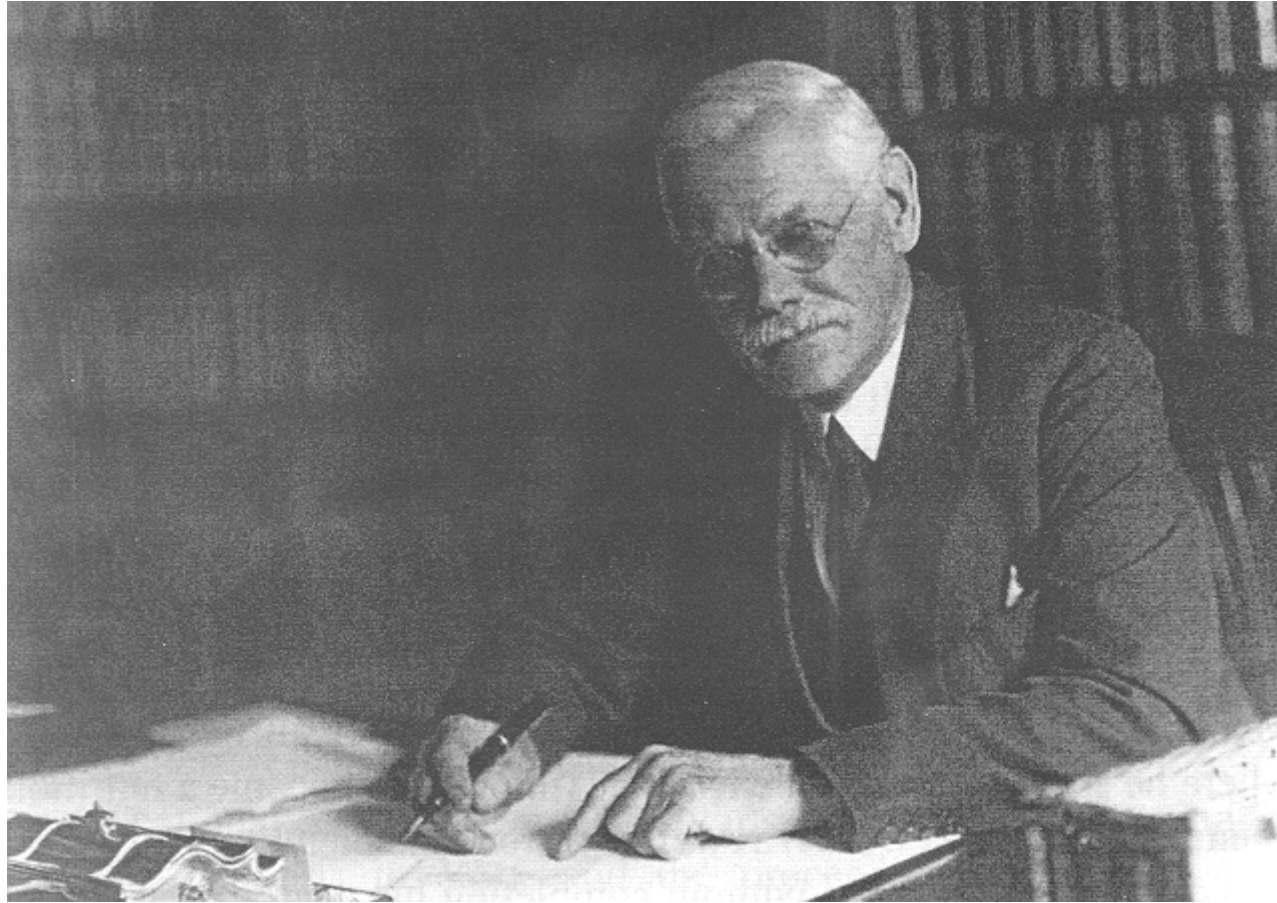
ADAM.



Health risks due to cold

Literature Review

Rowntree B. S. (1902) *Poverty of Town Life*, MacMillan, London.



Rowntree (1902)

Table A1
Income and source of income 1970 to 2002-03 based on un-weighted data unless otherwise footnoted

	Grossed number of households	Number of households in the sample	Weekly household income ¹				Source of income					
			Current prices		Constant prices		Wages and salaries	Self employment	Annuities and pensions ²	Investments	Social security benefits ³	Other sources
			Disposable	Gross	Disposable	Gross						
(000s)	Number	£	£	£	£	Percentage of gross weekly household income						
1970		6,393	28	34	269	324	77	7	3	4	9	1
1980		6,944	115	140	299	366	75	6	3	3	13	1
1990		7,046	253	317	363	447	67	10	5	6	11	1
1995-96		6,797	307	381	363	450	64	9	7	5	14	2
1996-97		6,415	325	397	375	458	65	9	7	4	14	1
1997-98		6,409	343	421	383	470	67	8	7	4	13	1
1998-99 ⁴	24,660	6,630	371	457	402	495	68	8	7	4	12	1
1999-2000	25,340	7,097	391	480	417	512	66	10	7	5	12	1
2000-01	25,030	6,637	409	503	424	521	67	9	7	4	12	1
2001-02 ⁵	24,450	7,473	442	541	451	552	69	9	7	4	11	1
2002-03	24,350	6,927	453	552	453	552	68	8	7	3	12	1

1. Does not include imputed income from owner-occupied and rent-free households.
2. Other than social security benefits.
3. Excluding housing benefit and council tax benefit (rates rebate in Northern Ireland) and their predecessors in earlier years - see appendix D.
4. Based on grossed data from 1998-99
5. From 2001-02 onwards, weighting is based on the population figures from the 2001 census

Table A2
Household expenditure as a percentage of total expenditure 1978 to 2002-03 (revised4)

Year				1978	1980	1982	1984	1986	1988	1990	1992	1994	1995	1995	1996	1997	1998	1999	2000	2001	2002	2003	
Grossed number of households (thousands)														24,130	24,310	24,560	24,660	25,330	25,030	24,450	24,346	24,350	
Total number of households in sample				7,001	6,944	7,428	7,081	7,178	7,265	7,046	7,418	6,853	6,797	6,797	6,415	6,409	6,630	7,097	6,637	7,473	6,927	6,927	
Total number of persons				19,019	18,844	20,022	18,557	18,330	18,280	17,437	18,174	16,617	16,586	16,586	15,732	15,430	16,218	16,786	15,925	18,122	16,586	16,586	
Average number of persons per household				2.7	2.7	2.7	2.6	2.6	2.5	2.5	2.5	2.4	2.4	2.4	2.5	2.4	2.4	2.3	2.4	2.4	2.4	2.4	
Commodity or service				Percentage of total expenditure										Percentage of total expenditure									
1	Housing (Net)			15	15	17	16	17	18	18	17	16	17	16	16	15	16	16	17	17	17	17	
2	Fuel and power			6	6	6	6	6	5	4	5	5	4	4	4	4	3	3	3	3	3	3	
3	Food and non-alcoholic drinks			24	23	21	21	20	19	18	18	18	18	18	18	17	17	17	16	16	16	16	
4	Alcoholic drink			5	5	5	5	5	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
5	Tobacco			3	3	3	3	3	2	2	2	2	2	2	2	2	2	2	2	1	1	1	
6	Clothing and footwear			8	8	7	7	8	7	6	6	6	6	6	6	6	6	6	6	6	5	5	
7	Household goods			8	8	7	8	8	7	8	8	8	8	8	9	8	8	9	8	8	8	8	
8	Household services			3	4	4	4	5	5	5	5	5	5	5	5	5	5	5	6	6	6	6	
9	Personal goods and services			3	4	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
10	Motoring			11	12	12	13	12	12	14	13	13	13	13	13	14	15	15	14	15	15	15	
11	Fares and other travel costs			3	3	3	2	2	2	3	3	2	2	2	2	3	2	3	2	2	2	2	
12	Leisure goods			4	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	
13	Leisure services			6	7	7	7	7	9	9	10	11	11	11	11	12	12	12	13	13	13	13	
14	Miscellaneous			1	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	
Jan-14	All expenditure groups			100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	

Table A3
Household expenditure by gross income decile group (based on weighted data and including children's expenditure)

		Lowest	Second	Third	Fourth	Fifth	Sixth	Seventh	Eighth	Ninth	Highest	All	2003
		Ten	decile	decile	Decile	Decile	decile	decile	decile	decile	ten	house-	
		Per	group	group	group	Group	group	group	group	group	per	holds	
		cent									cent		
Lower boundary of group (£ per week)			123	188	259	341	435	541	662	821	1,085		
Grossed number of households (thousands)		2,440	2,430	2,440	2,430	2,440	2,440	2,430	2,440	2,430	2,430	24,350	
Total number of households in sample		701	724	732	715	719	697	687	672	655	625	6,927	
Total number of persons in sample		901	1,278	1,412	1,590	1,718	1,807	1,967	1,917	1,974	2,022	16,586	
Total number of adults in sample		774	967	1,155	1,189	1,282	1,295	1,409	1,407	1,477	1,495	12,450	
Weighted average number of persons per household		1.3	1.7	1.9	2.2	2.4	2.5	2.8	2.8	3	3.2	2.4	
Commodity or service		Average weekly household expenditure (£)						Average weekly household expenditure (£)					
1	Food & non-alcoholic drinks	21.3	27.7	34.5	36.3	40.1	42.8	49	52	56.9	66.9	42.7	44.8
2	Alcoholic drinks, tobacco & narcotics	5.9	6.3	8.8	10.5	10.6	12	14.1	13.1	15.8	16.4	11.4	13.2
3	Clothing & footwear	6.2	7.9	11.4	15.8	18	20.2	26.2	31	36.6	49.6	22.3	50
4	Housing ¹ , fuel & power	20.9	28.4	29.3	34.8	37.6	38.1	37.1	39.7	42.8	60	36.9	11.9
5	Household goods & services	11	12.7	17.6	23.2	22	29.8	33	36.7	44.4	71.9	30.2	5.6
6	Health	1.6	2.2	4.4	3.1	4.5	5.5	5.2	6.5	5.3	9.9	4.8	17.4
7	Transport	13.1	16.1	22.6	36.9	46.9	58.1	66.3	84.8	107.8	139.6	59.2	23.1
8	Communication	5.4	5.9	7.5	8.8	9.5	10.7	11.3	13.6	15	18.3	10.6	15.4
9	Recreation & culture	16.6	21.3	28.6	43.1	46.7	54.7	68.4	79	89.3	116.4	56.4	11
10	Education	..	1.7	0.7	1.2	2	3.6	3.4	4.3	8.5	25.8	5.2	36.3
11	Restaurants & hotels	10.2	11.6	15.5	22.3	28.5	32.5	40.8	48.1	61.5	83.4	35.4	6.9
12	Miscellaneous goods & services	10.2	12.7	16	22.1	25.9	33.8	37.4	45.1	51.2	76.5	33.1	13.3
01-Dec	All expenditure groups	123.1	154.5	196.9	258.3	292.3	341.8	392.3	454.1	535.2	734.9	348.3	276.7
13	Other expenditure items	12.5	14.9	22.5	36.9	42.7	58.6	63.6	80	99	148.5	57.9	
Total expenditure		135.6	169.4	219.4	295.2	335	400.4	455.9	534.1	634.2	883.4	406.2	
Average weekly expenditure per person (£)													
Total expenditure		106	98.4	114.4	134.8	140.8	157.4	161.6	192.2	213.6	274.4	170.5	
Note: The commodity and service categories are not comparable to those in publications before 2001-02													
1 Excludes mortgage interest payments, council tax and NI rates													

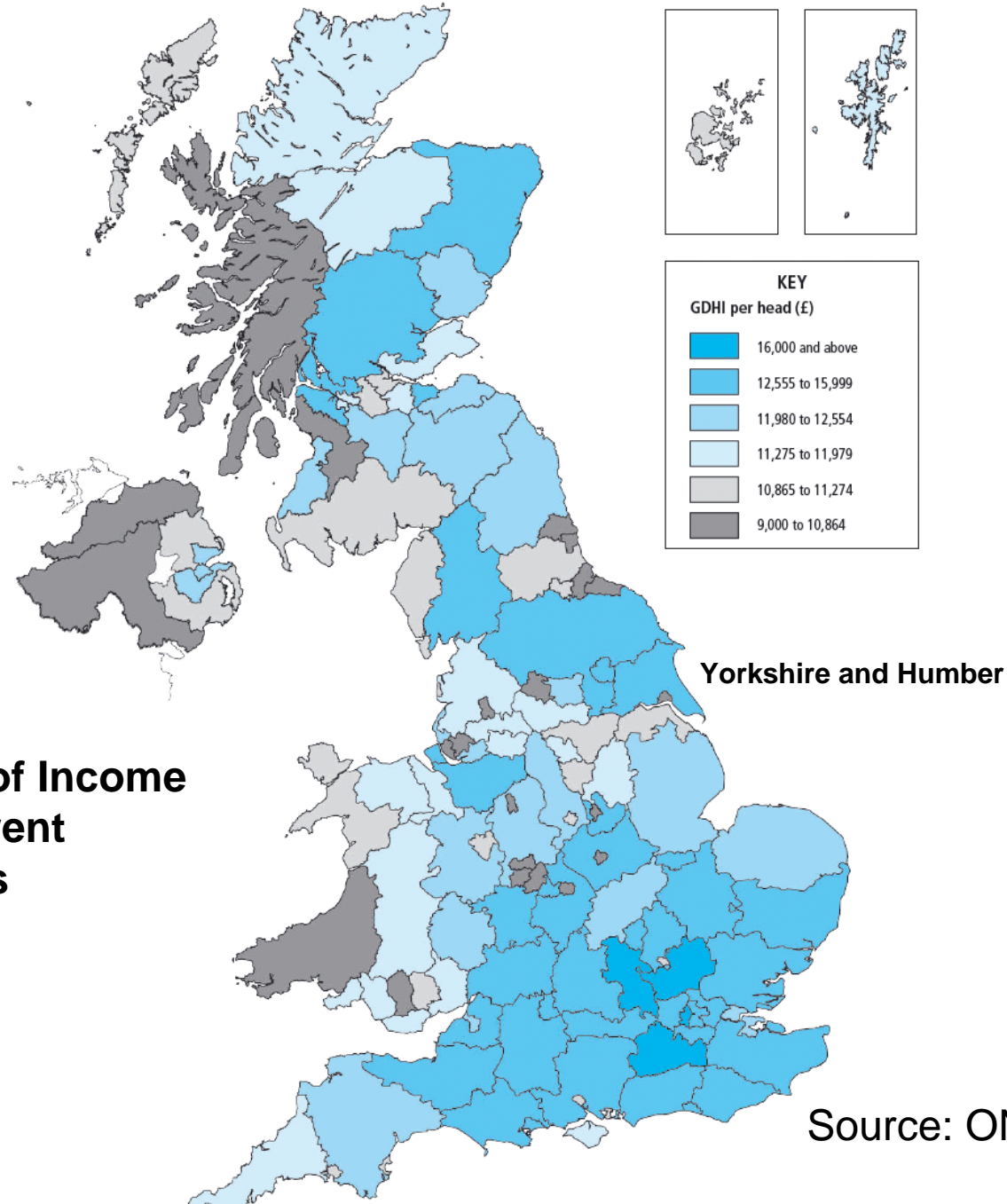
Source: <http://www.statistics.gov.uk/statbase/Expodata/Spreadsheets/D7561.xls> (accessed Feb 3, 2005)



Why Fuel Poverty?

- Higher energy prices
 - Rising prices of
 - Gas, electricity and oil
- Low income
 - Pensioners
 - Unemployed/low skilled
- Housing structure
 - Old and large
 - isolated
- Household composition
 - Single person
 - Single parent

Gross Disposable Household Income per head, 2004, by NUTS 3 area

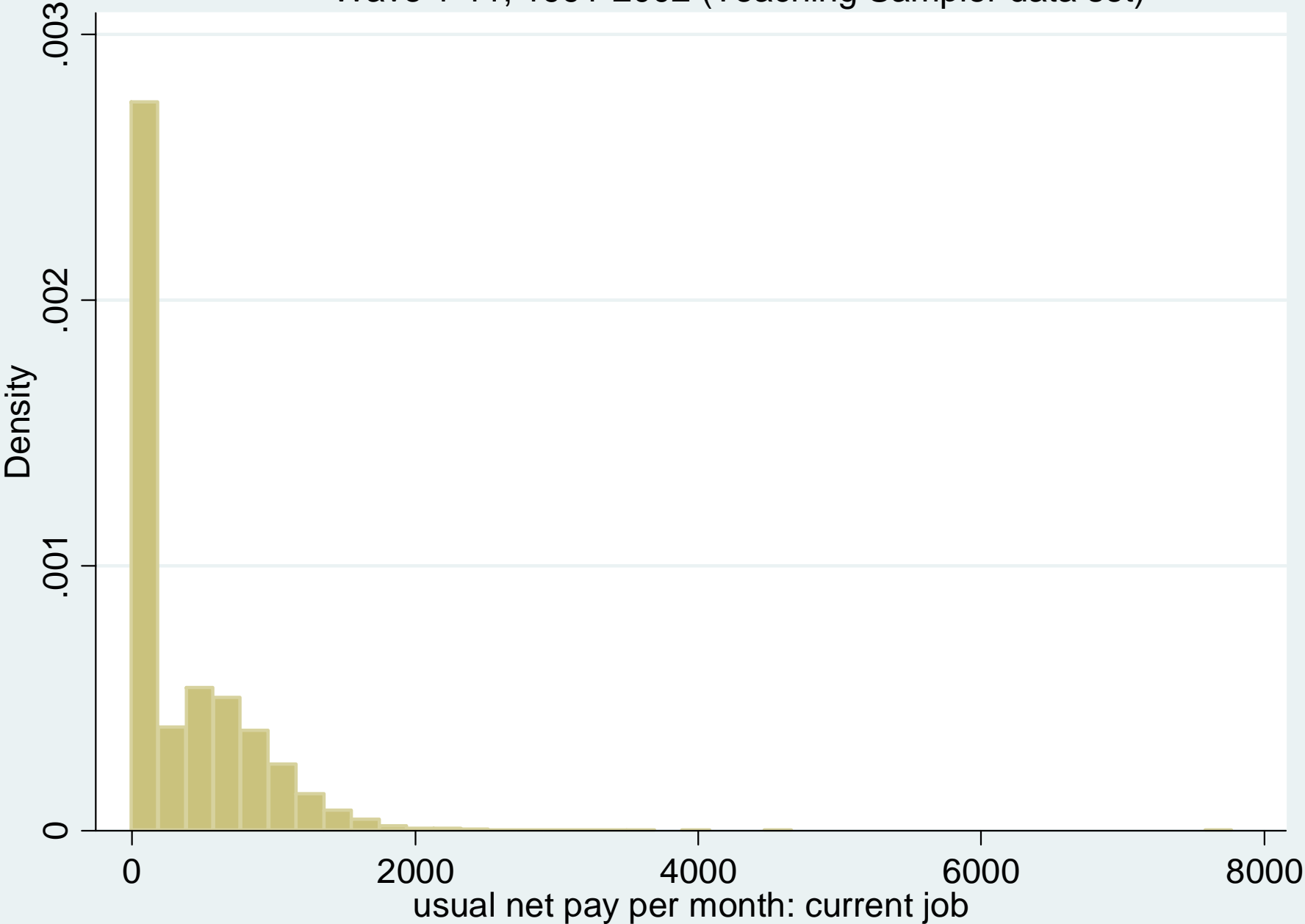


It has something to do with income distribution.

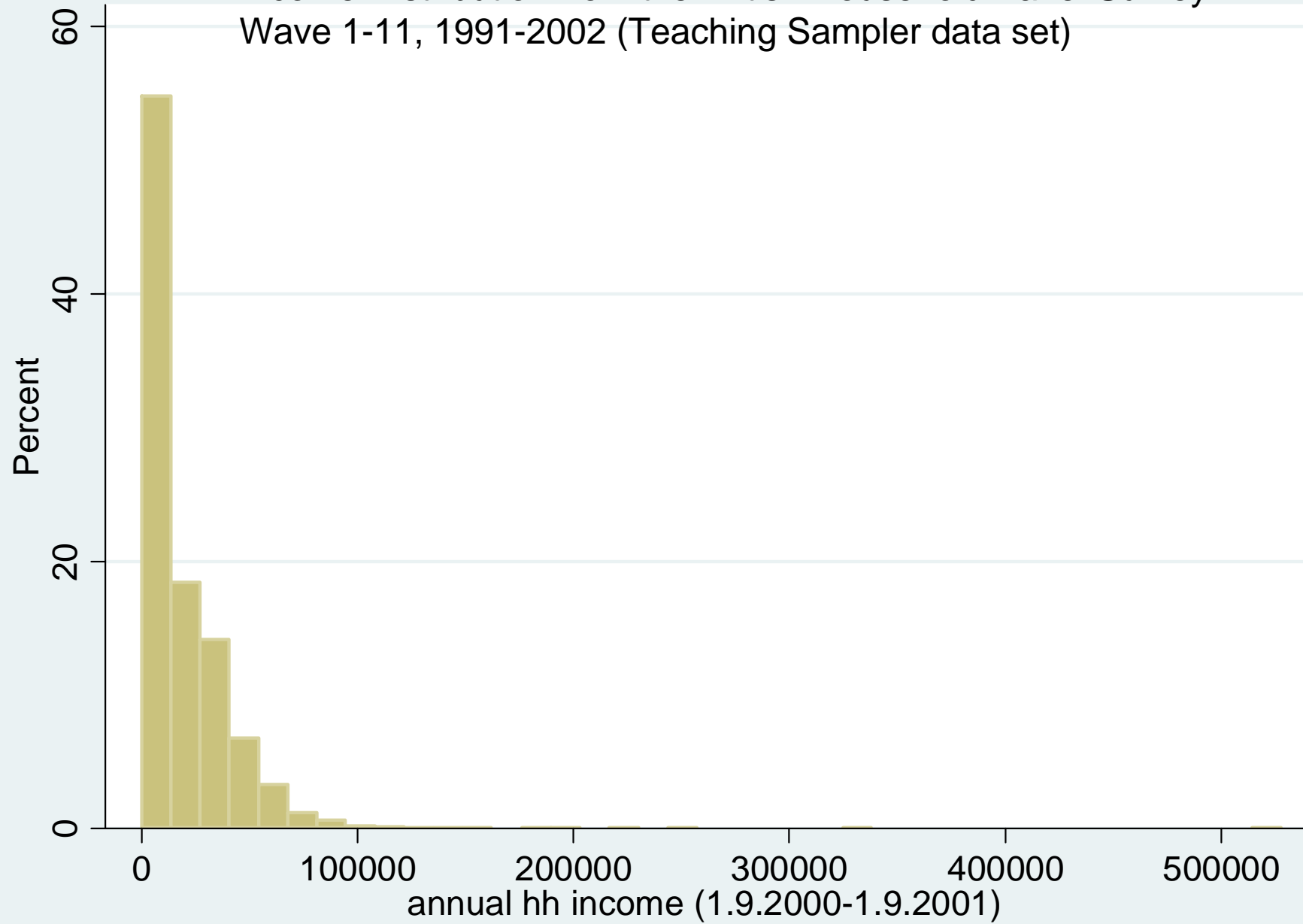
Levels of Income in Different Regions Of UK

Source: ONS

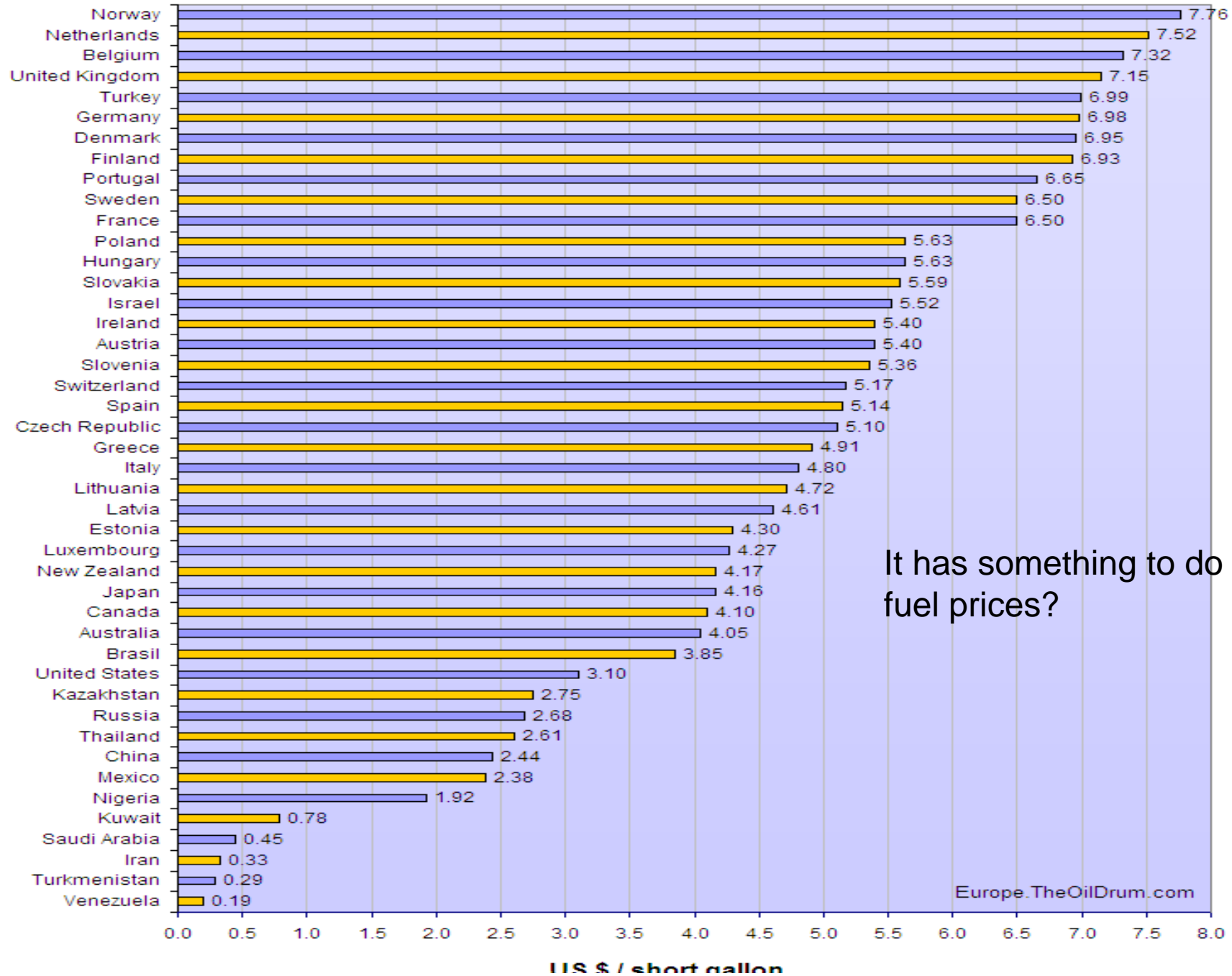
Income Distribution from the British Household Panel Survey
Wave 1-11, 1991-2002 (Teaching Sampler data set)



Income Distribution from the British Household Panel Survey
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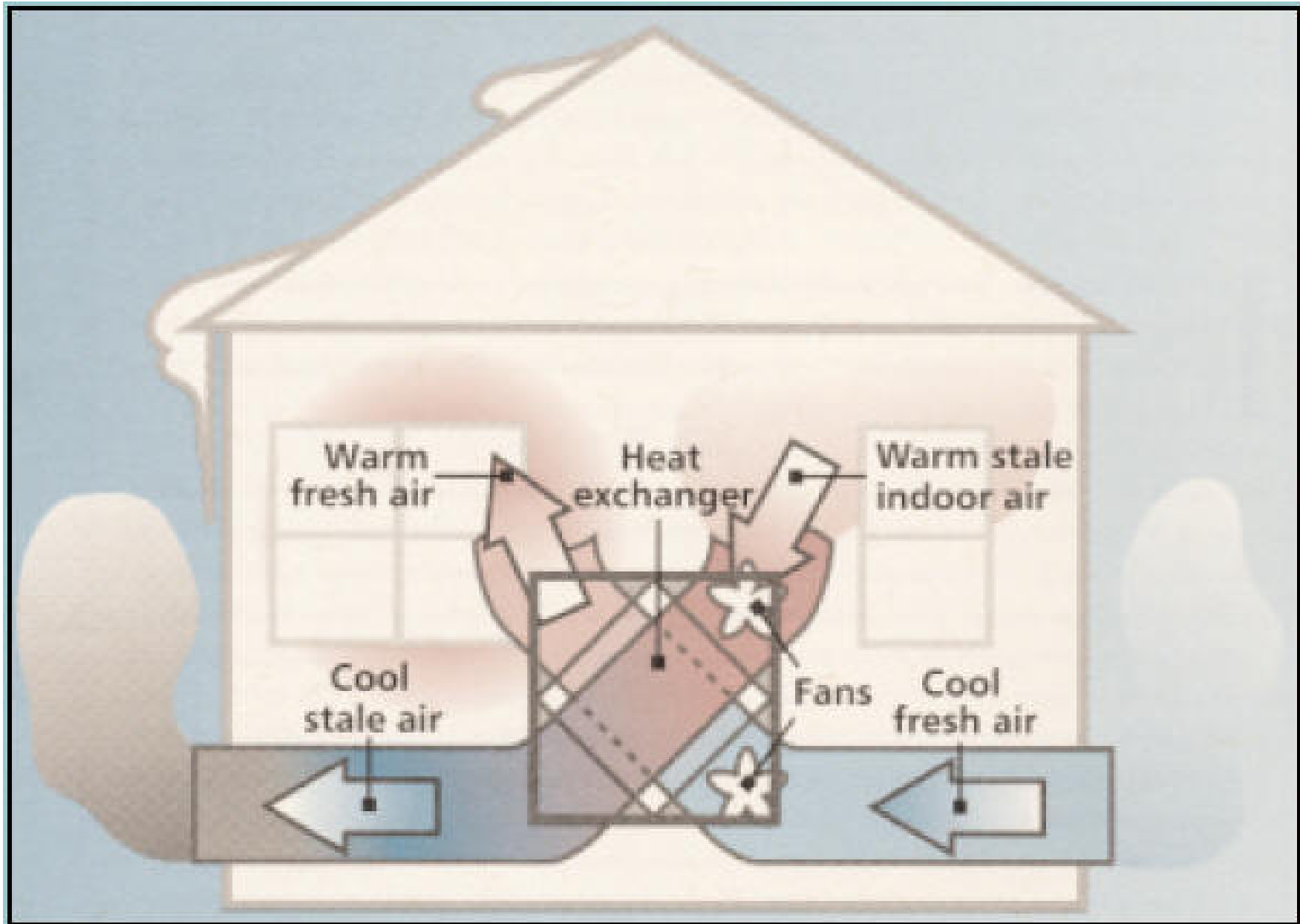


Gasoline Prices (\$/gal)



It has something to do with fuel prices?

It has something to do with fuel efficiency of houses



Fuel poverty can be studied in the perspective on literature on Poverty

and household income and expenditure survey

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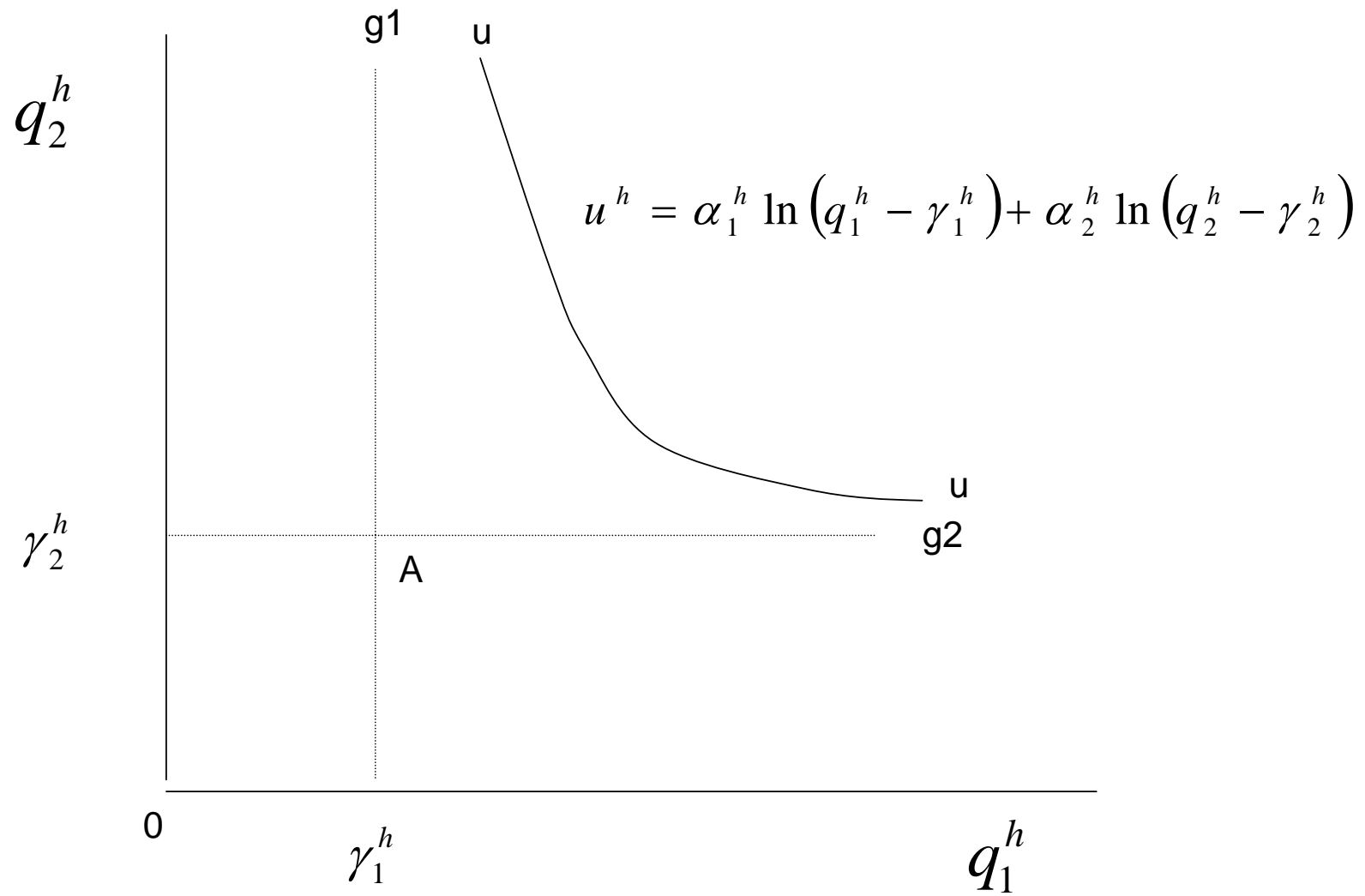
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 Camelia Minoiu (2007) Kernel Density approach to poverty

Basic Microeconomic Demand Side Model with Minimum Need

Stone Geary Preference for
Analysing Fuel Poverty

Figure 1
Stone Geary Preferences for Fuel and Other Goods



Stone-Geary preferences

$$u^h = \alpha_1^h \ln(q_1^h - \gamma_1^h) + \alpha_2^h \ln(q_2^h - \gamma_2^h)$$

$$u^{h'} = B_1^h \ln(q_1^h - \gamma_1^h) + B_2^h \ln(q_2^h - \gamma_2^h) \text{ with } B_1^h = \frac{\alpha_1^h}{\alpha_1^h + \alpha_2^h} \text{ and } B_2^h = \frac{\alpha_2^h}{\alpha_1^h + \alpha_2^h}$$

$$\text{Max } u^{h'} = B_1^h \ln(q_1^h - \gamma_1^h) + B_2^h \ln(q_2^h - \gamma_2^h) \quad (1)$$

Subject to

$$y^h = p_1 q_1^h + p_2 q_2^h \quad (2)$$

The Lagrangian constrained optimisation function for this problem becomes:

$$L(q_1^h, q_2^h, \lambda^h) = B_1^h \ln(q_1^h - \gamma_1^h) + B_2^h \ln(q_2^h - \gamma_2^h) + \lambda^h (y^h - p_1 q_1^h - p_2 q_2^h) \quad (3)$$

Optimization

$$\frac{\partial L(q_1^h, q_1^h, \lambda^h)}{\partial q_1^h} = \frac{B_1^h}{(q_1^h - \gamma_1^h)} - \lambda^h p_1 = 0 \quad (4)$$

$$\frac{\partial L(q_1^h, q_1^h, \lambda^h)}{\partial q_2^h} = \frac{B_2^h}{(q_1^h - \gamma_1^h)} - \lambda^h p_2 = 0 \quad (5)$$

$$\frac{\partial L(q_1^h, q_1^h, \lambda^h)}{\partial \lambda^h} = y^h - p_1 q_1^h - p_2 q_2^h = 0 \quad (6)$$

Rearrange (4) to get

$$p_1 q_1^h = p_1 \gamma_1^h + \frac{B_1^h}{\lambda^h} \quad (7)$$

Similarly rearrange (5) to get

$$p_2 q_2^h = p_2 \gamma_2^h + \frac{B_2^h}{\lambda^h} \quad (8)$$

Now using (7) and (8) in (6) to get

$$y^h - p_1 \gamma_1^h - \frac{B_1^h}{\lambda^h} - p_2 \gamma_2^h - \frac{B_2^h}{\lambda^h} = 0$$
$$\frac{B_1^h + B_2^h}{\lambda^h} = y^h - p_1 \gamma_1^h - p_2 \gamma_2^h \quad \text{or} \quad \frac{1}{\lambda^h} = y^h - p_1 \gamma_1^h - p_2 \gamma_2^h \quad (9)$$

Demands for Fuel and Non-Fuel Products

Put (9) into (7) $p_1 q_1^h = p_1 \gamma_1^h + B_1^h (y^h - p_1 \gamma_1^h - p_2 \gamma_2^h)$ tot get

$$q_1^h = \gamma_1^h + \frac{B_1^h}{p_1} (y^h - p_1 \gamma_1^h - p_2 \gamma_2^h) \quad (10)$$

Similarly put (9) into (8) $p_2 q_2^h = p_2 \gamma_2^h + B_2^h (y^h - p_1 \gamma_1^h - p_2 \gamma_2^h)$ to get

$$q_2^h = \gamma_2^h + \frac{B_2^h}{p_2} (y^h - p_1 \gamma_1^h - p_2 \gamma_2^h) \quad (11)$$

1) Household h below the point A in above diagram if $(y^h - p_1 \gamma_1^h - p_2 \gamma_2^h) < 0$.

This household faces fuel poverty and is in vulnerable situation.

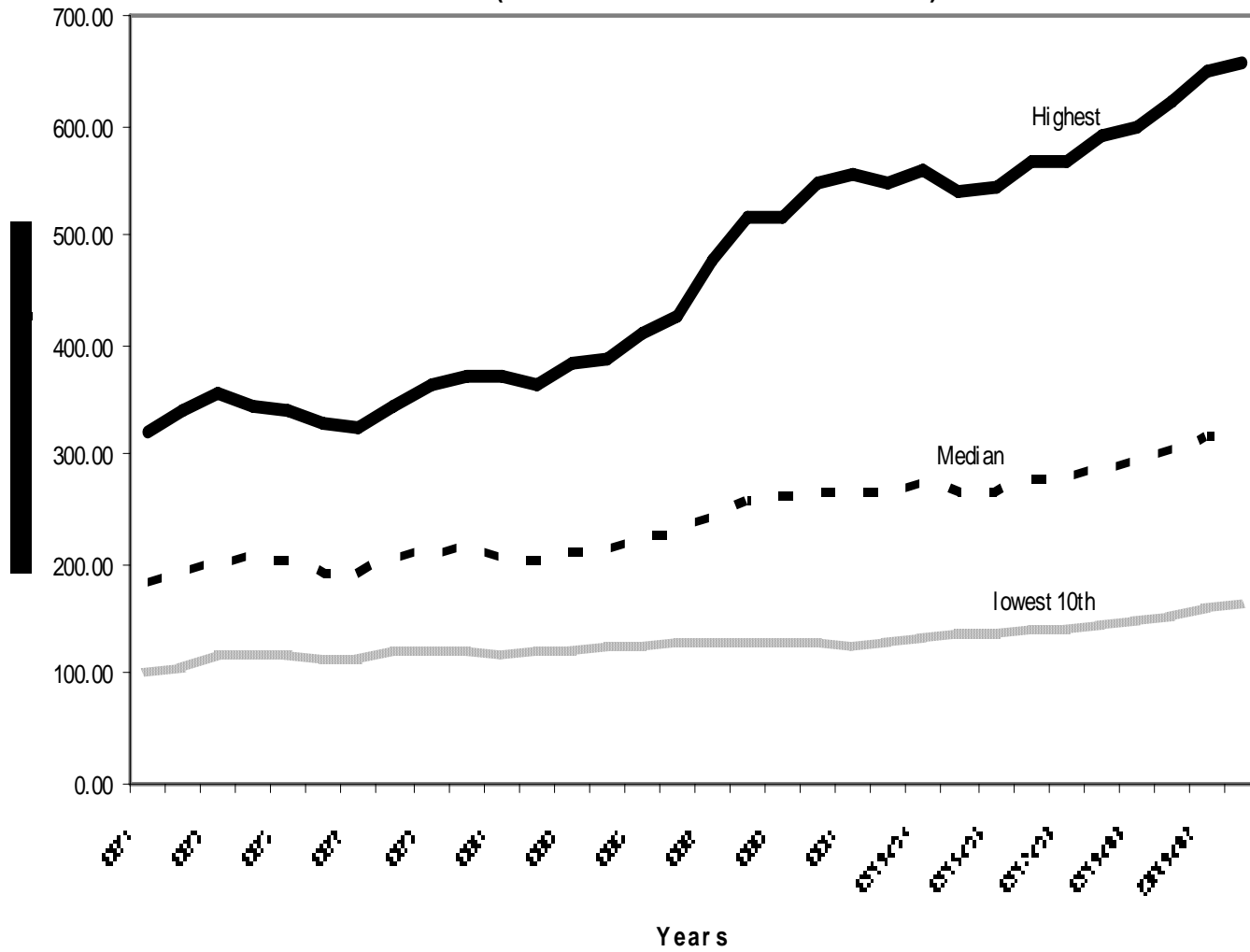
2) The household's budget constraint just allows to meet minimum requirement if

$(y^h - p_1 \gamma_1^h - p_2 \gamma_2^h) = 0$. It is at point A in the above diagram. Such household just manages to be out of fuel poverty.

3) Household is above the point A in the above diagram if $(y^h - p_1 \gamma_1^h - p_2 \gamma_2^h) > 0$,

where the household needs not to bother about the minimum needs as this household consumes above the basic needs.

Figure 2
Income Growth Process of Lowest, Median and Highest Income Decile Households
(Office of National Statistics)



Solutions of Fuel Poverty

- Transfer payments
 - winter fuel payments
 - Fuel subsidies
- Home improvement measures
 - Draught proofing (£100)
 - Cavity insulation (£300)
 - Loft insulation (£200)
 - Gas central heating (£2000)
 - Boiler replacement (£1000)
 - Oil fired central heating (£3500)
 - CHP Community heating (£5000)
 - Solid wall external £4000
 - Electric storage (£900)

government estimates on
fuel poverty white paper:

but

Seems to be too
conservative

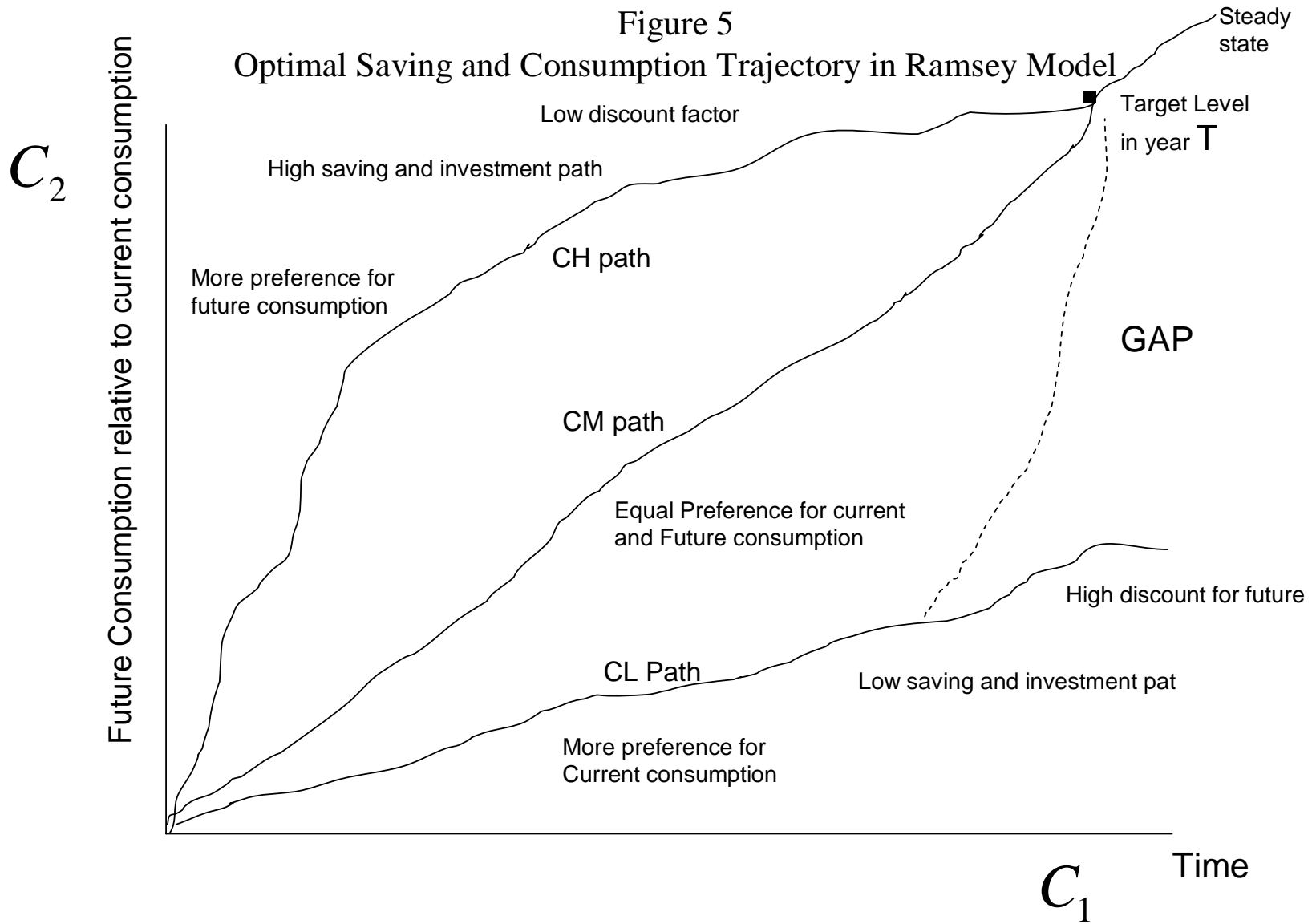
from the perspective of the
British Gas.

Dynamic Macroeconomic Supply Side Model of Fuel Poverty

Dynamic Programming for
Determining the Optimal
accumulation and Welfare

Energy White Paper aims to eliminate fuel poverty in UK by 2016

Figure 5



What is Optimal Saving and Consumption to Maximise Life Time Utility?

Application of Dynamic Programming Model for Analysing Fuel Poverty

$$\text{Max } U = \sum_t^{\infty} \beta^t \ln C_t \quad 0 < \beta < 1$$

subject to

$$K_{t+1} + C_t = AK_t^\alpha$$

In the context of fuel poverty C_t is composite of q_1^h and q_2^h , quantities of fuel and non fuel products. Similarly the output $Y_t = AK_t^\alpha$ also is composite of these two products, $q_{1,t}$ and $q_{2,t}$.

$$0 < \alpha < 1$$

The market clearing condition implies that $\sum_h q_{1,t}^h = q_{1,t}$ and $\sum_h q_{2,t}^h = q_{2,t}$. Capital stock is similarly divided in producing fuel and non-fuel products, $K_t = K_{1,t} + K_{2,t}$.

Value Function Iterations

$$V_1(K) = \max_k \{ \ln C + \beta \ln(V_0(K')) \}$$

$$K_{T+1} = 0 \quad C_t + K' = AK^\alpha \quad C_t = AK^\alpha$$

$$V_1(K) = \ln C = \ln(AK^\alpha) = \ln A + \alpha \ln K$$

$$V_2(K) = \ln C + \beta \ln(V_1(K')) = \ln C + \beta(\ln A + \alpha \ln K) = \ln(AK^\alpha - K') + \beta(\ln A + \alpha \ln K)$$

$$V_2(K) = \ln(AK^\alpha - K') + \beta(\ln A + \alpha \ln K)$$

Dynamic Programming Model : Optimality Conditions

$$\frac{\partial V_2(K)}{\partial K} = -\frac{1}{AK^\alpha - K'} + \frac{\beta\alpha}{K'} = 0$$

$$\frac{1}{AK^\alpha - K'} = \frac{\beta\alpha}{K'}$$

$$K' = \beta\alpha (AK^\alpha - K')$$

$$K'(1 + \beta\alpha) = \beta\alpha AK^\alpha$$

$$K' = \frac{\beta\alpha}{(1 + \beta\alpha)} AK^\alpha$$

Dynamic Programming Model : Second Iteration of Value Function

$$C = AK^\alpha - \frac{\beta\alpha}{(1 + \beta\alpha)} AK^\alpha$$

$$C = \frac{1}{(1 + \beta\alpha)} AK^\alpha$$

$$V_2(K) = \ln C + \beta V_1(AK^\alpha) = \ln \left[\frac{1}{(1 + \beta\alpha)} AK^\alpha \right] + \beta(\ln A + \alpha \ln K')$$

$$V_2(K) = \ln \left[\frac{1}{(1 + \beta\alpha)} AK^\alpha \right] + \beta \ln A + \beta\alpha \ln \left(\frac{\beta\alpha}{(1 + \beta\alpha)} AK^\alpha \right)$$

$$V_2(K') = \ln \left[\frac{1}{(1 + \beta\alpha)} A \right] + \beta \ln A + \beta\alpha \ln \left(\frac{\beta\alpha}{(1 + \beta\alpha)} A \right) + \alpha(1 + \alpha\beta) \ln K'$$

Third Iteration of the Value Function

$$V_3(K) = \ln C + \beta V_2(K')$$

$$V_3(K) = \ln(AK^\alpha - K') + \beta(\alpha(1 + \alpha\beta)\ln K')$$

$$\frac{\partial V_3(K)}{\partial K} = -\frac{1}{AK^\alpha - K'} + \frac{\beta\alpha(1 + \alpha\beta)}{K'} = 0$$

$$\frac{1}{AK^\alpha - K'} = \frac{\beta\alpha(1 + \alpha\beta)}{K'} \quad K' = \frac{(\beta\alpha + \alpha^2\beta^2)}{(1 + \beta\alpha + \alpha^2\beta^2)} AK^\alpha$$

$$C = (AK^\alpha - K') = \left[AK^\alpha - \frac{(\beta\alpha + \alpha^2\beta^2)}{(1 + \beta\alpha + \alpha^2\beta^2)} AK^\alpha \right] \quad C = \frac{1}{(1 + \beta\alpha + \alpha^2\beta^2)} AK^\alpha$$

$$V_3(K') = \ln \left[\frac{1}{(1 + \beta\alpha + \alpha^2\beta^2)} AK^\alpha \right] + \beta \left[\ln \left[\frac{1}{(1 + \beta\alpha)} A \right] + \beta \ln A + \beta\alpha \ln \left(\frac{\beta\alpha}{(1 + \beta\alpha)} A \right) + \alpha(1 + \alpha\beta)\ln K' \right]$$

$$V_3(K') = \beta \ln \left[\frac{A}{(1 + \beta\alpha)} \right] + \beta^2 \ln A + \beta^2 \alpha \ln \left(\frac{\beta\alpha A}{(1 + \beta\alpha)} \right) + \ln \left(\frac{A}{(1 + \beta\alpha + \alpha^2\beta^2)} \right) + \beta\alpha(1 + \alpha\beta) \ln \left[\frac{(\beta\alpha + \alpha^2\beta^2)A}{(1 + \beta\alpha + \alpha^2\beta^2)} \right] + \alpha(1 + \beta\alpha + \alpha^2\beta^2)\ln K'$$

Forth Iteration of Value Function

$$V_4(K) = \ln C + \beta V_3(K') = \ln(AK^\alpha - K') + \alpha(1 + \beta\alpha + \alpha^2\beta^2)\ln K'$$

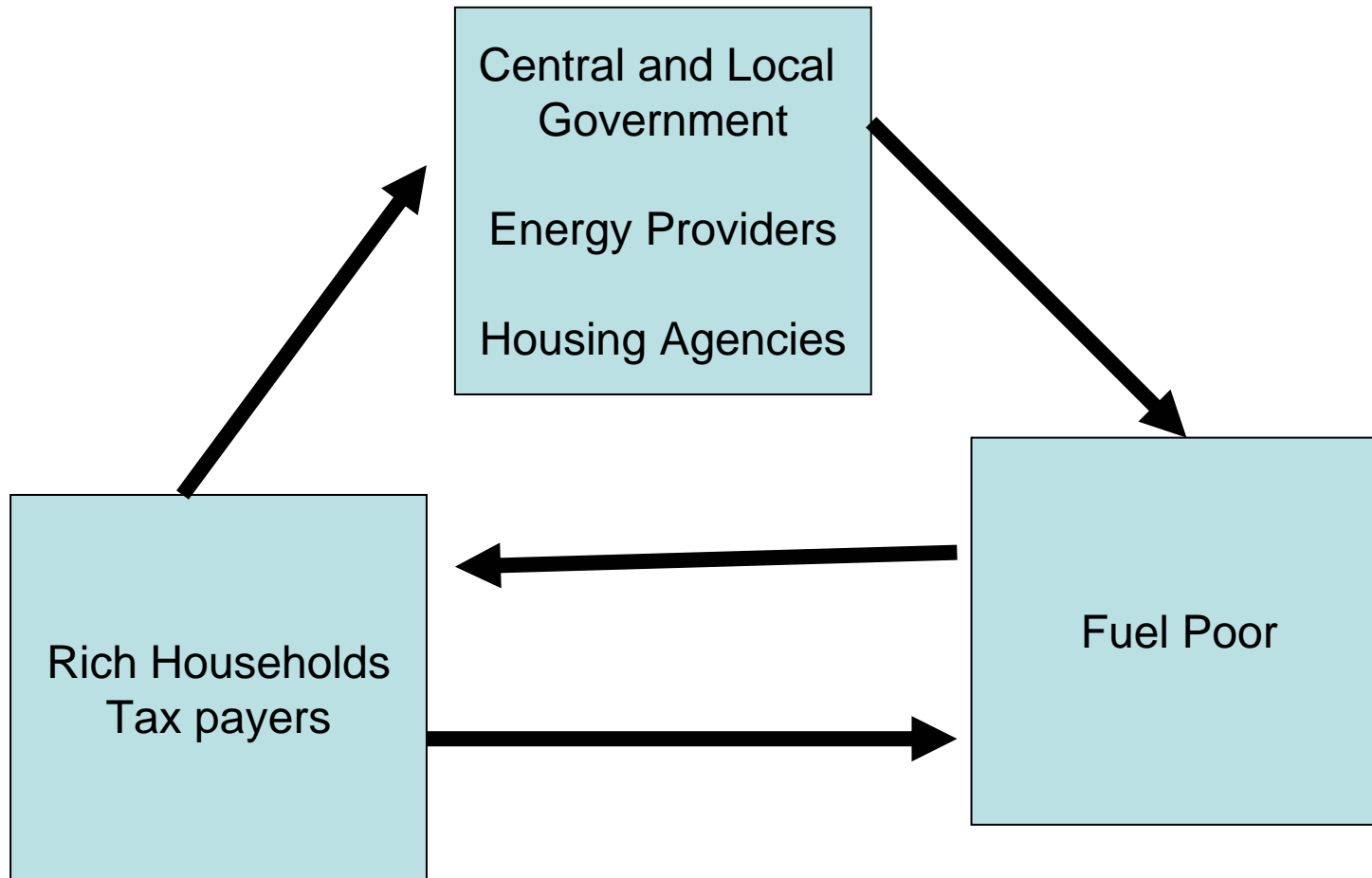
$$C = \frac{1}{1 + \alpha\beta + \alpha^2\beta^2 + \alpha^3\beta^3} AK^\alpha$$

$$V_4(K') = \ln \left[\frac{1}{1 + \alpha\beta + \alpha^2\beta^2 + \alpha^3\beta^3} AK^\alpha \right] + \beta \left[\begin{aligned} & \beta \ln \left[\frac{A}{(1 + \beta\alpha)} \right] + \beta^2 \ln A + \beta^2 \alpha \ln \left[\frac{\beta\alpha A}{(1 + \beta\alpha)} \right] \\ & + \ln \left(\frac{A}{(1 + \beta\alpha + \alpha^2\beta^2)} \right) + \beta\alpha(1 + \alpha\beta) \ln \left[\frac{(\beta\alpha + \alpha^2\beta^2)A}{(1 + \beta\alpha + \alpha^2\beta^2)} \right] \\ & + \alpha(1 + \beta\alpha + \alpha^2\beta^2) \ln \left[\frac{(\beta\alpha + \alpha^2\beta^2 + \alpha^3\beta^3)}{1 + \alpha\beta + \alpha^2\beta^2 + \alpha^3\beta^3} AK^\alpha \right] \end{aligned} \right]$$

$$\begin{aligned} V_4(K') = & \ln \left[\frac{1}{1 + \alpha\beta + \alpha^2\beta^2 + \alpha^3\beta^3} A \right] + \beta \ln \left(\frac{A}{(1 + \beta\alpha + \alpha^2\beta^2)} \right) + \beta^2 \left[\frac{\ln A}{(1 + \beta\alpha)} \right] + \beta^3 \ln A \\ & + \beta\alpha(1 + \beta\alpha + \alpha^2\beta^2) \ln \left[\frac{(\beta\alpha + \alpha^2\beta^2 + \alpha^3\beta^3)\alpha\beta A}{1 + \alpha\beta + \alpha^2\beta^2 + \alpha^3\beta^3} \right] + \beta \left\{ \beta\alpha(1 + \alpha\beta) \ln \left[\frac{(\beta\alpha + \alpha^2\beta^2)A}{(1 + \beta\alpha + \alpha^2\beta^2)} \right] \right\} + \beta^2 \left\{ \beta\alpha \ln \left[\frac{\alpha\beta A}{(1 + \beta\alpha)} \right] \right\} \\ & + \alpha \left[1 + \beta\alpha(1 + \beta\alpha + \alpha^2\beta^2 + \alpha^3\beta^3) \right] \ln K' \end{aligned}$$

Can this process can continue forever.....Need Strategic considerations 34

Fuel Poverty Game



Fuel Poverty Game

Players: Fuel poor, rich and government strategy profiles = (s, l, k,)

State contingent income of poor $y_t^P (s, l, k)$

State contingent income of rich $y_t^R (s, l, k)$

Transition probability of being rich $\pi_t^P (s, l, k)$

Probability of being poor $\pi_t^R (s, l, k)$

Expected Utility Maximisation

Proposition 1: The state contingent money metric utility of fuel poor is less than that of rich

$$\sum_{s=1}^s \sum_{l=1}^l \sum_{k=1}^k \sum_t^T \pi_t^P(s, l, k) \cdot \delta_t^P u(y_t^P(s, l, k)) < \sum_{s=1}^s \sum_{l=1}^l \sum_{k=1}^k \sum_t^T \pi_t^R(s, l, k) \cdot \delta_t^R u(y_t^R(s, l, k))$$

Proposition 2: Transfer raises money metric expected utility of fuel poor and reduces the utility of rich

$$\sum_{s=1}^s \sum_{l=1}^l \sum_{k=1}^k \sum_t^T \pi_t^P(s, l, k) \cdot \delta_t^P u(y_t^P(s, l, k) + T_t^P(s, l, k)) < \sum_{s=1}^s \sum_{l=1}^l \sum_{k=1}^k \sum_t^T \pi_t^R(s, l, k) \cdot \delta_t^R u(y_t^R(s, l, k) - T_t^P(s, l, k))$$

Condition 3: Participation and incentive compatibility requires

$$\sum_{s=1}^s \sum_{l=1}^l \sum_{k=1}^k \sum_t^T \pi_t^P(s, l, k) \cdot \delta_t^P u(y_t^P(s, l, k) + T_t^P(s, l, k)) > \sum_{s=1}^s \sum_{l=1}^l \sum_{k=1}^k \sum_t^T \pi_t^P(s, l, k) \cdot \delta_t^P u(y_t^P(s, l, k))$$

$$\sum_{s=1}^s \sum_{l=1}^l \sum_{k=1}^k \sum_t^T \pi_t^R(s, l, k) \cdot \delta_t^R u(y_t^R(s, l, k) - T_t^P(s, l, k)) < \sum_{s=1}^s \sum_{l=1}^l \sum_{k=1}^k \sum_t^T \pi_t^R(s, l, k) \cdot \delta_t^R u(y_t^R(s, l, k))$$

Condition 4: Growth requires that income of both poor and rich are rising over time:

$$Y_t^P(s, l, k) < Y_{t+1}^P(s, l, k) < Y_{t+2}^P(s, l, k) < \dots < Y_{t+T}^P(s, l, k)$$

$$Y_t^R(s, l, k) < Y_{t+1}^R(s, l, k) < Y_{t+2}^R(s, l, k) < \dots < Y_{t+T}^R(s, l, k)$$

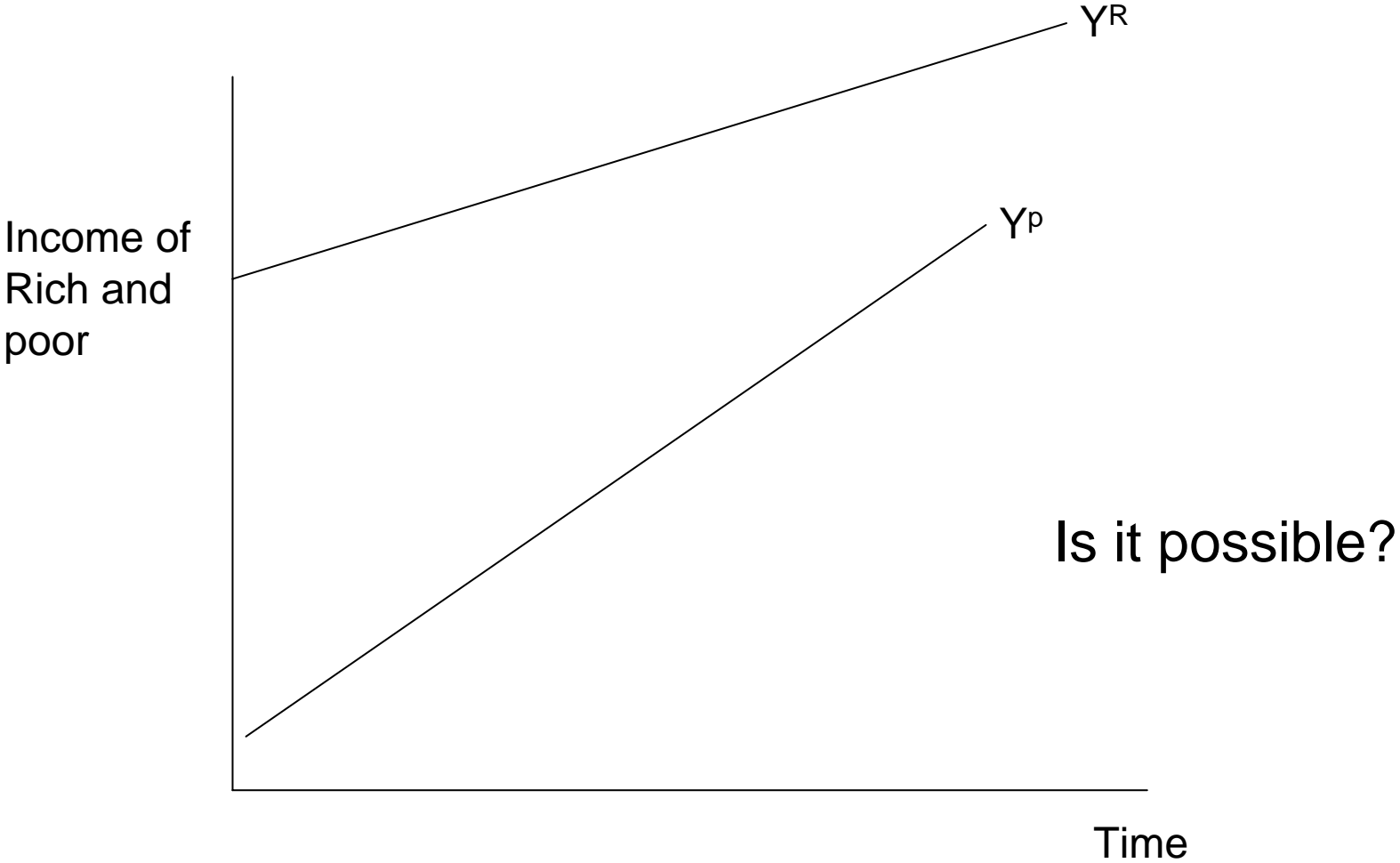
$$T_t^P(s, l, k) < T_{t+1}^P(s, l, k) < T_{t+2}^P(s, l, k) < \dots < T_{t+T}^P(s, l, k)$$

$$T_t^R(s, l, k) > T_{t+1}^R(s, l, k) > T_{t+2}^R(s, l, k) > \dots > T_{t+T}^R(s, l, k)$$

It partly depends on the state contingent poverty line:

$$Y_{t+T}^P(s, l, k) \geq \frac{1}{2} \sum_{p=1}^p Y_{t+T}^P(s, l, k)$$

Cooperative Solutions of Fuel Poverty Problem



Working and poor??

How government is solving the poverty problem in UK?

$$B_i = E_i - t_b (y_i - \bar{y})$$

B_i is amount of annual benefit,

E_i is the total entitlement that constitutes of child tax credit and working family tax credit and

t_b is the tax back rate,

y_i is the annual household income that includes income of husband y_i^{hs} and wife y_i^W and other incomes such as the interest rate earning y_i^O and \bar{y} the threshold income.

CTC and WFTC components of benefit entitlements for a family with dependent children

$$E_i = E_{F,i} + E_{CH,i}N_{CH,i} - 0.7CC_i + E_{B,i} + (E_{CP,i} \text{ or } E_{LP,i}) + E_{30}$$

where $E_{F,i}$ is the family entitlement,

$E_{CH,i}$ the entitlement per child

$N_{CH,i}$ the number of dependent children in the family, CC child care cost, E_B entitlement for family,

$E_{CP,i}$ or $E_{LP,i}$ the entitlement for couples or lone parent and $E_{30h,i}$ the entitlement for working more than 30 hours.

For a family with y_i^{hs} equal to £15,000, y_i^W equal to £10,000 and net y_i^O of £60 with three dependent children the DWP's benefit entitlement for fiscal year 2007 is calculated as

$$E_i = E_{F,i} + E_{CH,i} N_{CH,i} - 0.7 CC_i + E_{B,i} + (E_{CP,i} \text{ or } E_{LP,i}) + E_{30h,i}$$

$$= 545 + 3 \times 1690 + 1620 + 1595 + 660 = \text{£}9490 .$$

Thus the annual amount of benefit

$$\text{is } B_i = E_i - t_b (y_i - \bar{y}) = 9490 - 0.37 (25060 - 5220) = \text{£}2149 .20$$

which amounts to £41.33 per week.

Generally WFTC provides more benefits to a couple with many children and implicitly encourages at least one parent to remain at home to take care of children.

Various Transfers to Alleviate Poverty in the UK

Armed force allowance (£62.25-£41.65),
bereavement entitlement (£84.25),
care taker allowance (£46.95),
disability allowance (£62.25),
housing benefit (7.5% to 25%),
incapacity benefit (£78.50),
income support (single £57.45; couple £90.10),
hospital rates (£46.75),
industrial injuries (£127.10),
job seekers' allowance (£34.60 to £45.58),
maternity allowance (£108.85),
pension credit (£114.05), state pension (£84.25),
severe disablement allowance (£47.45),
widow benefit (£84.25),
winter fuel allowance (lump sum £200),
national insurance (£84.01 - £97.00)

Households: $i = 1, 2, \dots, N.$

Income: $y_i \neq y_j \quad \forall_i$

Income distribution: $y_1 < y_2 < \dots < y_N$

Average income: $\bar{y} = \sum_i^N \frac{y_i}{N}$

Poverty line $z = \frac{1}{2} \bar{y}$

Depth of Poverty: $I = \frac{\sum_i^n (y_i - z)}{z \cdot n}$

Measuring Poverty in a hypothetical economy

y	N	cy	cp	yshre	cyshre	pshre	cpshre	Triangle	Rectangle	Area	ygap
10	1	10	1	0.01	0.01	0.1	0.1	0.0005	0.000	0.0005	-90
20	1	30	2	0.02	0.03	0.1	0.2	0.001	0.001	0.0020	-80
30	1	60	3	0.03	0.06	0.1	0.3	0.0015	0.003	0.0045	-70
40	1	100	4	0.04	0.10	0.1	0.4	0.0020	0.006	0.0080	-60
50	1	150	5	0.05	0.15	0.1	0.5	0.0025	0.010	0.0125	-50
60	1	210	6	0.06	0.21	0.1	0.6	0.0030	0.015	0.0180	-40
90	1	300	7	0.09	0.30	0.1	0.7	0.0045	0.021	0.0255	-10
100	1	400	8	0.10	0.40	0.1	0.8	0.0050	0.030	0.0350	0
200	1	600	9	0.20	0.60	0.1	0.9	0.0100	0.040	0.0500	100
400	1	1000	10	0.40	1.00	0.1	1.0	0.0200	0.060	0.0800	300

Sen(1996) Measure of Poverty

$$P = H.I + H(1 - I)G$$

$$z = \frac{1}{2} \bar{y} = 50$$

$$I = \frac{\sum_i^n (y_i - z)}{z \cdot n} = \frac{40 + 30 + 20 + 10}{50 \cdot 4} = \frac{100}{200} = 0.5$$

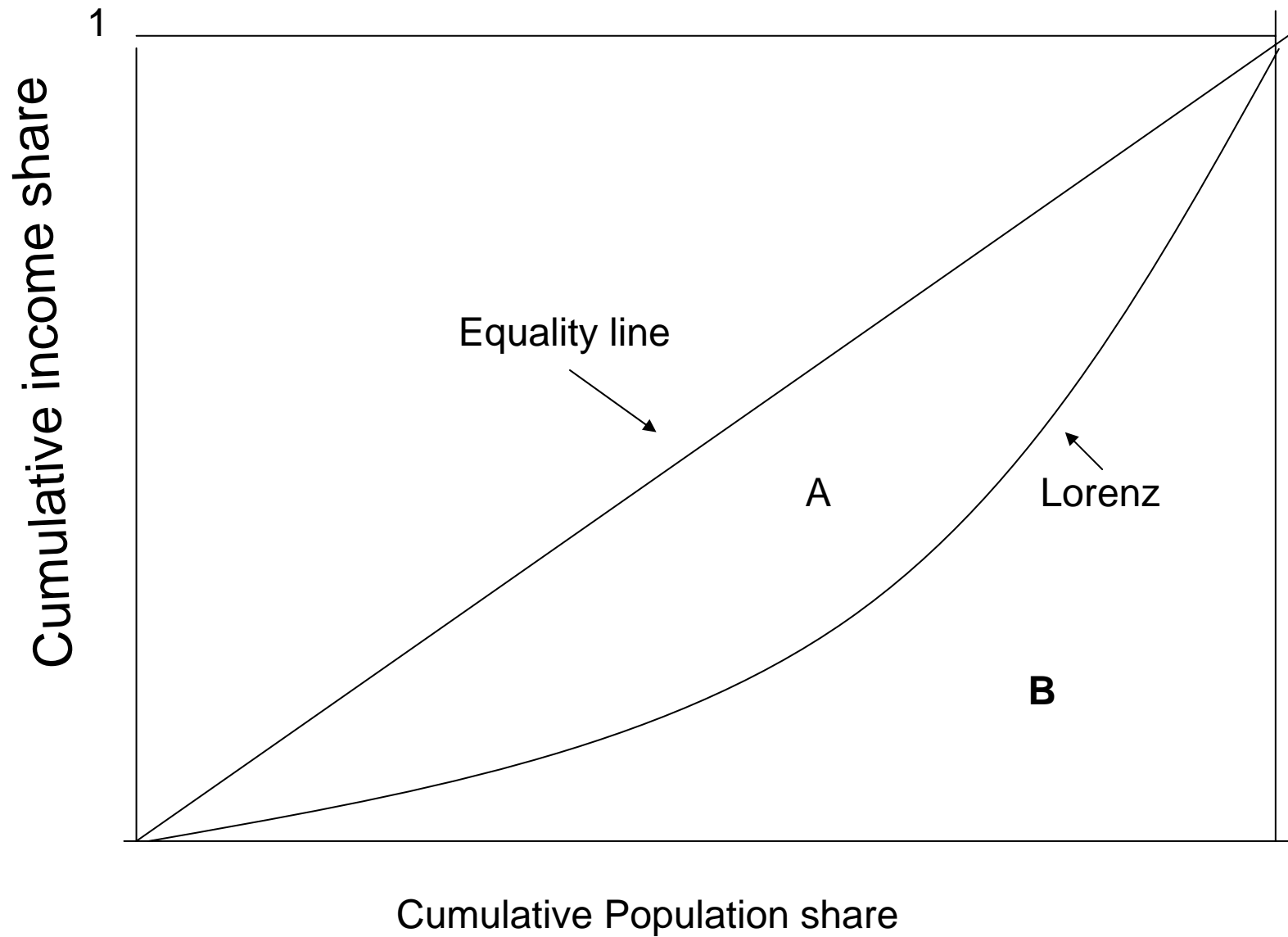
$$P = H.I + (1 - I)G = 0.4 \times 0.5 + 0.4(1 - 0.5)0.528 = 0.2 + 0.106 = 0.306$$

Poverty elimination

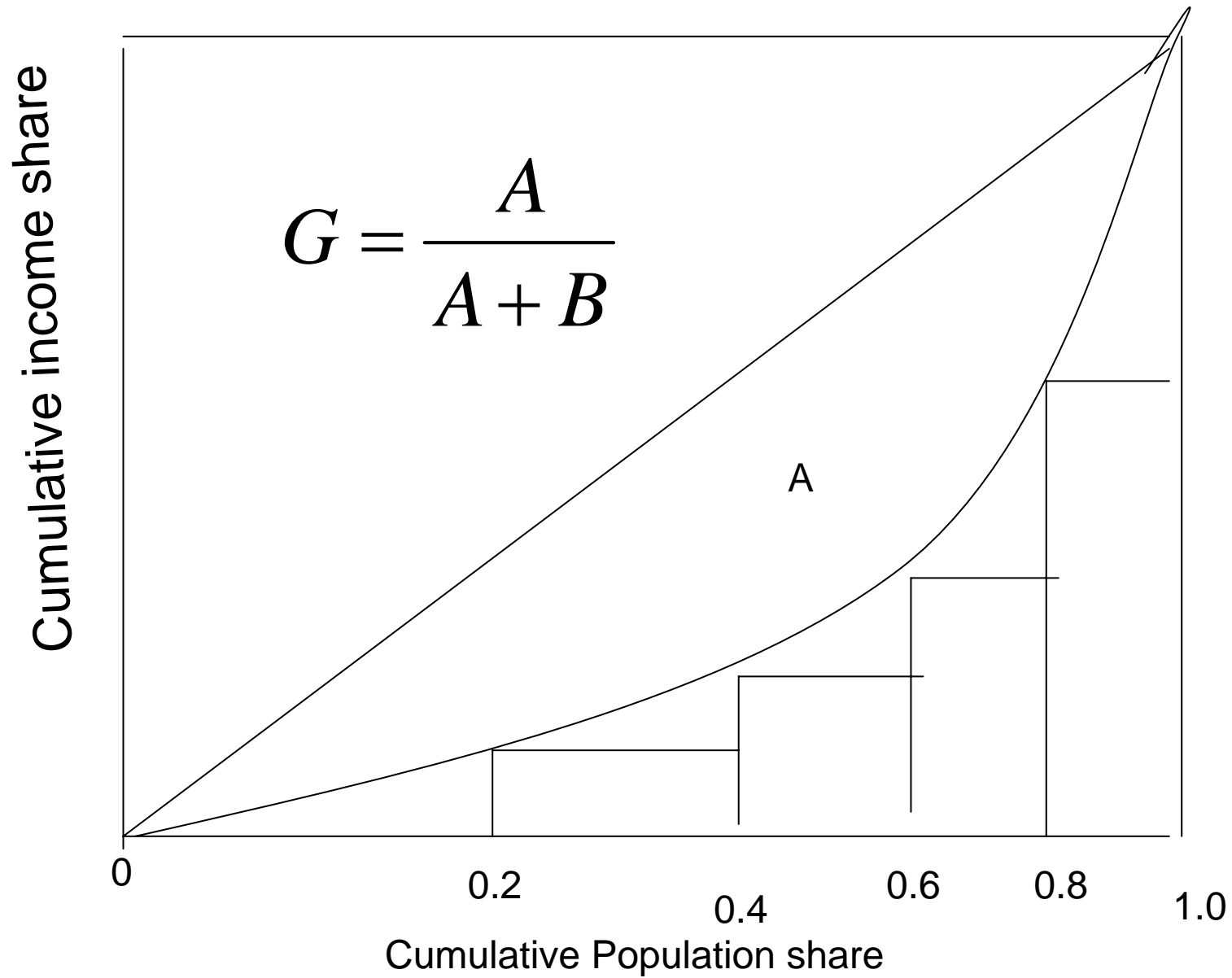
$$T_1 = 40 \quad T_2 = 30 \quad T_3 = 20 \quad T_4 = 10$$

Tax 9th and 10th decile 20 and 80

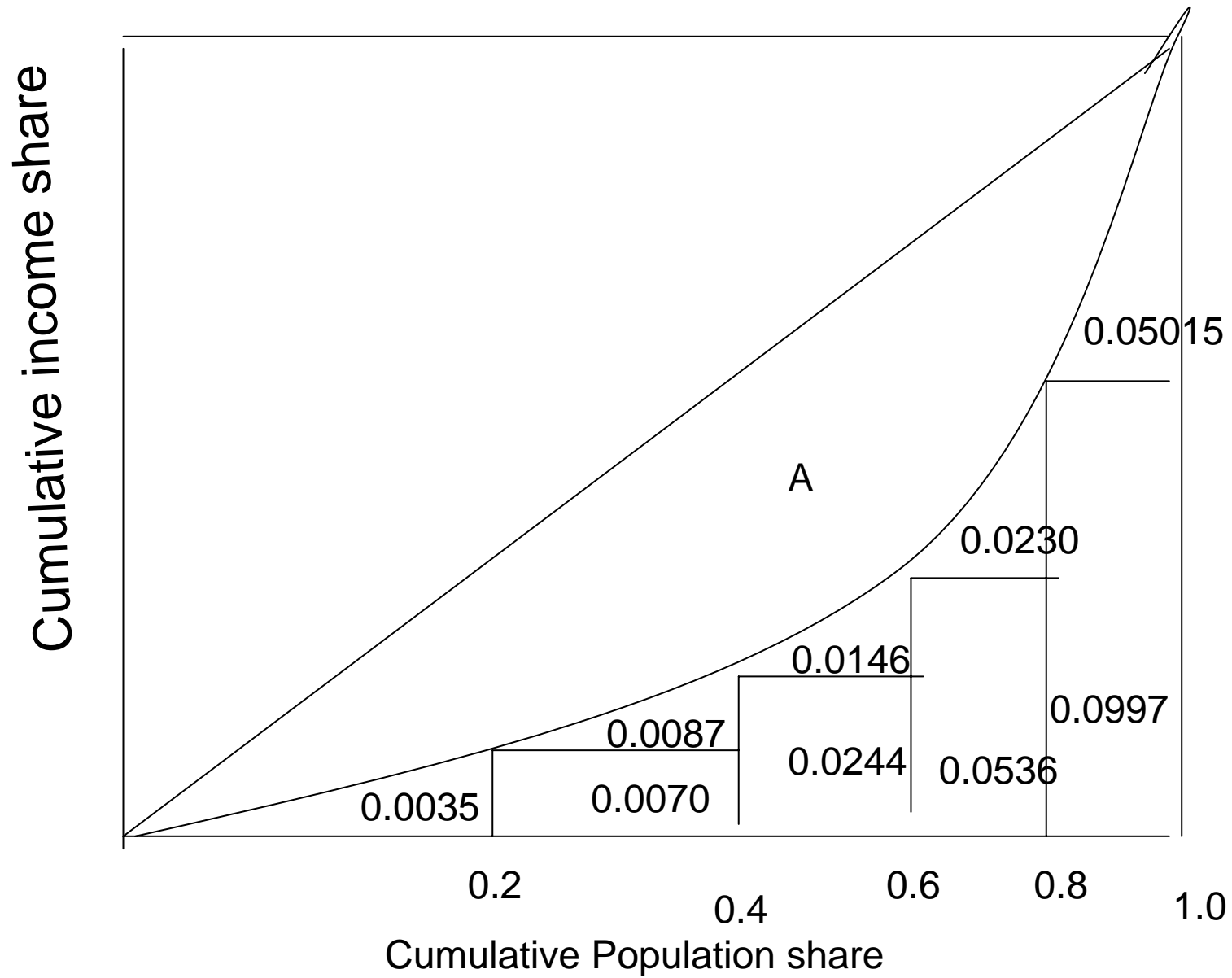
Income Inequality and Lorenz Curve



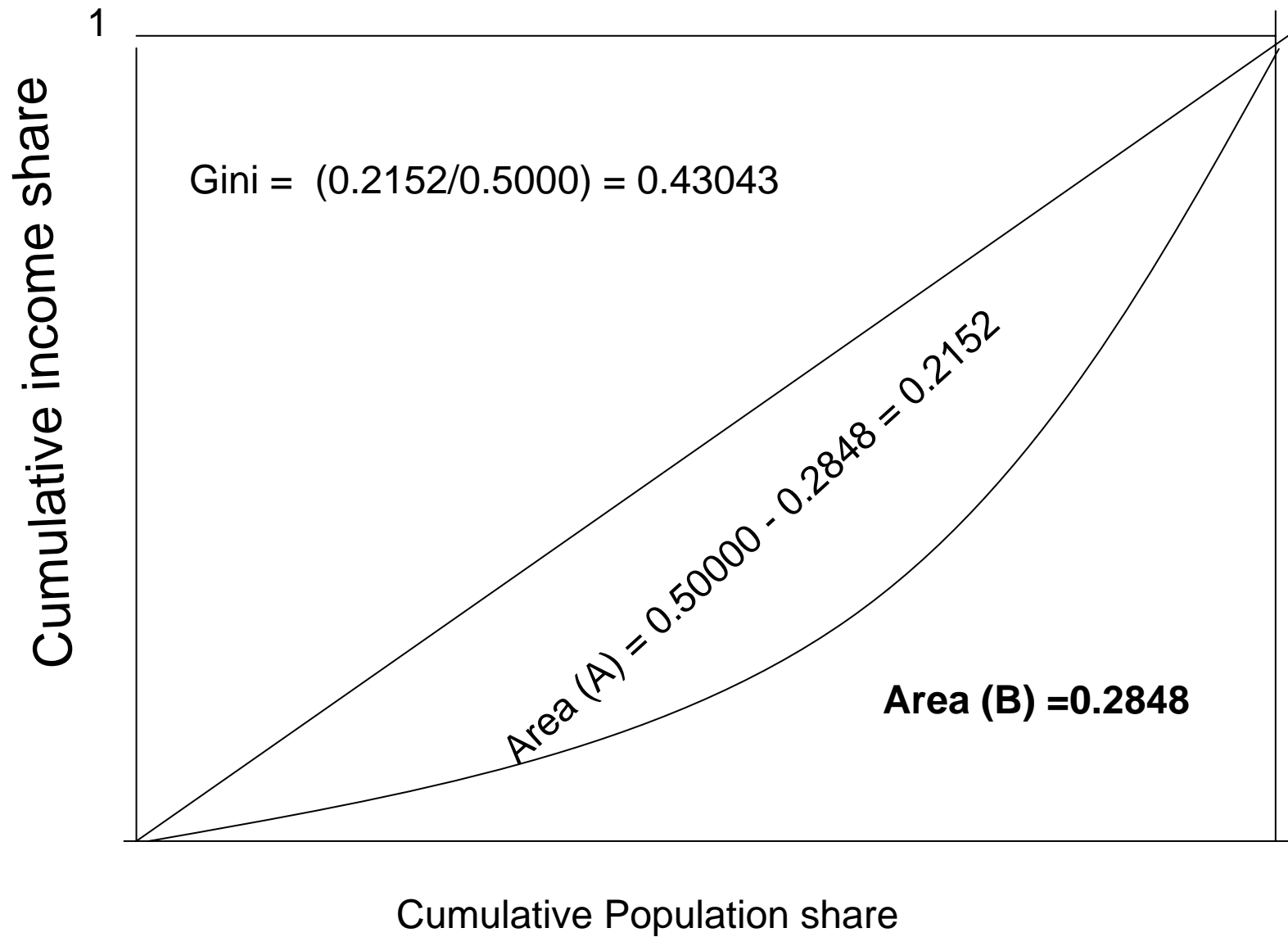
Approximation of Area Under the Lorenz Curve



Approximation of Area Under the Lorenz Curve



Computation of Gini Coefficient



Income group	Income
1st (lowest)	10186
2nd	25321
3rd	42492
4th	66939
5th (highest)	145811
Total	290749

Ishare	CMIshare	Equality	pop	cmpop	Rectangle	Triangle	Total area
0.035034	0.035034	0.2	0.2	0.2	0	0.003503	0.003503
0.087089	0.122123	0.4	0.2	0.4	0.007007	0.008709	0.015716
0.146147	0.268269	0.6	0.2	0.6	0.024425	0.014615	0.039039
0.23023	0.498499	0.8	0.2	0.8	0.053654	0.023023	0.076677
0.501501	1	1	0.2	1	0.0997	0.05015	0.14985

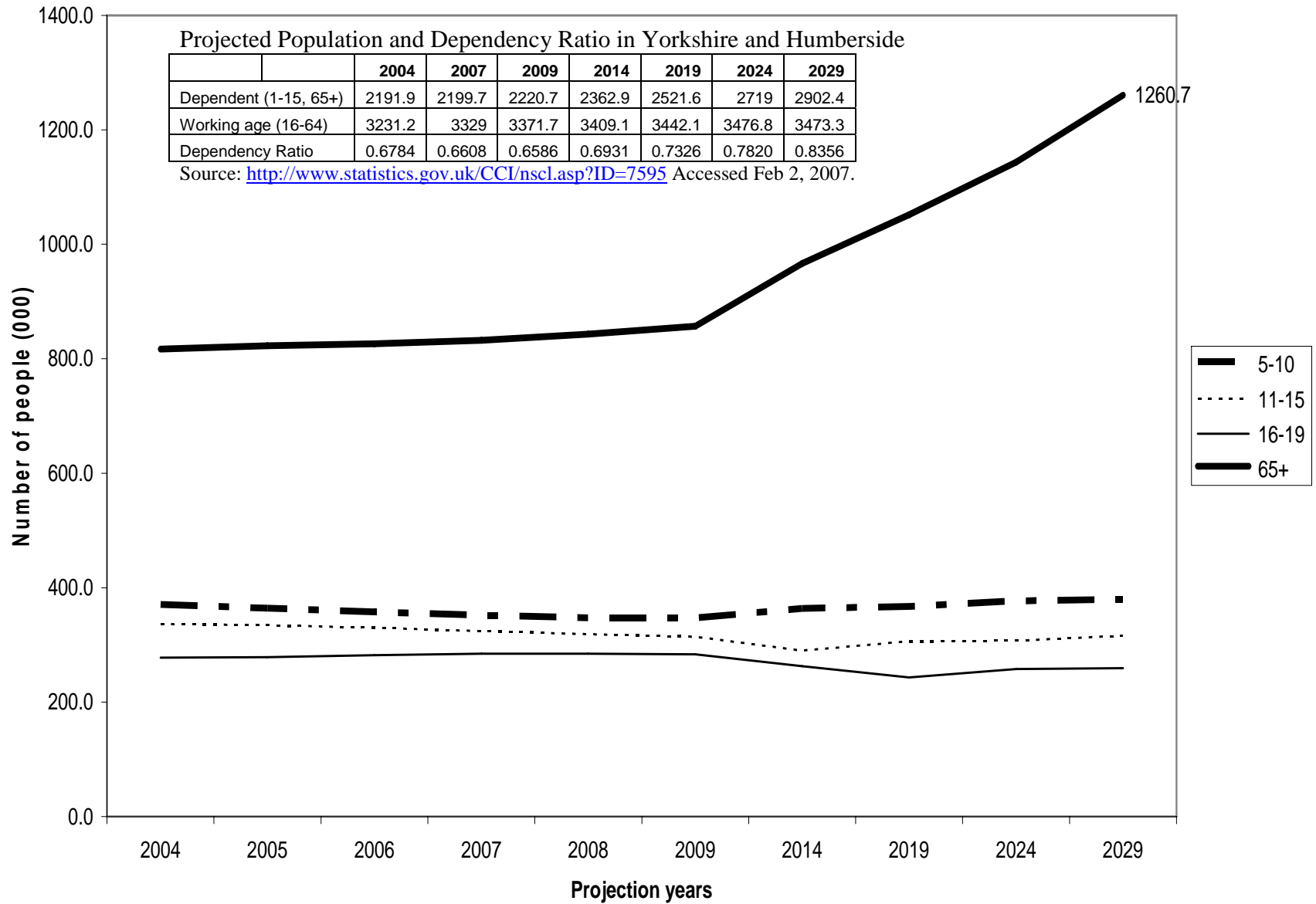
Total area under Lorenz

0.2848

Gini

0.43043

Projection of Old and young People in Yorksshire and Humber





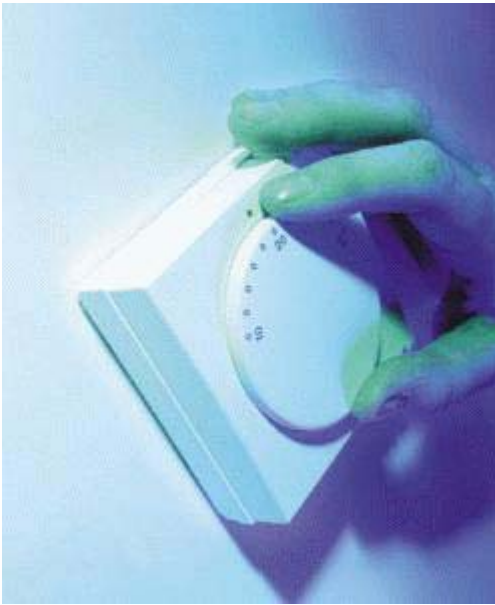
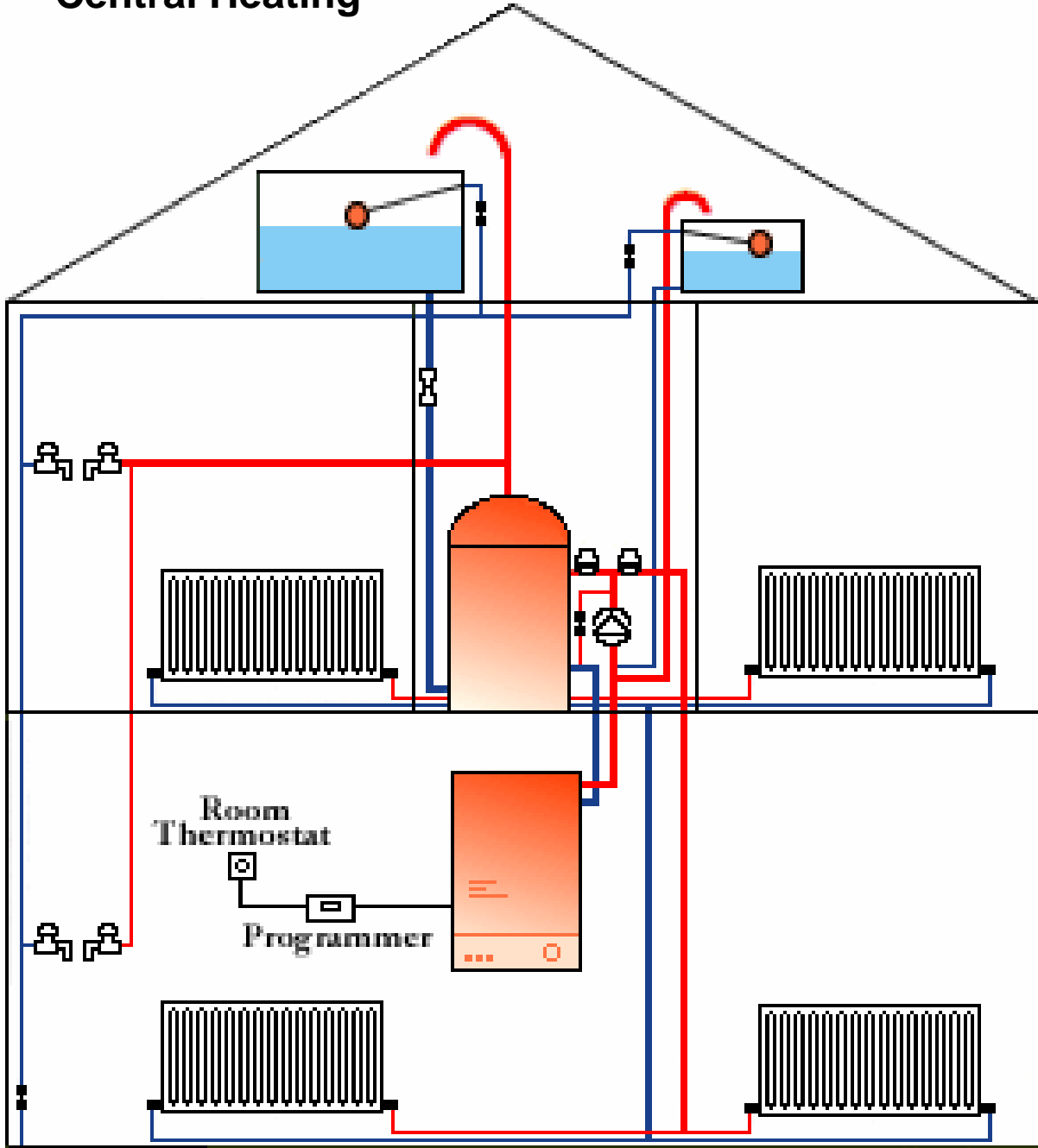
Gas fired central heating



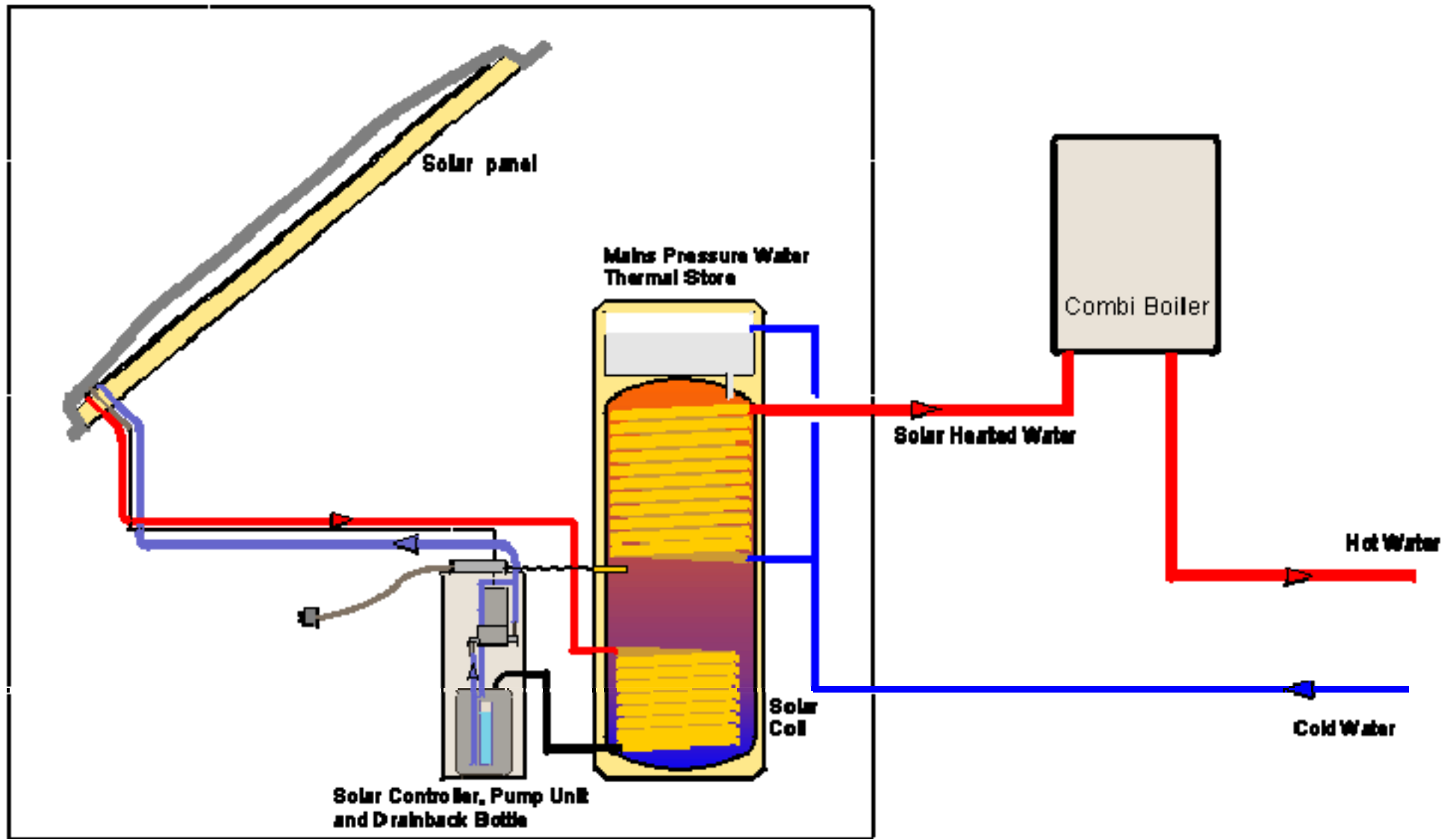


Woods and coals

Central Heating



Combi-Boilers





Current practices

Meters and fuel costs



© Mandy Barrow



Interior of House

