

# More On Trade Closure<sup>1</sup>

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## I. INTRODUCTION

Over his career, T.N. has made contributions in a wide array of areas, and not always only through his written works. In the 1970s T.N. attended meetings and conferences where numerical general equilibrium modellers presented their trade models some of which sought to analyze the effects of trade liberalization on various economies. T.N. picked up on the issue of trade closure, how the specification of the rest of the world and their trade behaviour influences the model outcome. T.N. highlighted the seeming inconsistency of having financial variables (exchange rates) in what were presented as real side models, and other points.

This present paper, as its title suggests, offers more on trade closure beyond the immediate debate that T.N. initiated. It takes as its point of departure the claim by modellers who responded to T.N. that the only exchange rate at issue in their closure treatment is the real exchange rate (the relative price of traded and non traded goods), and that in a formulation with product differentiation on both the demand and supply sides the issues that T.N. identified no longer apply. We argue that a new set of issues arise such as perversely sloped offer curves in small open economy trade models and ill-defined Nash equilibria in case of two country trade models. In small open economy models with Cobb-Douglas preferences trade shocks impacts only on demand side seemingly independently of the production side specifications. In two country models with perverse offer curves each trading country tends to reach extreme point of the partners offer curve and hence Nash equilibrium may not exist. T.N. has long argued that there is no short cut to well defined theoretical structures, and the debate on model closure is a direct reflection of his earlier contributions. Our contribution is merely a further twist on the earlier development that T.N. started.

## II. TRADE CLOSURE

The need for trade closure arises from the development of detailed single economy models for the analysis of trade liberalization, or other trade policy changes. As Figure 1 suggests, substantial modelling effort is typically put into the development of a detailed representation of the economy in question. Some treatment is then needed of the world that the single economy trades with, and in particular its trade behaviour, through the export demand and import supply functions that the country faces in world markets.

The pitfalls with trade closure are well illustrated by the simple homogenous good trade structure discussed in Whalley and Yeung (1984), which in turn is reflective of closure structures actually in use in some of the applied models in the early 1980's. For the two good case, they set out a system of closure as representing the behaviour of the rest of the world through three equations. The first is an export demand function

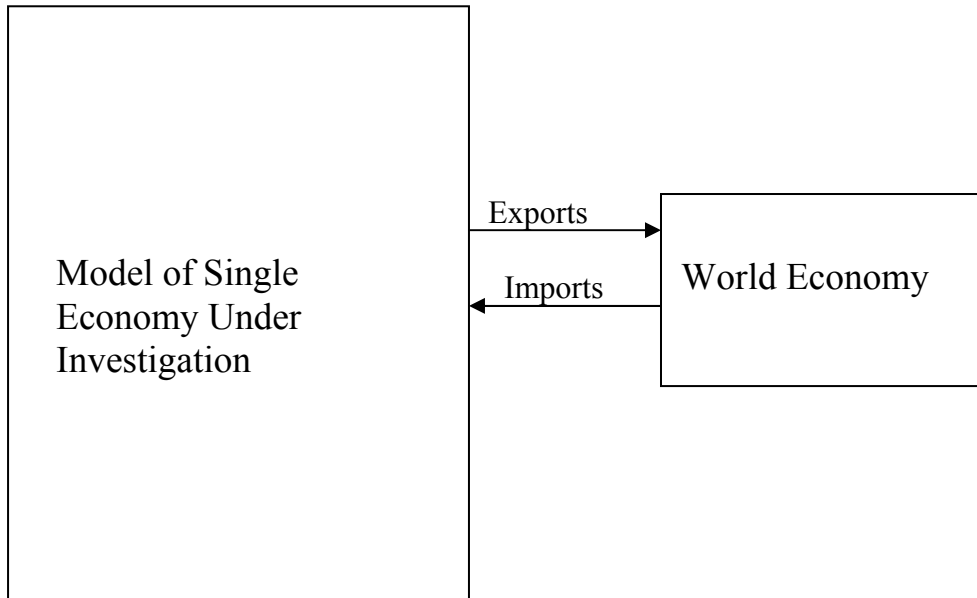
$$E = E_o \left( \frac{P_E}{e} \right)^\eta \quad -\infty < \eta < 0 \quad (1)$$

where  $E_o$  represents base period exports and  $P_E$  is the endogenously model determined price of exportable in own country currency. As foreign buyers of exports purchase these at world prices, this is divided by an exchange rate term  $e$  to yield the prices paid by buyers  $P_e / e$ ,  $\eta$  seemingly defines the price elasticity of the export demand function, and varying the parameter  $\eta$  allows modellers to vary the export demand elasticity that the country in question faces.

A related equation can be written down for the import supply function for the world to the domestic market, but with a change in sign for the elasticity parameter

**Figure 1**

**Closure in General Equilibrium Trade Models**



$$M = M_o \left( \frac{P_M}{e} \right)^\mu \quad \infty > \mu > 0 \quad (2)$$

where  $M_o$  is base period imports,  $P_M$  is the own country price of importables,  $(P_M/e)$  is the world price of imports for world suppliers to the country, and  $\mu$  is a further elasticity term, now interpreted as the import supply elasticity.

A trade balance condition (at either world or own country prices) completes the specification

$$P_M M = P_E E \quad (3)$$

As T.N. pointed out, for such structures there cannot be any meaningful exchange rate term in this closure system if the own country model structure to which the closure is appended is (as is typical) a real side barter economy. This follows directly by substituting (1) and (2) into (3) and solving out for  $e$ . This solution can, in turn, be substituted back into (1) and (2) to give export demand and import supply functions as

$$E = E_o P_E^{-1} \left( \frac{M_o}{E_o} \right)^{-\eta/(\mu-\eta)} P_M^{-(\mu+1)\eta/(\mu-\eta)} P_E^{\mu(\eta+1)/(\mu-\eta)} \quad (4)$$

$$M = M_o P_M^{-1} \left( \frac{M_o}{E_o} \right)^{-\mu/(\mu-\eta)} P_M^{-(\mu+1)\eta/(\mu-\eta)} P_E^{\mu(\eta+1)/(\mu-\eta)} \quad (5)$$

No exchange rate really exists within the structure; its presence is an illusion created by the closure structure. Moreover, as Whalley and Yeung show,  $\eta$  and  $\mu$  also define the true export demand and import supply elasticities. These come from equations (4) and (5) as

$$\varepsilon_E^{FD} = \frac{\partial E}{\partial P_E} \frac{P_E}{E} = \frac{\eta(1+\mu)}{(\mu-\eta)} \quad (6)$$

$$\varepsilon_M^{FS} = \frac{\partial M}{\partial P_M} \frac{P_M}{M} = \frac{-\mu(1+\eta)}{(\mu-\eta)} \quad (7)$$

and are in fact interconnected in the two good case by the relationship between the two export demand and import supply elasticities and the elasticity of the offer surface. Thus, for this system, modellers may believe that they control two independent elasticity parameters ( $\eta$  and  $\mu$ ), while in reality they only control one parameter  $\varepsilon^{OC}$  which implies the two jointly determined model elasticities.

$$\varepsilon^{OC} = \frac{\eta}{1 + \eta} \frac{1 + \mu}{\mu} \quad (8)$$

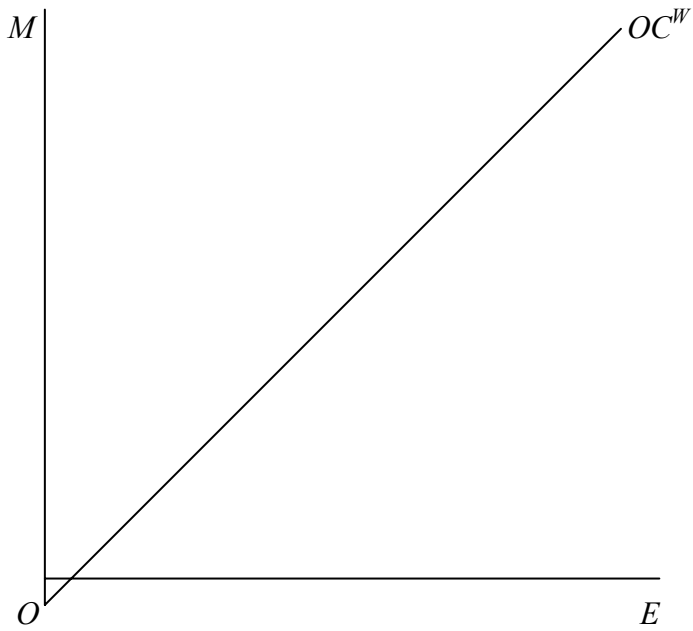
Modellers, therefore can easily misinterpret their model specification on the elasticity front if they adopt this form of closure treatment.

Another closure issue identified by T.N. involved the claim by some modellers to be separately incorporating price-making behaviour, say, on the export side and price-taking behaviour on the import side through their closure specification. As Figure 2 makes clear, in the simple two good case, only one relative price is involved in trade and the domestic economy has to be either a price taker in both imports and exports (a straight line offer curve), or a price maker in both (bowness in the offer curve). Mixed price-making/price-taking rule in the simple two good case would thus seem to be inconsistent.

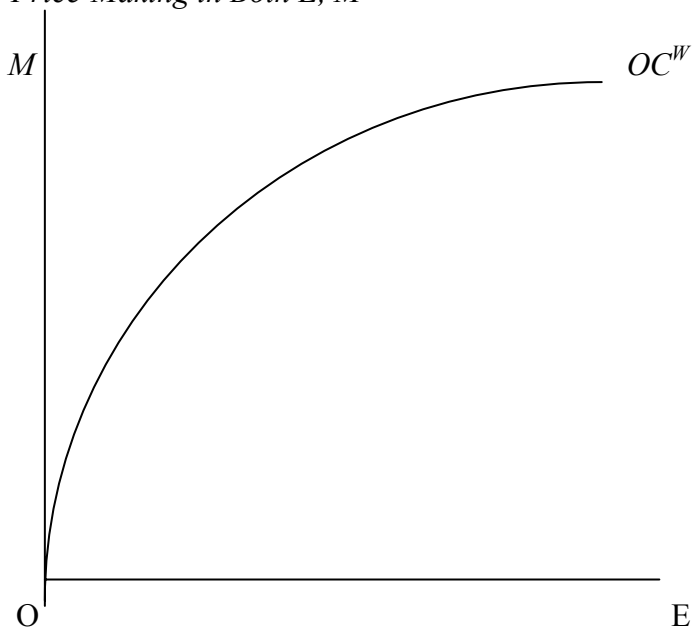
**Figure 2**

**World Offer Curves as Closure, and Price-Making  
Price-Taking Behaviour**

A. *Price Taking in Both E, M*



B. *Price-Making in Both E, M*



### III MORE ON TRADE CLOSURE<sup>3</sup>

In more recent numerical modelling trade policy literature, it is common to use the so-called Armington assumption of product heterogeneity; domestic products (which are both produced and consumed) treated as qualitatively different from products imported from abroad or exported to abroad. Such specification is useful in studying cross hauling phenomena in trade data in place of complete specialization characteristic of the Neoclassical trade models. This essentially involves using a constant elasticity of transformation (CET) frontier to separate each domestically produced good into exported and domestically consumed goods and a CES function to show imperfect substitution between domestic products and foreign imports. The price-taking behaviour is commonly used both for imports and exports, prices of domestically produced and consumed goods are delinked from world prices which now only apply to consumed imports and produced exports. Any difference between the value of imports and exports are met by a net foreign transfer in the trade balance equation (see, de Melo, Dervis and Robinson (1985), Devarajan and Lewis (1990), Robinson (1991), Harrison, Rutherford and Tarr (1993), and Rutherford (1995) and papers in the volume edited by Mercinier and Srinivasan (1995)). We argue that this widely used trade closure has some unattractive properties.

A trade balance condition specified as above is unusual to most trade models where external sector balance is a property of the equilibrium, deriving from a domestic version of Walras Law, and not an element of the model. Here, we note some peculiar features of behaviour that arise from simple versions of this structure. If, for instance, preferences are Cobb-Douglas, then independently of the specification of production (transformation) elasticities, the model will generate fixed export volumes under any world price change. World price changes have no effect on either domestic production or consumption of the

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<sup>3</sup> This section heavily relies on our earlier paper “On Some Unusual Properties of Trade Closure Widely Used

domestically produced good. If preferences are CES and elasticities are less than one, higher world prices for imports generate lower imports but higher exports; a perverse offer curve which asymptotically approaches the import axis and is truncated at an upper bound on exports in the two traded good case.

Using the similar structure in a 2 country model, we can also show cases where a Nash equilibrium of the two country tariff game does not exist, because of the perverse shape of the offer curves. It could also be shown that global equilibria may be obtained under which a country that reduces its tariffs increases its imports but reduces its exports.

De Melo and Robinson (1989) exploring this structure concluded that (p. 62) “An external closure with symmetric product differentiation for imports and exports is theoretically well behaved, and gives rise to normally shaped offer curves”. Our results suggest that this need not be the case; though some of our observations about the properties of this structure might be implicit in the analytics that de Melo and Robinson present.

## Model Specification

For illustration first we consider a small open 2 good economy which is a price taker for both exports (E) and imports (M) and also uses tariffs on imports.  $\bar{P}_M$  and  $\bar{P}_E$  are the fixed world prices at which the small economy trades internationally. Thus the economy faces a perfectly elastic supply function for imports and a perfectly elastic demand for its exports.

A representative consumer gets utility by consuming a domestically produced good, D and an imported good M and hence the utility function is given by

$$U = U(D, M) \tag{9}$$

A commonly used functional form for the utility function in (9) is constant elasticity of substitution (CES).

Total production,  $T$ , is allocated between domestic sales and exports by using a transformation frontier given by

$$T = T(E, D) \quad (10)$$

Most often such transformation frontier is explained by a CET (constant elasticity of transformation) function.

From optimizing behaviour on the demand side implies that the ratio of the marginal rate of substitution between  $D$  and  $M$  in preferences should equal the ratio of prices of domestic goods to imported goods (inclusive of tariffs).

$$\frac{U_D}{U_M} = \frac{P_D}{\bar{P}_M(1+t)} \quad (11)$$

where  $U_D$  and  $U_M$  are the first derivatives of the utility function with respect to  $D$  and  $M$ ,  $P_D$  is the price of the domestic good, and  $t$  is the tariff rate on imports.

Economy wide income,  $I$ , is given by the sum of the value of domestic sales plus exports and tariff revenue.

$$I = P_D D^D + \bar{P}_E E + R \quad (12)$$

where  $R$  is tariff revenue. Given  $t$ ,  $\bar{P}_M, \bar{P}_E$  (11) and (12) determine demands for the domestically produced and consumed commodity, which we denote as  $D^D$ , and if  $E$  is known, import demands  $M$ . From (10), and prices  $P_D$  and  $\bar{P}_E$ , profit maximizing behaviour gives

$$\frac{T_D}{T_E} = \frac{P_D}{\bar{P}_E} \quad (13)$$

where  $T_D$  and  $T_E$  are the first derivatives of the transformation frontier with respect to  $D$  and  $E$ . Equations (13) and (10) in combination give the export supplies  $E$ , and the supply of the domestically produced and consumed good, which we denote as  $D^S$ .

To both solve for and characterize an equilibrium in this structure, external sector balance needs to be specified

$$\bar{P}_M M - \bar{P}_E E = B \quad (14)$$

where  $B$  is the exogenous trade imbalance ;  $B$  will equal zero if there is zero trade balance.

This treatment of trade balance is unusual since trade balance is usually not implied by Walras Law (domestically), a property of an international trade equilibrium. In this set up trade balance must be specified as a model feature, rather than it emerges as a property of an equilibrium. This resolves the simultaneity between income, exports, and hence import demands.

An equilibrium for this system is given by the price of the domestic good,  $P_D$ , such that there is market clearing in the domestically produced and consumed good, i.e.

$$D^D = D^S \quad (15)$$

We illustrate that a range of problems in comparative static properties of trade structure can arise for particular set of functional forms and parameters, often used in the trade literature. If, for example, Cobb-Douglas preferences in consumption are used, then independently of the specification of the transformation frontier, the implied offer curve for the domestic economy involves constant exports<sup>4</sup>. This reflects the constant expenditure shares implied by Cobb-Douglas, as in this case changes in world prices can be accommodated by changes in  $M$ , with no further change in  $E$  or  $D$ . Such properties would not characterize a homogenous good trade model with unitary elasticities in preferences, and would not occur independently of the specification of production.

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<sup>4</sup> In an appendix, de Melo and Robinson (1989) analytically derive the elasticity of the offer curve for this system and note that their formula implies an offer curve elasticity  $\frac{\partial E}{\partial M} \frac{M}{E}$  of  $\infty$  implying the same behaviour we note above.

Even with CES preferences, which introduces bowness to the home country offer curve, depending upon the value of elasticity of substitution in preferences two alternative regimes may appear, one with perversely sloped offer curves and the other with more usual slope.

We compute coordinates of offer curves for different values of  $\sigma_U$  (the elasticity of substitution in preferences), and for given share and shift parameters for an arbitrarily specified numerical example, and display results in Table 1 and Figure 3. Here, the perverse offer curves for the low elasticity specification imply falling  $E$  with rising  $M$ , opposite to conventionally sloped offer curves in traditional trade models. An offer curve approaching the  $M$  axis implies infinite amounts of low priced imports for an infinitesimally small amount of high priced exports. Similarly an offer curve approaching the  $E$  axis implies infinite amounts of low priced exports financing an infinitesimally small amount of high priced imports. In reality,  $E$  will face an upper bound reflecting the maximal  $E$  value in production given finite resources and the production frontier (10).

**Table 1**

**Offer Curves Generated by a Simple Product Differentiation Model for Alternative Substitution Elasticities in Preferences**

*A. Numerical Specification of Model*

Production specification	T= 100; E share = 0.4; D share = 0.6; Transformation elasticity = 0.8
Demand specification	D share = 0.7; M share = 0.3; Demand Elasticity (varied as below)
World price ratio	Varied to generate offer curve

*B. Model behaviour*

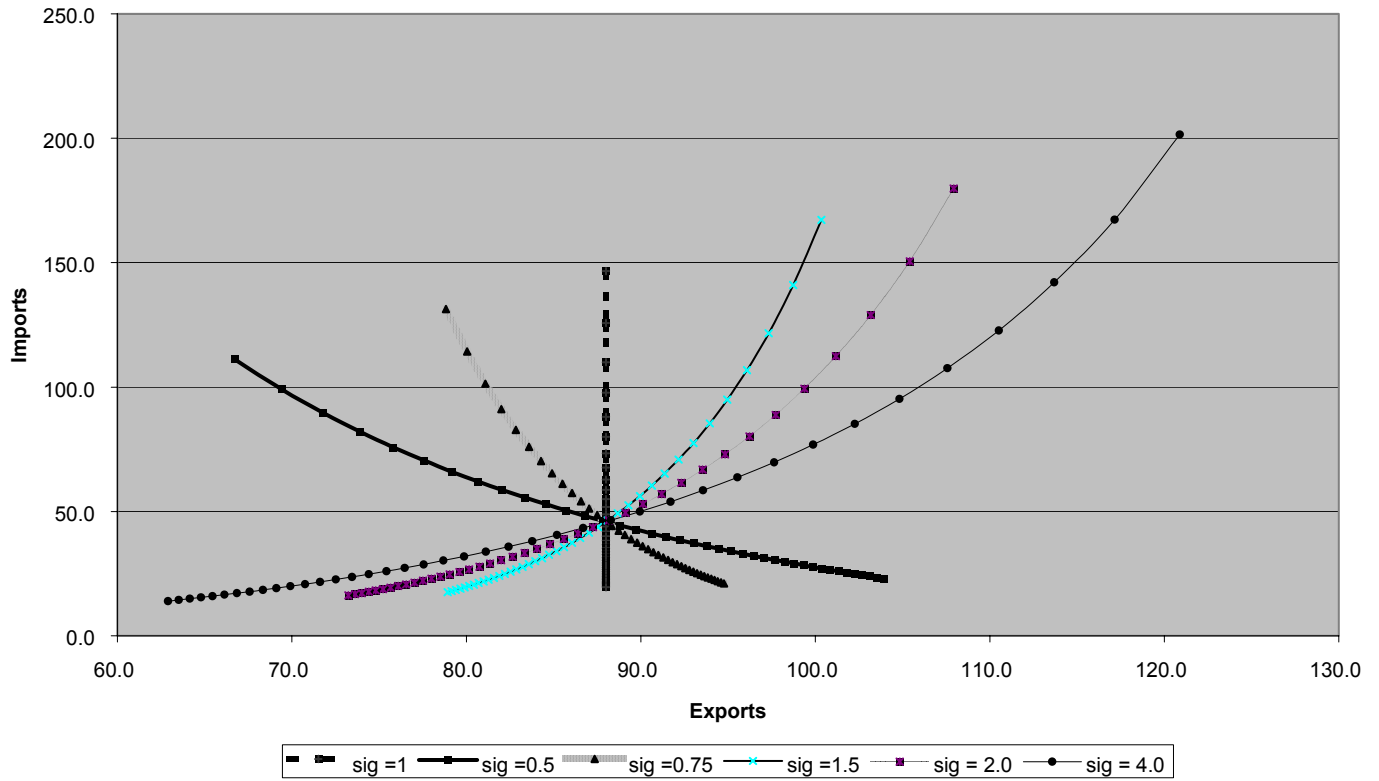
**Perverse cases**

$\left(\frac{P_M}{P_E}\right)$	$\sigma_U = 0.5$		$\sigma_U = 0.75$		$\sigma_U = 1.0$	
	Imports	Exports	Imports	Exports	Imports	Exports
0.5	127.2	63.6	154.8	77.4	176.0	88.0
1.0	75.8	75.8	82.8	82.8	88.0	88.0
2.0	44.4	88.8	44.2	88.3	44.0	88.0
3.0	32.2	96.5	30.5	91.6	29.3	88.0
4.0	25.4	101.8	23.5	93.8	22.0	88.0

**Non-perverse cases**

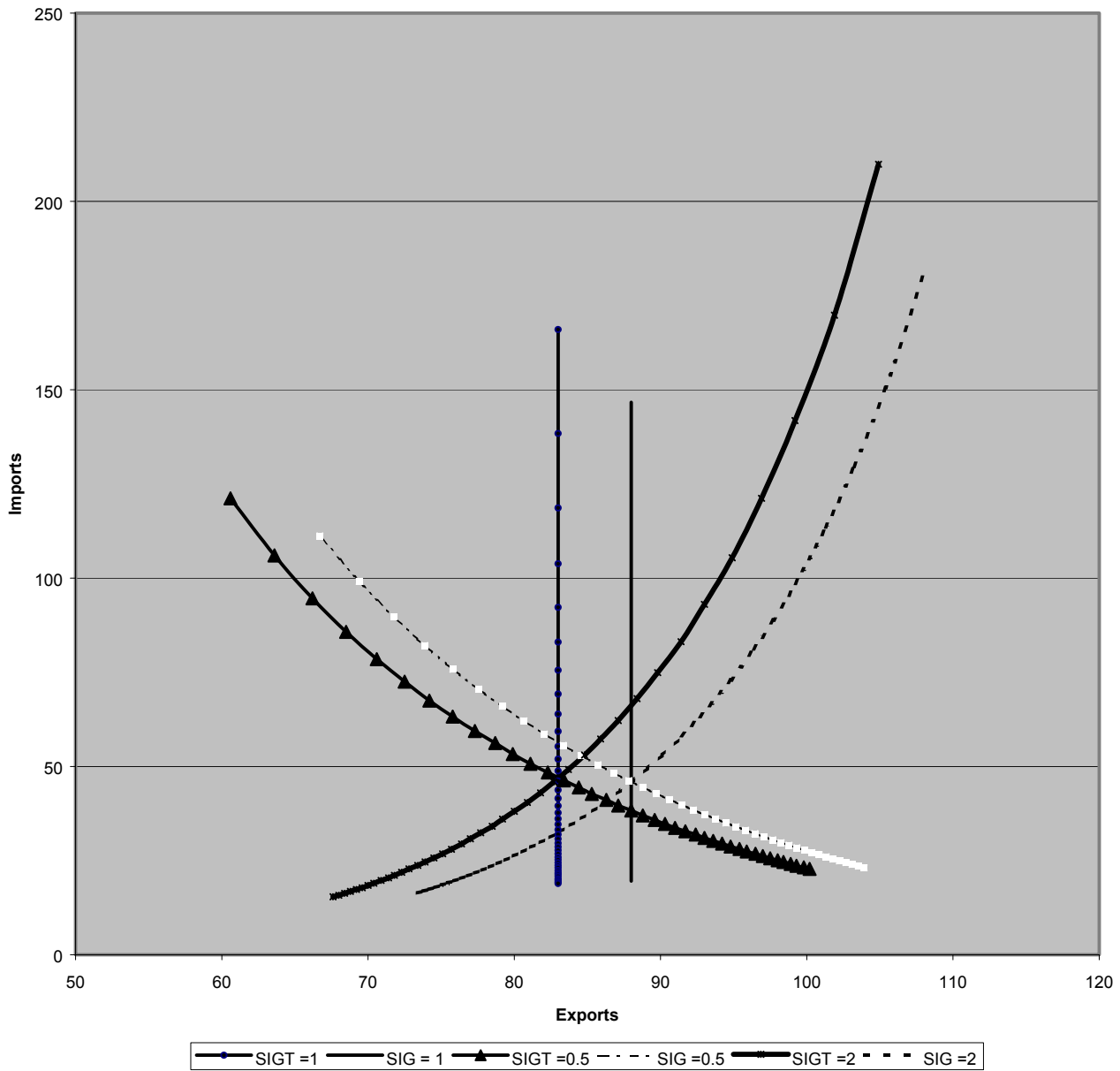
$\left(\frac{P_M}{P_E}\right)$	$\sigma_U=1.5$		$\sigma_U=2.0$		$\sigma_U = 4.0$	
	Imports	Exports	Imports	Exports	Imports	Exports
0.5	204.5	102.2	221.7	221.7	249.9	125.0
1.0	95.0	95.0	99.4	99.4	107.6	107.6
2.0	43.8	87.5	43.6	43.6	43.3	86.7
3.0	27.7	83.2	26.7	26.7	24.8	74.4
4.0	20.0	80.2	18.8	18.8	16.5	66.1

**Figure 3**  
**Graphical Representation of Perverse and Non-perverse Offer Curves**  
**from the Model in Table 1**



**Note:** SIG refers to a non-distorted offer curve with substitution elasticity as specified.

**Figure 4**  
**Tariff distorted perverse and non-perverse offer curves**



**Note:** SIGT refers to a tariff distorted offer curve with substitution elasticity as specified. SIG refers to a non-distorted offer curve with substitution elasticity as specified.

Offer curves with and without tariffs are drawn in Figure 4 from the computational results in Table 2. In the perverse case, imposition of tariff shifts the offer curve to the left, as do the non perverse and perfectly inelastic offer curves.

We modify the above model to represent a multi-country model<sup>5</sup> by allowing changes in  $P_M$  and  $P_E$  endogenously to clear global markets. Under this formulation, preferences and technology (equations (9) and (10)) for each country, and trade balance (15)) are again specified for each country. The Armington assumption implies that domestically produced goods for domestic consumption are different from that produced for export sales and that for imported for domestic consumption. This structure generates two separate country offer curves; and given the discussion above three possibilities appear (1) where both offer curves are non perverse, an intersection of two conventionally sloped offer curves characterizes equilibrium country behaviour with respect to induced changes in world prices as is conventional (either due to foreign country tariff changes, or technology or demand shifts), and a Nash tariff game (as in Johnson (1954) and Gorman (1957)) has a well defined equilibrium ; (2) either of two countries have perverse offer curves; and (3) both countries have perverse offer curves.

In case 2) and 3) peculiar behaviour can be exhibited by the model. In case 2) with one perverse and one non perverse offer curve, increases in the tariff of country with the perverse offer curve will reduce the volume of trade in both  $E$  and  $M$ , but increases in the tariff of the country with the non perverse offer curve will reduce  $E$  while  $M$  increases. In addition, a Nash equilibrium will not exist since when maximizing welfare subject to the

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<sup>5</sup> Harrison and Rutherford (1997) use a multi country structure of this form to evaluate international burden sharing under global carbon tax initiatives.

other country's offer curve wishes, the non perverse country wishes to go to an extremal point of the country's offer curve.

We again provide some numerical simulation results to illustrate the possible perverse behaviour resulting from the price taking Armington structure. Table 2 reports on a two country case, where both countries can have perverse offer curves. In this case, a reduction in tariffs by country 1 reduces imports but increases country 1 exports, with the same also being true of country 2. As noted above no meaningful Nash equilibrium exists in case of (2) and (3).

**Table 2**

**Two Country Cases of Product Variation Trade Models with Perverse Behaviour**

*A. Specification*

	Shares		Elasticities	
	M	D	Perverse	Non-perverse
Preferences in 1	0.5	0.5	0.5	1.5
Preferences in 2	0.5	0.5	0.5	1.5
Technology in 1	0.5	0.5	-3.5	-1.5
Technology in 2	0.5	0.5	-3.5	-1.5
Economywide resources in 1	1000			
Economywide resources in 2	500			

*B. Model behaviour*

	Perverse case				Non-Perverse case			
	Country 1		Country 2		Country 1		Country 2	
	M1	X1	M2	X2	M1	X1	M2	X2
No tariff	36.5	125.1	125.1	36.5	53.4	92.9	92.9	53.4
10% tariff	36.8	122.8	122.8	36.8	53.2	89.6	89.6	53.2
20% tariff	37.0	120.7	120.7	37.0	53.0	86.6	86.6	53.0
50% tariff	37.7	115.0	115.0	37.7	52.5	78.9	78.9	52.5

In Armington trade models, commonly values in excess of one are used for elasticities, suggesting that the problems we highlight may be avoided. Undesirable properties noted above appear only when elasticity configurations are 1 or below 1. However, typically applied trade models are used with elasticities in preferences which are calibrated to import price elasticities in the neighbourhood of one (the current literature consensus, even though such values strike many trade economists as implausibly low<sup>6</sup>) possible presence of perverse behaviour is a source of unease. Applied trade models need to be specified in the way that modellers believe that the offer curves generated from them be in term of their implied model responses.

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<sup>6</sup> See the discussion of these elasticities in Perroni and Whalley (1994).

#### **IV. CONCLUSION**

Closure rules used in applied Armington trade models involve price-taking behaviour for both imports and exports, and product differentiation between domestic and traded goods. In such models, most often a CET function describes transformation between domestic sales and exports and a CES function explains the substitution between imports and domestically produced goods for domestic consumption. A trade balance condition is imposed to solve the model, rather than trade balance being a property of an international equilibrium via Walras Law (which is more usual). When the elasticity of substitution equals one or is less than one implied offer curves can be perverse with exports falling as imports rise with falling import prices. The Nash equilibria of tariff games may not exist in the two country case. When domestic products and imports have unit elasticity of substitution, changes in world prices merely result in changes in import volumes, with no change in exports and consumption and production of domestic goods in the new equilibrium. Trade disturbances do affect nothing but imports.

We do not claim that all Armington trade models generate perverse offer curves; but modellers using trade closure should be aware of the somewhat unfortunate properties that can result in some situations, which they did not intend it to be. This is a point long stressed by T.N., and simply re-emphasized here.

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