

Role of Financial Markets in an Economy

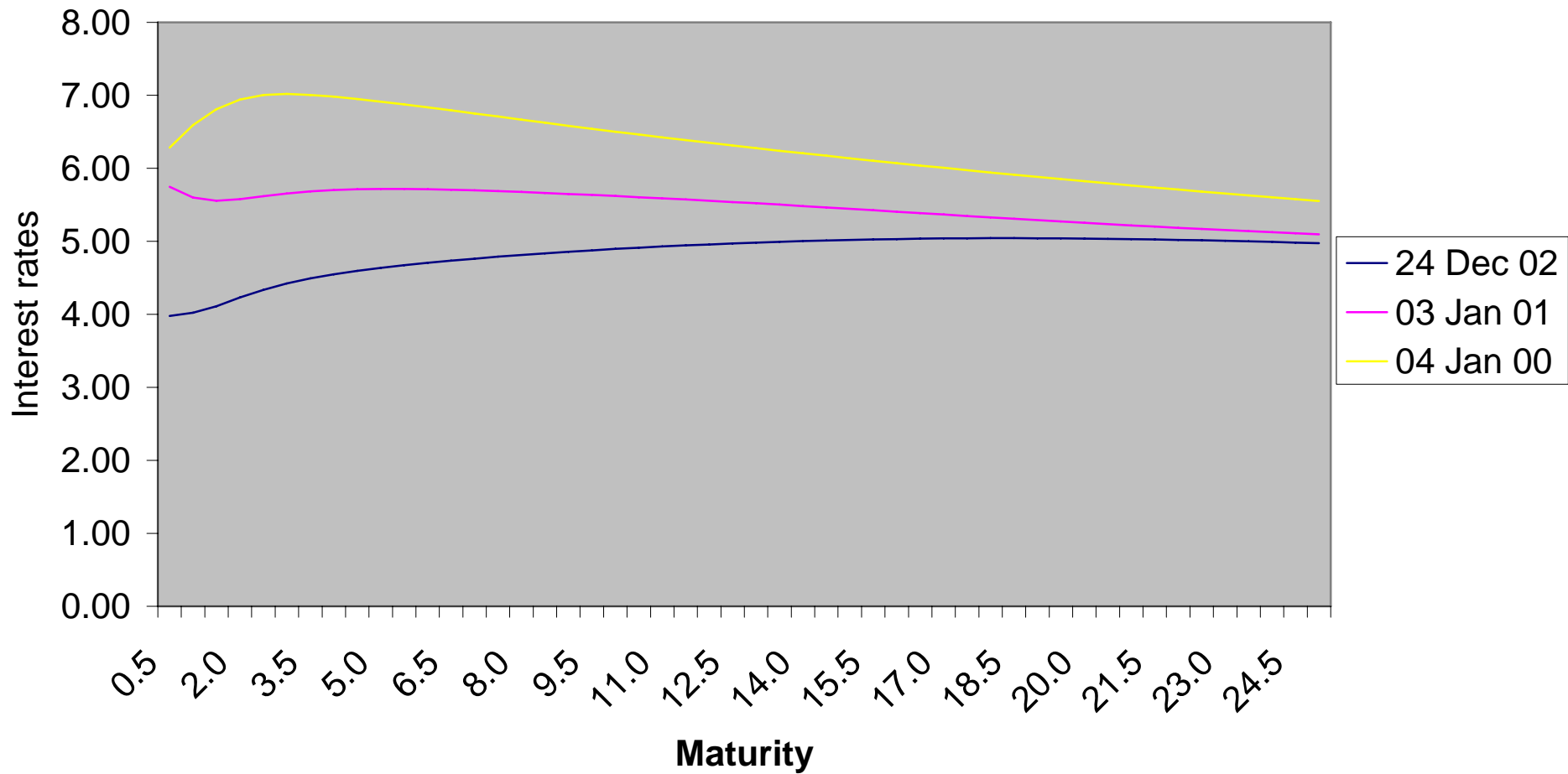
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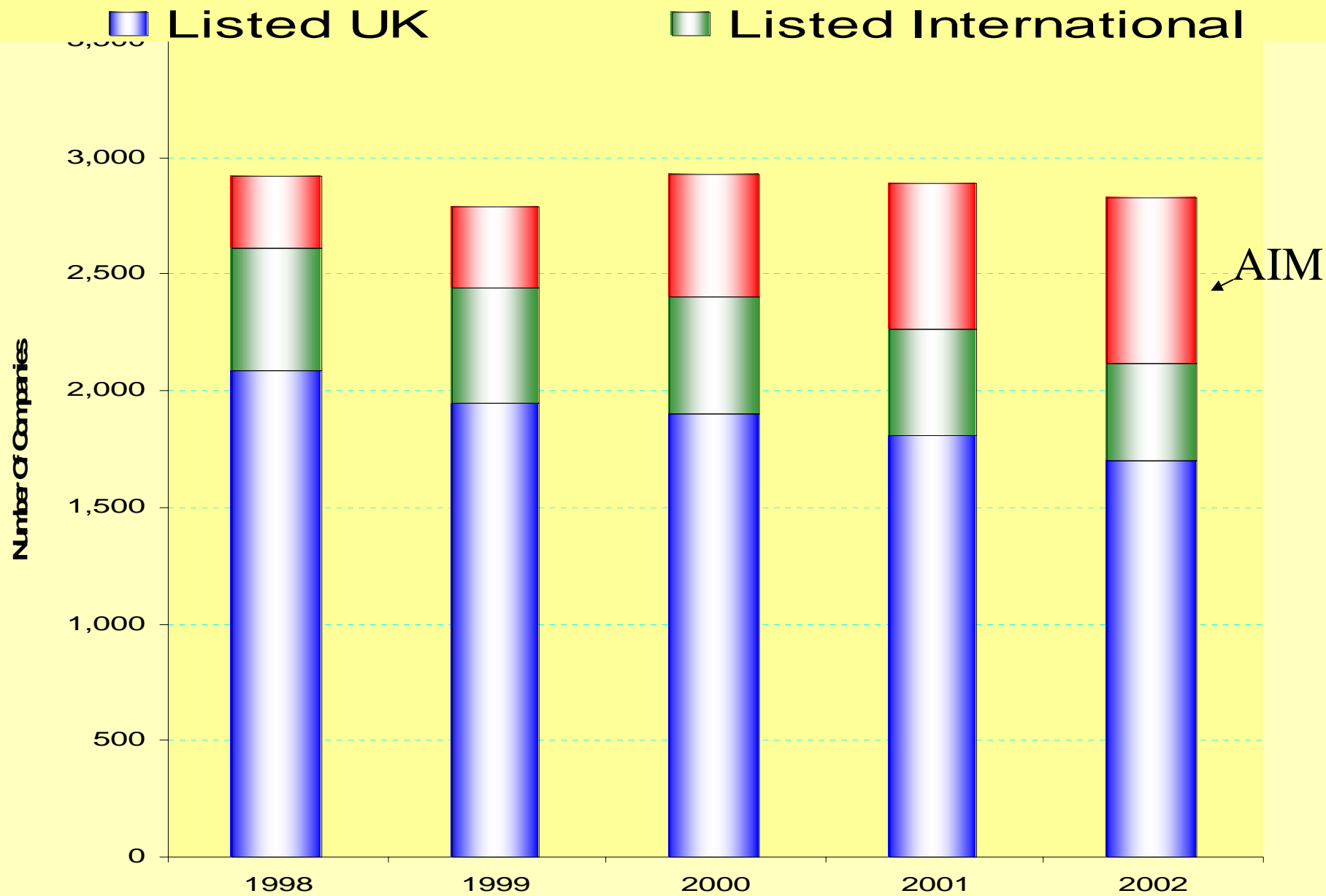
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Higher Long Run Interest Rates are Better for Investment

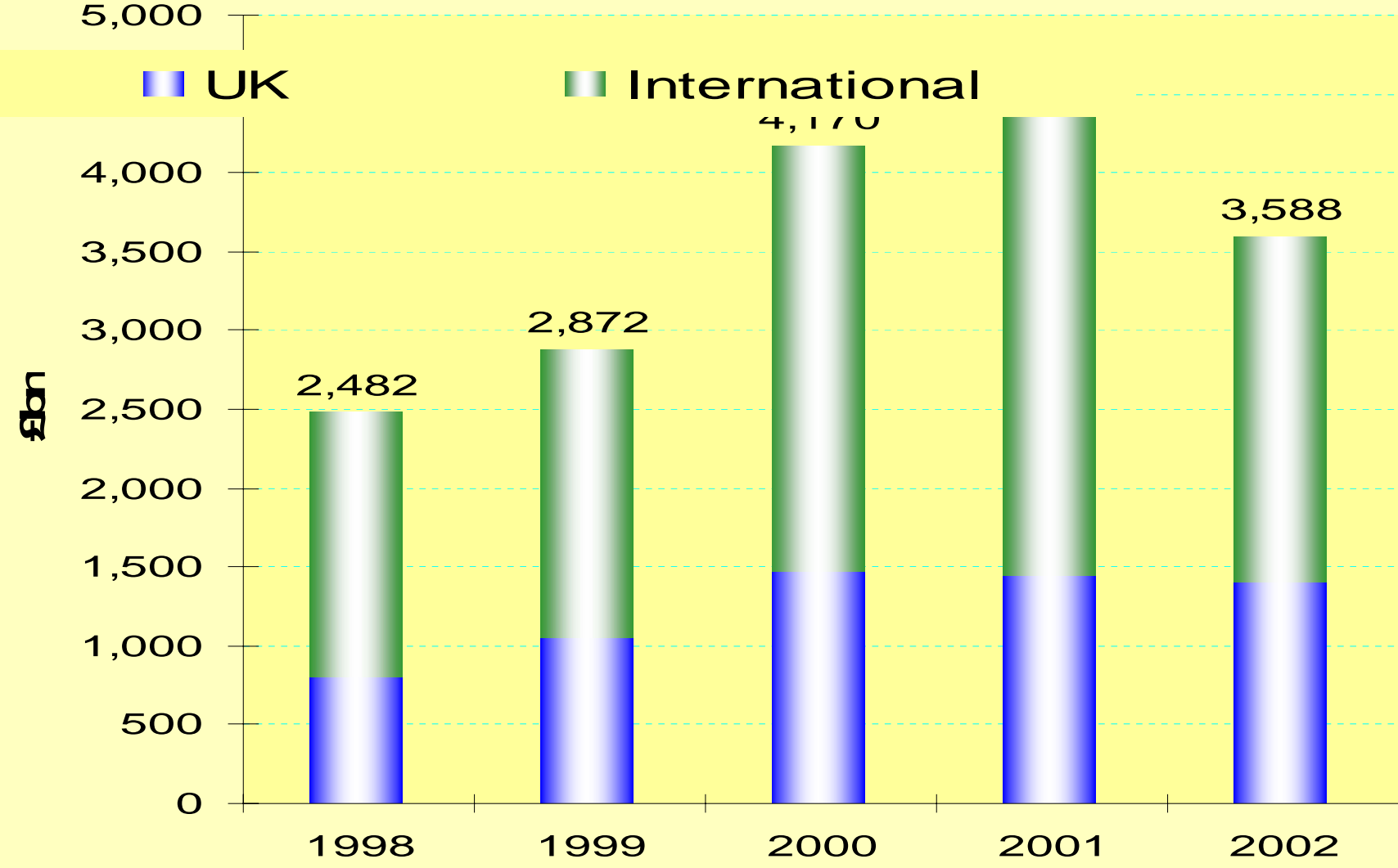
Yield Curve for Government Bonds: Long Term and Short Term Interest Rates in the UK



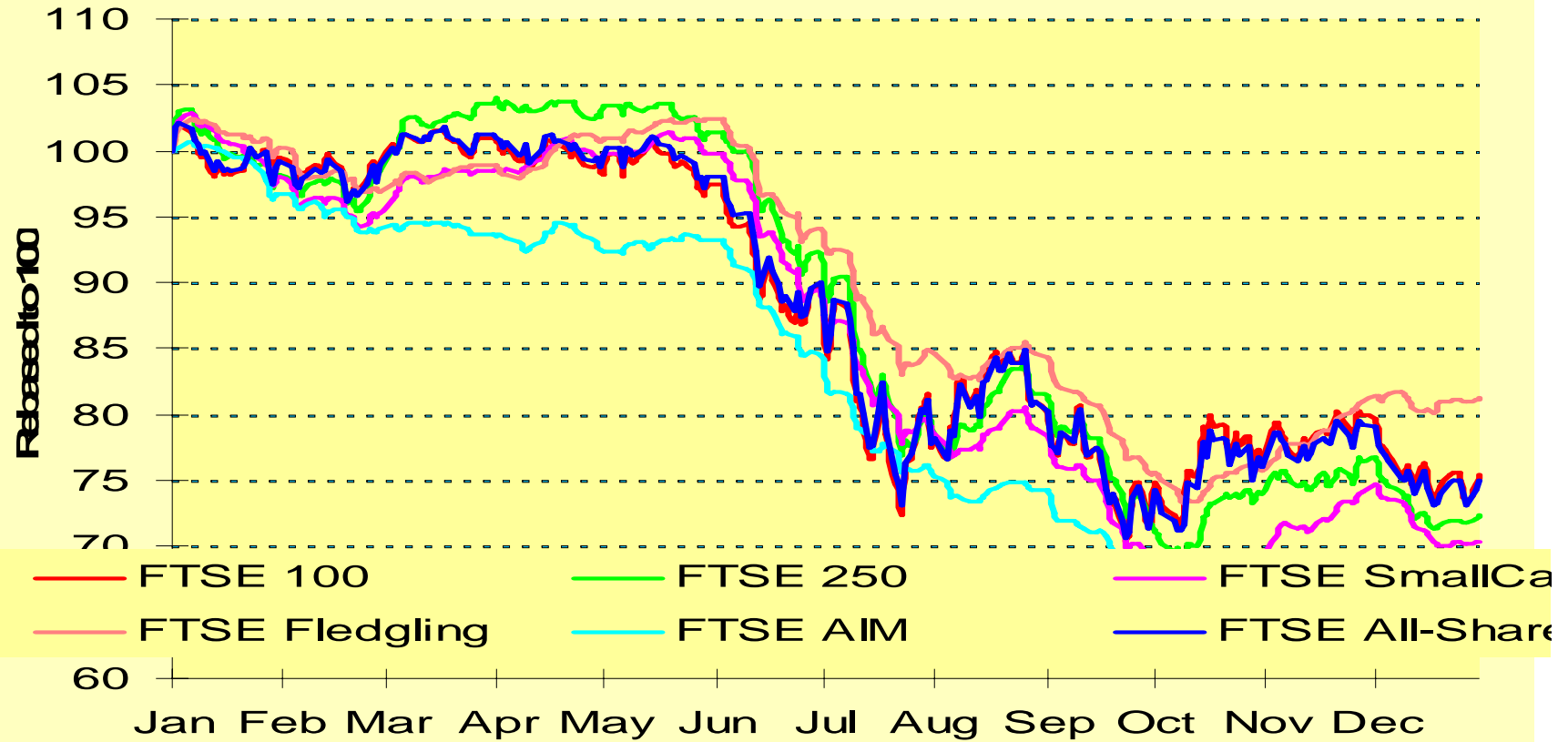
Total number of UK and International companies
at end December



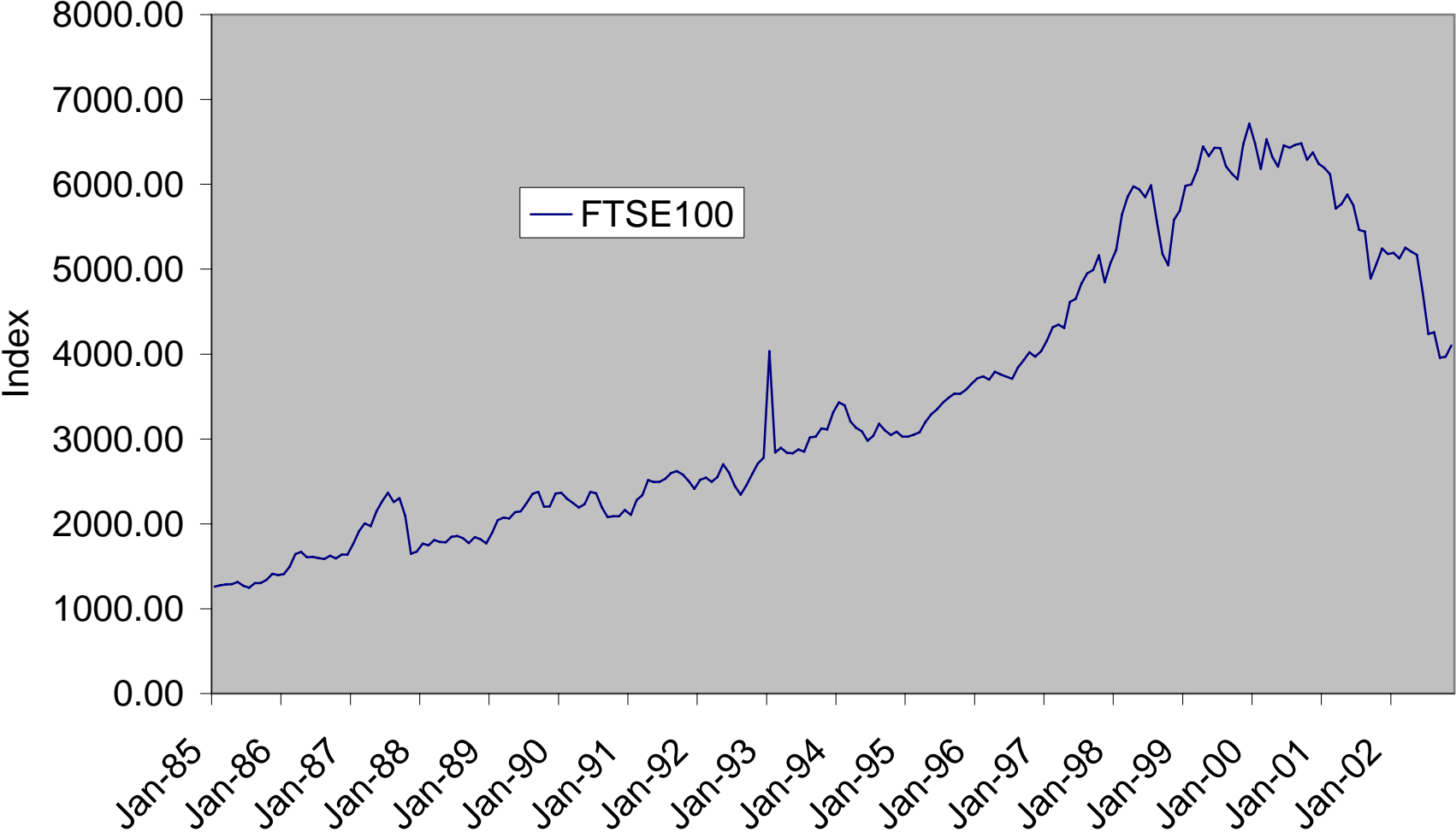
Total equity turnover value as at end September



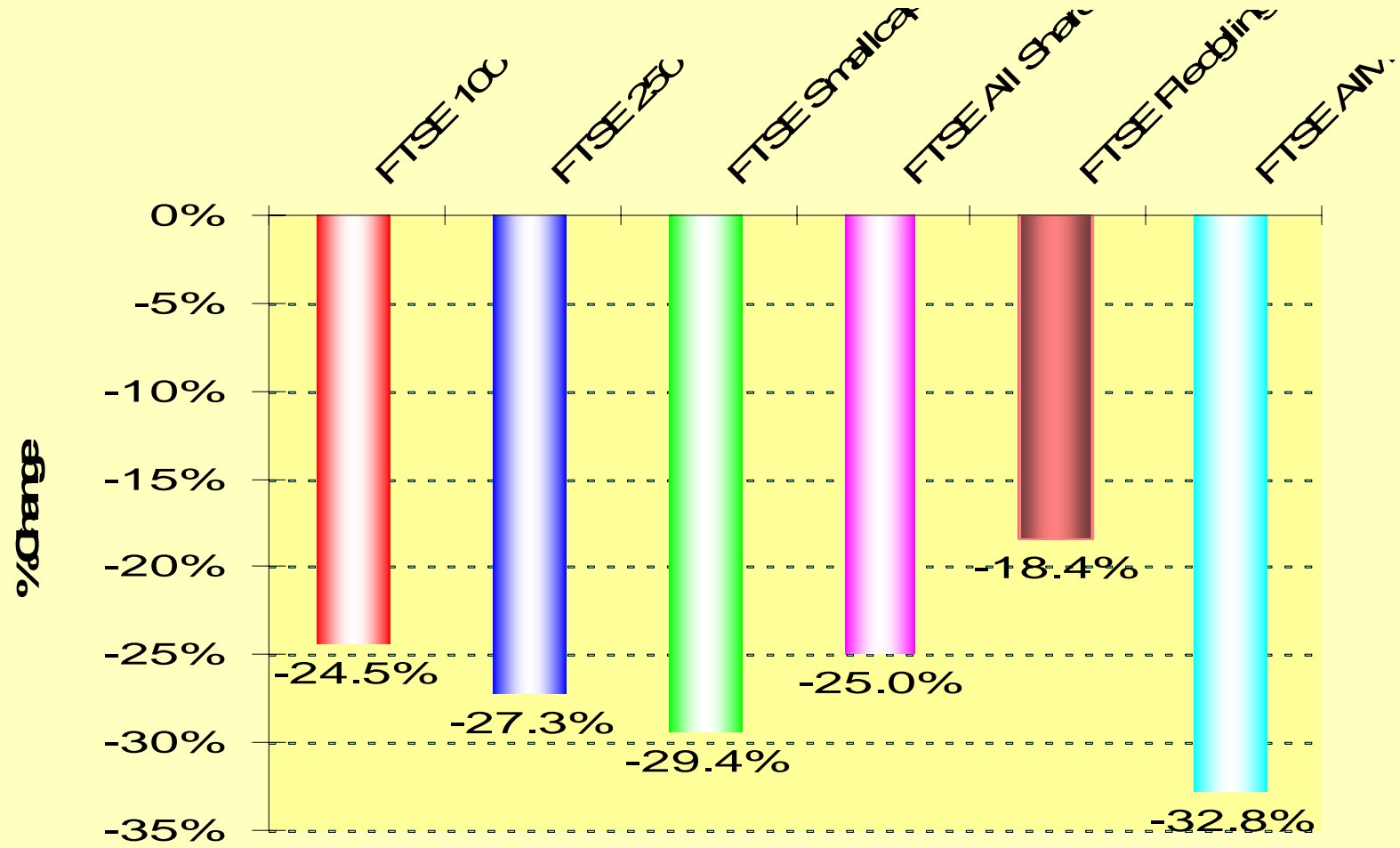
UK indices - daily index movements January to December 2002



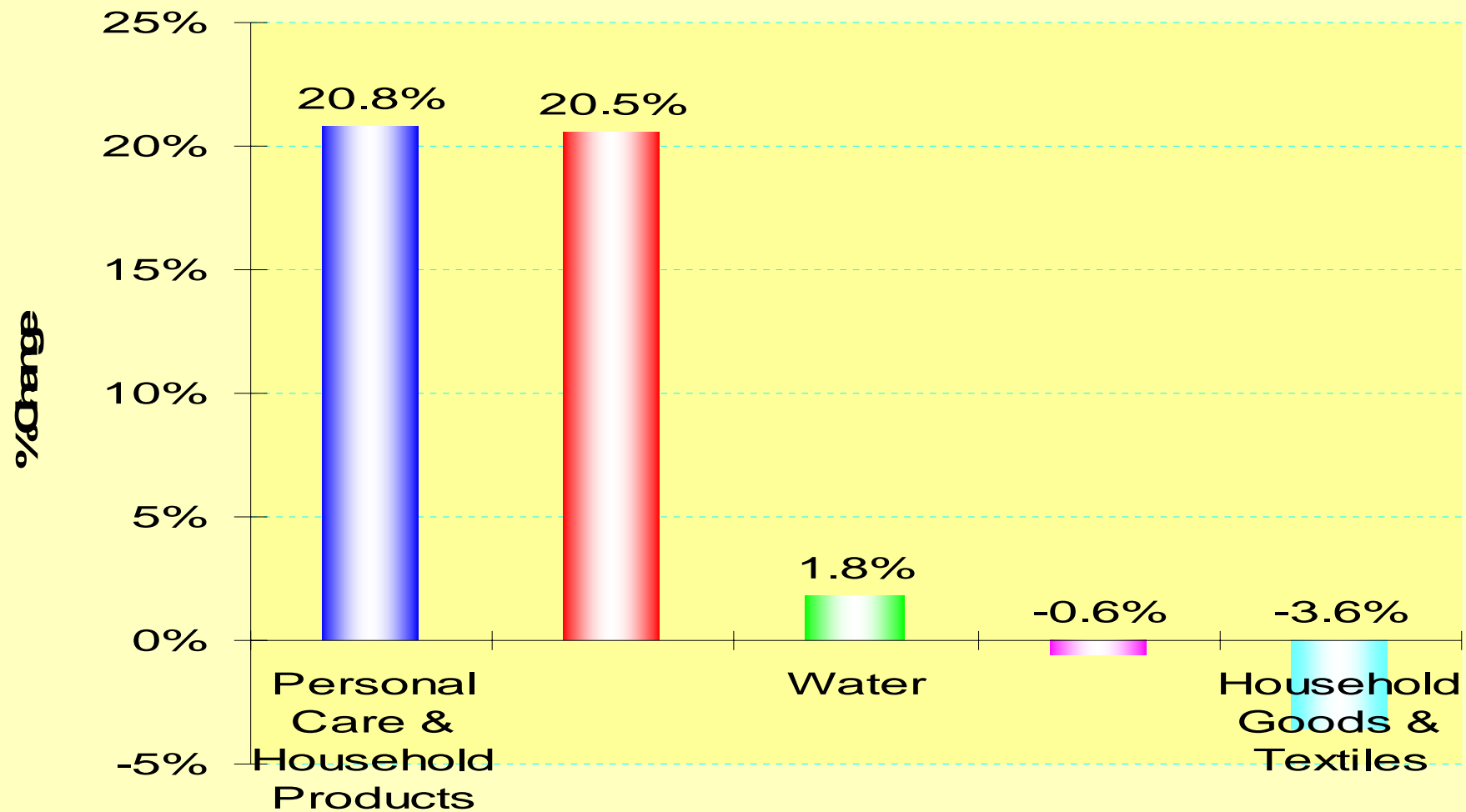
FTSE100 Index



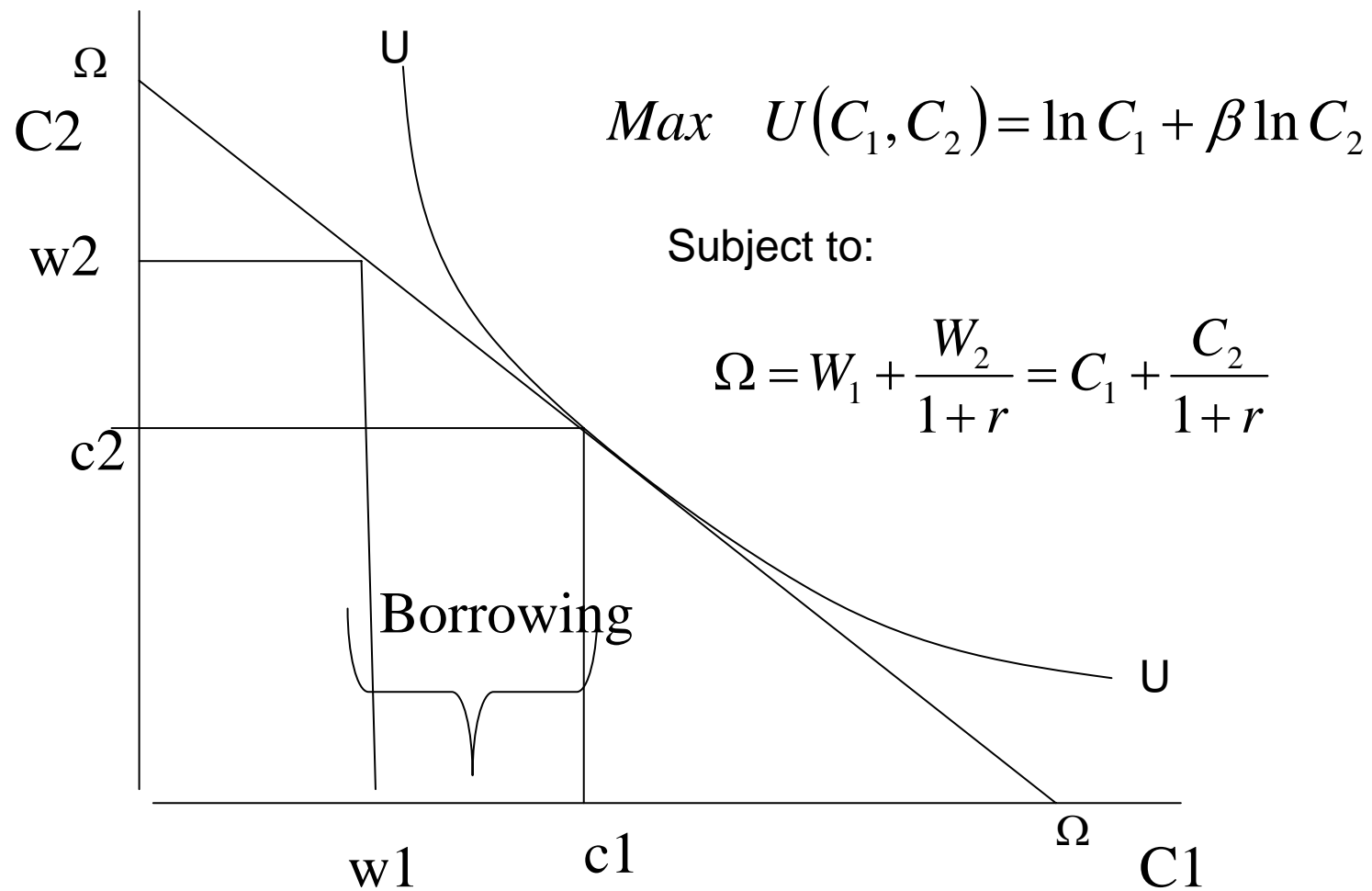
Comparative UK indices' Performance - January to December 2002



Five Largest UK sector indices' movements - January to December 2002



Two Period Model of Consumption and Saving



Consumption-Saving Problem in a Two Period Model

$$\text{Max } U(C_1^i, C_2^i) = \ln C_1^i + \beta \ln C_2^i \quad (1)$$

Subject to

$$\begin{aligned} C_1^i + b^i &\leq W_1^i && \text{budget constraint in period 1} \\ C_2^i &\leq b^i(1+r) + W_2^i && \text{budget constraint in period 2} \end{aligned} \quad (2)$$

The inter temporal budget constraint is:
(4)

$$C_1^i + \frac{C_2^i}{1+r} = W_1^i + \frac{W_2^i}{1+r} \quad (3)$$

$$L^i = \ln C_1^i + \beta \ln C_2^i + \lambda \left[C_1^i + \frac{C_2^i}{1+r} - W_1^i - \frac{W_2^i}{1+r} \right] \quad (4)$$

Demand for period 1 and 2 consumptions

$$C_1^i = \frac{1}{1+\beta} \left[W_1^i + \frac{W_2^i}{1+r} \right] \quad (5)$$

$$C_2^i = (1+r)\beta C_1^i \Rightarrow C_2^i = (1+r) \frac{\beta}{1+\beta} \left[W_1^i + \frac{W_2^i}{1+r} \right] \quad (6)$$

Market clearing conditions in period 1 and 2

$$C_1^A + C_1^B = W_1^A + W_1^B$$

$$\frac{1}{1+\beta} \left[W_1^A + \frac{W_2^A}{1+r} \right] + \frac{1}{1+\beta} \left[W_1^B + \frac{W_2^B}{1+r} \right] = W_1^A + W_1^B \quad (7)$$

Market clearing condition for period 2:

$$C_2^A + C_2^B = W_2^A + W_2^B$$

$$\frac{\beta(1+r)}{1+\beta} \left[W_1^A + \frac{W_2^A}{1+r} \right] + \frac{\beta(1+r)}{1+\beta} \left[W_1^B + \frac{W_2^B}{1+r} \right] = W_2^A + W_2^B \quad (8)$$

The gross interest rate that clears the market

$$\frac{1}{1+\beta} \left[W_1^A + \frac{W_2^A}{1+r} \right] + \frac{1}{1+\beta} \left[W_1^B + \frac{W_2^B}{1+r} \right] = W_1^A + W_1^B$$

or

$$\frac{1}{1+\beta} \left[W_1^A + W_1^B \right] + \frac{1}{1+\beta} \left[\frac{W_2^A}{1+r} + \frac{W_2^B}{1+r} \right] = W_1^A + W_1^B$$

$$\frac{1}{1+\beta} + \frac{1}{1+\beta} \left(\frac{W_2^A + W_2^B}{W_1^A + W_1^B} \right) = 1+r \quad (9)$$

Numerical Example

If endowments and preferences were $\{W_1^A, W_2^A, W_1^B, W_2^B\} = \{100, 0, 0, 200\}$ and $\beta = 0.9$

respectively the interest rate that clears these two markets is given by:

$$\frac{1}{1+0.9} + \frac{1}{1+0.9} \left(\frac{200}{100} \right) = 1+r = 2.222 \Rightarrow r = 1.222 \quad (10)$$

Same result can be obtained using the period 2 budget constraint

$$1+r = \frac{1}{\beta} \left(\frac{W_2^A + W_2^B}{W_1^A + W_1^B} \right) = \frac{1}{0.9} \left(\frac{200}{100} \right) \Rightarrow 2.222 \quad (10)$$

Scenarios of Equilibrium Allocation in Pure Exchange Model

Supply of savings in Two period Model with Various parameters

Parameters of Consumption Saving Model							
	I	II	III	IV	V	VI	VII
w1a	100	100	100	100	100	500	1000
w2a	0	0	0	50	200	200	200
w1b	0	0	0	100	100	400	2000
w2b	200	200	200	200	200	200	100
Beta	0.9	0.95	1	0.9	0.9	0.9	0.9
Equilibrium Interest Rate and Consumption							
1+r	2.222222	2.105263	2	1.388889	2.222222	0.493827	0.111111
c1a	52.63158	51.28205	50	71.57895	100	476.3158	1473.684
c2a	105.2632	102.5641	100	89.47368	200	211.6959	147.3684
Ua	8.154133	8.336304	8.517193	8.315351	9.373656	10.98572	11.78916
Uanb	4.60517	4.60517	4.60517	8.125991	9.373656	10.98309	11.67624
c1b	47.36842	48.71795	50	128.4211	100	423.6842	1526.316
c2b	94.73684	97.4359	100	160.5263	200	188.3041	152.6316
Ub	7.953948	8.236282	8.517193	9.425926	9.373656	10.76324	11.85584
Ubnb	4.768486	5.033401	5.298317	9.373656	9.373656	10.75995	11.74556
Equilibrium Savings (Borrowing or Lending)							
s1a	47.36842	48.71795	50	28.42105	0	23.68421	-473.684
s2a	-105.263	-102.564	-100	-39.4737	0	-11.6959	52.63158
s1b	-47.3684	-48.7179	-50	-28.4211	0	-23.6842	473.6842
s2b	105.2632	102.5641	100	39.47368	0	11.69591	-52.6316

Consumption Saving Decisions of Agent A

$$C_1^A = \frac{1}{1+\beta} \left[W_1^A + \frac{W_2^A}{1+r} \right] = \frac{1}{1.9} (100) = 52.63$$

$$C_2^A = (1+r)\beta \frac{1}{1+\beta} \left[W_1^A + \frac{W_2^A}{1+r} \right] = 2.222 \left(\frac{0.9}{1.9} \right) 100 = 105.25$$

Amount of savings in period 1 and period 2 respectively are

$$S_1^A = W_1^A - C_1^A = W_1^A - \frac{1}{1+\beta} \left[W_1^A + \frac{W_2^A}{1+r} \right] = 100 - 52.63 = 47.37$$

Consumption Saving Decisions of Agent B

$$C_1^B = \frac{1}{1+\beta} \left[W_1^B + \frac{W_2^B}{1+r} \right] = \frac{1}{1.9} \left(\frac{200}{2.222} \right) = 47.37$$

$$C_2^B = (1+r) \frac{\beta}{1+\beta} \left[W_1^B + \frac{W_2^B}{1+r} \right] = 2.222 \left(\frac{0.9}{1.9} \right) \frac{200}{2.222} = 94.73$$

$$S_1^B = W_1^B - C_1^B = -47.37; \quad S_2^B = W_2^B - C_2^B = 200 - 94.73 = 105.3$$

Calculations of Life Cycle Income

$$v(Y_{Lt}^e - T_t^e) = (1-t) \left[1 + (1+g) + (1+g)^2 + \dots + (1+g)^{36} \right] Y_0 \quad (11)$$

Apply sum of geometric series $s = 1 + X + X^2 + \dots + X^n = \frac{1 - X^{n+1}}{1 - X}$ to calculate this sum

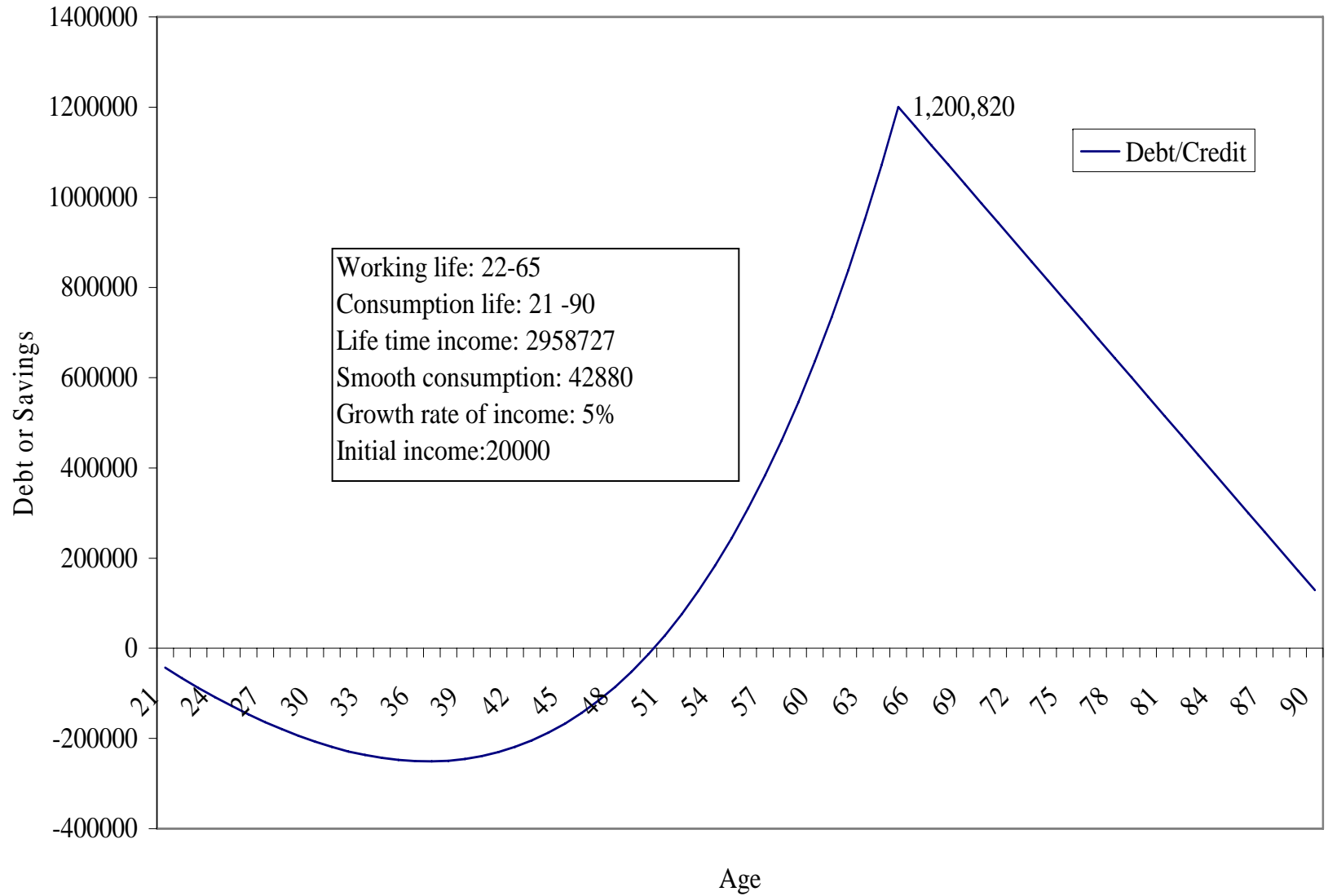
as: $v(Y_{Lt}^e - T_t^e) = 0.75 \left[1 + 1.03 + (1.03)^2 + \dots + (1.03)^{36} \right] \text{€}40000 = 1985227.$

Table 2

Variation in Life Time Income and Consumption by Initial Income and Tax Rates

Individuals	Initial income	tax rate	Growth	working life	Cons life	Life time y	Consumption per period
A	1000000	0.4	1.03	36	56	39704534	709009.5
B	50000	0.3	1.04	36	56	2859579	51063.9
C	40000	0.25	1.03	20	56	860295	15362.4
D	40000	0.2	1.04	30	56	1898507	33901.91
E	20000	0.15	1.03	40	60	1337276	22287.93
F	10000	0.1	1.05	40	65	1150558	17700.89

Figure 1
Debt and Savings in Life Cycle Model



Output

$$Y = F(K) \quad (12)$$

Capital stock accumulation

$$K_t = K_{t-1}(1 - \delta) + I_t \quad \text{Or} \quad I_t = K_t - K_{t-1}(1 - \delta) \quad (13)$$

Profit function can be represented as:

$$\Pi = \frac{F(K)}{(1+r)} - P^k K + \frac{(1-\delta)P_2^K K}{1+r} \quad (14)$$

Profit maximisation

$$\frac{\partial \Pi}{\partial K} = \frac{F'(K)}{(1+r)} - P^k + \frac{(1-\delta)P_2^K}{1+r} = 0 \quad \text{or} \quad MPK = (1+r)P_1^k - (1-\delta)P_2^K \quad (15)$$

$$P_2^k = P_1^k (1 + \pi^K), \text{ where } \pi^K = \frac{P_2^K}{P_1^K} - 1$$

$$MPK = [(1+r) - (1-\delta)(1 + \pi^K)]P_1^k \rightarrow MPK = [r + \delta - \pi^K]P_1^k.$$

$$Y = AK^\alpha L^{1-\alpha} \text{ and } MPK = \alpha AK^{\alpha-1} L^{1-\alpha} \text{ or } MPK = \frac{\alpha Y}{K} \text{ or } K = \frac{\alpha Y}{MPK} \quad (16)$$

$$K \cong \frac{\alpha Y}{(r + \delta - \pi^K)P_1^K}.$$

An investment tax credit leads to more accumulation of capital by reducing its cost

as: $K \cong \frac{\alpha Y}{(1-\tau)(r + \delta - \pi^K)P_1^K}$ where $\tau > 0$ represents the

Table 3									
Optimal Capital Stock and its Marginal Product and Output with investment tax credit									
Parameters in the Marginal Productivity of Investment Model									
	I	II	III	IV	V	VI	VII	VIII	IX
Technology (A)	1.2	1.2	1.2	1.2	2.2	2.2	2.2	2.2	2.2
Share of capital (α)	0.4	0.7	0.8	0.2	0.2	0.4	0.4	0.4	0.4
interest rate (r)	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Depreciation(δ)	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.08	0.08
Capital gain (π^K)	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Labour input (L)	100	100	100	100	100	100	100	100	200
Optimal Capital Stock, Output and Marginal Product of Capital with and without investment tax credit									
Price of capital at period 1	1	1	1	1	1	1	1	1	1
at period 2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
tax credit rate	0	0	0	0	0	0	0.3	0.3	0.3
Output, Capital stock and the Marginal Productivity of Capital									
K	4336	1214407	260919263	710	1516	11908	21579	5799	11597
Y	542	86743	16307454	178	379	1489	1888	1116	2232
MPK	0.05	0.05	0.05	0.05	0.05	0.05	0.035	0.077	0.077

Tobin's Q and Marginal Productivity

$$q_t = \frac{V(\Pi_t^e)}{P_t^k K} \quad \text{and} \quad q_t = \begin{cases} > 1 & \Rightarrow I_t > 0 \\ 1 & \Rightarrow I_t = 0 \\ < 1 & \Rightarrow I_t < 0 \end{cases} \quad (17)$$

$$V(\Pi_t^e) = \left[\frac{1}{1+r_t} \Pi_{t+1}^e + \frac{1}{(1+r_t)(1+r_t^e)} (1-\delta) \Pi_{t+1}^e + \dots + \right] \quad (18)$$

Profits in each period in turn depend upon the productivity of capital:

$$\Pi_t = \Pi \left(\frac{Y_t}{K_t} \right) \cdot I \left[V(\Pi_t^e) \right] = I \left[\frac{1}{1+r_t} \Pi_{t+1}^e + \frac{1}{(1+r_t)(1+r_t^e)} (1-\delta) \Pi_{t+1}^e + \dots + \right]$$

(19).

Samuelson's (1939) Multiplier-Accelerator Model

$$I_{n,t} = \alpha Y_t - \alpha Y_{t-1} = I_{n,t} = \alpha(\Delta Y_t)$$

$$C_t = a_0 + a_1 Y_{t-1} \text{ with } a_0 > 1 \text{ and } 0 < a_1 < 1$$

$$C_t = a_0 + a_1 Y_{t-1} \text{ with } a_0 > 1 \text{ and } 0 < a_1 < 1$$

$$\bar{Y} = \frac{a_0 + b_0 + G_0}{(1 - a_1)}$$

$$Y_t = K_1 \lambda_1^t + K_2 \lambda_2^t$$

$$Y_t = \bar{Y} + \left(\frac{Y_1 - 0.42Y_0}{1.85} \right) (2.37)^t + \left(\frac{1.43Y_0 - Y_1}{1.85} \right) (0.42)^t \quad (20)$$

Scenarios in Solow Model

Table 4
Role of Saving /Investment, Intermediation Cost and Technology in the Solow Model

$$Y = AK^\alpha$$

Technology	Capital Share	Saving rate	Depreciation	Capital Gain	Interest rate
A = 1.000	$\alpha = 0.500$	s = 0.200	$\delta = 0.010$	$\pi^K = 0.020$	R = 0.040
Role of Investment and Saving: Initial Scenario					
Time prd	Capital Stock	Output	MPK	Investment	Consumption
1	100.000	10.000	25.000	2.000	8.000
2	101.000	10.050	25.250	2.010	8.040
3	102.000	10.100	25.500	2.020	8.080
.
1761	399.941	19.999	99.985	4.000	15.999
Impact of financial Intermediation : $\phi = 0.800$					
	Capital Stock	Output	MPK	Saving	Consumption
1	100.000	10.100	25.000	2.020	8.080
2	100.616	10.127	25.154	2.025	8.102
3	101.230	10.154	25.308	2.031	8.123
.
1761	255.972	15.999	63.993	3.200	12.799
Role of Technical Progress : 1.010					
	Capital Stock	Output	MPK	Investment	Consumption
1	100.000	10.100	25.000	2.020	8.080
2	101.020	10.253	25.255	2.051	8.202
3	102.060	10.409	25.515	2.082	8.327
.
1761	7.325E+16	1.1024E+16	1.8312E+16	2.2048E+15	8.8193E+15

Optimal Growth Models

One Sector Ramsey Model

Role of the Financial Sector

Ramsey (1928), Cass (1965) Koopmans(1965) type Optimal Growth Model

$$U_t = \sum_{t=0}^{\infty} \beta^t \ln(C_t) \quad 0 < \beta < 1$$

$$Y_t = A_t K_t^\alpha \quad 0 < \alpha < 1$$

$$K_{t+1} = I_t$$

$$C_t + I_t = Y_t \quad K_0 = K_0$$

$$K_{t+1} = I_t \quad K_{t+1} = K_t(1 - \delta) + I_t$$

$$C_t = Y_t - I_t \quad C_t = AK_t^\alpha - \phi\{K_{t+1} - K_t(1 - \delta)\}$$
$$0 < \phi < 1$$

Steady State in an Optimal Growth Model

$$U_t = \sum_{t=0}^{\infty} \beta^t \ln(AK_t^\alpha - K_{t+1}) \quad \delta=1$$

$$U_t = +\beta^t \ln(AK_t^\alpha - K_{t+1}) + \beta^{t+1} \ln(AK_{t+1}^\alpha - K_{t+2}) + \dots$$

$$\frac{\partial U_t}{\partial K_{t+1}} = -\frac{\beta^t}{C_t} + \frac{\beta^{t+1}}{C_{t+1}} \alpha AK_{t+1}^{\alpha-1} = 0 \quad \frac{C_{t+1}}{C_t} = \frac{\beta^{t+1}}{\beta^t} \alpha AK_{t+1}^{\alpha-1}$$

$$\frac{C_{t+1}}{C_t} = \beta \alpha AK_{t+1}^{\alpha-1}$$

$$U_t = \ln(A\bar{K}^\alpha - \bar{K}) \sum_{t=0}^{\infty} \beta^t$$

$$\frac{C_{t+1}}{C_t} = \frac{\bar{C}}{\bar{C}} = \beta \alpha A \bar{K}^{\alpha-1}$$

$$\dots = K_{t-1} = K_t = K_{t+1} = \dots = \bar{K}$$

$$\dots = C_{t-1} = C_t = C_{t+1} = \dots = \bar{C}$$

$$\bar{K} = \left(\frac{1}{\beta \alpha A} \right)^{\frac{1}{\alpha-1}} = (\beta \alpha A)^{\frac{1}{1-\alpha}}$$

$$\bar{Y} = A\bar{K}^\alpha \quad \bar{Y} = A(\beta \alpha A)^{\frac{\alpha}{1-\alpha}}$$

$$\bar{C} = \bar{Y} - \bar{I} = A\bar{K}^\alpha - \bar{K} = A(\beta \alpha A)^{\frac{\alpha}{1-\alpha}} - (\beta \alpha A)^{\frac{1}{1-\alpha}} = A^{\frac{1}{1-\alpha}} (\beta \alpha)^{\frac{1}{1-\alpha}} [(\beta \alpha)^\alpha - 1]$$

Optimal Growth Model with less than 100% depreciation

$$0 < \delta < 1$$

$$K_{t+1} = K_t(1 - \delta) + I_t \quad K_{t+1} - K_t(1 - \delta) = I_t$$

$$C_t = Y_t - I_t \quad C_t = AK_t^\alpha - K_{t+1} - K_t(1 - \delta)$$

$$U_t = \sum_{t=0}^{\infty} \beta^t \ln(AK_t^\alpha - K_{t+1} + K_t(1 - \delta))$$

$$U_t = +\beta^t \ln(AK_t^\alpha - K_{t+1} + K_t(1 - \delta)) + \beta^{t+1} \ln(AK_{t+1}^\alpha - K_{t+2} + K_{t+1}(1 - \delta)) + \dots +$$

$$\frac{\partial U_t}{\partial C_t} \frac{\partial C_t}{\partial C_{t+1}} \frac{\partial C_{t+1}}{\partial K_{t+1}} = -\frac{\beta^t}{C_t} + \frac{\beta^{t+1}}{C_{t+1}} (\alpha AK_{t+1}^{\alpha-1} + (1 - \delta)) = 0$$

$$\frac{C_{t+1}}{C_t} = \beta (\alpha AK_{t+1}^{\alpha-1} + (1 - \delta)) \quad \dots = K_{t-1} = K_t = K_{t+1} = \dots = \bar{K}$$

$$\dots = C_{t-1} = C_t = C_{t+1} = \dots = \bar{C} \quad U_t = \ln(A\bar{K}^\alpha - \bar{K} + \bar{K}(1 - \delta)) \sum_{t=0}^{\infty} \beta^t$$

Steady State in an Optimal Growth Model with less than 100% depreciation

$$0 < \delta < 1$$

$$\frac{C_{t+1}}{C_t} = \frac{\bar{C}}{\bar{C}} = \beta(\alpha A \bar{K}^{\alpha-1} + (1-\delta)) \quad (\alpha A \bar{K}^{\alpha-1} + (1-\delta)) = \left(\frac{1}{\beta}\right)$$

$$(\bar{K}^{\alpha-1}) = \frac{1}{\alpha A} \left(\frac{1}{\beta} - (1-\delta) \right) \quad (\bar{K}^{\alpha-1}) = \frac{1}{\alpha A} \left(\frac{1 - \beta(1-\delta)}{\beta} \right)$$

$$\bar{K} = \left(\frac{1 - \beta(1-\delta)}{\alpha A \beta} \right)^{\frac{1}{\alpha-1}} \quad \bar{K} = \left(\frac{\alpha A \beta}{1 - \beta(1-\delta)} \right)^{\frac{1}{1-\alpha}}$$

$$\bar{Y} = A \bar{K}^{\alpha} \quad \bar{Y} = A^{\frac{2-\alpha}{1-\alpha}} \left(\frac{\alpha \beta}{1 - \beta(1-\delta)} \right)^{\frac{\alpha}{1-\alpha}}$$

$$\bar{I} = \bar{K} - (1-\delta)\bar{K} \quad \bar{I} = \delta \bar{K} \quad \bar{I} = \delta \bar{K} = \delta \left(\frac{\alpha A \beta}{1 - \beta(1-\delta)} \right)^{\frac{1}{1-\alpha}}$$

$$\bar{C} = \bar{Y} - \bar{I} \quad \bar{C} = \left(\frac{\alpha A \beta}{1 - \beta(1-\delta)} \right)^{\frac{\alpha}{1-\alpha}} - \delta \left(\frac{\alpha A \beta}{1 - \beta(1-\delta)} \right)^{\frac{1}{1-\alpha}}$$

Optimal Growth Model with Financial Intermediation

$$\phi S_t = I_t$$

$$C_t = AK_t^\alpha - \phi\{K_{t+1} - K_t(1 - \delta)\}$$

$$U_t = \sum_{t=0}^{\infty} \beta^t \ln[AK_t^\alpha - \phi\{K_{t+1} - K_t(1 - \delta)\}]$$

$$U_t = +\beta^t \ln[AK_t^\alpha - \phi\{K_{t+1} - K_t(1 - \delta)\}] + \beta^{t+1} \ln[AK_{t+1}^\alpha - \phi\{K_{t+2} - K_{t+1}(1 - \delta)\}] + \dots$$

$$\frac{\partial U_t}{\partial C_t} = -\frac{\phi\beta^t}{C_t} + \frac{\beta^{t+1}}{C_{t+1}}(\alpha AK_{t+1}^{\alpha-1} + \phi(1 - \delta)) = 0 \quad \frac{C_{t+1}}{C_t} = \frac{\beta}{\phi}(\alpha AK_{t+1}^{\alpha-1} + \phi(1 - \delta))$$

$$\dots = K_{t-1} = K_t = K_{t+1} = \dots = \bar{K} \quad \dots = C_{t-1} = C_t = C_{t+1} = \dots = \bar{C}$$

$$U_t = \ln(A\bar{K}^\alpha - \phi\bar{K} + \bar{K}\phi(1 - \delta)) \sum_{t=0}^{\infty} \beta^t$$

Steady State in Optimal Growth Model with Financial Intermediation

$$\frac{C_{t+1}}{C_t} = \frac{\bar{C}}{C} = \frac{\beta}{\phi} (\alpha A \bar{K}^{\alpha-1} + \phi(1-\delta))$$

$$(\alpha A \bar{K}^{\alpha-1} + \phi(1-\delta)) = \left(\frac{\phi}{\beta}\right) \quad (\bar{K}^{\alpha-1}) = \frac{1}{\alpha A} \left(\frac{\phi}{\beta} - \phi(1-\delta)\right) \quad (\bar{K}^{\alpha-1}) = \frac{1}{\alpha A} \left(\frac{\phi - \beta\phi(1-\delta)}{\beta}\right)$$

$$\bar{K} = \left(\frac{\phi - \beta\phi(1-\delta)}{\alpha A \beta}\right)^{\frac{1}{\alpha-1}} \quad \bar{K} = \left(\frac{\alpha A \beta}{\phi - \beta\phi(1-\delta)}\right)^{\frac{1}{1-\alpha}}$$

$$\bar{Y} = A \bar{K}^{\alpha} \quad \bar{Y} = \left(\frac{\alpha A \beta}{\phi - \beta\phi(1-\delta)}\right)^{\frac{\alpha}{1-\alpha}} \quad \bar{I} = \bar{K} - (1-\delta)\bar{K}$$

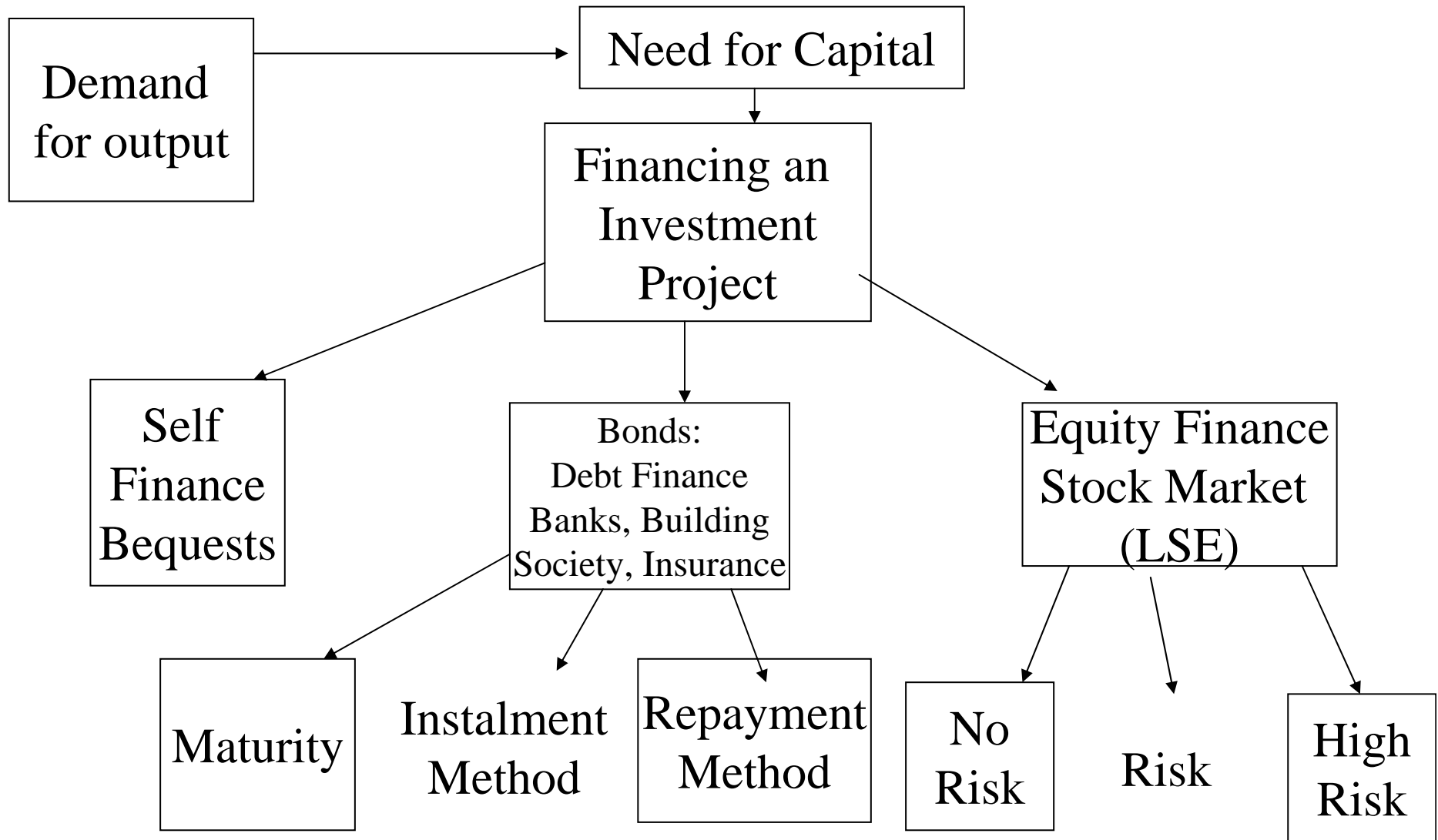
$$\bar{I} = \delta \bar{K} = \delta \left(\frac{\alpha A \beta}{\phi - \beta\phi(1-\delta)}\right)^{\frac{1}{1-\alpha}}$$

$$\bar{C} = \bar{Y} - \bar{I} \quad \bar{C} = \left(\frac{\alpha A \beta}{\phi - \beta\phi(1-\delta)}\right)^{\frac{\alpha}{1-\alpha}} - \delta \left(\frac{\alpha A \beta}{\phi - \beta\phi(1-\delta)}\right)^{\frac{1}{1-\alpha}}$$

Table 5
Capital Stock, Output, Consumption and Investment in the Steady State

Parameters of the Infinite Horizon Model									
	I	II	II	IV	V	VI	VII	VIII	IX
Technology	44.025	44.025	44.025	44.025	44.025	44.025	100	100	100
Capital share: alpha	0.4	0.4	0.2	0.6	0.6	0.6	0.4	0.4	0.6
Beta	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
Initial capital K0	100	100	100	100	100	100	100	100	100
Delta	1	0.05	0.05	0.05	0.05	0.05	1	0.05	0.05
Intermediation cost	1	1	1	1	1.2	1.05	1	1	1.05
Infinite Horizon Economy in the Steady State									
Capital Stock	100	2,499	149	344,202	218,202	304,677	392	9,807	2,369,142
Output	278	9,750	2,472	420,017	319,518	390,376	1,090	62,607	4,214,584
Consumption	178	7,251	2,323	75,815	101,315	85,699	698	52,800	1,845,441
Investment	100	2,499	149	344,202	218,202	304,677	392	9,807	2,369,142

Financing of an Investment Project



Key Points about Prices Financial Assets

Prices depend on

- Expected earnings from these assets
 - Expected returns from bonds
 - Expected stream of dividends from stocks
- Expected interest rates
 - Interest rates discount future earnings to present values
- Expected resale values of those assets
 - short run vs. long run
- Arbitrage implies investors reshuffle assets until the rate of return equalise s in every asset

Market Price of a bond (console)

What is the market price (value) of a console that pays 100 each year forever at the interest rate, r ?

$$PV = \frac{100}{(1+r)^1} + \frac{100}{(1+r)^2} + \frac{100}{(1+r)^3} + \dots + \frac{100}{(1+r)^n}$$

$$PV = \frac{100}{(1+r)^1} \left[\frac{1 - \frac{1}{(1+r)^{n+1}}}{1 - \frac{1}{1+r}} \right] \quad \text{as } n \rightarrow \infty \quad \frac{1}{(1+r)^{n+1}} \rightarrow 0$$

$$PV = \frac{100}{(1+r)^1} \left[\frac{1+r}{r} \right] = \frac{100}{r} = \frac{100}{0.1} = 1000.$$

Market Prices of Bonds By Maturity

One Period Bond:
$$P_{1,t} B = \frac{\text{Face value}}{\left(1+i_{1,t}\right)}$$

Two Period Bond:
$$P_{2,t} B = \frac{\text{Face value}}{\left(1+i_{1,t}\right)\left(1+i_{1,t+1}^e\right)}$$

Derivation

$$P_{2t} = \frac{P_{1,t+1}^e}{1+i_{1,t}}$$

$$P_{1,t+1}^e = \frac{1}{1+i_{1,t+1}^e}$$

$$P_{2t} B = \frac{P_{1,t+1}^e (\text{Face value})}{1+i_{1,t}} = \frac{\text{Face value}}{\left(1+i_{1,t}\right)\left(1+i_{1,t+1}^e\right)}$$

Yield to maturity on Bonds

Face Value 100 Maturity 2 years

Market Price $P_{2t}^B = 90$

$$\left(1 + i_{2t}\right) = \sqrt{100/90} = 1.054$$

Yield to maturity 5.4%

Note

$$P_{2t}^B = \frac{\text{Face value}}{\left(1 + i_{2t}\right)^2} = \frac{100}{\left(1 + i_{2t}\right)^2}$$

Arbitrage Between Short and Long Run Yields

Long Term

Short term

$$(1 + i_{2t})^2 = (1 + i_{1t})(1 + i_{1,t+1}^e)$$

Approximations:

$$(1 + 2i_{2t}) = (1 + i_{1t} + i_{2t}^e) \Rightarrow i_{2t} = \frac{1}{2}(i_{1t} + i_{2t}^e)$$

$$i_{nt} = \frac{1}{n}(i_{1t} + i_{2t}^e + i_{3t}^e + \dots + i_{nt}^e)$$

Long-term interest rate (yield) is average of expected short run interest rates.

Market Price of Stocks

Arbitrage between bonds and stock: $(1+i_{1,t}) = \frac{D_{t+1}^e + P_{S,t+1}^e}{P_{S,t}}$

By Rearranging: $P_{S,t} = \frac{D_{t+1}^e}{(1+i_{1,t})} + \frac{P_{S,t+1}^e}{(1+i_{1,t})}$

Stock Price depends on expected stream of dividends and resale:
(by iterating forward the above equation)

$$P_{S,t} = \frac{D_{t+1}^e}{(1+i_{1,t})} + \frac{D_{t+2}^e}{(1+i_{1,t})(1+i_{1,t}^e)} + \dots + \frac{D_{t+n}^e}{(1+i_{1,t})(1+i_{1,t}^e) \dots (1+i_{n,t}^e)}$$

$$+ \frac{P_{S,t+n}^e}{(1+i_{1,t})(1+i_{1,t}^e) \dots (1+i_{n,t}^e)}$$

Price of a Stock: An example

$$P_{Share} = \left(\frac{D}{r - g + x} \right)$$

P_{Share} = Price of a Stock

D = expected Dividend

r = interest rate

g = growth rate of dividend

x = risk premium

D = 1000 and r = 5% and g = 3% has a risk x = 0;

$$P_{Share} = \left(\frac{1000}{0.05 - 0.03 + 0} \right) = \frac{1000}{0.02} = 50,000$$

If r = 8%

$$P_{Share} = \left(\frac{1000}{0.08 - 0.03 + 0} \right) = \frac{1000}{0.05} = 20,000$$

Price of Stock with Risks: An example

D =1000 and r =5% and g=3% has a risk x =8%

$$P_{Share} = \left(\frac{D}{r - g + x} \right) = 1000 \left(\frac{1}{0.05 - 0.03 + 0.08} \right) = \frac{1000}{0.1} = 10000$$

when r=8%

$$P_{Share} = \left(\frac{D}{r - g + x} \right) = 1000 \left(\frac{1}{0.08 - 0.03 + 0.08} \right) = \frac{1000}{0.13} = 7692.31$$

Table 7

Value of Stock by Dividends, Interest Rate, Dividend Growth and Risk

Dividend	1000	1000	1000	1000	1000	1000	2000
Interest rate	0.05	0.05	0.05	0.08	0.08	0.03	0.03
Growth rate	0	0.03	0.03	0.03	0.03	0.03	0.03
Risk	0	0	0.08	0	0.08	0.08	0.08
Market Value of share	20000	50000	10000	20000	7692	12500	25000

Observations From the above Analysis of Stock Markets

- Lower the market interest rate, higher is the value of stock. Because future earnings are discounted at lower rate.
- Higher the growth rate of dividend higher the value of stock. As dividend grows earning from the share rises and hence price rise.
- Higher the risk premium lower is the value of the share. A decrease in the risk premium will increase the market value of a stock.
- Arbitrage implies same rate of risk adjusted returns in both stocks and bonds.
- (in the short run) Higher the resale value of the stock higher is its price.

Households Inter-temporal Budget Balance

Change in the net wealth $B_t - B_{t-1} = Y_t - C_t + rB_{t-1}$ (B.1)

$$B_{t-1} = \frac{B_t - Y_t + C_t}{1+r} \quad (B.1)$$

By iteration forward

$$B_t = \frac{B_{t+1} - Y_{t+1} + C_{t+1}}{1+r} \quad (B.2)$$

$$B_{t+1} = \frac{B_{t+2} - Y_{t+2} + C_{t+2}}{1+r} \quad (B.3)$$

Substituting (2) and (3) in (1)

$$B_{t-1} = \frac{C_t - Y_t}{1+r} + \frac{C_{t+1} - Y_{t+1}}{(1+r)^2} + \frac{B_{t+1}}{(1+r)^2} \quad (B.4)$$

with the higher order iterations

$$B_{t-1} = \frac{C_t - Y_t}{1+r} + \frac{C_{t+1} - Y_{t+1}}{(1+r)^2} + \dots + \frac{B_{t+n}}{(1+r)^{n+1}} \quad \text{No-Ponzi condition implies that}$$

$$\lim_{n \rightarrow \infty} \frac{B_{t+n}}{(1+r)^{n+1}} = 0 \quad \sum_{t=0}^{\infty} \frac{C_t}{(1+r)^{t-1}} = (1+r)B_0 + \sum_{t=0}^{\infty} \frac{Y_t}{(1+r)^{t-1}} \quad (B.5)$$

Firms Inter-temporal Budget Balance

Two period case (from the budget constraints of households and firms):

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r} + \frac{F(K_1 + I_1)}{1+r} - I_1 - I_1 \phi\left(\frac{I_1}{K_1}\right) \quad (\text{B.5})$$

I_1 is investment in period 1, $I_1 \phi\left(\frac{I_1}{K_1}\right)$ is the installation cost with

$$\phi'\left(\frac{I_1}{K_1}\right) > 0 \quad \phi''\left(\frac{I_1}{K_1}\right) < 0 \quad (\text{B.6})$$

Investment adds on capital stock and capital stock depreciates 100 percent next period. Consumption decisions are separated from the production decisions.

$$\frac{F'(K_1 + I_1)}{1+r} = 1 + \phi\left(\frac{I_1}{K_1}\right) + I_1 \phi'\left(\frac{I_1}{K_1}\right) \quad (\text{B.7})$$

LHS is the ration of marginal value of capital to its cost (Tobin's q) and the RHS is the unit cost plus the instalment cost.. Let $\psi\left(\frac{I_1}{K_1}\right) = \phi\left(\frac{I_1}{K_1}\right) + I_1 \phi'\left(\frac{I_1}{K_1}\right)$.

$$\psi\left(\frac{I_1}{K_1}\right) = (q-1) \text{ or its inverse function } \frac{I_1}{K_1} = \psi^{-1}(q-1).$$

$$I_1 = K_1 \psi^{-1}(q-1) \quad (\text{B.8})$$

No investment occurs when $q = 1$.

Multiple period case

$$\text{Max}_{I, K} \Pi_t = F(K_t) - I_t - I_t \phi\left(\frac{I_t}{K_t}\right) \quad (\text{B.9})$$

$$\text{s.t } K_{t+1} = (1-\delta)K_t + I_t \quad (\text{B.10})$$

where Π_t is the profit, $F(K)$ is the output, $P \cdot F(K)$ is the revenue of the firm, assume

$P=1$, I_t is investment, $I_t \phi\left(\frac{I_t}{K_t}\right)$ is the installation cost with $\phi'\left(\frac{I_t}{K_t}\right) > 0$ and

$\phi''\left(\frac{I_t}{K_t}\right) < 0$. The law of motion of the capital stock is $V_t = \sum_{i=0}^{\infty} \frac{\Pi_{t+i}}{(1+r)^i}$

$$L_t(I_t, K_t, q_t) = \sum_{i=0}^{\infty} \frac{\Pi_t}{(1+r)^i} - \sum_{i=0}^{\infty} \frac{q_{t+i}}{(1+r)^i} [K_{t+i-1} - (1-\delta)K_{t+i} - I_t] \quad (\text{B.11})$$

$$\frac{\partial L_t(I_t, K_t, q_t)}{\partial I_{t+i}} = \frac{-1 - \phi\left(\frac{I_{t+i}}{K_{t+i}}\right) - \frac{I_{t+i}}{K_{t+i}} \phi'\left(\frac{I_{t+i}}{K_{t+i}}\right) + q_{t+i}}{(1+r)^i} = 0 \quad (\text{B.12})$$

$$q_{t+i} - 1 = \phi\left(\frac{I_{t+i}}{K_{t+i}}\right) + \frac{I_{t+i}}{K_{t+i}} \phi'\left(\frac{I_{t+i}}{K_{t+i}}\right); \text{ let } \psi\left(\frac{I_{t+i}}{K_{t+i}}\right) = \phi\left(\frac{I_{t+i}}{K_{t+i}}\right) + \frac{I_{t+i}}{K_{t+i}} \phi'\left(\frac{I_{t+i}}{K_{t+i}}\right) \quad (\text{B.13})$$

$$q_{t+i} - 1 = \psi\left(\frac{I_{t+i}}{K_{t+i}}\right) \rightarrow \frac{I_{t+i}}{K_{t+i}} = \psi^{-1}(q_{t+i} - 1) \rightarrow \text{investment occurs if } q > 1.$$

$$\frac{\partial L_t(I_t, K_t, q_t)}{\partial K_{t+i}} = \frac{I_{t+i}/K_{t+i}}{(1+r)^i} + \frac{(1-\delta)q_{t+i}}{(1+r)^i} - \frac{q_{t+i-1}}{(1+r)^i} = 0 \quad (\text{B.14})$$

$$\frac{I_{t+i}/K_{t+i}}{(1+r)^i} + \frac{(1-\delta)q_{t+i}}{(1+r)^i} = \frac{q_{t+i-1}}{(1+r)^i} \rightarrow q_{t+i-1} = \frac{I_{t+i}/K_{t+i}}{(1+r)} + \frac{(1-\delta)q_{t+i}}{(1+r)} \quad (\text{B.15})$$

From successive iterations $q_T = \left[\frac{1-\delta}{1+r} \right]^T q_{t+T} + (1+r)^{-1} \sum_{t=0}^{\infty} \pi_{t+i} \left[\left[\frac{1-\delta}{1+r} \right]^T \right]^i$ (B.16)

$$\lim_{n \rightarrow \infty} \left[\frac{1-\delta}{1+r} \right]^T q_{t+T} = 0 \Rightarrow q_T = (1+r)^{-1} \sum_{t=0}^{\infty} \pi_{t+i} \left[\left[\frac{1-\delta}{1+r} \right]^T \right]^i$$

$$\lim_{n \rightarrow \infty} \frac{B_{t+n}}{(1+r)^{n+1}} = 0 \quad \sum_{t=0}^{\infty} \frac{C_t}{(1+r)^{t-1}} = (1+r)B_0 + \sum_{t=0}^{\infty} \frac{Y_t + F(K_{t-1}) - I_t}{(1+r)^{t-1}} \quad (\text{B.18})$$

$$\sum_{t=0}^{\infty} \frac{C_t}{(1+r)^{t-1}} = (1+r)B_0 + \sum_{t=0}^{\infty} \frac{Y_t}{(1+r)^{t-1}} + V_t \quad (\text{B.19})$$

Change in the government borrowing $D_t - D_{t-1} = T_t - G_t + r_g D_{t-1}$ (B. 20)

$$D_{t-1} = \frac{D_t - T_t + G_t}{1 + r_t} \quad D_{t-1} = \frac{G_t - T_t}{1 + r_g} + \frac{D_t}{1 + r_g} \quad (\text{B. 21})$$

By iteration forward

$$D_{t-1} = \frac{G_t - T_t}{1 + r_g} + \frac{G_{t+1} - T_{t+1}}{(1 + r_g)^2} + \dots + \dots + \frac{D_{t+n}}{(1 + r_g)^{n+1}} \quad \text{No-Ponzi condition implies that}$$

$$\lim_{n \rightarrow \infty} \frac{D_{t+n}}{(1 + r_g)^{n+1}} = 0 \quad \sum_{t=0}^{\infty} \frac{T_t}{(1 + r_g)^{t-1}} = (1 + r)D_0 + \sum_{t=0}^{\infty} \frac{G_t}{(1 + r_g)^{t-1}} \quad (\text{B.22})$$

Consolidating the public sector the household budget constraint changes to

$$\lim_{n \rightarrow \infty} \frac{B_{t+n}}{(1 + r)^{n+1}} = 0 \quad \sum_{t=0}^{\infty} \frac{C_t}{(1 + r)^{t-1}} = (1 + r)B_0 + \sum_{t=0}^{\infty} \frac{Y_t - T_t + F(K_{t-1}) - I_t}{(1 + r)^{t-1}} \quad (\text{B.23})$$

$$\sum_{t=0}^{\infty} \frac{C_t}{(1 + r)^{t-1}} = (1 + r)B_0 + \sum_{t=0}^{\infty} \frac{Y_t - T_t}{(1 + r)^{t-1}} + V_t \quad (\text{B.24})$$

Define a primary current account deficit as: $CA_t = PCA_t + rF_t = (X_t - M_t) + rF_t$ (B.25)

The budget should balance over period $PCA_t + \frac{PCA_{t+1}}{1 + r} = 0$ (B.26)

Steady State and Transitions Dynamics in an Overlapping Generation model

The transition dynamics in the neoclassical model is given by the law of motion of the capital stock. Consider an economy inhabited by N number of individuals.

At period 0 each of them is endowed by k_0 capital stock and aggregate capital stock is K_0 . The level of technical know how is denoted by A .

Production technology is standard Cobb-Douglas production function; $Y_t = AK_t^\beta L_t^{1-\beta}$. This implies per capita output to be $y_t = A_t k_t^\beta$. Let the labour force L_t be fixed to N in each period.

The remuneration to capital is according to its marginal productivity;

$$r_t = \frac{\partial y_t}{\partial k_t} = \beta A_t k_t^{\beta-1}. \text{ Labour is paid the residual amount } w_t = \frac{\partial y_t}{\partial L_t} = (1-\beta)A_t k_t^\beta.$$

There are two types of people living in this economy, young and old. Young work and earn labour income and consume a α fraction of income $c_{yt} = \alpha w_t$ and save $(1-\alpha)$ for their old age.

Consumption Saving and Capital Accumulation of Young and Old

There are two types of people living in this economy, young and old. Young work and earn labour income and consume a α fraction of income $c_{yt} = \alpha w_t$ and save $(1 - \alpha)$ for their old age.

The life time budget constraint is given by $C_{Y,t} + \frac{C_{O,t}}{(1 + r_{t+1})} = W_t$. The old people earn interest in their asset and consume all of their income, $c_{ot} = a_t(1 + r_t)$.

The capital stock of period t is result of the saving of old people; $a_{t+1} = (1 - \alpha)w_t$. Next periods capital stock equals the assets saved today as given by the equation of accumulation;

$K_{t+1} = (1 - \alpha)w_t = (1 - \alpha)(1 - \beta)A_t k_t^\beta$. Aggregate saving equals total output minus the consumption of young and old, this also it the market clearing condition in this model $S_t = Y_t - Nc_{yt} - Nc_{ot}$.

Saving equals investment in each period $S_t = I_t$ and investment adds to the capital stock $I_t = K_{t+1} - K_t$.

Numerical Results of Steady State and Transitional Dynamics in an Overlapping Generation Model

Growth, Capital Accumulation and Consumption in an Overlapping Generation Model

Time	K	K	Y	w	r	cy	co	S	I
0.0	3.000	300.000	1390.389	9.733	1.390	4.866	7.171	186.636	186.636
1.0	4.866	486.636	1607.538	11.253	0.991	5.626	9.689	76.002	76.002
2.0	5.626	562.638	1679.070	11.753	0.895	5.877	10.664	25.036	25.036
3.0	5.877	587.675	1701.144	11.908	0.868	5.954	10.980	7.726	7.726
4.0	5.954	595.400	1707.822	11.955	0.861	5.977	11.077	2.338	2.338
5.0	5.977	597.738	1709.831	11.969	0.858	5.984	11.107	0.703	0.703
6.0	5.984	598.441	1710.434	11.973	0.857	5.987	11.116	0.211	0.211
..
40.0	5.987	598.742	1710.693	11.975	0.857	5.987	11.120	0.000	0.000
41	5.987	598.742	1710.693	11.975	0.857	5.987	11.120	0.000	0.000

α	0.5	Mpc	k = capital labour ratio	cy = consumption of young
N	100	Population	K = capital stock	co = consumption of old
β	0.3	productivity of capital	Y = aggregate output	S=aggregate saving
A	10	technological index	w=wage rate	I= net investment
K0	300	Initial capital stock	r= interest rate	a = asset position

For details on this type of model see Auerbach and Kotlikoff (1998) *Macroeconomics: An Integrated Approach*, MIT Pres p.63-71.

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