

Input Output Analysis Using Linear Programming

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I. Linear programming: Primal and Dual

Every primal problem has corresponding dual problem.

Primal problem with quantity variables, x_1 and x_2 : how much one should produce to maximise revenue.

$$\text{Max } R = 10x_1 + 5x_2$$

Subject to:

$$\text{Skilled labour constraint: } 25x_1 + 10x_2 \leq 1000$$

$$\text{Unskilled labour constraint: } 20x_1 + 50x_2 \leq 1500$$

$$\text{Non-negativity constraints: } x_1 \geq 0 \quad x_2 \geq 0$$

Dual problem with price variables y_1 and y_2 : what should be the prices of skilled and unskilled labour to minimise the cost of production?

$$\text{Min } C = 1000y_1 + 1500y_2$$

Subject to:

$$\text{Cost and price for good 1: } 25y_1 + 20y_2 \geq 10$$

$$\text{Cost and price for good 2: } 10y_1 + 50y_2 \geq 5$$

$$\text{Non-negativity constraints: } y_1 \geq 0 \quad y_2 \geq 0$$

II. Linear Programming approach to input-output model

$$\text{Production function } X_j = \min \left(\frac{X_{1,j}}{a_{1,j}}, \frac{X_{2,j}}{a_{2,j}}, \dots, \frac{X_{n,j}}{a_{n,j}}, \frac{r_{1,j}}{b_{1,j}}, \frac{r_{2,j}}{b_{2,j}}, \dots, \frac{r_{n,j}}{b_{n,j}} \right) \text{ for } j = 1, 2, \dots, n$$

Assuming positive prices of all goods and services all arguments in the min (...) function are equal and equal to total output

$$X_{k,j} = a_{k,j} X_j \quad \text{for } j, k = 1, 2, \dots, n$$

$$r_{i,j} = b_{i,j} X_j \quad i = 1, 2, \dots, m \quad j = 1, 2, \dots, n$$

Therefore the material balance equation

$$X_k = \sum_{j=1}^n X_{k,j} + F_k \quad \text{for all } k = 1, 2, \dots, n$$

$$X_k = \sum_{j=1}^n a_{k,j} X_j + F_k \quad \text{for all } k = 1, 2, \dots, n$$

Writing in matrix gives the Leontief equation $X = AX + F$

Where

$$X = \begin{pmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ X_n \end{pmatrix} \geq 0 ; \quad A = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdot & \cdot & \cdot & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdot & \cdot & \cdot & a_{2,n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{n,1} & a_{n,2} & \cdot & \cdot & \cdot & a_{n,n} \end{pmatrix} \geq 0 ; \quad F = \begin{pmatrix} F_1 \\ F_2 \\ \cdot \\ \cdot \\ F_n \end{pmatrix} \geq 0$$

Consider an economy with n different sectors with following material balance equations:

$$X_1 = a_{1,1}X_1 + a_{1,2}X_2 + \dots + a_{1,n}X_n + F_1$$

$$X_2 = a_{2,1}X_1 + a_{2,2}X_2 + \dots + a_{2,n}X_n + F_2$$

.....

$$X_n = a_{n,1}X_1 + a_{n,2}X_2 + \dots + a_{n,n}X_n + F_n$$

Collecting terms $X = AX + F$

$$X(I - A) = F$$

The output required for a given vector of final demand is then given by

$$X = (I - A)^{-1} F$$

Here term $(I - A)^{-1}$ is the multiplier matrix which gives the total of backward and forward linkages in the economy of a change in final demand as

$$\Delta X = (I - A)^{-1} \Delta F$$

For primary factors

$$r_{i,j} = b_{i,j} X_j \quad i = 1, 2, \dots, m \quad j = 1, 2, \dots, n$$

$$\sum_{j=1}^n r_{i,j} = \sum_{j=1}^n b_{i,j} X_j$$

$$\sum_{j=1}^n b_{i,j} X_j \leq r_i \quad i = 1, 2, \dots, m$$

in Matrix notation $BX \leq r$

$$B = \begin{pmatrix} b_{1,1} & b_{1,2} & \dots & b_{1,n} \\ b_{2,1} & b_{2,2} & \dots & b_{2,n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ b_{m,1} & b_{m,2} & \dots & b_{m,n} \end{pmatrix} \geq 0; \quad r = \begin{pmatrix} r_1 \\ r_2 \\ \cdot \\ \cdot \\ r_m \end{pmatrix} \geq 0$$

vectors of product prices $p = (p_1 \quad p_2 \quad \dots \quad p_n) \geq 0$

vector of input prices $w = (w_1 \quad w_2 \quad \dots \quad w_n) \geq 0$

zero profit condition in equilibrium implies

$$\sum_{k=1}^n p_k a_{k,j} + \sum_{i=1}^m w_i b_{i,j} \geq p_j$$

or in matrix notation

$$pA + wB \geq p$$

$$p(I - A) \leq wB$$

General equilibrium with the linear input output technology can be presented in terms of primal and dual.

$$\max_x \quad pc = F \quad \text{subject to} \quad X = AX + F \quad BX \leq r \quad X \geq 0$$

Where pc is the value of final demand (consumption, investment, government spending and net exports: GDP)

$$F = pc = \sum_{j=1}^n p_j c_j$$

This problem can be instated in the standard linear programme by using

$$X = AX + F \quad \text{or} \quad X(I - A) = F$$

$$\text{Primal } \max_x \quad p(I - A)X \quad \text{subject to} \quad BX \leq r \quad X \geq 0$$

It has dual as (minimising the national income, value added)

$$\min_w \quad wr \quad \text{subject to} \quad pA + wB \geq p \quad w \geq 0$$

$$wr = \sum_{i=1}^m w_i r_i$$

$$\min_w \quad wr \quad \text{subject to} \quad wB \geq p(I - A) \quad w \geq 0$$

Existence of equilibrium

This requires derivation of household demand and supply function of factors

In general $F_j = F_j(p_1 \quad p_2 \quad \dots \quad p_n; w_1 \quad w_2 \quad \dots \quad w_n)$ for $j, k = 1, 2, \dots, n$

$$r_i = r_{ij}(p_1 \ p_2 \ \dots \ p_n; w_1 \ w_2 \ \dots \ w_m) \text{ for } i = 1, 2, \dots, m$$

or in vector notation $F_i = F(p, w) \ r = r(p, w)$

If there exist optimal values of p^*, w^*, X^*, r^*, F^* that satisfy the following equations

$$\begin{aligned} X^* &= AX^* + F^* \\ BX^* &\leq r^* \\ p^*A + w^*B &\geq p^* \\ F^* &= F(p^*, w^*) \\ r^* &= r(p^*, w^*) \end{aligned}$$

Kakutani fixed point theorem is applied to prove the existence of general equilibrium with price normalisation $S = \left\{ (p, w) \middle/ \sum_{j=1}^n p_j + \sum_{i=1}^m w_i = 1 \ p \geq 0, w \geq 0 \right\}$

In equilibrium X^* solves the linear programming problem and w^* solves the dual linear programming problem. In equilibrium $PF^* = wr^*$.

Input-output model for this system involve solving the output for each sectors in terms of the vectors of the final demand and the technology coefficient matrix:

$$\begin{bmatrix} X_1 \\ X_2 \\ \cdot \\ X_n \end{bmatrix} = \begin{bmatrix} 1 - a_{11} & -a_{12} & \cdot & -a_{1n} \\ -a_{21} & 1 - a_{22} & \cdot & -a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ -a_{n1} & -a_{n2} & \cdot & 1 - a_{nn} \end{bmatrix}^{-1} \begin{bmatrix} F_1 \\ F_2 \\ \cdot \\ F_n \end{bmatrix}$$

Or in the matrix notation

$$X = (I - A)^{-1} F$$

where X and F are $n \times 1$ vectors of output and the final demand and I is the identity matrix of order $n \times n$ and A is the $n \times n$ matrix of the Leontief technology coefficients. The explicit solution of the above model is obtained either by application of the Cramer's rule or by the matrix inverse method in which

$$(I - A)^{-1} = \frac{1}{|(I - A)|} \text{adj}((I - A)) \text{ and then with the matrix multiplication (note the}$$

order of resulting from the matrix multiplication $n \times n \times n \times 1 = n \times 1$). To illustrate the case of Cramer's rule for X_1

$$X_1 = \frac{\begin{vmatrix} F_1 & -a_{12} & \cdot & -a_{1n} \\ F_2 & 1 - a_{22} & \cdot & -a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ F_n & -a_{n2} & \cdot & 1 - a_{nn} \end{vmatrix}}{\begin{vmatrix} 1 - a_{11} & -a_{12} & \cdot & -a_{1n} \\ -a_{21} & 1 - a_{22} & \cdot & -a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ -a_{n1} & -a_{n2} & \cdot & 1 - a_{nn} \end{vmatrix}}$$

The above solution can be applied to all other sectors.

Demand for primary factors, such as capital and labour in this system is written as:

$$L_0 = l_1 X_1 + l_2 X_2 + \dots + l_n X_n$$

$$K_0 = k_1 X_1 + k_2 X_2 + \dots + k_n X_n$$

No where in the above solution is explicitly mentioned that above system is also an optimal solution in a linear programming problem. In fact it can be proved that the above solution generated by an input-output model is equivalent to a solution given by the optimisation process with a linear programming. The above problem need to be restated including an objective function such as the minimising the requirement of labour or capital input in the following way.

Minimize $L_0 = l_1 X_1 + l_2 X_2 + \dots + l_n X_n$

Subject to the resource balance constraint

$$X_1 \geq a_{1,1} X_1 + a_{1,2} X_2 + \dots + a_{1,n} X_n + F_1$$

$$X_2 \geq a_{2,1} X_1 + a_{2,2} X_2 + \dots + a_{2,n} X_n + F_2$$

.....

$$X_n \geq a_{n,1} X_1 + a_{n,2} X_2 + \dots + a_{n,n} X_n + F_n$$

or taking the Leontief inversion a constraint.

$$\begin{bmatrix} X_1 \\ X_2 \\ \cdot \\ X_n \end{bmatrix} \geq \begin{bmatrix} 1 - a_{11} & -a_{12} & \cdot & -a_{1n} \\ -a_{22} & 1 - a_{22} & \cdot & -a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ -a_{n1} & -a_{n2} & \cdot & 1 - a_{nn} \end{bmatrix}^{-1} \begin{bmatrix} F_1 \\ F_2 \\ \cdot \\ F_n \end{bmatrix}$$

and non-negativity constraint: $X_1 \geq 0, X_2 \geq 0, \dots, X_n \geq 0$.

These constraints can be read either horizontally or vertically. When read horizontally they describe the feasibility set in $n \times n$ Euclidean space. From the algebra of metric space or vector space the optimal point is obtained at the maximum of these feasible sets. The objective function $L_0 = l_1 X_1 + l_2 X_2 + \dots + l_n X_n$ gives the isobar hyperplane to produce the optimal output of n sectors as given by $\bar{X} = (\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n)$ where \bar{X}_i is the optimal output.

When read vertically columns of the above constraint matrix represent activity for that sector. Then $X \geq (I - A)^{-1} F$ is spanned in $n \times n$ metric space of final demand, The solution is contained in the convex polyhedral set with F_1, F_2, \dots, F_n giving the convex cone of n dimension.

Exercise:

Convert a three sector input output table into 3×3 input-output model. Find the solution to the LP model using both horizontal and vertical constraints of the input-output constraint. Represent the solution using a polyhedral set.

Input Output and Social Accounting Matrix to Calibrate the Benchmark

A general equilibrium model is valid when a given set of parameters can replicate the base year quantities and prices of a model economy. Parameters of a general equilibrium model are calibrated using initial values of variables obtained from a social accounting matrix of the model economy.

Social Accounting Matrix (SAM)

A social accounting matrix (SAM) is designed to present an overall picture of the circular flow of resources in a market economy. It brings consistently the transactions of the various actors whose behavior is being modeled. Generally SAM follows the conventions for national accounts established by the United Nations Statistical Office.

A careful study of a SAM reveals salient features of an economy: saving - investment gap, trade gap, public deficit, household income and savings, domestic production and absorption. The rows of a SAM are interpreted as receipts or revenue accounts. The columns on the other hand represent the expenditure accounts. In an economy expenditure of one sector is income of the other sector. The accounting principle that revenue should equal expenditure implies that the sum of corresponding rows and columns should be equal to each other in a SAM table. A brief discussion of the structure of SAM is given below.

Structure of Social Accounting Matrix-1

Expenditure									
Receipt	Activities	Commodities	Factors	Households	Tourists	Government	Capital Acct.	Row	Total
Activities		Domestic Sales				Export Tax or Subsidy		Exports	Domestic Output
Commodities	Intermediate inputs			HH consumption	Tourist Consumption	Government Consumption	Public and Private Investment		Absorption
Factors	Value Added								Value Added
Households			HH Allocation			Transfer		Remittances and Factor Income	Household Income
Tourists								Tourist Expense	Tourist Income
Government	Indirect Taxes	Tariffs	Income Tax	Household Tax	Tourist Taxes		Public Borrowing from Private HH	Foreign Aid and Loans	Public Revenue
Capital Acct.			Retained Profit	Household Saving		Government Saving			National Saving
Row		Imports	Repatriation	Transfer Abroad		Payment of foreign debt			Import Expenses
Total	Domestic Output	Absorption	Uses of Factor Income	Household Expenditure	Tourist Expenditure	Government Expenditure	National Investment	Export Income	634773

The first two accounts in the SAM, the activities and commodities, represent the production and supply system of the economy. The activities account represents the domestic production by producers and its disposition between domestic markets and exports. The value of intermediate inputs is the value of inter-industry flows; these intermediate inputs are received from the commodity accounts. When combined

with value added, wages and rental payments, and excise taxes, it gives the value of output at market price. This value of output, in turn, should equal the revenue received from the domestic sales and exports.

The commodities account gives the total demand and supply of goods and services in the economy. Supply includes outputs sold by domestic industries and goods imported from foreign market plus tariffs paid on those goods. This total value of imports should equal the total absorption in the economy; the intermediate demand, consumption of households, government, tourists, investment.

The income in the factors account is payment made by domestic producers to labour and capital employed in production. These incomes are then flow to households after deducting taxes to the government, retained earnings of the corporations, and repatriation of profits to foreigners for their contribution in the production process.

Households receive income from supplying the factors of production to the domestic producers, or transfer payment from the government or remittances from the foreign account. The households spend their income in consumption, paying income and property taxes to the government, and transferring a part to citizens abroad. Then a part of their income is left for consumption.

The government account records the revenue and expenditure of the government. Government's revenue consists of indirect taxes paid by the domestic producers, tariffs, income taxes, taxes on labour and capital income. In addition government receives aid from the foreigners and borrows from the domestic capital market.

The capital account is equivalent to the bank's account in the model. There are three sources of funds to the banks; household savings, government savings, and corporate savings.

Finally the foreign account records the inflows and outflows of foreign exchange in the economy. The foreign sector pays for the exports made by the domestic producers, for remittances paid to households by their relatives and friends abroad, foreign aid received by the government and foreign savings (or withdrawal from) in the banking sector. In return foreigner are paid for imports of goods and services, repatriation of profits by foreigners, payment by households to their relatives and friends abroad, debt servicing by the government and repayment of private foreign debt.

Aggregated NEPAL SAM 1990/91 (Millions of Rs.)									
	Activities	Commodities	Factors	Households	Tourism	Government	Capital Account	Row	Total
Activities		156536				-78.499		9366.94	165824
Commodities	64904.1			87950	3298.5	22463.3	8397.6		187014
Factors	98296								98296
Households			92273.8					2984.8	95258.6
Tourism								3587.6	3587.6
Government	2624.3	4354.2	1194.5	1977.7	289.1		4398.5	8633.04	23471.3
Capital Account			4827.67	3365.13		0		5875.6	14068.8
Row		26123.4		1965.8		1086.5	1272.3		30448
Total	165824	187014	98296	95258.6	3587.6	23471.3	14068.8	30448	

Source: Maxwell Stamp, ADB model 1992; See appendix for a disaggregated SAM.

Activities and commodities sectors are further disaggregated in order to capture the major sectors of the economy. The ADB SAM incorporates twelve different sectors (Appendix V). Intersectoral linkages among these sectors in the base year is explained by the input-output table contained in the social accounting matrix.

Production sectors included in the model use investment goods from various sectors. The composition of inter-sectoral investment-flows is given in terms of capital coefficient matrix for the base year Table 4.9. In the dynamic version of the model, the amount of intersectoral investment flow changes as there will be a change in the total sectoral investment depending on the rate of return on such investment.

Sectoral investment cost index is assumed to be unity in the benchmark scenario. Financial policies favoring one sector against the another would bring a change in this index. Terminal conditions of the model are assigned such that level of investment beyond the model horizon is enough to cover the growth rate and the depreciation rate of the capital.

Financial Accounting Matrix (FAM)

A financial accounting matrix is a simplified representation of the major financial linkages in the model economy. Like SAM, FAM is a square, in which columns represent liabilities and rows represent assets. Accounting identity is maintained when sum of a column equals sum of the corresponding row.

The Structure of a Simple Financial Accounting Matrix

	Δ in Assets	Assets of Institutions				Total Asset
	From SAM	Households	Government t	Foreign Sector	Banks	in Period t
Households	$S_{h,t} - \Delta RA_{h,t}$				FA_{t-1}	FA_t
Government	$B_t = S_{g,t}$			$\sum_{s=0}^t FLG_s$	DB_{t-1}	D_t
Foreign Sector	ΔFR_t		$\sum_{s=0}^t FLG_s$		FR_{t-1}	FR_t
Banks	$(S_{h,t} - \Delta RA_{h,t})$ + $(B_t + \Delta FR_t)$	$\sum_{s=0}^t (S_{h,s} - \Delta RA_s)$	$\sum_{s=0}^t B_s$	$\sum_{s=0}^t \Delta FR_s$		Total Loans
Total	Savings	Household Savings	Public Debt	Foreign Reserves	Deposits	

**A Simple Financial Accounting Matrix of Nepal 1990/91
(Millions of Rs.)**

	From SAM	Financial Accounts				
	Δ in Assets	Households	Government	Foreign	Banks	Total Liabilities
Households	3365				36134	39499
Government	0			59398	13907	73305
Foreign	5876		59398		12014	77288
Banks	-9241	39499	13907	17890		62055
Total Assets		39499	73305	77288	62055	

source: ADB SAM for Savings and Economic Survey Tables 6.6, 7.3, 7.1, 8.10 and 8.11; Table 4.6: Foreign exchange reserve of banks; Table 7.1: Currency holding ; Table 7.3: Total deposit; Table 8.10: Foreign Debt; Table 8.11: Internal Borrowing of Government.

A simple financial accounting matrix of the formal financial system of Nepal is constructed from data available in the economic survey. It shows that in 1990/91, financial assets of household are given by their deposits in the banks (FA_t), saving for a years is ($S_{h,t} - \Delta RA_{h,t}$) which represents a change in the financial asset of households. Households accumulate financial assets by making deposits over the years,

$\sum_{s=0}^t (S_{h,s} - \Delta RA_t)$. The effects of leakage of savings, ΔRA_t , is approximated by flow of income going to unproductive assets which does not return to the economic system. In essence such a flow increases under financial repression and decreases as the financial sector become more competitive.

The government borrows from the foreign sector and banks (B_t) and owes a debt stock $\sum_{s=0}^t FLG_s$ to foreigners and debt stock DB_t to domestic banks and D_t is the total debt stock of the government at the end of the year. While the government borrowing represents assets of the foreign sector, the foreign exchange reserve with the banking system represents the liabilities to the foreign sector. Finally the banking sector reconciles the financial account of the economy. Its assets are represented by the funds received from the households, the government bonds and the foreign exchange reserves, while its liabilities are made of their deposits. In turn financial assets of households are bank's liabilities.

The savings entries in the first column represent flows from current accounts to capital accounts. This essentially is injections of savings to loanable funds market. These saving entries are sensitive to interest rates and time preference of consumers. The economy wide equilibrium requires that total savings in the formal sector be equal to total investment. The rest of FAM describes the transformation of these loanable funds accumulated over years to different types of financial assets.

The balance sheets of the financial institutions is an outcome of the portfolio allocation decision of households, firms, government and the foreign sector. Financial assets net of government bonds and foreign reserves are used to make loans firms. The capital stock resulting from such loans is given in the last column of the table. Changes in the foreign exchange account affect the capital account and the current account simultaneously. Balance of international payment requires that a deficit in the current account be financed by a net inflow foreign capital. The government can crowd out in the loanable funds market through domestic borrowing.

The data structure obtained from SAM and FAM can be used to calibrate the parameters of the model. Information on share parameters on demand functions and production functions and associated wage rates, direct and indirect tax rates, foreign exchange rates, remittances, is based on base year SAM. Some basic parameters of the model such as utility discount factors of the households, growth rate of the labor force, elasticity of substitutions and transformation in tradable-goods-producing sectors are set after a sensitivity analysis. The rate of return on capital are obtained by imposing equilibrium steady state condition on the base year.

Calibration vs. Econometric Estimation of Parameters

There is a debate among economic policy analysts about the appropriate method of deriving model parameters. Some prefer econometric estimates; others can live with "point estimate" or calibration of the model parameters. Since Lucas'

critique on policy evaluation (Lucas 1976) economists prefer forecasting technique based on rational expectation: the method that predicts the value of target variables at $t+1$ period conditional on the information set available at period t . In Lucas' world, a change in parameter set (λ) affects the behavior of the system $[y_{t+1} = F(y_t, x_t, \theta(\lambda), \varepsilon_t)]$ either by changing the behavior of a policy variable, x_t , or by changing the policy response function, $\theta(\lambda)$ ¹. Earlier econometricians treated λ as fixed once it is estimated by a linear or non-linear technique. Lucas opposed the idea of treating x_t as exogenous, λ as fixed and ε_t representing the all stochastic elements in a model arguing that once people know relations between x_t , and λ , they change behavior according to $\theta(\lambda)$. Over time x_t , λ , and ε_t become interdependent and this affects the system represented by y_{t+1} . Econometric techniques for estimating parameters of a multi-sectoral forward looking general equilibrium model taking account of rational expectation remains still a challenge among the econometricians.

The calibration technique assumes equilibrium in the base year and derives parameters from equilibrium conditions in goods and factor markets. Properly calibrated parameters in light of the intuition from economic theories produce sensible results over an intermediate or long-term period, they can stand out even if judged according to the mean absolute deviation(MAD) and mean square error (MSE) criteria. Therefore the calibration technique is widely used in applied general equilibrium models.

It should be noted, however, that there are several criticisms against the calibration approach. Some argue that calibrated parameters are simply “drawn out of hat” or based on subjective prior beliefs of the modeler or copied from the results of other empirical studies (Lau 1984 commenting on Mansur and Whalley 1984). In the calibration approach, parameters are estimated with a functions $F(y, x, \lambda, \varepsilon) = 0$, where the stochastic random term, ε , is simply set equal to zero and the parameter λ , is found from the resulting system of equations using a single observation of endogenous variables, y , and exogenous variables, x . This means that no factors other than those already included in the model affects the values of the endogenous variable in the benchmark period and none are expected to affect them in any future period. Lau (1984) outlines three problems with such approach. First, the calibration approach is prone to underidentification - “that is, the inability to determine all parameters of the model given the data (ibid)”. In econometrics, if one has a problem in determining whether a parameter obtained by observations on prices and quantities belongs to a demand or supply curve, a standard procedure is to introduce another exogenous variables that shifts one curve without shifting the other in order to identify the function corresponding to the parameter. In contrast, in calibration techniques used in general equilibrium analysis, the number of independent unknown components of λ , cannot exceed the number of equations. “Adding exogenous variables whose effects are not known a priori increases the number of independent

¹ One implementation of this philosophy on policy evaluation is due to Fair (1984, 1994), who has developed a timeseries full information maximum likelihood (FIML) estimates and uses this method to do stochastic simulation of rational expectation model such as $f_t(y_t, y_{t-1}, \dots, y_{t-p}, E_{t-1}y_{t+1}, \dots, E_{t-1}y_{t+h}, X_t, \alpha_t) = u_{it}$ and $u_{it} = \rho_1 u_{it-1} + \varepsilon_{it}$ ($i = 1, \dots, n$). The y_t endogenous variables, E_{t-1} Expectation operator based on information available at period $t-1$, u_{it} , ε_{it} are error terms, ρ_1 is the serial correlation coefficient for the error term u_{it} . This means economic agents utilize all the relevant information available up to period t , to predict the values of parameters that determine the path of model variables in periods $t+1$ and onwards. The model is nonlinear in variables, parameters and expectations. For a forward looking general equilibrium model with infinite horizon, this method can be very unpersuasive and it is not very appropriate for analysis of issues in the long run.

unknown components of λ that must be estimated and thus worsens the underidentification problems (ibid)". The second problem with calibration techniques is that in a two commodity general equilibrium model "it is easy to see that given only one observation, an unaccountably infinite number of pairs of curves can serve as the calibration candidate of indifference and production possibility frontier. Thus it is not possible to identify either the indifference curve or the production possibility frontier without additional a priori non-data-based restrictions on these curves". The applied CGE model somehow need to be calibrated to estimates of elasticity parameters existing in the literature. The third problem of calibration approach is the "lack of measure of the degree of reliability of the model and its parameters. If the parameters are determined completely by the solution of equations for the benchmark period, they might be quite sensitive to the choice of the benchmark period (ibid)". While the econometric method provides measures of the reliability of model in terms of variance-covariance matrix of estimator parameters and other within-sample goodness of fit statistics to judge the stationarity and stability of the model, these are not available in the calibration approach.

In spite of shortcomings outlined above calibration of parameters is almost universal in empirical general equilibrium models. I will rely on calibration as the virtues of this approach - such as parsimony of data requirement, the ease with which the values of the independent unknown parameters can be determined, and possibility of sensitivity analysis to investigate appropriateness of alternative key parameters of the model - outweigh the problems of econometric methods as mentioned above. After all, there is very little difference using calibrated parameters against econometrically estimated parameters. In theory any model that can be calibrated can be econometrically estimated but not vice versa (Lau 1984, see Jorgenson 1984 for econometric estimation for Applied General Equilibrium models). Some authors have suggested a routine for conditional or unconditional systematic sensitivity analysis (Harrison, Jones, Kimbell and Wigle 1993), CSSA or USSA, to improve the robustness of model outcomes. Given the fact that the performance of well specified CGE models in tracking the evolution of model economies, as measured by the mean absolute deviation (MAD) or mean square error (MSE) of model predictions, AGE or CGE models are widely used for policy analysis.

Another important point to be noted is the construction of a reference path of the model. Generally static models are solved for a base year and when the model reproduces the base year data, then model is considered to be a valid representation of the economy. Model validation is more challenging in dynamic models, as we do not yet have any realized information about the future of the economy. Modelers solve this problem by choosing a model horizon, a balanced growth rate and investment rate in the terminal year along with the intertemporal maximization problem to compute a benchmark equilibrium that serves as a reference path of the model (Rutherford 1995; Mercenier and Michel 1994). This essentially means looking at the time path of model variable over the model horizon assuming all sorts of policies and structure of economy be continued in the future. Then a modeler studies the evolution of the economy associated with one or more policy instruments. Following this procedure we have solved the forward-looking CGE model of Nepal explained in detail in chapter five with initial values for the base year 1990 and various terminal conditions relating to the growth rate of the economy, labor force, and investment (Appendix IV). The benchmark equilibrium path of variables in this model are computed assuming that the economic policy structure of the base year will be continued in the

following years. Then some other counter-factual simulations are made to study the general equilibrium effect of changes in financial sector policies.

4.4 Modeling Language and Algorithm

The data structure and calibration techniques adopted by model builders depend upon the solution algorithm available to solve the general equilibrium model. In fact in the past, solution techniques have been limiting factors to implement more complete models numerically. The models of Harberger (1962, 1966), Scarf (1967, 1973), Shoven and Whalley (1976) were modest and essentially for pedagogical purposes. The applied equilibrium models had some shortcoming regarding the treatment of time. Using static expectations to determine the investment rates some models built a recursive dynamic structure using a sequence of static equilibria (Robinson (1978), Bourguignon et. al (1991) and the ADB model of Nepal). The approach is basically inconsistent with the dynamic equilibrium concept; such type of dynamic equilibrium do not maximize agents' inter-temporal welfare. The major reason for updating of static equilibrium was the difficulty of implementation of models with infinitely lived consumers (Ballard 1983). The situation has significantly improved after the development of more efficient languages in recent years, after the development of more efficient solution algorithms.

4.4.1 MPSGE/GAMS

The forward looking CGE model of Nepal discussed in chapter 5 is formulated in Mathematical Programming System for General Equilibrium (MPSGE: Rutherford 1993) and uses PATH (Ferris and Dirkse 1994) as the solution algorithm. The computer program of this model is presented in appendix I to III. Both MPSGE and PATH are active research areas at the moment. MPSGE is the most useful, transparent and relatively painless way to write down and analyze the complicated system of nonlinear inequalities contained in a multi-sectoral model. Originally it was designed for solving Arrow-Debreu economic equilibrium models, allowing very concise specification of complicated non-linear models in four classes of variables namely, sectors, commodities, consumers and auxiliary, and nested CES or CET functions for preferences and technologies². MPSGE uses the General Algebraic Modeling System (GAMS: Meeraus et al. 1988) at the front end and back end to facilitate data handling and report writing (Rutherford 1993). The essential steps in MPSGE/GAMS include declaration and definition of sets, parameters, scalars, input data, variables, equations and model, and solving them with linear, non-linear or mixed integer programming algorithms such as MINOS 5, PATH, MILES and others (Meeraus et al. 1988, and Deverajan. Lewis and Robinson 1991). One advantage of using MPSGE over algebraic GAMS is that given the values of elasticity and functional forms of production or utility functions, MPSGE automatically generates model equations for variables to the base year data in terms of unit functions. In other words, using benchmark quantities and prices, MPSGE automatically calibrates function coefficients and generates non-linear equations and Jacobians. This

² Essential steps in MPSGE are benchmarking or model specification in GAMS, model declaration, benchmark replication, counter-factual calculation and report writing (Rutherford 1994, 38). While MPSGE is appropriate for a specific class of nonlinear equations, the GAMS is capable of representing any system of equations.

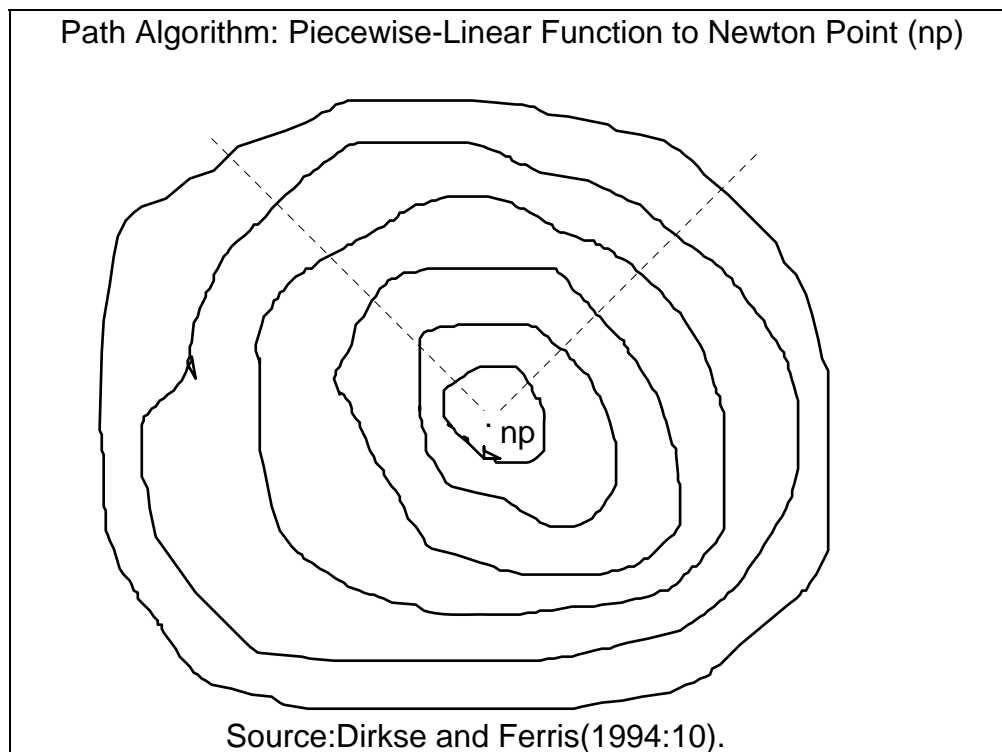
simplicity inherent in MPSGE is a virtue if one intends to incorporate a forward looking behavior of the households in the economy.

4.4.2 PATH Algorithm

The PATH algorithm designed by Dirkse and Ferris (1994) is a damped Newton method for solving mixed complementarity and variational inequality problems. It is robust and efficient in computing general equilibria. It consists of two parts; the front end and the solver. The front end basically is an interface to the routines which evaluate the function F and its Jacobean J , the initial values and lower and upper bounds for the problems variables z , and other data-specific problem. The solver is called by the front end to solve the mixed complementary problem (MCP) defined on F and bounds B . The process of solution is carried out in two steps: the construction of path and stabilization technique to determine a pathsearch. In the path construction phase the basis updating scheme is used in order to find the sequence of directions. The path construction phase of the algorithm terminates at a Newton point, at the base of a ray or as the result of an iteration interrupt.

The PATH algorithm (Dirkse and Ferris 1994) uses pivotal techniques to construct a path $p^k(\cdot)$ parameterized by t , from the current point x^k to the Newton point x_N^k of the nonsmooth equation $F_b(x) = 0$; $F_b(x^*) = 0$. A Newton point x_N^k is given by $x_N^k = x^k + d^k$, where d^k is Newton direction. The next iteration in the Newton process is determined by a linear search along this direction such that the new point $[x^{k+1} = x^k + \lambda d^k]$ is chosen to satisfy some descent criteria in $\|F\|$ with the ultimate goal finding a point x^* such that $\|F(x^*)\| = 0$. At this point also $F(x^*) = 0$. The watchdog stabilization technique is used to determine whether the path should be searched for a point $p^k(t)$ satisfying monotone descent condition, if the Newton point should be accepted without searching the path. The PATH solver, written primarily in C language (also in FORTRAN) is an implementation of the PATH algorithm. The process of the algorithm can explained in terms of the above figure. The contour lines shown represent the Euclidean norm of the piecewise-linear functions F_b while the paths to the Newton point for each mapping are given by dashed lines.

Fig. 4.3



Dirkse and Ferris(1994) warn of some possible problems in constructing the path, e.g., the initial basis may not be invertible, the value of t may not increase monotonically on the path ($0 \leq t \leq 1$; at Newton point $t=1$) due either to a cycling of bases or ray termination. The stabilization technique determines whether a path search is necessary using information stored on the stack. If necessary a backtrace of the path is performed in which the path is reconstructed in reverse order from its computation (see Dirkse and Ferris *ibid*:13).

4.5 Model Outcomes and Quality of Data

Consider a forward looking general equilibrium a decentralized market economy presented in GAMS/MPSGE in the next page. Conditions on the market of goods and services, of factors of production and of financial assets determine sectoral outputs and sectoral price indices which helps in evaluating the dynamic effects of alternative economic policies. Policy makers are mainly interested in the allocation of economic resources across various sectors and the distribution of income. It generates an optimal level of consumption and savings and associated utility and welfare indices for consumers living in urban and rural areas. Given the rate of depreciation model solves for the stock of capital for each sector as a function of equilibrium rental rate of capital.

In spite of the comprehensive pictures of the economy which emerge from this model, we should note that the outcomes of the model very much depend upon the quality of data we have used for the base year. The limited amount of time and resources during this study did not permit us to refine the data more extensively. Refinement of the social and financial accounting matrix using macro- and micro-level data sources deserves a serious research.

@Keshab Bhattarai, 1995

\$ontext

Suppose we have a following social accounting matrix for an economy.

Markets X1 X2 E M W Cons

p1	150	-100	50	-100		
p2		50	-25	75	-100	
pl	-100	-20				120
pk	-50	-30				80
pm1						
pm2						
pw				200	-200	
px		125	-125			

Here X1 and X2 are activities; E and M are exports and imports

W is welfare index and cons is representative consumer.

\$offtext

\$TITLE A SMALL TWO SECTOR DYNAMIC GENERAL EQUILIBRIUM MODEL

SET

F Primary factors /L, K/

I Sectors /s1, s2/;

ALIAS (I,J);

SET TP /2000*2020/, TFIRST(TP) /2000/, TLAST(TP);

TLAST(TP) = YES\$(ORD(TP) EQ CARD(TP));

DISPLAY TP, TFIRST, TLAST;

TABLE ZZ(*,J) Initial data of the economy

	s1	s2
Y0	150.0	50.0
D0	50.0	70.0
I0	50.0	30.0
M0	50.0	75.0
X0	100.0	25.0
VX0	100.0	25.0
VL0	100.0	20.0
VK0	50.0	30.0
DEP0	20.0	8.0
ESUB	0.5	0.5;

PARAMETER ETRNDX(I) / s1 8.0, s2 5.0/;

ETRNX(I) / s1 10.0, s2 7.50/;

TABLE IMA(I,J) Investment matrix

```
      s1  s2
s1  30.0 20.0
s2  20.0 10.0;
SCALAR  W0, INCADJ;
PARAMETER  Y0(I)
           D0(I)
           I0(I)
           INV0(J)
           M0(I)
           X0(I)
           VX0(I)
           K0(I)
           VK0(I)
           VL0(I)
           DEP0(I)
           ENDOW(F)
           EXOGFX0
           S0(I)
           PX0(I)
           PM(I)
           PM0(I)
           VM0(I)
           TX(I)
           TM(I)
           MKT(I);
Y0(I) = ZZ("Y0",I);
D0(I) = ZZ("D0",I);
I0(I) = ZZ("I0",I);
M0(I) = ZZ("M0",I);
X0(I) = ZZ("X0",I);
*K0(I) = ZZ("K0",I);
VK0(I) = ZZ("VK0",I);
VL0(I) = ZZ("VL0",I);
MKT(I) = Y0(I)+ M0(I)- D0(I)-X0(I)-I0(I);
S0(I) = Y0(I)+M0(I)-X0(I);
ENDOW("L") = SUM(I,VL0(I));
ENDOW("K") = SUM(I, VK0(I));
TX(I) = 0;
TM(I) = 0;
```

```

PM0(I) = 1+TM(I);
PX0(I) = 1+TX(I);
VX0(I) = PX0(I)*X0(I);
VM0(I) = PM0(I)*M0(I);
DISPLAY MKT, S0;
*IMA(I)=SUM(J,IMA(I,J));
*IMA(I,J) = IMA(I,J)/INV0(I);
*DISPLAY IMA;
EXOGFX0 = SUM(I,VM0(I)-VX0(I));
DISPLAY EXOGFX0;
SCALAR G    POTENTIAL GROWTH RATE /0.02/
      R    REFERENCE INTEREST RATE
      DEPR  DEPRECIATION RATE /0.07/
      ESUBT ELASTICITY OF SUBSTITUTION
            /0.99 /;
PARAMETER  QREF(TP)    STEADY STATE QUANTITY INDEX
            PREF(TP)   PRESENT VALUE PRICE
            QLABOR(TP) GROWTH RATE OF LABOR
            NR(I)      IMPLIED NET RATE OF INTEREST
            RK0(I)     BASEYEAR GROSS COST OF CAPITAL;
*computation of benchmark interest rate
NR(I)=(VK0(I)*(G+DEPR)-DEPR*SUM(J,IMA(I,J)))
/(SUM(J,IMA(I,J))+VK0(I)*(G+DEPR)-DEPR*SUM(J,IMA(I,J)));
RK0(I) = (NR(I)/(1-NR(I)) + DEPR);
K0(I) = VK0(I)/RK0(I);
IO(I) = K0(I)*(G+DEPR);
DISPLAY NR, RK0, K0, IO, Y0, D0, IO, X0, M0, VK0, VL0;
PARAMETER INVRATIO;
INVRATIO(I) = SUM(J,IMA(J,I))/VK0(I);
PARAMETER IKRATIO;
IKRATIO(I) = SUM(J, IMA(I,J))/K0(I);
* WHEN INVRATIO(I) = 1 THEN THE STEADY STATE BENCHMARK RATE OF
INTEREST IS:
R = G/(1+G);
DISPLAY INVRATIO, IKRATIO, R;
* IF IO(I) = VK0(I), THEN WE NEED TO COMPUTE DIFFERENT INTEREST RATE.
*Steady state growth path quantity and price indices:
      QREF(TP) =(1+G)**(ORD(TP)-1);
      PREF(TP) =(1-R)**(ORD(TP) -1);
      QLABOR(TP) =QREF(TP);

```

DISPLAY QREF, PREF;
 \$ONTEXT
 \$MODEL:SMALL3
 *Declaration of variables in the model. There are three blocks
 * production block, demand block and agent block.
 \$SECTORS:
 Y(I,TP) ! Production index
 K(I,TP) ! Capital stock
 INV(I,TP) ! Investment
 X(I,TP) ! Export index
 A(I,TP) ! Armington supply index
 U(TP) ! Itra-period utility index
 W ! Welfare index over period
 \$COMMODITIES:
 P(I,TP) ! Price index
 PD(I,TP) ! price index
 PX(I,TP) ! Export price index
 PL(TP) ! wage index
 RK(I,TP) ! Rental price of capital
 PK(I,TP) ! Index of rental rate
 PTK(I) ! Terminal investment premium
 PVPFX ! Real exchange rate index
 PU(TP) ! Price of consumption each period
 PW ! Welfare price index
 \$CONSUMERS:
 RA ! Representative agent
 \$AUXILIARY:
 TK(I) ! Terminal capital demand
 * Production function is of a Cobb-Douglas type
 \$PROD:Y(I,TP) t:ETRNDX(I) VA:1
 O:PD(I,TP) Q:(Y0(I)-VX0(I))
 O:PX(I,TP) Q:VX0(I)
 I:PL(TP) Q:VL0(I) VA:
 I:RK(I,TP) Q:K0(I) P:RK0(I) VA:
 \$PROD:K(I,TP) s:0
 O:RK(I,TP) Q:K0(I)
 O:PK(I,TP+1) Q:(K0(I)*(1-DEPR))
 O:PTK(I)\$TLAST(TP) Q:(K0(I)*(1-DEPR))
 I:PK(I,TP) Q:K0(I)

```

$PROD:INV(I,TP) s:0
    O:PK(I,TP+1)    Q:(SUM(J,IMA(J,I)))
    O:PTK(I)$TLAST(TP) Q:(SUM(J,IMA(J,I)))
    I:P(J,TP)      Q:IMA(J,I)
$PROD:X(I,TP) t:ETRNXX(I)
    O:PVPFX    Q:(X0(I)*PREF(TP)) P:(PX0(I)/PREF(TP)) A:RA T:TX(I)
    I:PX(I,TP) Q:X0(I)
$PROD:A(I,TP) s:0
    O:P(I,TP) Q:S0(I)
    I:PD(I,TP) Q:(Y0(I)-X0(I))
    I:PVPFX    Q:(M0(I)*PREF(TP)) P:(PM0(I)/PREF(TP)) A:RA T:TM(I)
$PROD:U(TP) s:1
    O:PU(TP) Q:(SUM(I,D0(I)))
    I:P(I,TP) Q:D0(I)
$PROD:W s:ESUBT
    O:PW    Q:W0
    I:PU(TP) Q:(QREF(TP)*SUM(I,D0(I))) P:PREF(TP)
$DEMAND:RA
    E:PL(TP)    Q:(QLABOR(TP)*ENDOW("L"))
    E:PK(I,TFIRST) Q:K0(I)
    E:PW        Q:INCADJ
    E:PTK(I)    Q:(-K0(I)) R:TK(I)
    E:PVPFX    Q:(EXOGFX0 + SUM((I,TP),(TX(I)*X0(I)*PREF(TP)+
TM(I)*M0(I)*PREF(TP))))
    D:PW        Q:W0
$CONSTRAINT:TK(I)
    PTK(I)*I0(I) =E= SUM((J,TLAST),P(J,TLAST)*IMA(I,J));
$OFFTEXT
$SYSINCLUDE mpsgeset SMALL3

```

```

P.L(I,TP) = PREF(TP);
PD.L(I,TP) = PREF(TP);
PL.L(TP) = PREF(TP);
PX.L(I,TP) = PREF(TP);
PK.L(I,TP) = PREF(TP)/(1-R);
RK.L(I,TP) = RK0(I)*PREF(TP);
PU.L(TP) = PREF(TP);
Y.L(I,TP) = QREF(TP);
X.L(I,TP) = QREF(TP);

```

$INV.L(I,TP) = QREF(TP);$
 $K.L(I,TP) = QREF(TP);$
 $A.L(I,TP) = QREF(TP);$
 $U.L(TP) = QREF(TP);$

$W0 = SUM(TP, PREF(TP)*QREF(TP)*SUM(I, D0(I)));$
 $PTK.L(I) = SUM(TLAST, PK.L(I,TLAST)*(1-R));$
 $TK.L(I) = (1+G)*SUM(TLAST, K.L(I,TLAST));$
 DISPLAY W0, PTK.L, TK.L;

$INCADJ = PW.L*W0$
 $+SUM(I, PTK.L(I)*K0(I)*TK.L(I))$
 $-SUM(I, TX(I)*(SUM(TP, X.L(I,TP)*PX.L(I,TP)*PVPFX.L)))$
 $-SUM(I, TM(I)*VM0(I)*PVPFX.L)$
 $-SUM(TP, QREF(TP)*PL.L(TP)*ENDOW("L"))$
 $-SUM((I,TFIRST),(PK.L(I,TFIRST)*K0(I)));$
 DISPLAY INCADJ;
 *SMALL.WORKSPACE =10;
 SMALL3.OPTFILE=1;
 SMALL3.ITERLIM =0;
 \$INCLUDE SMALL3.GEN
 SOLVE SMALL3 USING MCP;
 PARAMETER REPORT, GDP;
 $REPORT(I,TP, "Y") = Y.L(I,TP);$
 $GDP(tp,i) = y.l(i,tp);$
 $GDP(tp,i) = inv.l(i,tp);$
 DISPLAY REPORT;

*Do not compute counter factual experiment until you have calibrated the model

*Counter factual experiment

$TM("s2") = 0.05;$
 \$INCLUDE SMALL3.GEN
 SOLVE SMALL3 USING MCP;
 $TM("s2") = 0.1;$
 \$INCLUDE SMALL3.GEN
 SOLVE SMALL3 USING MCP;

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INPUT-OUTPUT AND GENERAL EQUILIBRIUM MODELS FOR
HULL AND HUMBER REGION IN ENGLAND

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ABSTRACT

This paper shows how one can construct an input output table for four Humber sub regions in England with information on levels and share of employment and output provided by the Humber Forum using coefficients from the national input-output table of UK. It then illustrates how these can be applied to construct multisectoral general equilibrium models specific to Hull, East Riding, North Lincolnshire and North East Lincolnshire regions situated in two sides of Humber estuary and a regional model that takes these four inter-dependent economies constituting the Humber economy. A dynamic model is constructed for Hull to assess the prospects in next hundred years based on micro consistent dataset in which households and firms are assumed to have perfect foresight in making their consumption and production decisions. These models are then applied to evaluate impacts of tax policies that can distort relative prices of commodities and factors of production and thus can distort the efficient allocation of scarce economic resources and on welfare of households in the Humber region. To my knowledge this is the first study of this type for this region.

***Key words:* Hull and Humber, input output model, general equilibrium, growth, efficiency, welfare, tax policy, regional model**

***JEL Classification:* C67, C68, O15, R13**

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INPUT-OUTPUT AND GENERAL EQUILIBRIUM MODELS FOR HULL AND HUMBER REGION IN ENGLAND

Introduction

Humber region, consisting of Hull, East Riding, North Lincolnshire and North East Lincolnshire counties in the North and South sides of Humber estuary as shown in the map below, is one of the dynamic sub-regions in the North Eastern part of England. This estuary has been an active route for international trade via the North Sea in the Northern England for Ireland-Liverpool-Manchester-Leeds-Hull-Rotterdam or the Zeebrugge segment of EU trade⁴ for a long time. With well developed railways, roads, canal systems and air transports from the Humberside International Airport, it is well integrated to the Yorkshire and Lincolnshire and the other parts of UK, EU and the Global economy. The regional authority of Humberside, the Humber Forum (HF (2003)) has recently announced a plan for transforming Humber to a self reliant, confident, creative, world- class, prosperous and sustainable economy. It has published some information on the structure of business and employment for this region that includes Hull, East Riding, North East Lincolnshire and North-Lincolnshire with its vision for an advanced Humber economy(www.humberforum.com).

Based on estimates of sectoral shares and aggregate levels of employment and output, the HF has set sectoral growth targets for nine sectors - agriculture, energy and water, manufacturing, construction, distribution, transport and communication, public administration, and other services - for up to 2012⁵. I argue that the sectoral growth rates chosen by the HF can be only tentative, based on intuitive judgments of professionals and politicians involved in regional planning but these growth rates need to be determined scientifically after realistic evaluation of full knock on effects of economic events to be more accurate in economic planning and projections. Household and government spending or business investments or trade activities have more than first round effects that accumulate over time until the regional economy settles down to a new steady state because of such initial push. The HF can improve its growth projections using more integrated input-output or general equilibrium models which account for price based

⁴ Ports of Humberside and Immingham also have direct connections to commercial ports of Oslo in Norway, Gothenburg in Sweden, Esbjerg in Denmark, and Hamburg and Bremerhaven in Germany.

⁵ See Humberside feasibility study from the Central Unit for Environment Planning in 1969 for earlier plans and Manners et. al. (1972) for comparative regional analyses of Humber with the Rest of UK from regional planning perspective, Hull City and County Council (1970), Wood (1998) about the regeneration.

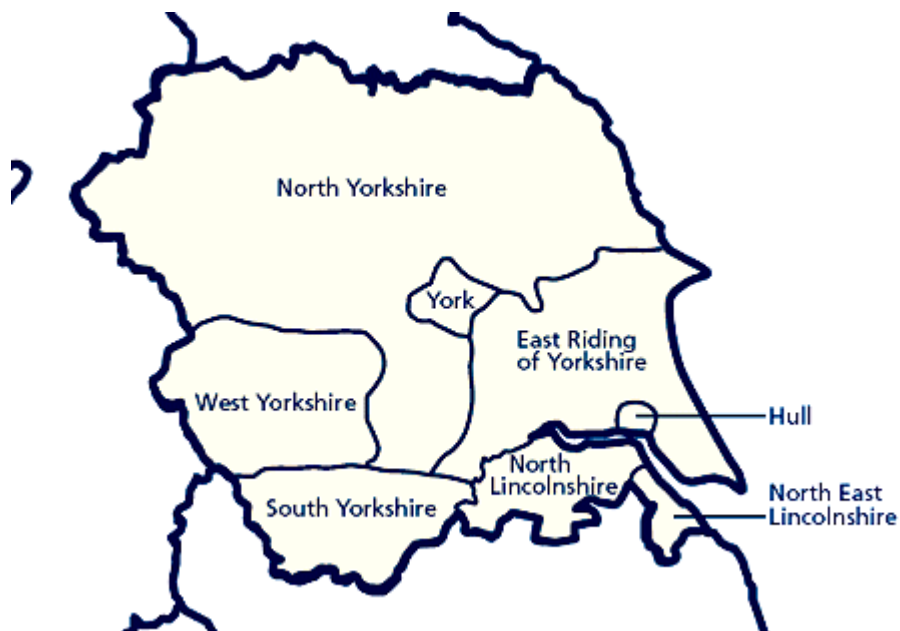
income and substitution effects on decisions of households and firms aiming for efficient allocations of scarce economic resources under their disposal.

Map 1: Yorkshire and Humber Region in the United Kingdom



http://www.20millionvotes.org.uk/images/content/map_yh.gif

Map 2: Composition of Yorkshire and Humber Region



<http://www.ice-yorkshireandhumber.org.uk/yorkshirehumber/images/yorks.gif>

The major objective of this paper is to develop a consistent and coherent micro-founded macroeconomic general equilibrium model for each of the four sub-regions and to build

a regional and a dynamic general equilibrium model for Hull and Humber. In the process of research, I will construct an input-output table for all four sub-regions separately and apply the micro-consistent data set contained in those input output tables to build a general equilibrium model for each sub region as well as for the entire Humber region based on dynamic choices for a set of preferences of households and the technology of production of firms operating in the region. The price based income and substitution effects are important for allocation of scarce economic resources, for investment and for capital accumulation in such models. These aspects are vital for smooth and speedier rates of growth according to the aspirations expressed in advanced economy policy foresights for sustainable development of Humber (HF(2003)). Section II and III provide details on the underlying Leontief model for the input-output table of Hull, which then is applied to derive input-output tables for East Riding, North Lincolnshire and North East Lincolnshire. Uses of these input-output tables for impact analyses and manpower projection and planning are briefly explained in section IV. A brief discussion of applied general equilibrium models for each sub-region and Humber region with those benchmark data is in sections V followed by a brief note on results of a dynamic general equilibrium model in section VI. Conclusions and references are given in sections VII and VIII.

Main Elements of the Input -Output Model of the Hull Economy

On the supply side the gross output by sectors, reflect the cost of production that is divided into intermediate inputs, value added, tax and import components. On the demand side, each sector sells its product for intermediate use by itself and by other sectors and for final demand. Consistency requires equality between supply and demand for each sector. The sum of labour income, capital income, and production taxes should equal final demand of households for consumption, of producers for investment, of government for public spending and of the Rest of the World Sector (ROW) for net exports. A nine sector input output model can be constructed with a standard Leontief (1949) model for each of four Humber regions as following:

$$X_1 = a_{1,1}X_1 + a_{1,2}X_2 + \dots + a_{1,9}X_9 + F_1$$

$$\begin{aligned}
X_2 &= a_{2,1}X_1 + a_{2,2}X_2 + \dots + a_{2,9}X_9 + F_2 \\
&\dots \dots \dots \\
X_9 &= a_{9,1}X_1 + a_{9,2}X_2 + \dots + a_{9,9}X_9 + F_9
\end{aligned} \tag{1}$$

where X_i represents gross output of sector i , the intermediate inputs are linked by

a Leontief technology coefficient such as $a_{i,j} = \frac{X_{i,j}}{X_j}$, where $a_{i,j}$ shows how

much sector i commodity is required to produce one unit of sector j commodity

with $X_{i,j}$ being the intermediate input from sector i to sector j .

This essentially is a simultaneous equation model with nine equations which gives the solutions for gross output of all nine sectors, X_1 to X_9 and can be applied to assess the impact of changes in the final demand on gross output, employment, investment or for manpower required by these industries for smooth functioning of these economies.

$$\begin{aligned}
(1 - a_{1,1})X_1 - a_{1,2}X_2 - a_{1,3}X_3 - \dots - a_{1,9}X_9 &= F_1 \\
-a_{2,1}X_1 + (1 - a_{2,2})X_2 - a_{2,3}X_3 - \dots - a_{2,9}X_9 &= F_2 \\
&\dots \dots \dots \\
-a_{9,1}X_1 - a_{9,2}X_2 - a_{9,3}X_3 - \dots - (1 - a_{9,9})X_9 &= F_9
\end{aligned} \tag{2}$$

It is better represented by a matrix system as:

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \\ X_8 \\ X_9 \end{bmatrix} = \begin{bmatrix} (1 - a_{1,1}) & -a_{1,2} & -a_{1,3} & -a_{1,4} & -a_{1,5} & -a_{1,6} & -a_{1,7} & -a_{1,8} & -a_{1,9} \\ -a_{2,1} & (1 - a_{2,2}) & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & (1 - a_{i,j}) & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & (1 - a_{8,8}) & \cdot \\ -a_{9,1} & -a_{9,1} & \cdot & \cdot & -a_{9,j} & \cdot & \cdot & \cdot & (1 - a_{9,9}) \end{bmatrix}^{-1} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \\ F_8 \\ F_9 \end{bmatrix} \tag{3}$$

In a standard matrix notation, the input output model is given by $X = (I - A)^{-1} F$.

Solution for this model exists when the determinant of $(I - A)$ is nonzero, or when

this matrix is non-singular, i.e. $|I - A| \neq 0$ and it can easily be checked using matrix routine in Excel.

Construction of an Input-Output Table for Hull Economy

A balanced and consistent input-output table provides the basis for assessment of economic policies taking account of entire structure of Hull economy. It also provides benchmark data for a general equilibrium model. This section shows a procedure to construct such an input-output table for Hull based on existing data on per capita GDP, level of employment and output and their sectoral compositions from the HF with some production and demand side coefficients taken from the national input-output table for UK until such information becomes available for the Hull and Humber region. It is expected that a detailed survey could be conducted to gather necessary information on the cost of production of firms and the patterns of expenditure of households, investment structure of firms, net exports from the Rest of the World sector and the spending of the government sector. For the time being it is assumed that the production technology and preferences of economic agents in Hull are not significantly different than those of the national economy, which might not be too unrealistic given the freedom of choices and movement of goods, factors and assets based on market signals. With proper consideration of above points, one can construct a nine sector Hull economy input output table completing six steps as following.

1. Take the total employment and output and sectoral composition of these employments and output from the existing data from the HF (Table 1).
2. Split the total output in labour and capital income using employment and income per capita information. Sectoral employment is obtained by multiplying the sectoral share by total employment. Then sectoral labour income is obtained by multiplying the sectoral employment by the gross value added per worker. Sectoral output is obtained by multiplying the sectoral share of output by total output. Capital income is the difference between total output and the labour income. All these numbers are presented in Table 2.

3. Analyses of inter-sectoral linkages require information on input-output coefficients which do not exist for the Hull and Humber. If the structure of production for each sector is not very different in Hull than in the national economy the Hull economy input-output coefficients can be approximated by the national input-output coefficients. Some adjustment is necessary for education and public services sector, and the mining and utility sector from the existing nine sector IO table for the national economy as given in Table 3. This needs to be refined when better data becomes available. It may not be too unrealistic if the regional coefficients do not vary significantly from the national coefficients.
4. Derive the gross output of Hull using the above Leontief coefficients assuming that final demand to be equal to total value added as in Table 4. Then decompose this final demand into consumption, investment, government consumption and export components again based on approximations from the national input-output table as in Table 5 until such information is obtained from the primary survey in Hull.
5. Derive tax or imports components as residuals in the supply side - gross output minus intermediate, labour and capitals costs altogether.
6. For consistency of demand should equal supply for each sector as shown in constructed input output tables for Hull, East Riding, North Lincolnshire and North East Lincolnshire in Tables 6a to 6d.

Table 1
Hull Economy: Composition of Employment and Output by Sectors

	Employers	Employment	Output 2002 (ml £)	Output 2012
Agriculture and Fishing	0.3%	0.01%	0.10%	0.10%
Energy and Water	0.1%	0.60%	1.30%	1.30%
Manufacturing	11.7%	20.70%	28.20%	28.20%
Construction	7.3%	4.10%	4.30%	4.30%
Distribution, hotels and restaurants	35.3%	24.30%	17.00%	17.00%
Transport and communication	5.4%	5.40%	9.70%	9.70%
Banking, Finance and insurance	19.6%	12.60%	12.40%	12.40%
Public administration, education and health	11.8%	27.40%	23.50%	23.50%
Other services	8.6%	4.90%	3.40%	3.40%
Total	7562	120856	3125881	3,125,881,000

Source: www.humberforum.com

Table 2
Hull Economy: Capital and Labour Income by Sectors

	Capital Income	Employees	Labour Income	Total Income
Agriculture and Fishing	2,967,039.9592	12.09	158,841.04	3,125,881.00
Energy and Water	31,105,990.5520	725.14	9,530,462.45	40,636,453.00
Manufacturing	552,697,487.5440	25,017.19	328,800,954.46	881,498,442.00
Construction	69,288,056.2720	4,955.10	65,124,826.73	134,412,883.00
Distribution, hotels and restaurants	145,416,040.8560	29,368.01	385,983,729.14	531,399,770.00
Transport and communication	217,436,294.9680	6,526.22	85,774,162.03	303,210,457.00
Banking, Finance and insurance	187,469,532.5920	15,227.86	200,139,711.41	387,609,244.00
Public administration, education and health	299,357,583.2080	33,114.54	435,224,451.79	734,582,035.00
Other services	28,447,844.0080	5,921.94	77,832,109.99	106,279,954.00
Total	1,534,185,869.96	120,868.09	1,588,569,249.04	3,122,755,119.00

Derivation capital and labour income obtained following steps mentioned above. See excel file for details.

Table 3
Input-Output Coefficients for Hull

	Agric	Enrwtr	Manu	Const	Distb	Trans	Busi	Edupub	OthSect
Agric	0.086583	0.000604	0.035177	4.74E-05	0.003639	0.000487	5.68E-05	0	0.000832
Enrwtr	0.000826	0.120376	0.016645	0.004751	0.000677	0.000173	3.03E-05	0.13229	0.000321
Manu	0.203363	0.068337	0.202288	0.152745	0.102854	0.083622	0.043803	0.040461	0.056192
const	0.007105	0.00526	0.000636	0.249816	0.003891	0.001532	0.022375	0	0.000821
Distb	0.041515	0.017505	0.037544	0.016244	0.026867	0.025066	0.008611	0.008509	0.004443
Trans	0.010121	0.044796	0.027601	0.010509	0.09595	0.158739	0.065376	0.004386	0.017857
Busi	0.080511	0.049237	0.073745	0.124203	0.14469	0.125707	0.249916	0.045159	0.075562
Edupub	0.011525	0.006898	0.019606	0.003223	0.007749	0.008697	0.004566	0.294183	0.003965
OthSect	0.015615	0.001811	0.012468	0.002867	0.006459	0.013893	0.015504	0.004291	0.043622

Source: Input Output Table, ONS; Aggregation as in Bhattarai (2006).

The gross output taking account of all intermediate transactions can be obtained using

an input output model⁶ as $X = (I - A)^{-1} F$. For Humber region the technology

coefficient matrix A is taken from the national input-output table as in Table 3.

Leontief inverse times the final demand equals the gross output of the economy in

Table 4.

Table 4
Gross Output to Hull by Production Sectors

Agric	Ener	Manu	const	Distb	Trans	BusiFin	Edupub	OthSect	Total
62,648,983	242,181,122	1,452,733,597	218,010,053	647,427,906	587,275,170	1,025,074,800	1,107,405,717	166,690,690	5,509,448,037

Table 5
Structure of Final Demand for Hull IO Table

	Consumption	Investment	Public spending	Exports	Total
Agric	0.772232	0.000115	0.004819	0.222834	1
Enrwtr	0.040784	0.00012	0.005654	0.953441	1
Manu	0.219586	0.097767	0.04372	0.638926	1
Const	0.921867	5.64E-05	0.074581	0.003495	1
Distb	0.062893	0.858263	0.078844	0	1
Trans	0.863897	0.020094	0.00955	0.106459	1
BusiFin	0.558103	0.022052	0.07465	0.345195	1
Edupub	0.727305	0.078453	0.078222	0.11602	1
OthSect	0.27582	6.32E-06	0.695715	0.028458	1

The input-output table for Hull emerging from above six-step derivations appears in Table 6a. This is a very preliminary table and in my knowledge first such table for Hull. It should be taken very cautiously as a lot of information is taken from the

⁶ [Pyatt and Round](#) (1979) Robinson (1991) Jackson (1998) Dietzenbacher and Stage (2006) have more on input-output model

national input-output table with only primary information on employment and output from the Humber Forum. For instance this estimation suggests gross output of Hull region to be equal to 5.5 billion in 2003. More work is required for splitting the residual costs between taxes and imports. Treating them as difference between the gross output and sum of labour, capital and intermediate costs is not very satisfactory. This requires company level data on taxes and imports. Similarly more work is required in determining the input-output coefficients and coefficients of final demand. Nevertheless, it gives some indication on how the different bits of Hull economy fit together and how the inter-sectoral relations should look like for a comprehensive understanding of the Hull economy. This can be important in a comprehensive, consistent and realistic assessment of economic policies in Hull. Similar methods are employed to construct the input-output tables for East Riding, North Lincolnshire and North East Lincolnshire as presented in Tables 6b to 6d, Excel files include these details.

The next sections will present how the input output model can be applied to assess the impacts of changes in the composition of demand or for manpower projections with the information on skill-industry matrices. That will be followed by some discussions on regional and dynamic general equilibrium models that can be applied based on the micro-consistent data set constructed above that reflect on economic behaviours based on the real price system for goods and factor markets of these four economies.

Table 6a: Input-Output Table of Hull (preliminary version September 2006)

	Agric	Ener	Manu	const	Distb	Trans	BusiFin	Edupub	OthSect	Total IntD	Cons	Inv	Gov	Exp	FinDem	Gross Output
Agric	5,424,334	146,182	51,103,260	10,332	2,356,000	286,072	58,173	0	138,751	59,523,102	2,413,905	359	15,064	696,553	3,125,881	62,648,983
Ener	51,759	29,152,785	24,181,417	1,035,782	438,617	101,317	31,026	146,498,529	53,438	201,544,669	1,657,334	4,889	229,778	38,744,453	40,636,453	242,181,122
Manu	12,740,455	16,549,844	293,870,093	33,299,988	66,590,412	49,108,956	44,901,826	44,806,943	9,366,636	571,235,155	193,565,032	86,181,836	38,539,146	563,212,427	881,498,442	1,452,733,597
const	445,127	1,273,868	924,286	54,462,477	2,518,915	899,934	22,935,688	0	136,876	83,597,170	123,910,811	7,577	10,024,705	469,790	134,412,883	218,010,053
Distb	2,600,885	4,239,266	54,540,953	3,541,288	17,394,296	14,720,767	8,826,788	9,423,261	740,631	116,028,136	33,421,309	456,080,801	41,897,660	0	531,399,770	647,427,906
Trans	634,047	10,848,762	40,096,965	2,291,118	62,120,697	93,223,579	67,015,336	4,857,625	2,976,586	284,064,713	261,942,717	6,092,623	2,895,527	32,279,591	303,210,457	587,275,170
BusiFin	5,043,906	11,924,241	107,131,988	27,077,550	93,676,055	73,824,349	256,182,410	50,009,645	12,595,412	637,465,556	216,326,008	8,547,703	28,934,907	133,800,626	387,609,244	1,025,074,800
Edupub	722,037	1,670,647	28,482,588	702,575	5,016,943	5,107,570	4,680,990	325,779,390	660,943	372,823,682	534,264,949	57,630,395	57,460,555	85,226,136	734,582,035	1,107,405,717
OthSect	978,243	438,545	18,112,753	625,085	4,181,482	8,159,000	15,892,873	4,751,447	7,271,307	60,410,736	29,314,185	672	73,940,538	3,024,559	106,279,954	166,690,690
Total IntDem	28,640,792	76,244,139	618,444,303	123,046,194	254,293,417	245,431,543	420,525,111	586,126,840	33,940,580							2,386,692,918
Labour	158,841	9,530,462	328,800,954	65,124,827	385,983,729	85,774,162	200,139,711	435,224,452	77,832,110							1,588,569,249
Capital	2,967,040	31,105,991	552,697,488	69,288,056	145,416,041	217,436,295	187,469,533	299,357,583	28,447,844							1,534,185,870
Taxes	31,265,941	118,511,672	-46,141,443	-28,339,275	-134,049,400	30,906,536	157,066,037	-109,267,613	18,187,479							38,139,935
Imports	-383,631	6,788,858	-1,067,705	-11,109,749	-4,215,881	7,726,634	59,874,408	-104,035,545	8,282,676							-38,139,935
Gross output	62,648,983	242,181,122	1,452,733,597	218,010,053	647,427,906	587,275,170	1,025,074,800	1,107,405,717	166,690,690	2,386,692,918	1,396,816,251	614,546,854	253,937,879	857,454,135	3,122,755,119	

Table 6b: Input-Output Table of East Riding (preliminary version September 2006)

	Agric	Ener	Manu	const	Distb	Trans	BusiFin	Edupub	OthSect	Cons	Inv	Gov	Exp	FinDem	Gross Output
Agric	25,267,934	123,704	42,455,598	14,280	2,277,232	245,991	50,538	0	110,998	170,886,181	25,392	1,066,452	49,310,693	221,288,717	291,834,991
Ener	241,106	24,670,047	20,089,453	1,431,577	423,953	87,122	26,954	104,707,348	42,749	2,170,601	6,403	300,939	50,743,395	53,221,337	204,941,645
Manu	59,348,301	14,005,023	244,141,580	46,024,656	64,364,117	42,228,370	39,008,879	32,025,005	7,493,089	144,545,774	64,356,770	28,779,323	420,582,039	658,263,905	1,206,902,925
const	2,073,514	1,077,989	767,878	75,273,803	2,434,701	773,845	19,925,592	0	109,498	183,340,677	11,211	14,832,735	695,109	198,879,733	301,316,553
Distb	12,115,589	3,587,407	45,311,567	4,894,493	16,812,759	12,658,261	7,668,354	6,735,117	592,487	32,415,454	442,354,481	40,636,698	0	515,406,632	625,782,665
Trans	2,953,551	9,180,580	33,311,782	3,166,605	60,043,836	80,162,155	58,220,196	3,471,905	2,381,199	217,789,452	5,065,645	2,407,455	26,838,518	252,101,070	504,992,879
BusiFin	23,495,803	10,090,685	89,003,180	37,424,486	90,544,215	63,480,924	222,560,848	35,743,548	10,076,034	171,964,767	6,794,854	23,001,323	106,362,585	308,123,530	890,543,254
Edupub	3,363,432	1,413,756	23,662,782	971,045	4,849,213	4,391,955	4,066,654	232,845,313	528,739	374,857,653	40,435,359	40,316,193	59,797,427	515,406,632	791,499,521
OthSect	4,556,908	371,111	15,047,724	863,944	4,041,684	7,015,854	13,807,081	3,396,017	5,816,875	21,632,997	496	54,565,917	2,232,035	78,431,444	133,348,642
Labour	9,647,823	9,647,823	184,380,610	48,239,113	249,771,407	55,742,975	100,766,147	377,337,062	35,375,350						1,070,908,309
Capital	211,640,894	43,573,514	473,883,295	150,640,620	265,635,225	196,358,095	207,357,383	138,069,570	43,056,094						1,730,214,691
imports	-63,650,855	82,475,457	34,059,348	-48,582,454	-131,286,684	33,477,865	157,170,426	-73,167,422	19,077,523						9,573,205
taxes	780,992	4,724,549	788,127	-19,045,613	-4,128,993	8,369,466	59,914,201	-69,663,942	8,688,007						-9,573,205
Gross output	291,834,991.5	204,941,645	1,206,902,925	301,316,553.2	625,782,665.5	504,992,878.8	890,543,254.2	791,499,521.1	133,348,642	1,319,603,554	559,050,611	205,907,034	716,561,801	2,801,123,000	

Table 6c: Input-Output Table of North East Lincolnshire (preliminary version September 2006)

	Agric	Ener	Manu	const	Distb	Trans	BusiFin	Edupub	OthSect	Total IntD	Cons	Inv	Gov	Exp	FinDem	Gross Output
Agric	3,662,300	80,906	28,077,835	7,676	1,399,997	178,139	31,282	0	65,900	33,504,035	6,791,111	1,009	42,381	1,959,634	8,794,135	42,298,170
Ener	34,946	16,134,984	13,286,077	769,476	260,638	63,091	16,684	78,823,409	25,381	109,414,685	1,004,258	2,962	139,233	23,477,124	24,623,578	134,038,263
Manu	8,601,863	9,159,724	161,462,031	24,738,378	39,569,776	30,580,468	24,145,357	24,108,337	4,448,724	326,814,659	103,505,463	46,084,206	20,608,124	301,167,843	471,365,636	798,180,295
const	300,532	705,039	507,833	40,459,874	1,496,805	560,394	12,333,360	0	65,010	56,428,849	97,284,282	5,949	7,870,550	368,839	105,529,620	161,958,469
Distb	1,756,017	2,346,276	29,966,619	2,630,803	10,336,149	9,166,718	4,746,488	5,070,178	351,766	66,371,013	20,021,831	273,226,065	25,099,791	0	318,347,687	384,718,700
Trans	428,084	6,004,387	22,030,610	1,702,059	36,913,753	58,050,932	36,036,602	2,613,641	1,413,742	165,193,809	173,216,847	4,028,915	1,914,747	21,345,770	200,506,278	365,700,087
BusiFin	3,405,450	6,599,625	58,861,888	20,115,763	55,664,777	45,970,905	137,758,670	26,907,647	5,982,245	361,266,969	106,013,577	4,188,921	14,179,955	65,570,863	189,953,316	551,220,285
Edupub	487,491	924,641	15,649,284	521,939	2,981,199	3,180,517	2,517,140	175,285,322	313,918	201,861,451	286,541,495	30,908,821	30,817,731	45,709,202	393,977,248	595,838,699
OthSect	660,472	242,718	9,951,751	464,372	2,484,747	5,080,663	8,546,180	2,556,512	3,453,538	33,440,953	12,613,132	289	31,814,692	1,301,389	45,729,502	79,170,455
Total IntDem	19,337,155	42,198,301	339,793,928	91,410,341	151,107,840	152,831,826	226,131,763	315,365,045	16,120,224	1,354,296,423	806,991,996	358,447,137	132,487,203	460,900,664	1,758,827,000	3,113,123,423
Labour	4,416,096	4,416,096	153,680,145	43,277,742	242,885,287	68,007,880	107,752,745	230,520,217	27,379,796	882,336,005						882,336,005
Capital	4,378,039	20,207,482	317,685,491	62,251,878	75,462,400	132,498,398	82,200,571	163,457,031	18,349,706	876,490,995						876,490,995
imp	14,342,866	63,574,560	-12,685,724	-25,129,902	-82,153,096	9,889,587	97,838,609	-58,143,850	11,900,966	19,434,015						19,434,015
tax	-175,986	3,641,824	-293,545	-9,851,590	-2,583,732	2,472,397	37,296,597	-55,359,744	5,419,764	-19,434,015						-19,434,015
Gross output	42298170.28	134038262.9	798180294.6	161958468.5	384718700.2	365700087.2	551220284.7	595838699.5	79170455.36	3,132,557,438	806,991,996	358,447,137	132,487,203	460,900,664	1,758,827,000	

Table 6d: Input-Output Table of North Lincolnshire (preliminary version September 2006)

	Agric	Ener	Manu	const	Distb	Trans	BusiFin	Edupub	OthSect	Total IntD	Cons	Inv	Gov	Exp	FinDem	Gross Output
Agric	7,271,493	87,967	40,772,323	11,184	1,700,571	226,136	43,015	0	97,661	50,210,350	26,080,291	3,875	162,760	7,525,695	33,772,620	83,982,970
Ener	69,384	17,543,091	19,292,948	1,121,233	316,596	80,090	22,941	68,976,307	37,612	107,460,203	1,561,049	4,605	216,429	36,493,553	38,275,636	145,735,839
Manu	17,078,989	9,959,097	234,461,881	36,047,206	48,065,243	38,820,014	33,201,980	21,096,576	6,592,745	445,323,732	156,724,940	69,779,355	31,204,217	456,019,524	713,728,036	1,159,051,768
const	596,706	766,568	737,434	58,955,581	1,818,163	711,386	16,959,450	0	96,341	80,641,629	143,215,785	8,758	11,586,527	542,982	155,354,052	235,995,681
Distb	3,486,570	2,551,037	43,515,059	3,833,441	12,555,277	11,636,582	6,526,836	4,436,780	521,296	89,062,878	23,789,476	324,640,878	29,822,990	0	378,253,344	467,316,222
Trans	849,960	6,528,393	31,991,040	2,480,133	44,838,983	73,692,070	49,553,483	2,287,129	2,095,082	214,316,272	215,902,975	5,021,767	2,386,602	26,606,045	249,917,388	464,233,660
BusiFin	6,761,517	7,175,577	85,474,392	29,311,423	67,615,776	58,357,222	189,430,231	23,546,179	8,865,331	476,537,649	157,071,763	6,206,386	21,009,294	97,151,056	281,438,500	757,976,149
Edupub	967,913	1,005,335	22,724,603	760,537	3,621,251	4,037,470	3,461,288	153,387,609	465,207	190,431,212	240,717,248	25,965,825	25,889,302	38,399,302	330,971,676	521,402,888
OthSect	1,311,367	263,900	14,451,114	676,654	3,018,211	6,449,587	11,751,746	2,237,137	5,117,939	45,277,655	19,872,383	455	50,125,039	2,050,379	72,048,256	117,325,911
Total IntDem	38,393,900	45,880,965	493,420,793	133,197,392	183,550,072	194,010,558	310,950,971	275,967,717	23,889,214	1,699,261,580	984,935,909	431,631,905	172,403,159	664,788,535	2,253,759,508	3,953,021,088
Labour	2,708,849	7,223,598	196,843,045	76,750,728	211,290,241	82,168,427	97,518,573	197,745,995	31,603,241	903,852,697						903,852,697
Capital	31,063,771	31,052,038	516,884,991	78,603,324	166,963,103	167,748,961	183,919,927	133,225,681	40,445,015	1,349,906,811						1,349,906,811
Taxes	11,963,238	58,242,838	-47,009,274	-37,754,856	-91,606,162	16,244,571	119,885,636	-43,817,306	14,695,865	844,552						844,552
Imports	-146,788	3,336,400	-1,087,786	-14,800,907	-2,881,032	4,061,143	45,701,041	-41,719,199	6,692,576	-844,552						-844,552
Gross output	83,982,970	145,735,839	1,159,051,768	235,995,681	467,316,222	464,233,660	757,976,149	521,402,888	117,325,911	3,953,021,088	984,935,909	431,631,905	172,403,159	664,788,535	2,253,759,508	3,953,021,088

Impact Analyses and Manpower Projections with an Input-Output Model

The input output tables estimated in Table 6a to Table 6d can provide a comprehensive framework to think about the much complicated economic relations among producers, consumers, investors and the public sector and the Rest of the World sectors and for the distribution of resources between workers and the owners of capital among production sectors in Humber. These can also be applied to estimate negative or positive impacts of public policy measures such as taxes and transfers that affect on output, employment and investment. The short run impacts of changes in consumption, government spending, investment or the net exports in output, employment, capital stock and distribution of resources in the economy are captured by a multiplier matrix, $\Delta X = (I - A)^{-1} \Delta F$. For instance, a 10 percent change in the consumption demand of manufacturing products will impact the gross output, employment and capital accumulation by $\Delta X = (I - A)^{-1} \Delta C_{man} (0.1)$ or in employment by $\Delta L = \sum_i l_i (I - A)^{-1} \Delta C_{man} (0.1)$ or in capital stock by $\Delta K = \sum_i k_i (I - A)^{-1} \Delta C_{man} (0.1)$ with l_i and k_i as labour and capital coefficients for production in sector i .

One can ask similar questions about the impacts of changes in other components of final demand. These can include increase or decrease in net exports, investment or the public spending for all sectors or for some combination of them or for one of them on the partial basis as required by policy makers. Similarly when ideal occupation industry matrix is known from technical relations in production, the above input-output tables can also be applied to estimate the gap in various categories of manpower in these economies. If $M_{i,s}$ denotes the manpower of s skill category required from i th industry where s denotes such as engineers, scientists, economists, teachers, professionals and managers, skilled non-manual workers, skilled and semiskilled workers and unskilled workers, $M_{i,s} = IOM_{i,s} X_i$ can give a required projection of manpower to produce X_i goods with the industry occupation matrix $IOM_{i,s}$ which shows how many s skill category workers are required for industry i to produce X_i amount of goods. Comparing such estimate

with the existing number of those workers gives an estimate of the manpower gap by skill categories. These projections can be important for indicating planning by schools, colleges and universities and training institutions. All of these analyses however are for short run because of constant relative price assumptions behind the input-output table.

Input-Output Table as a Micro-Consistent Dataset for Each Sub-Region

More important use of the IO table remains as a micro consistent data set to calibrate the general equilibrium model of an economy allowing income and substitution impacts to occur across sectors of any changes in relative prices and to illustrate how various components of the economy are coordinated by the market mechanism for allocation of scarce economic resources. The essence of such an economy is often reflected by a circular flow diagram in which demand for and the supply of goods and services respond to relative prices, income and expenditure balance for households and the public sector; saving equals investment; and net factor inflows from the Rest of the World sector equals the balance in the current account. More elaborately when one considers inter-dependence among regions, flows of trade in goods and services and factors of production among them can have significant influence in each of them. Such interactions have local as well as global consequences. Various scenarios behind the vision for a sustainable self-dependant, creative and productive advanced economy of Hull and Humber can be analysed more realistically in this framework. The consistency and coherency of an economic plan in this manner can spur innovative efforts required higher rates of economic growth.

Static Regional General Equilibrium Model for Humber Region

I have developed specific models for Hull, East Riding, North Lincolnshire and North East Lincolnshire sub regions with the input-output tables constructed above⁷. Each model includes a representative household that demands goods and services from all nine sectors according to its preferences and budget constraints and pays taxes to the government and provides its labour and capital endowment to the producers. Each firm produces the optimal level of output to maximise its profit hiring workers and capital on marginal productivity principle. Each economy imports goods and services to provide for internal demand and it also exports to the Rest of the World based on the ratio of prices of imported and exported commodities. The local government also uses its revenue to provide for public consumption. The equilibrium is attained by the Walrasian process in which relative prices adjust until the demand equals supply in each market. Each of these models is solved with GAMS/MPSGE software (Rutherford (1998)). Likely impacts of labour and capital income taxes in the level of employment, output and capital stock by sectors are shown in Table 7. The results show size and proportions in terms of labour, capital, regional output and aggregate supply for each of these four regions. These are results of optimal choices made by producers given their technology and by consumers given their budget constraints with the market clearing prices assuring the general equilibrium in the regional economic system. The first column lists sectors by regions; the next four columns represent labour, capital, domestic output and aggregate supply for each sector and region; the last four columns compare sizes of domestic output, aggregate supply, employment and capital stock across regions relative to the Hull sub-region. Comparisons can be made across regions and sectors based on these equilibrium results. For instance, the Hull is larger than all other regions in business and finance, in education and public services, in transport and communications, and in other sectors. All other regions are bigger than Hull in agricultural sector. The North East Lincolnshire is smaller than Hull in all except the agriculture sector. Knowing levels of variables in this manner can itself become an important factor for policy making in both public and private sectors.

⁷ See Scarf (1986) Kehoe (1985) Shoven, and Whalley (1992) Zou (1996), [Miller M.H. and Spencer J.E. \(1977\)](#) Whalley J. (1985) Elbers (1992) Rutherford (1998) Bhattarai (2001), Bhattarai and Whalley (2000, 2006) for some developments in national and regional applied general equilibrium models.

Table 7
Sectoral and Regional Comparison of Labour, Capital, Domestic Output and Supply:
Results from the Regional General Equilibrium Model of Humber Regions

	LABOUR	CAPITAL	DOMOUT	SUPPLY	DOMOUTR	SUPPLYR	LABOURR	CAPITALR
AGRIC .HULL	158841	2967040	62336060	61952430	1.000	1.000	1.000	1.000
AGRIC .ER	9647823	211640900	306175200	242524300	4.912	3.915	60.739	71.331
AGRIC .NL	2708849	31063770	76604060	76457270	1.229	1.234	17.054	10.470
AGRIC .NEL	4416096	4378039	25995670	40338540	0.417	0.651	27.802	1.476
ENER .HULL	9530462	31105990	196647800	203436700	1.000	1.000	1.000	1.000
ENER .ER	9647823	43573510	71722790	154198200	0.365	0.758	1.012	1.401
ENER .NL	7223598	31052040	105905900	109242300	0.539	0.537	0.758	0.998
ENER .NEL	4416096	20207480	46986580	110561100	0.239	0.543	0.463	0.650
MANU .HULL	328801000	552697500	890588900	889521200	1.000	1.000	1.000	1.000
MANU .ER	184380600	473883300	752261500	786320900	0.845	0.884	0.561	0.857
MANU .NL	196843000	516885000	704120000	703032200	0.791	0.790	0.599	0.935
MANU .NEL	153680100	317685500	509698200	497012500	0.572	0.559	0.467	0.575
CONST .HULL	65124830	69288060	228650000	217540300	1.000	1.000	1.000	1.000
CONST .ER	48239110	150640600	349203900	300621400	1.527	1.382	0.741	2.174
CONST .NL	76750730	78603320	250253600	235452700	1.094	1.082	1.179	1.134
CONST .NEL	43277740	62251880	186719500	161589600	0.817	0.743	0.665	0.898
DISTB .HULL	385983700	145416000	651643800	647427900	1.000	1.000	1.000	1.000
DISTB .ER	249771400	265635200	757069300	625782700	1.162	0.967	0.647	1.827
DISTB .NL	211290200	166963100	470197300	467316200	0.722	0.722	0.547	1.148
DISTB .NEL	242885300	75462400	466871800	384718700	0.716	0.594	0.629	0.519
TRANS .HULL	85774160	217436300	547268900	554995600	1.000	1.000	1.000	1.000
TRANS .ER	55742970	196358100	444676500	478154400	0.813	0.862	0.650	0.903
TRANS .NL	82168430	167749000	433566500	437627600	0.792	0.789	0.958	0.771
TRANS .NEL	68007880	132498400	334464700	344354300	0.611	0.620	0.793	0.609
BUSIFIN.HULL	200139700	187469500	831399800	891274200	1.000	1.000	1.000	1.000
BUSIFIN.ER	100766100	207357400	627010200	784180700	0.754	0.880	0.503	1.106
BUSIFIN.NL	97518570	183919900	615124100	660825100	0.740	0.741	0.487	0.981
BUSIFIN.NEL	107752700	82200570	387810800	485649400	0.466	0.545	0.538	0.438
EDUPUB .HULL	435224500	299357600	1126215000	1022180000	1.000	1.000	1.000	1.000
EDUPUB .ER	377337100	138069600	804869500	731702100	0.715	0.716	0.867	0.461
EDUPUB .NL	197746000	133225700	524722800	483003600	0.466	0.473	0.454	0.445
EDUPUB .NEL	230520200	163457000	608273300	550129500	0.540	0.538	0.530	0.546
OTHSECT.HULL	77832110	28447840	155383500	163666100	1.000	1.000	1.000	1.000
OTHSECT.ER	35375350	43056090	112039100	131116600	0.721	0.801	0.455	1.514
OTHSECT.NL	31603240	40445010	108583000	115275500	0.699	0.704	0.406	1.422
OTHSECT.NEL	27379800	18349710	65968100	77869070	0.425	0.476	0.352	0.645

Note: ER = East Riding, NL = North Lincolnshire, NEL = North East Lincolnshire

Input-Output Table and Dynamic General Equilibrium Model of Hull Economy

A dynamic general equilibrium model is developed for the Hull economy based on input-output table constructed above in which consumers and producers look far out in the future, up to hundred years. The consumption and production decisions are consistent to preferences of households and technology of firms and constraints they face in the

process of optimisation. In the balanced steady state investment plans are consistent to the underlying growth process as well as the sectoral rates of depreciation of capital stocks.

There can be an adjustment towards this steady state once the economy deviates from this bringing disequilibrium into the system though all economic relations are fully consistent in equilibrium in the steady state. Ultimately capital accumulation and the natural rate of population growth are the driving forces of the model economy⁸.

Tax distortions harm such economy and reduce the amount of capital accumulation and welfare level of households. For space regions the results of this dynamic model are presented in terms of growth rates of investment, capital stocks and output up to year 2100 in Figures 1 to 3. If the base year patterns continue, the growth rates of investment fall steadily in agriculture, construction, manufacturing, education and public sectors from around two percent in 2000 to about below 1.5 percent in year 2100, rise to about 2 percent rate for energy and water sector but fall continuously up to 2050 in business finance sector to pick up continuously by the second half of the century. Similar pattern is observed also in other sectors.

The growth rate of capital stock is shown in Figure 2, which tends to converge to the steady state growth rate of two percent except in the agriculture sector where it continues to decline until the end of the model horizon. The growth rates of output declines, like that of investment, in agriculture, manufacturing, construction, distribution and education and public services sectors, but rises continuously in the utility (energy and water) and business finance sectors and turns upward towards the steady state in the transportation and communication sectors.

⁸ See Pyatt (1963) Becker (1980), Chamley (1985) King and Robson (1993) Judd, Kubler and Schmedders (2000) for theoretical dynamic equilibrium models, Rutherford (1998) and Bhattacharai (2005) for applied dynamic multisectoral general equilibrium models.

Figure 1
Growth Rates of Investment by Sectors in Humber Region GE Model Over 21st Century

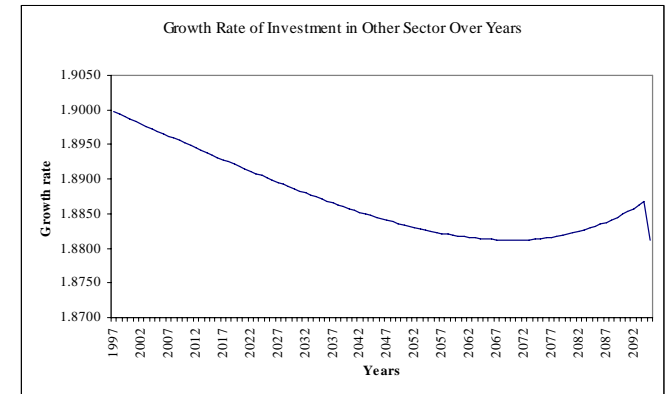
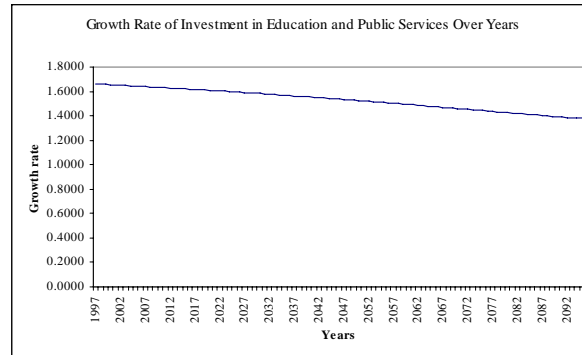
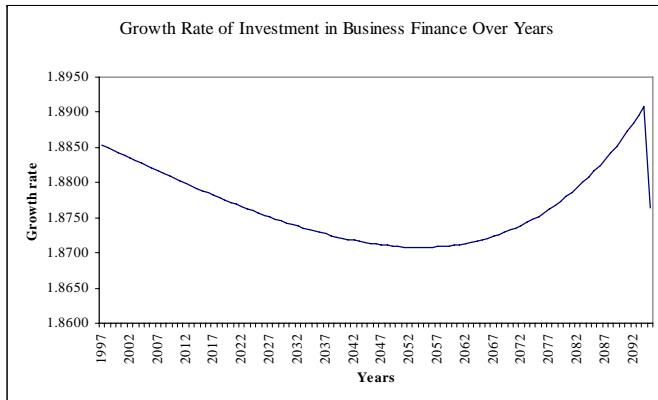
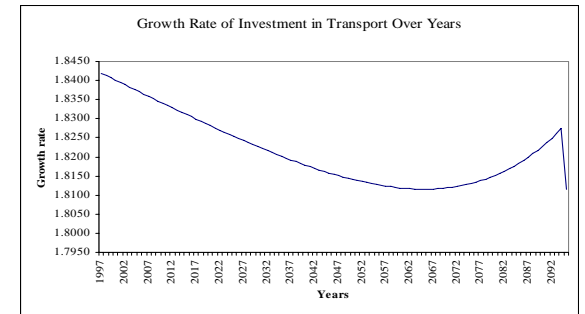
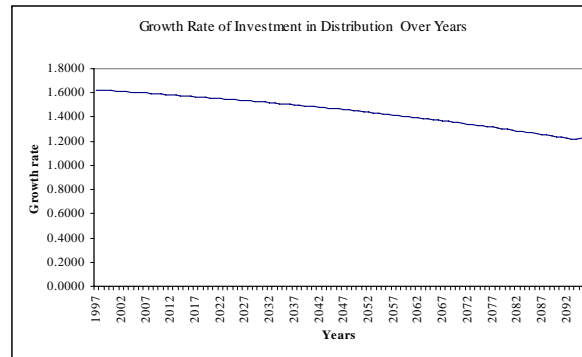
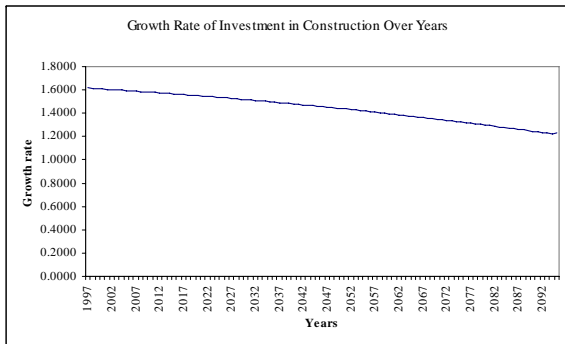
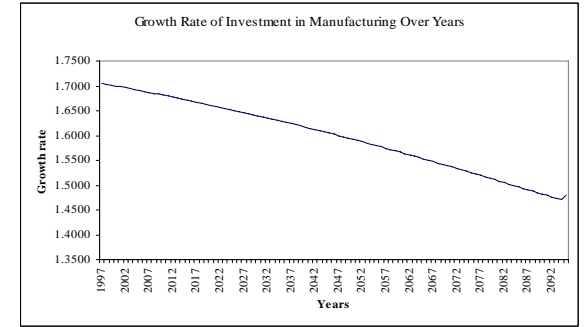
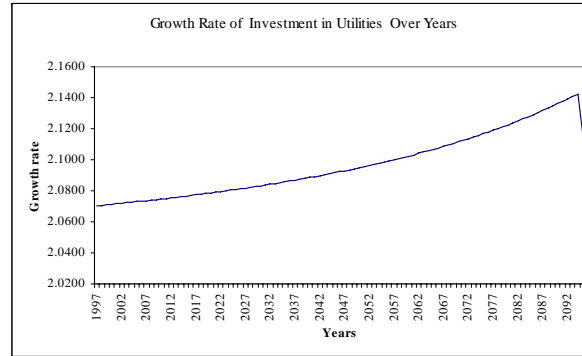
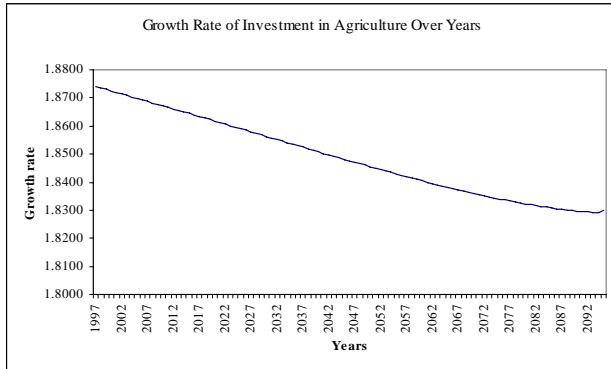


Figure 2
Growth Rates of Capital Stock by Sectors in Humber Region GE Model Over 21st Century

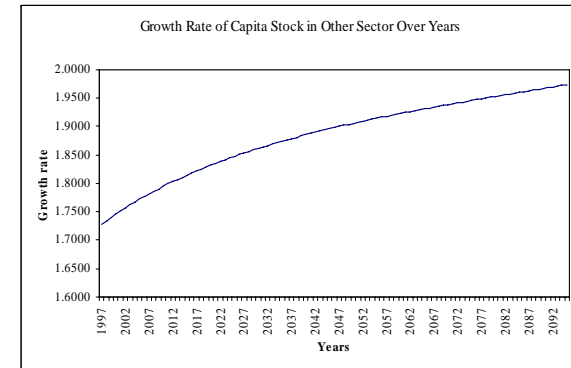
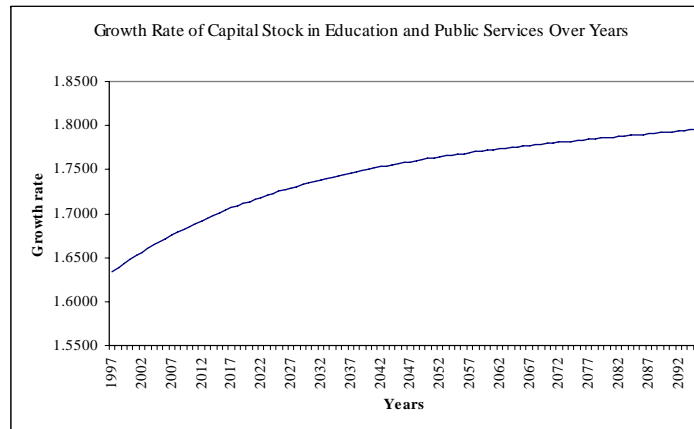
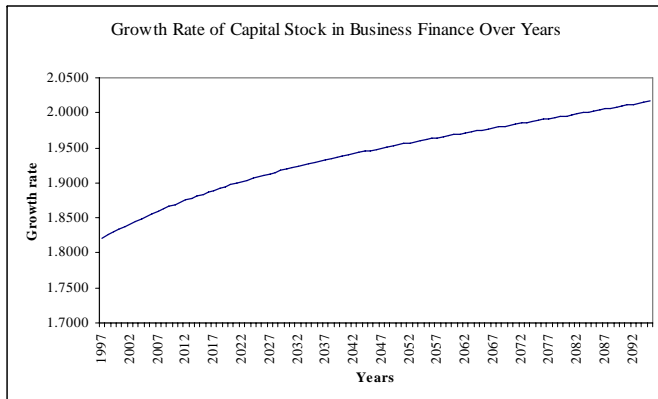
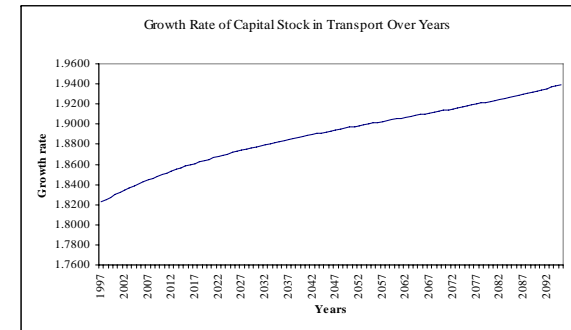
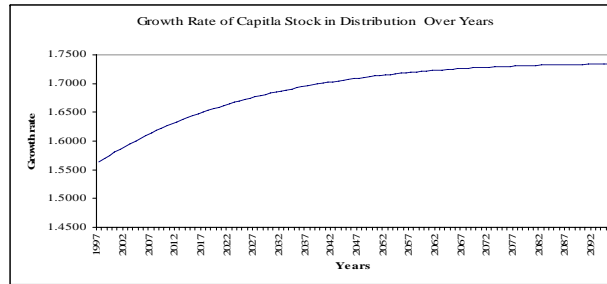
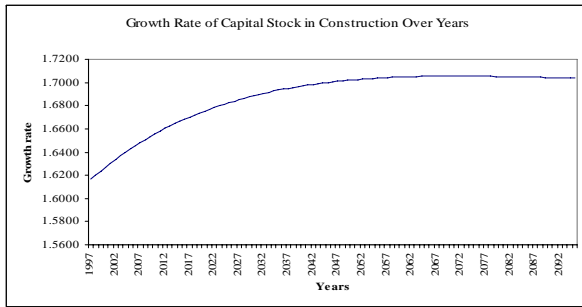
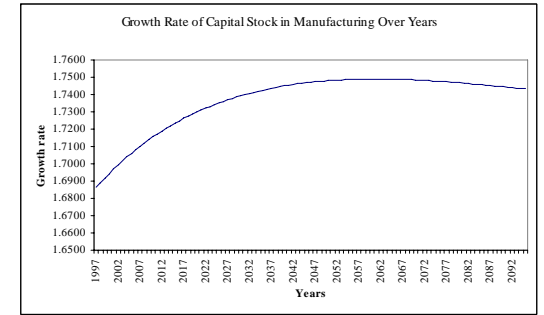
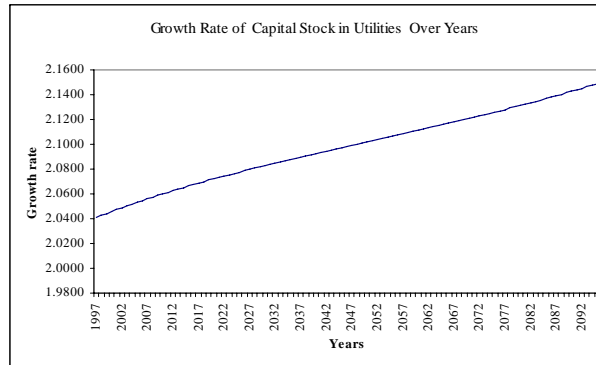
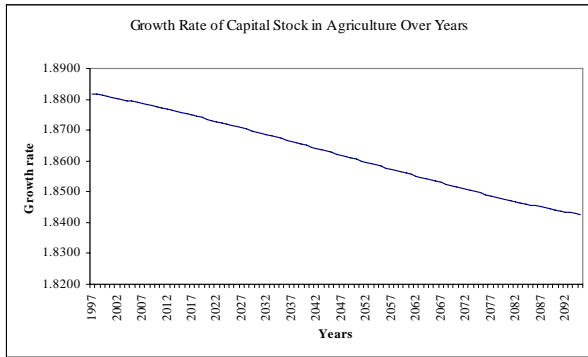
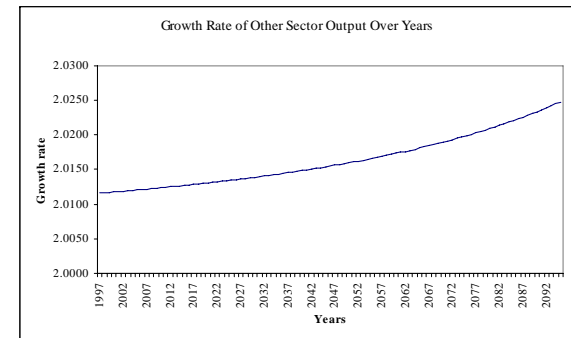
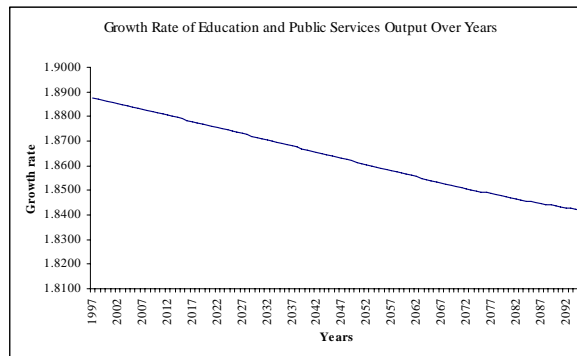
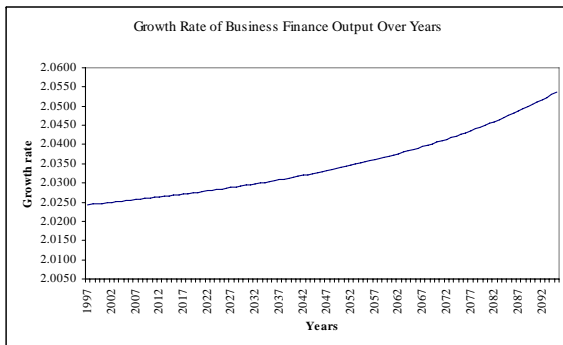
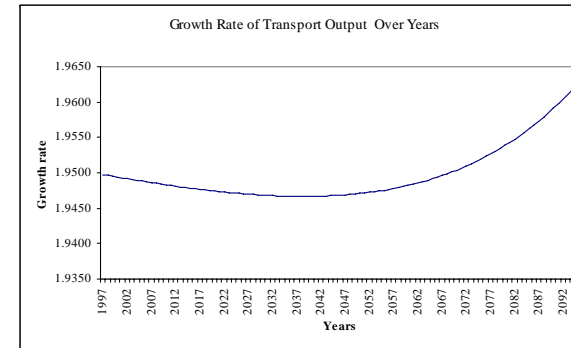
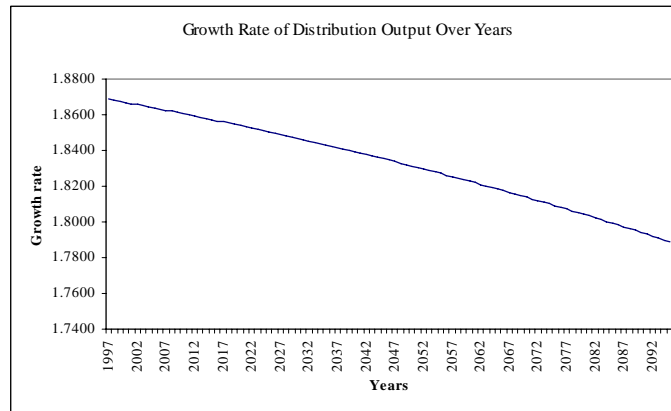
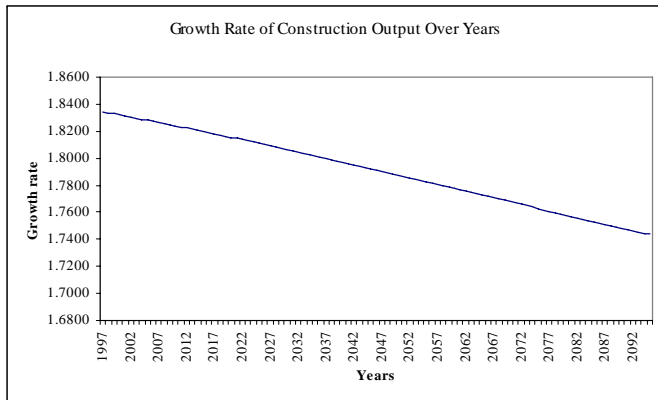
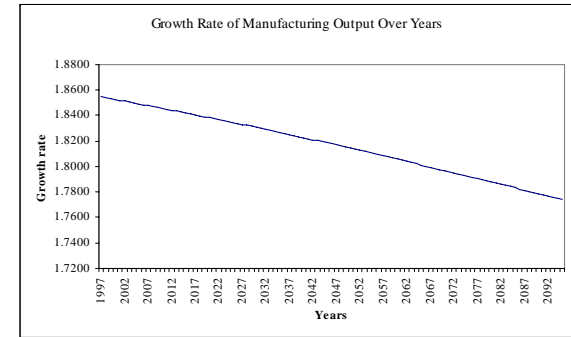
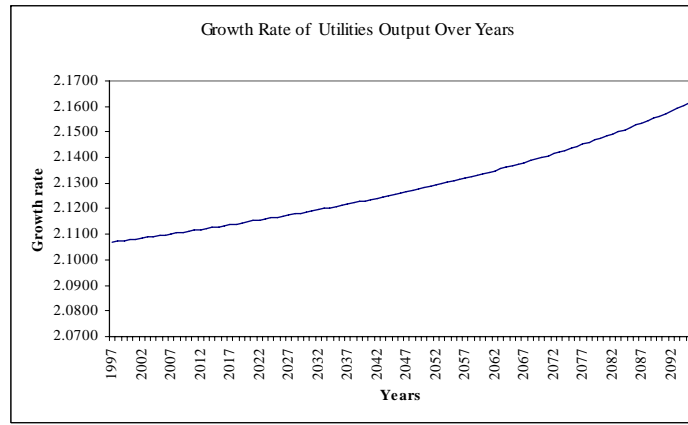
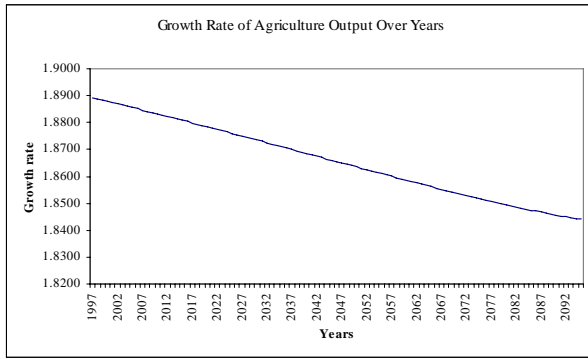


Figure 3
Growth Rates of Output by Sectors in Humber Region GE Model Over 21st Century



Similar projection can be made about the employment by sectors, relative prices of goods and factors of production, exports, imports, public revenue and expenditure and the level of welfare for households over the model horizon. It is not possible to discuss all of these results in one paper.

Conclusion

This paper shows how one can construct an input output table for four Humber sub regions with information on levels and share of employment and output provided by the Humber Forum using additional coefficients from the national input output table. It then illustrates how these can be applied to construct a multisectoral general equilibrium models specific for Hull, East Riding, North Lincolnshire and North East Lincolnshire regions situated in two sides of the Humber estuary in England. The regional model of Humber is made of these four inter-dependent economies. A dynamic model is constructed for Hull to assess the prospects in the next hundred years based on micro consistent dataset in which households and firms are assumed to have perfect foresight in their consumption and production decisions. These models are then applied to evaluate impacts of tax policies that distort relative prices of commodities and factors of production and thus are harmful for efficient allocation of scarce economic resources and on the welfare of households in the Humber economy over next 100 years. To my knowledge this is the first study of this type for this region.

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