Advanced Economic Theory
Lecture 3

Macroeconomic Model of Fluctuations

1. Keynes-Hicks-Samuelson-Mundel-Flemming Models for Analysis of Macroeconomic Fluctuations (Demand Side)

Macro-econometric Simulations and Forecasting

Policy Rules

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Macroeconomic modelling

- Keynes (1936), Hicks (1937), Samuelson (1939),
- Phillips (1958), Friedman (1968), Phelps (1968), Tobin (1969),
- Sargent and Wallace (1975), Lucas (1976), Fisher (1977), Kydland and Prescott (1977) ,Wallis (1980), King and Plosser (1984) Mankiw (1989), Prescott (1986), Taylor (1987)
- Blanchard and Kiyotaki (1987), Manning (1995), Rankin (1992)
- Barro and Gordon (1983), Sargent (1986) Goodhart (1989), Nickell (1990), Mankiw and Romer(1993), Lockwood Miller and Zhang (1998)
- Wallis (1989), MPC (1999), Pagan and Wickens (1989), Hendry (1995), Holly and Weale (2000)
- Taylor (1993), Sargent and Ljungqvists (2000), Minford and Peel (2002), Blake and Weal (2003), Garratt, Lee, Pesaran and Shin (2003)
- Solow (1956), Lucas (1988), Romer (1990), Mankiw, Romer and Weil (1992),
- Harrod (1939), Domar (1947) and Solow (1956), Parente and Prescott (1993)
- Fullerton, Shoven and Whalley (1983), Auerbach and Kotlikoff (1987), Perroni (1995), Rutherford (1995), Bank of England, NIESR) Kehoe, Srinivasan and Whalley (2005), Bhattarai (1997, 1999)

Classical View: Free Market and Minimum Government

(Ideas of Adam Smith (1776), Ricardo (1817), Say (1821) ,Malthus (1798), Mill (1873), Marshall (1925)

- Market is always in equilibrium: Demand = Supply both in goods and factor markets; No excess demand or no excess supply can persist.
- Perfectly flexible prices (Invisible hand) make this happen.
- No glut or shortages in goods market.
- No unemployment or labour pressure in the labour market.
- It is long run view (growth model)
- Prices proportional to money supply.
- Money is neutral (quantity theory of money).
- Balance budget recommended.
- Laisser faire: minimum government is the best government.
- Downward sloping aggregate demand and vertical supply curve

- See http://socserv2.socsci.mcmaster.ca/~econ/ugcm/3ll3/

Why Keynesian Economics?

- Automatic equilibrium is not guaranteed - animal spirits not the rational choices dominate the economy.

- Labour market imbalance: rigid labour market, nominal wages not flexible upwards

  – unemployment may persist for a long period if the deficiency in demand continues. Loss of welfare

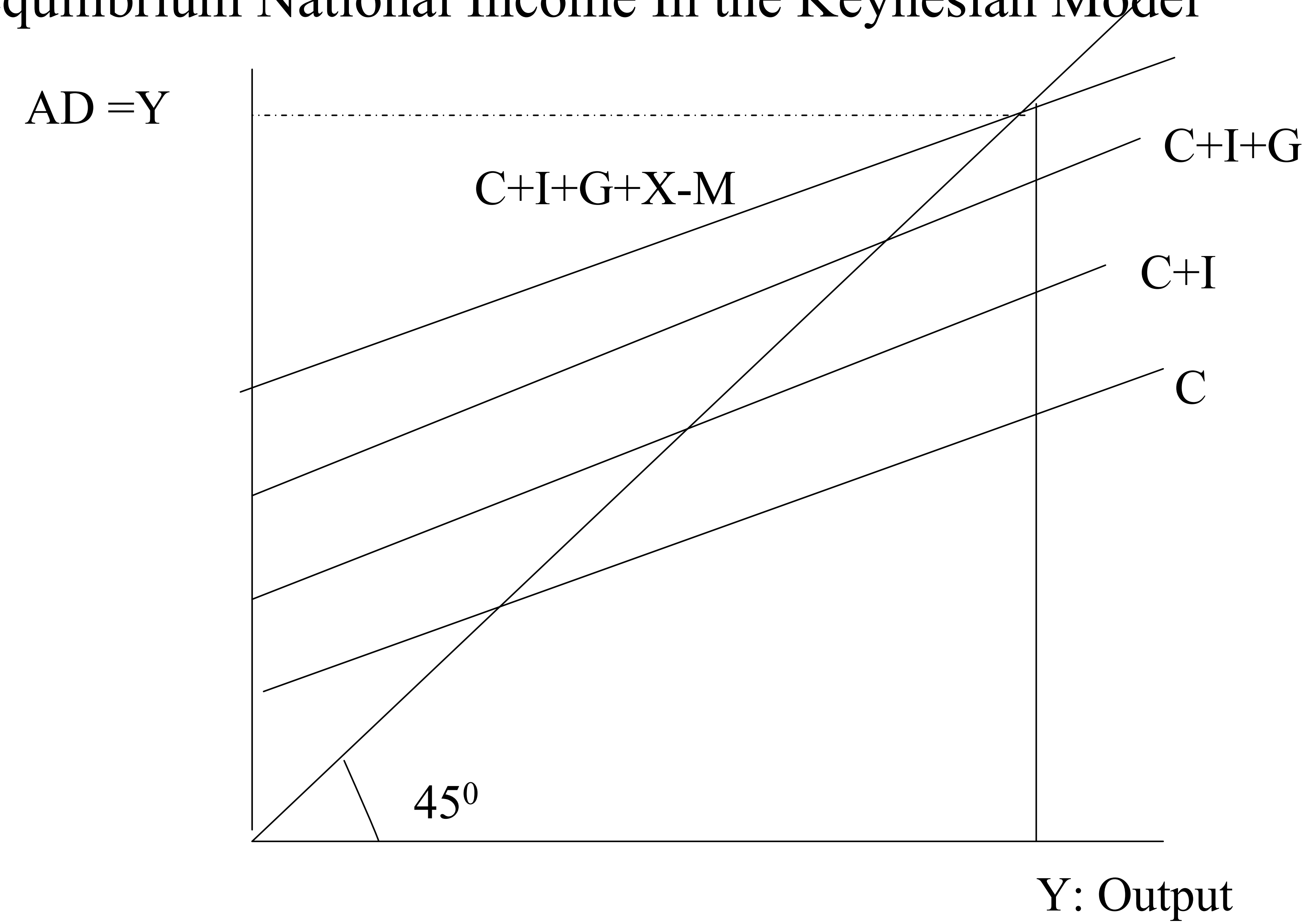
- Prices of commodities not flexible because of market power of firms; market fails

- Active fiscal and monetary policy can fine tune the economy

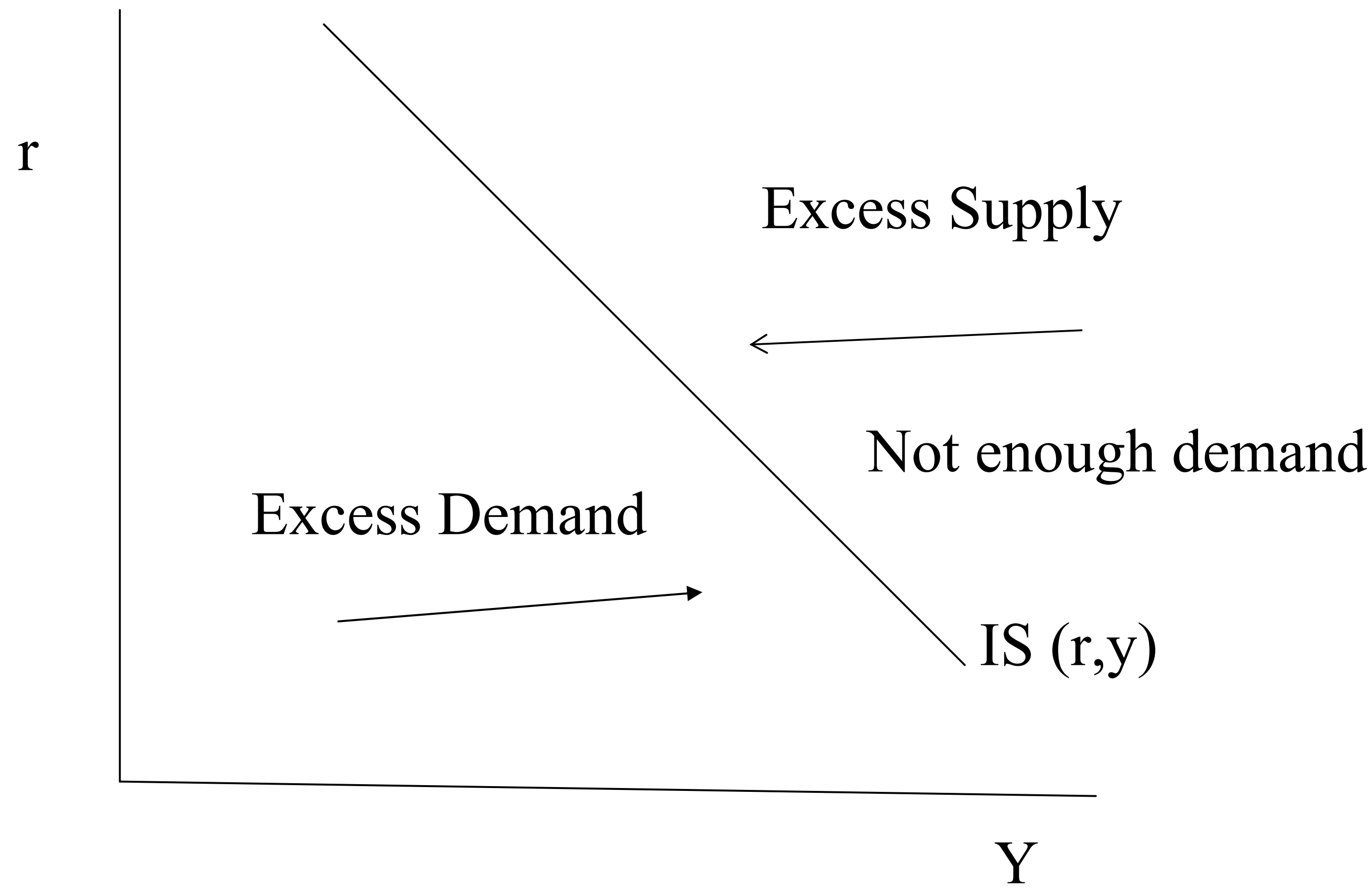
- “We are all dead in the long run” - Keynes

- In time of massive unemployment dig holes and fill them to create jobs and income

Equilibrium National Income In the Keynesian Model



Why the IS Curve Represents a Good Market Equilibrium?



Model Solutions

- Y = 900
- C = 840
- I = 90
- T=G= 100;X=50
- M = 180
- X-M = -130
- S=Y-T-C=-40
- S-I=-40-90=-130

$$C = 200 + 0.8(Y - 100)$$

$$I = 100 - 200(0.05)$$

$$Z = 0.2Y$$

A Simple Keynesian Model

MODEL

$$C = C_0 + a(Y - T)$$

$$I = b - dr$$

$$Z = mY$$

T=100 G =100 X =50

$$Y = C + I + G + X - Z$$

Numerical Example

$$C = 200 + 0.8(Y - 100)$$

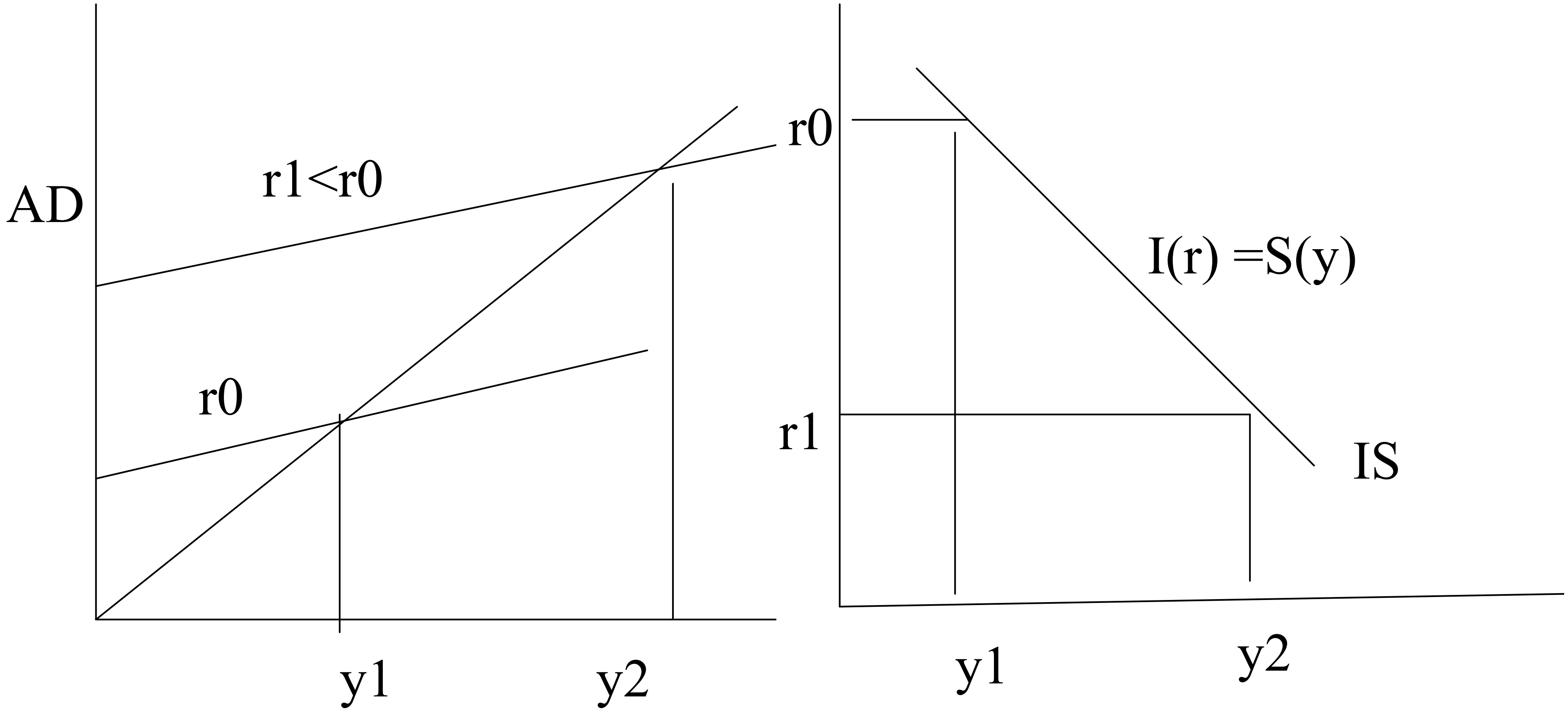
$$I = 100 - 200(0.05)$$

$$Z = 0.2Y$$

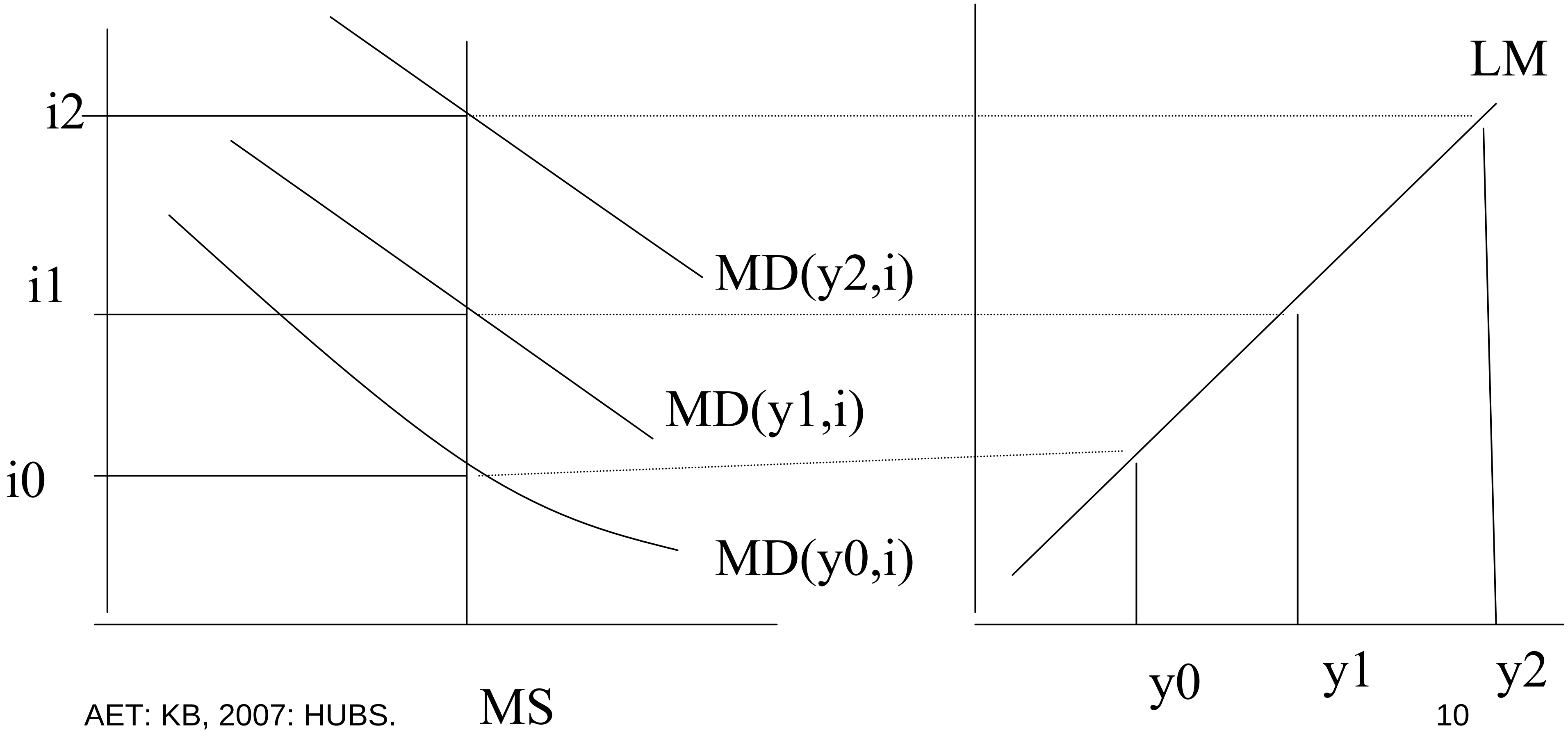
$$Y = \frac{C_0 - aT + b - dr + G + X}{1 - a + m}$$

$$Y = \frac{200 - 0.8 * 100 + 100 - 200 * 0.05 + 100 + 50}{1 - 0.8 + 0.2}$$

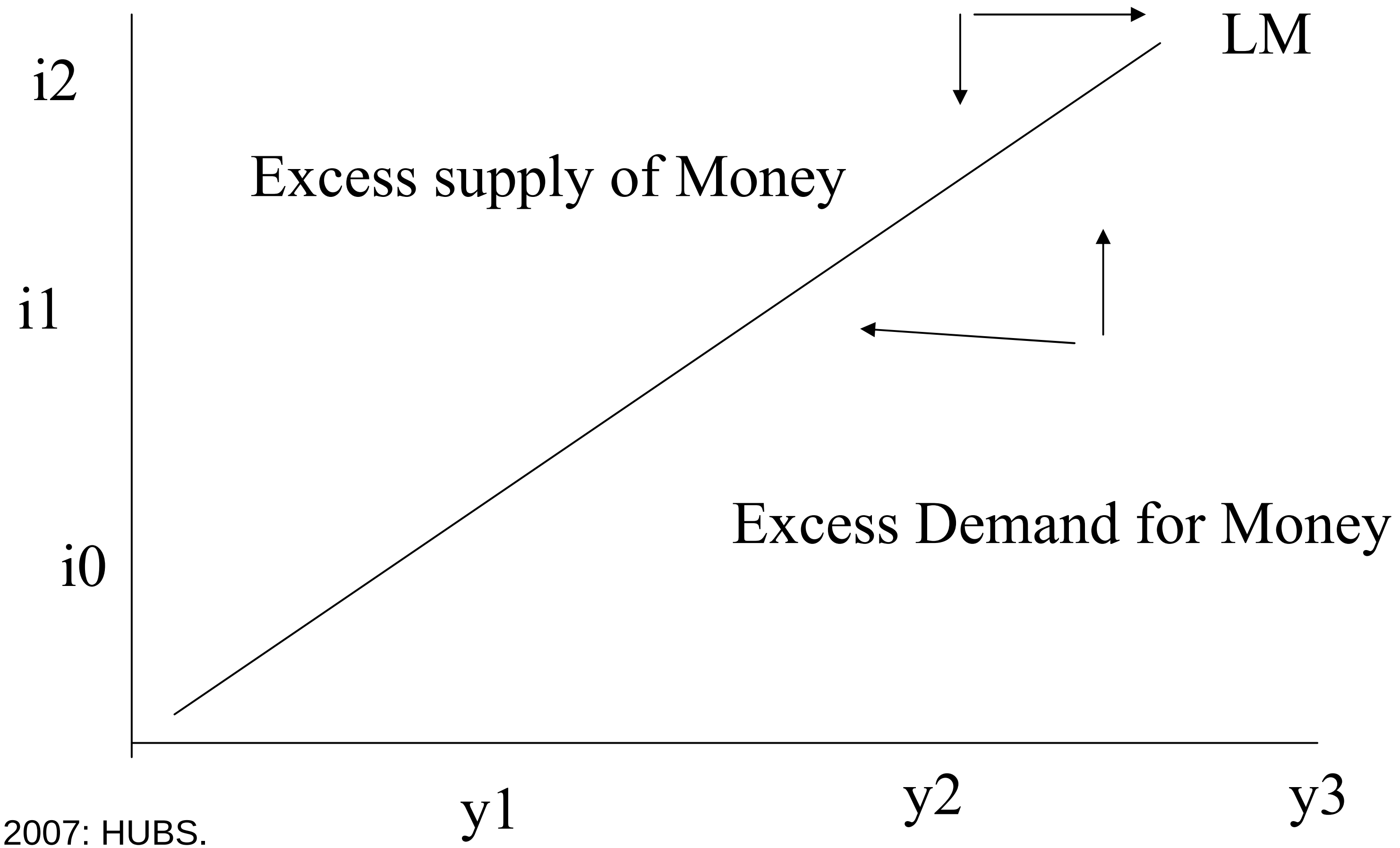
Goods Market Equilibrium in a Closed Economy
Saving = Investment
Derivation of the IS Curve in the Keynesian Model



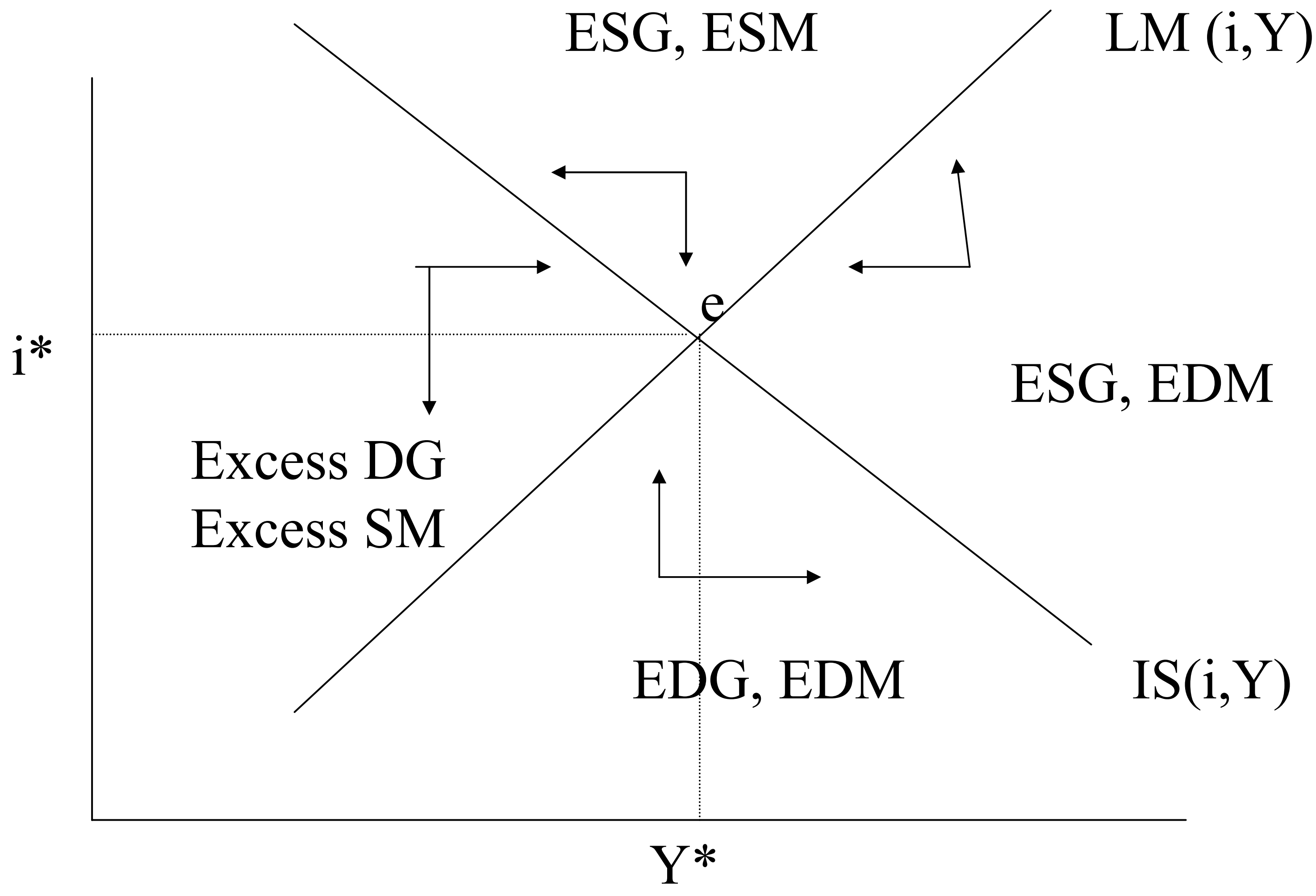
Money Market Equilibrium: LM Curve



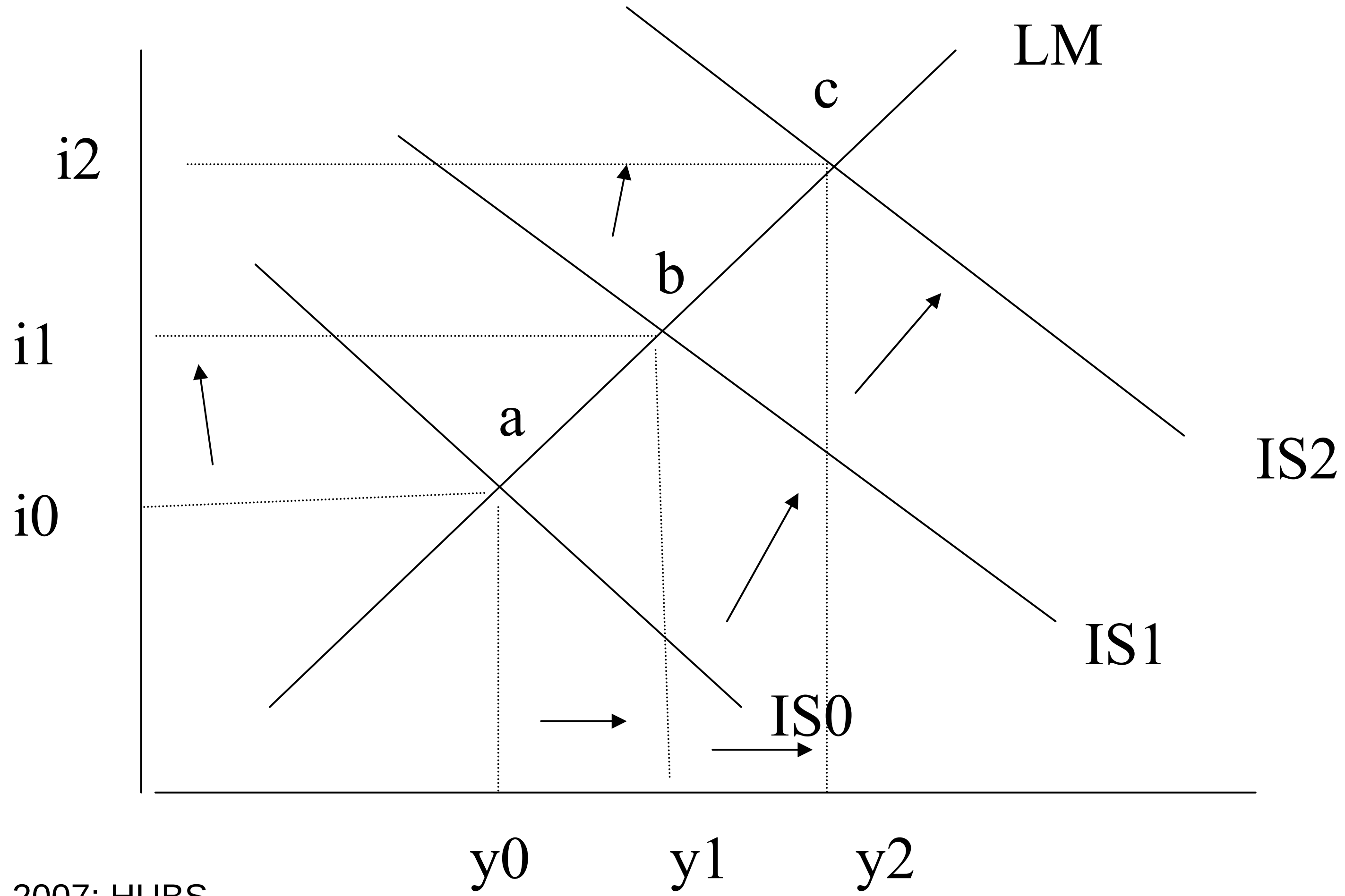
Equilibrium and Disequilibrium in the Money Market



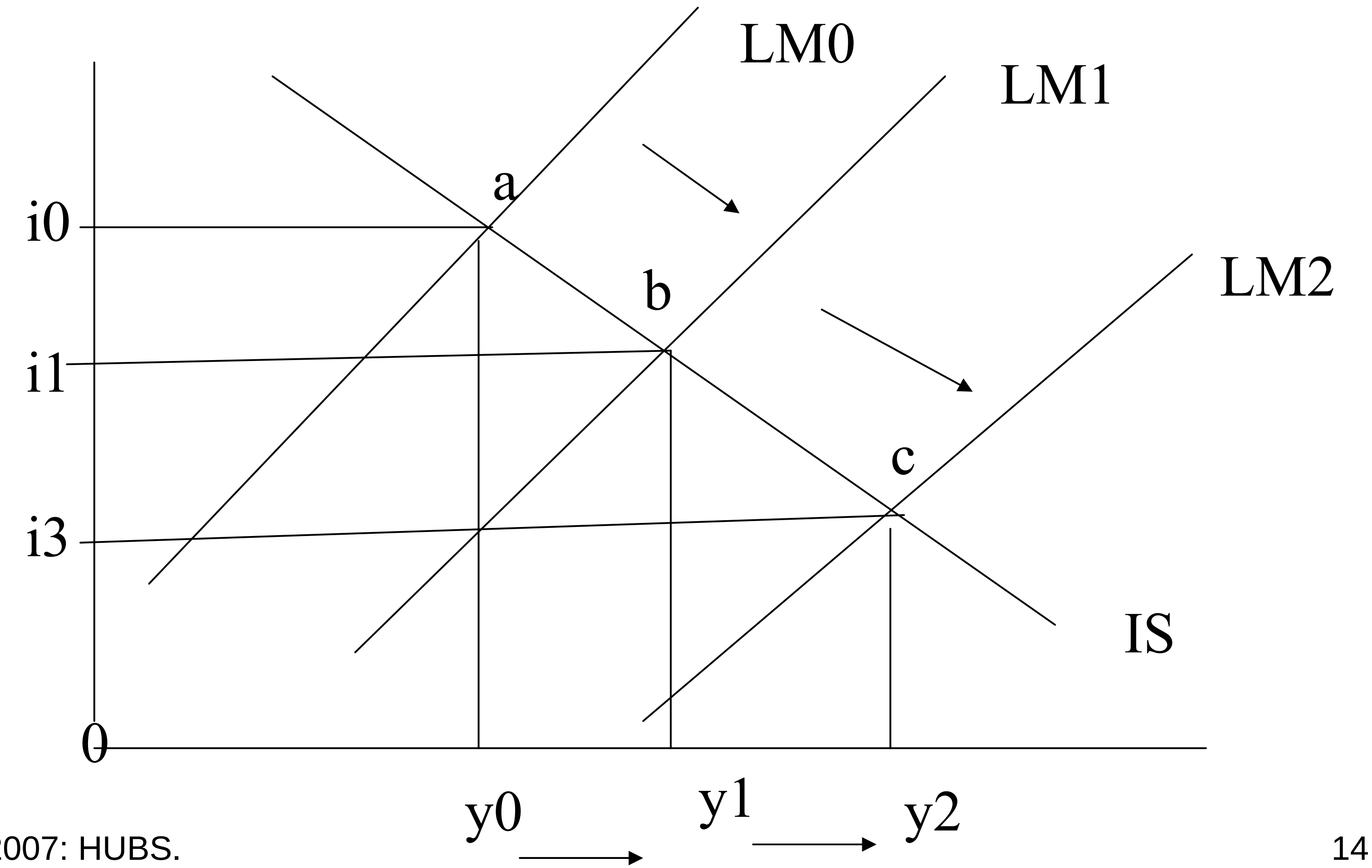
Economy Wide Equilibrium in a IS-LM Model



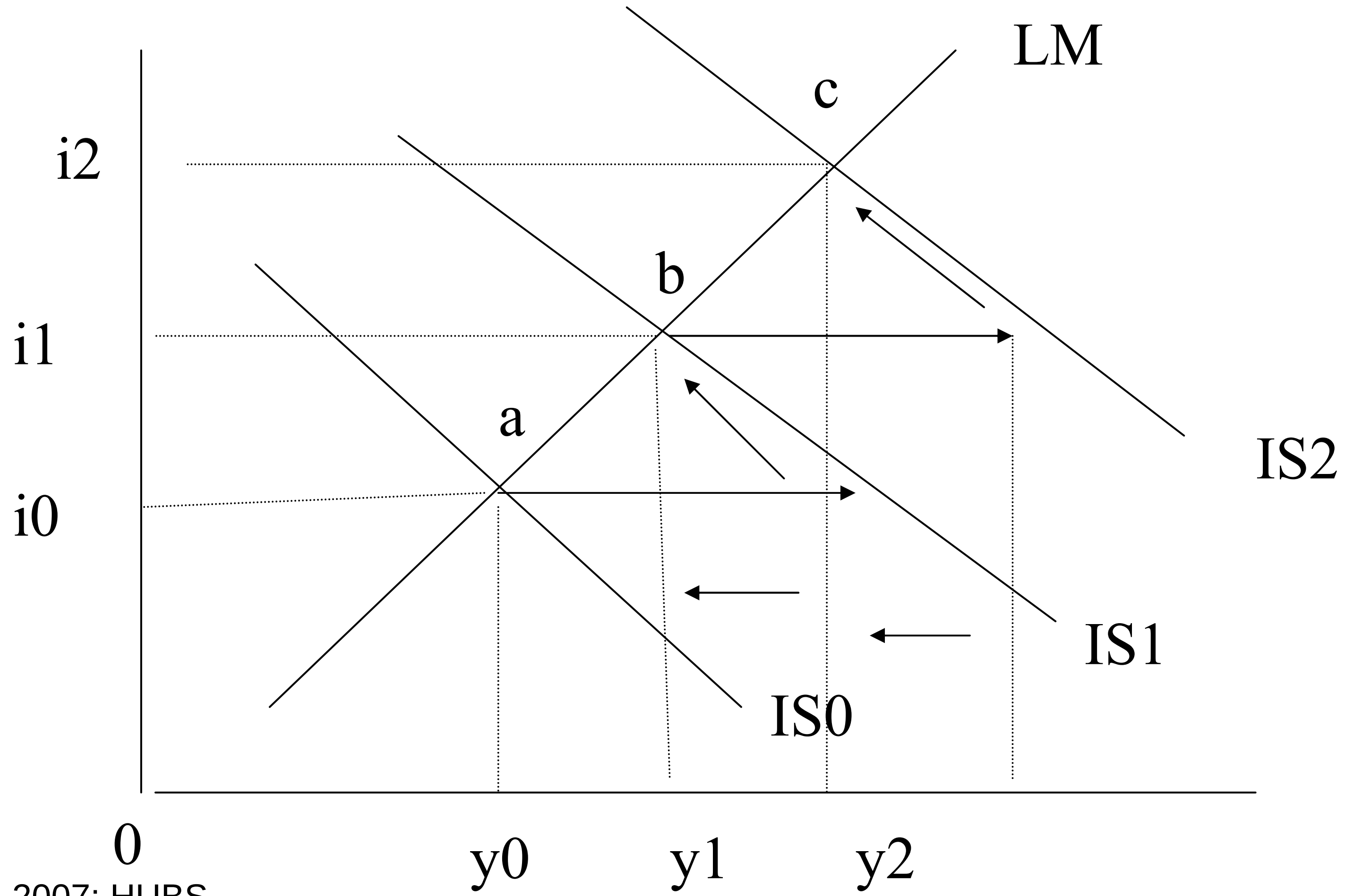
Impact of expansionary fiscal policy in the interest rate and output



Impact of expansionary monetary policy in the interest rate and output

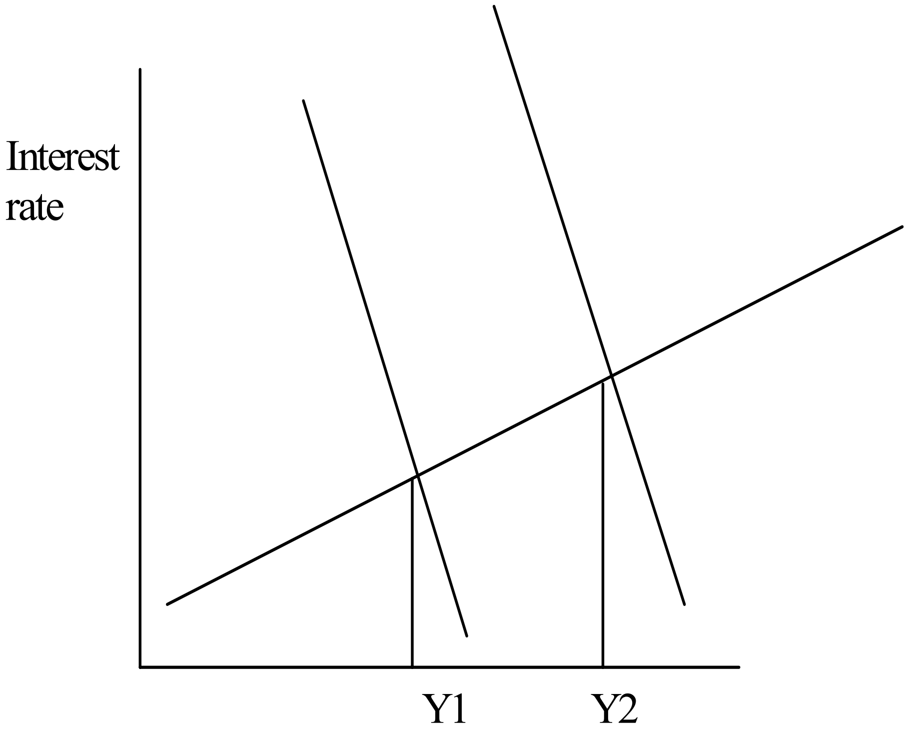


Crowding Out Effect of an Expansionary Fiscal Policy



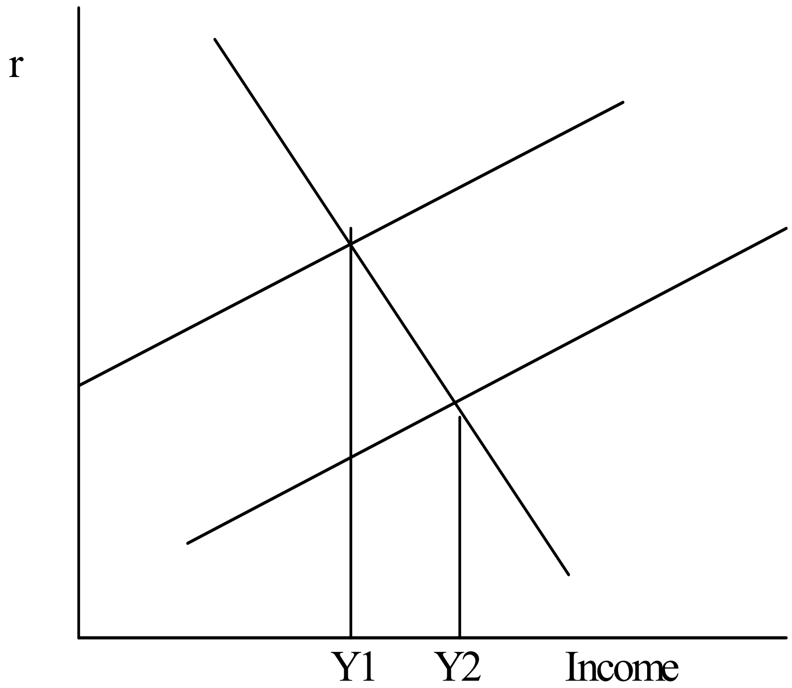
Keynesian View on Impacts of Fiscal and Monetary Policies

Keynesian view on Fiscal Policy



Fiscal policy is more effective (Flat LM)
Large output effect of fiscal policy (Steep IS)

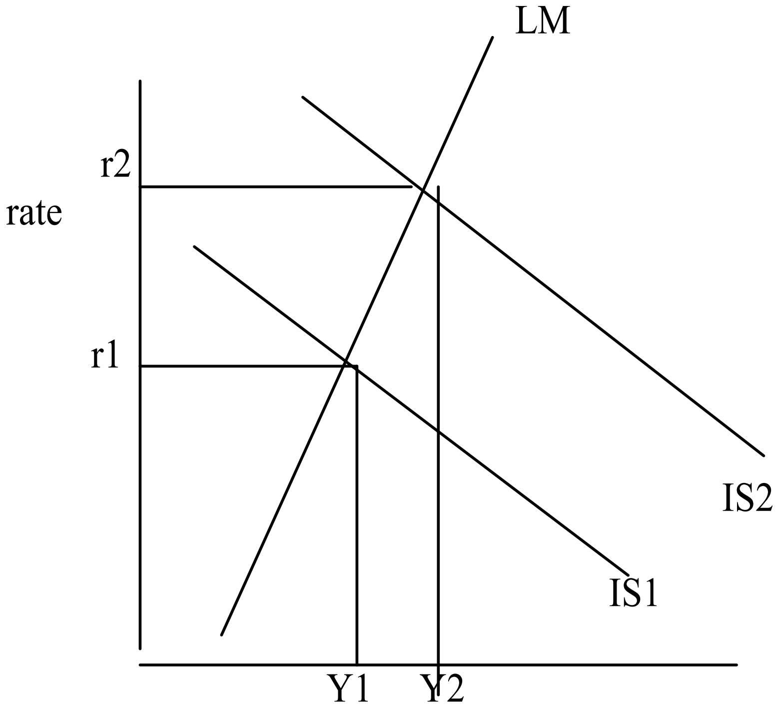
Keynesian view on Monetary Policy



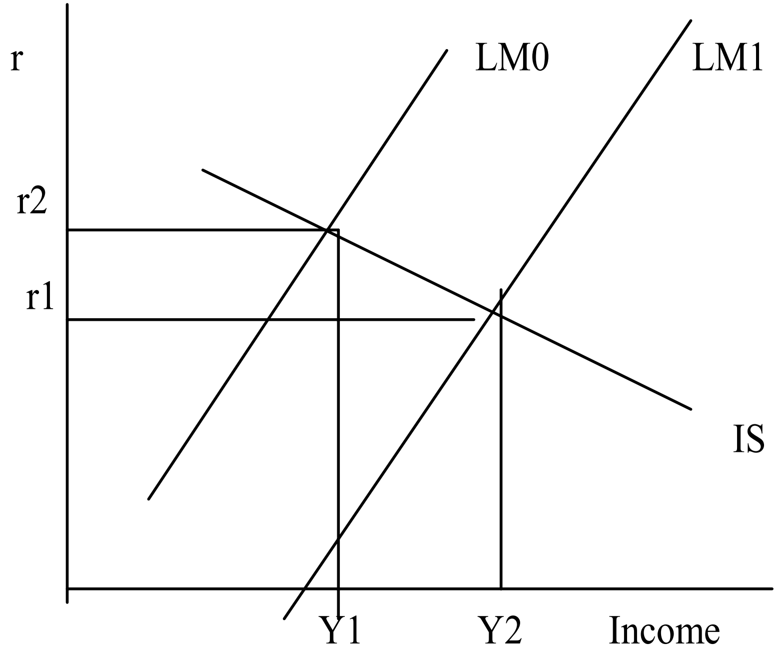
Monetary policy is effective (Flat LM) and
Small output effect of Monetary policy (Steep IS)

Monetarist's View on Impacts of Fiscal and Monetary Policies

Monetarist view on Fiscal Policy



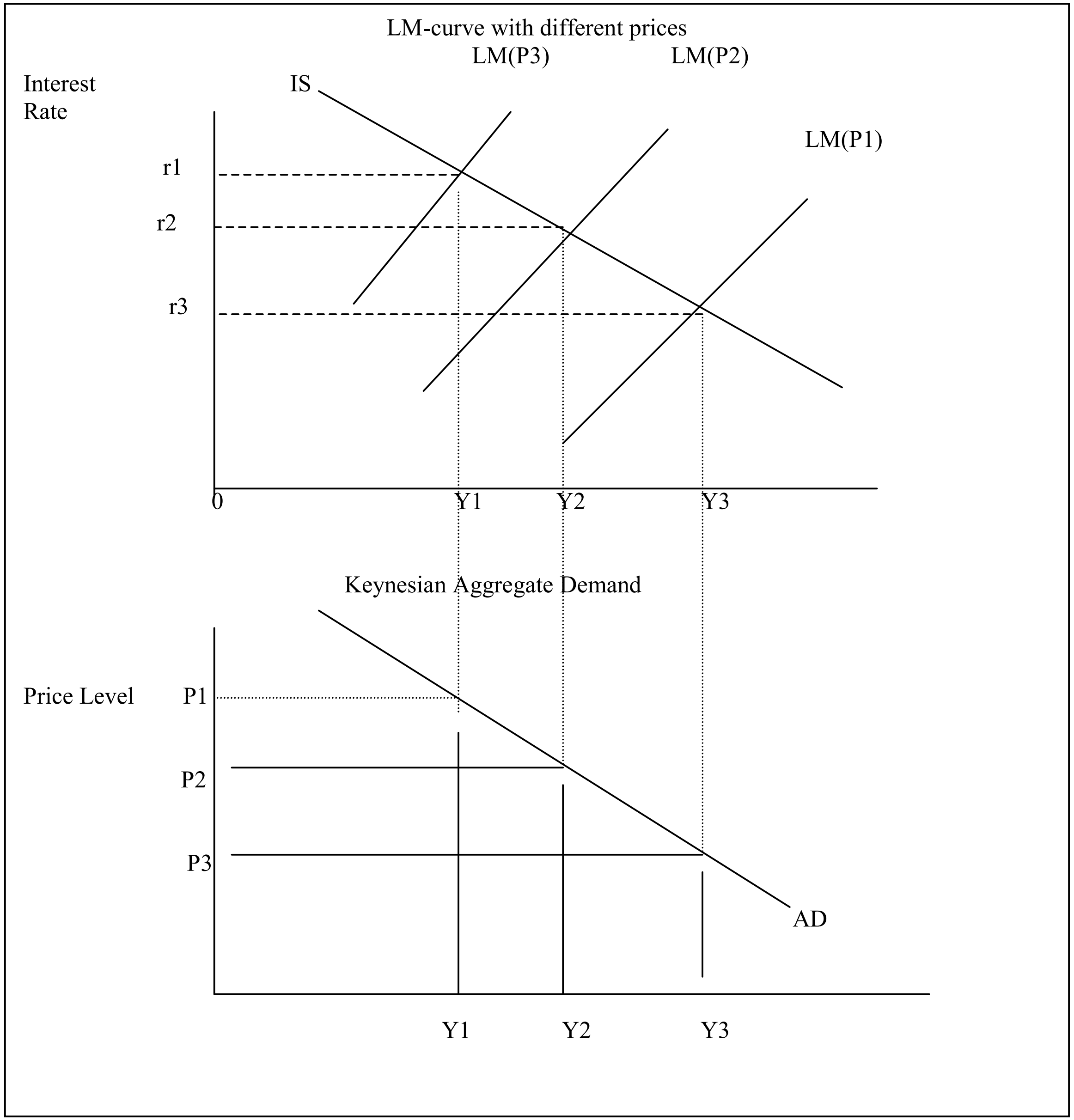
Monetarist view on Monetary Policy



Fiscal policy is in effective (Steep LM)
Small output large interest effect of fiscal policy (Flat IS)

Monetary policy is effective (Steep LM) and
Large output small interest rate effect of monetary policy (FLAT IS)

Derivation of Keynesian Aggregate Demand Curve



Derivation of AD form an ISLM Model: a Numerical Example

Supply-demand: $Y = C + I + G$ (12)

Consumption: $C = 250 + 0.75(Y - T)$ (13)

Investment: $I = 200 - 25r$ (14)

Balanced budget: $T = G = 100$ (15)

Derive IS curve:

$$Y = 250 + 0.75(Y - T) + 200 - 25r + 100 \quad (16)$$

$$Y = 1900 - 100r$$

$$\frac{\partial y}{\partial r} = -100 :$$

a negatively sloped IS curve (17)

ISLM Model: Example 2

Money demand: $(M/P)^d = Y - 100r$;

Money supply $\overline{M} = 1000$ (18)

Money market equilibrium:

$$1000 = Y - 100r \qquad (19)$$

Or $r = -10 + 0.01Y$

=> $\dfrac{\partial r}{\partial y} = 0.01 > 0$

Positive slope of the LM (20)

Economy wide Equilibrium

It is given by the intersection point of the IS and LM curves.

$$Y = 1900 - 100r$$

or $Y = 1900 - 100(-10 + 0.01Y)$

$$Y = \frac{2900}{2} \Rightarrow Y = 1450 \qquad (21)$$

$$r = -10 + 0.01Y \implies$$

$$r = -10 + 0.01(1450) = 4.5\% \qquad (22)$$

Impact of an Expansionary Fiscal Policy

G rises from 100 to 150

$$Y = 250 + 0.75(Y - T) + 200 - 25r + 150$$

$$0.25Y = 525 - 25r \rightarrow Y = 2100 - 100r \quad (16')$$

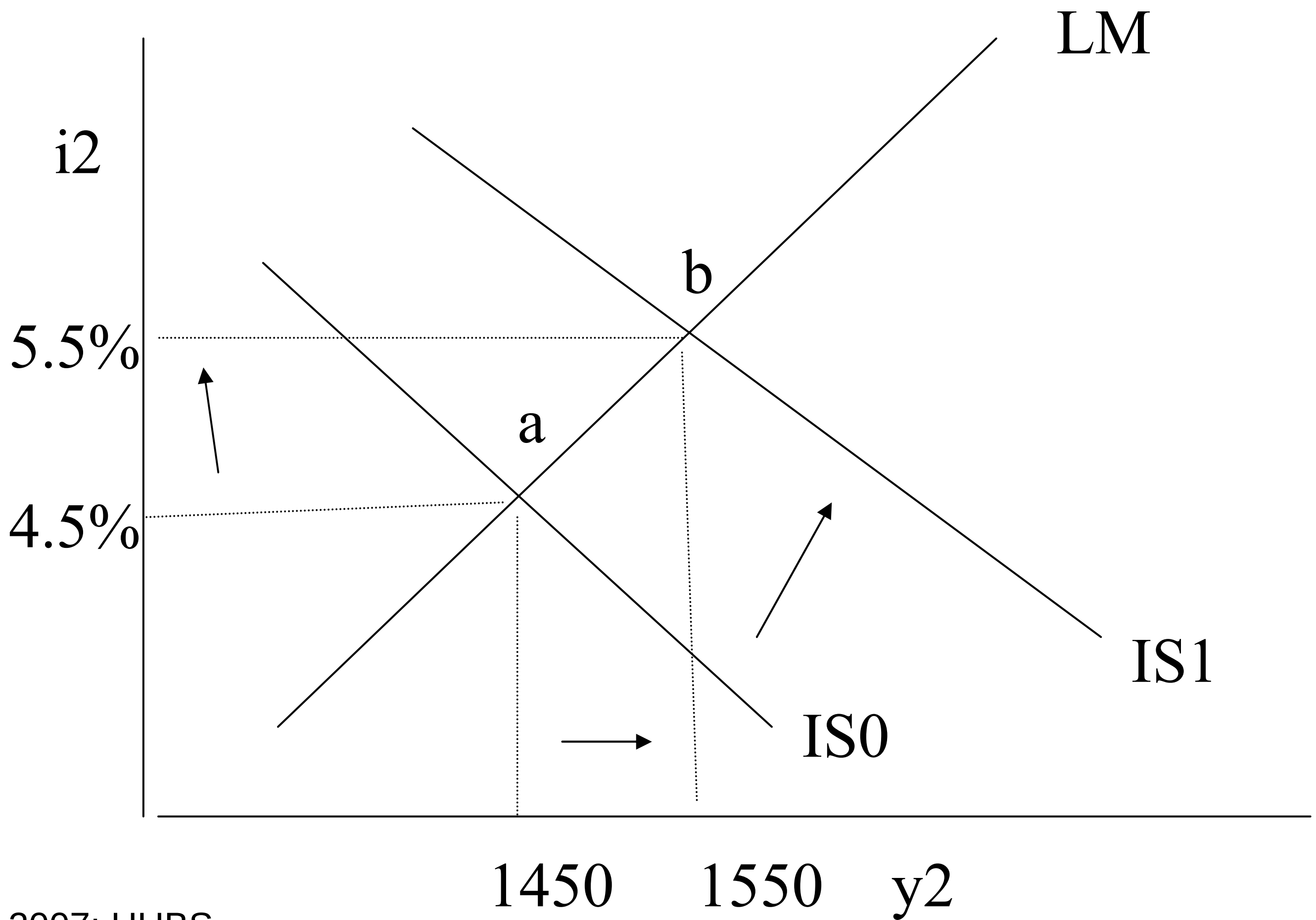
$$Y = 2100 - 100(-10 + 0.01Y) \rightarrow$$

$$Y = 3100 - Y \rightarrow Y = \frac{3100}{2} = 1550$$

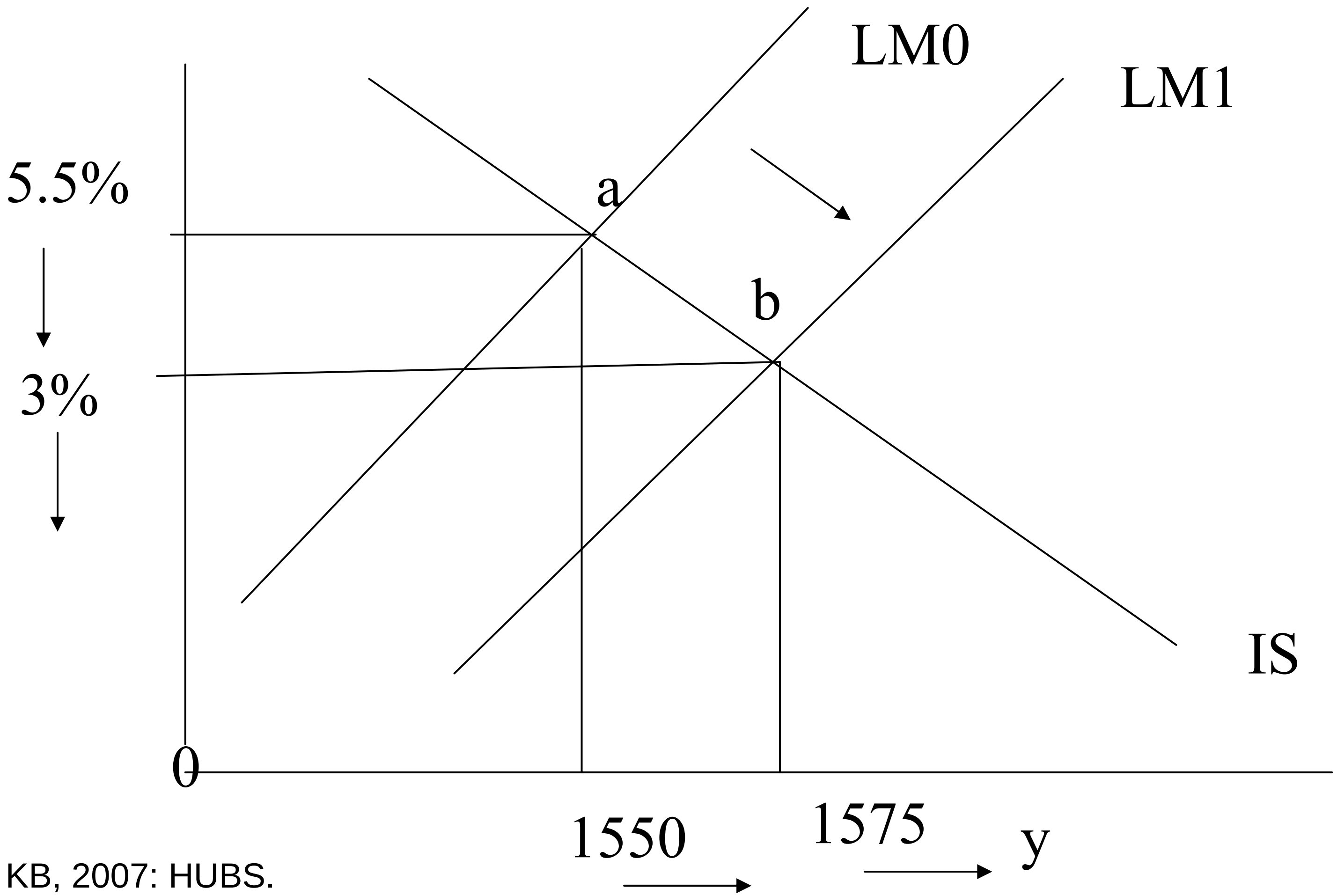
$$r = -10 + 0.01Y \rightarrow r = -10 + 0.01(1550)$$

$$\rightarrow \mathrm{r} = 5.5\%$$

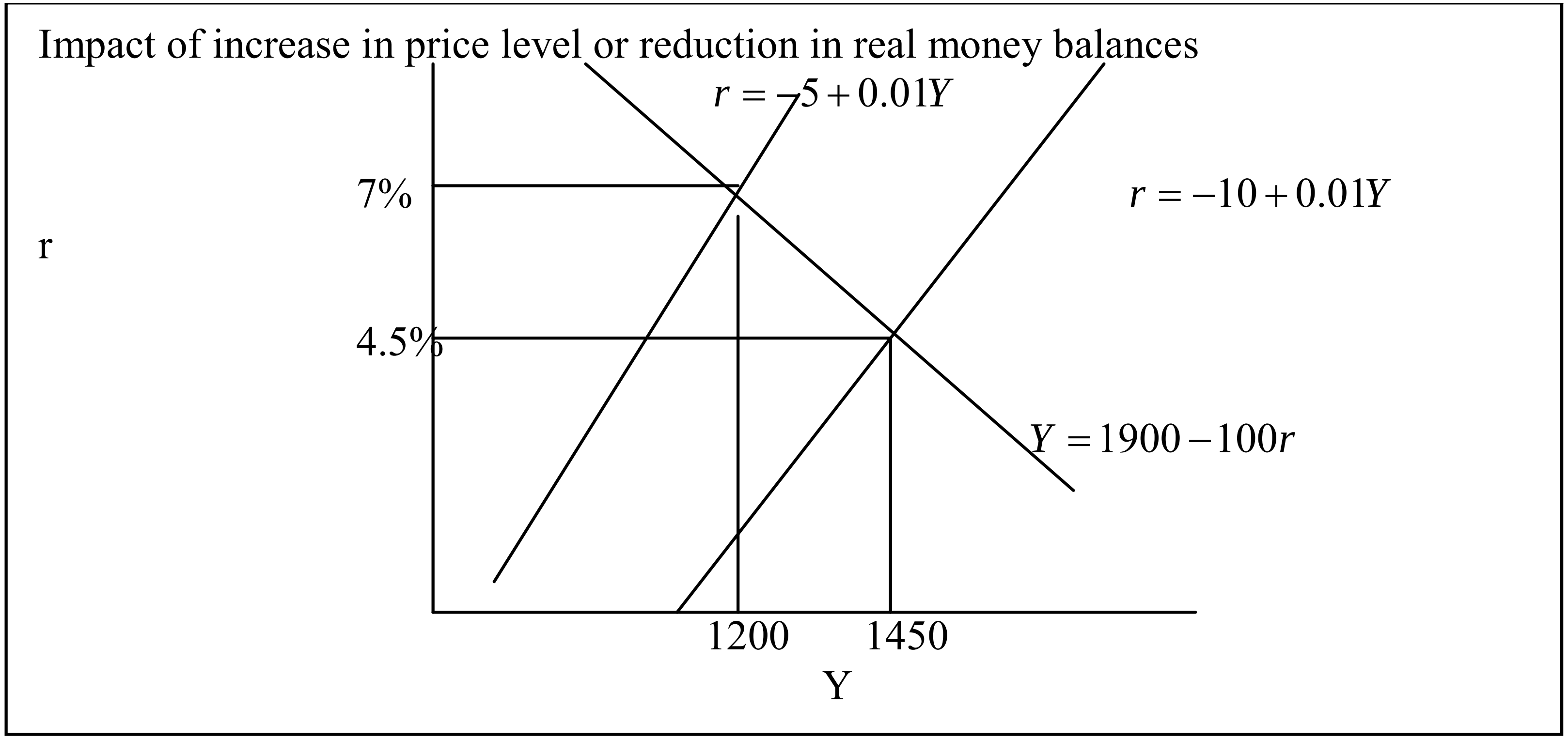
Impact of expansionary fiscal policy in the interest rate and output (G rises from 100 to 150)



Impact of an Increase in Money Supply from 1000 to 1250 on the interest rate and output



Real Balance Effect in the IS-LM Model with an increase in the price level from 1 to 2



Why is an AD Downward Sloping?

- Real balance effect (M/P)
- Interest rate effect (i)
- Exchange rate effect (£/$)
- Market expectations (Pe, Ye….)

Simulations with a basic Keynesian Model

$$C_t = \beta_0 + \beta_1 \left(Y_t - T_t \right)$$

$$I_t = \mu_0 + \mu_1 R_t + \phi \Delta Y_{t-1}$$

$$T_t = t_0 + t_1 Y_t$$

$$M_t = m_0 + m_1 Y_t + m_2 \lambda_t$$

$$C_t + T_t + S_t = Y_t = C_t + I_t + G_t + X_t - M_t$$

$$\left(T_t - G_t \right) + \left(S_t - I_t \right) = \left(X_t - M_t \right)$$

$$Y_t = \frac{\beta_0 - \beta_1 c_0 + \mu_0 - m_0 + G_t + X_t}{1 - \beta_1 + \beta_1 t_1 + m_1} + \frac{\mu_1 R_t}{1 - \beta_1 + \beta_1 t_1 + m_1} + \frac{\phi \Delta Y_{t-1}}{1 - \beta_1 + \beta_1 t_1 + m_1}$$

Parametric Specification of the Keynesian Model for Scenarios (Change one thing at a time (Excel file Keynesian1.xls))

	Parameters	Base Case	Tax cut	Spending	MPC	T &G	High X	High I	MMM
G	200	200	200	400	200	400	200	200	200
X	100	100	100	100	100	100	300	100	100
r	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
C0	300	300	300	300	300	300	300	300	300
b	0.8	0.8	0.8	0.8	0.9	0.8	0.8	0.8	0.8
I0	50	50	50	50	50	50	50	200	50
d	10	10	10	10	10	10	10	10	10
t0	30	30	30	30	30	30	30	30	30
t	0.3	0.3	0.2	0.2	0.3	0.2	0.3	0.3	0.3
m0	20	20	20	20	20	20	20	20	20
m1	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.4

Solutions of the Basic Keynesian Model from Keynesian1. XLS

	Y	T	C	I	G	X	M	S	T-G	X-M	S-I	Bal
Base case	876.8	293.0	767.0	49.0	200.0	100.0	239.2	-183.2	93.0	-139.2	-232.2	-139.2
Tax cut	991.8	228.4	910.8	49.0	200.0	100.0	268.0	-147.3	28.4	-168.0	-196.3	-168.0
Spending	1166.7	380.0	929.3	49.0	400.0	100.0	311.7	-142.7	-20.0	-211.7	-191.7	-211.7
MPC	971.0	321.3	884.7	49.0	200.0	100.0	262.7	-235.0	121.3	-162.7	-284.0	-162.7
T&G	1319.7	293.9	1120.6	49.0	400.0	100.0	349.9	-94.9	-106.1	-249.9	-143.9	-249.9
High X	1166.7	380.0	929.3	49.0	200.0	300.0	311.7	-142.7	180.0	-11.7	-191.7	-11.7
High I	1094.2	358.3	888.8	199.0	200.0	100.0	293.6	-152.8	158.3	-193.6	-351.8	-193.6
MMM	720.2	246.1	679.3	49.0	200.0	100.0	308.1	-205.2	46.1	-208.1	-254.2	-208.1

Keynes-Hicks IS-LM Model (Introduce Money Market)

$$\left(\frac{\overline{MM}}{P}\right)_t = b_0 + b_1 Y_t - b_2 R_t$$

$$R_t = \frac{b_0}{b_2} - \frac{1}{b_2}\left(\frac{\overline{MM}}{P}\right)_t + \frac{b_1}{b_2} Y_t$$

$$Y_t = \frac{b_2\left(\beta_0 - \beta_1 t_0 + \mu_0 - m_0 + G_t + X_t\right)}{\left(1 - \beta_1 + \beta_1 t_1 + m_1\right) b_2 - \mu_1 b_1} + \frac{b_2 \phi \Delta Y_{t-1}}{\left(1 - \beta_1 + \beta_1 t_1 + m_1\right) b_2 - \mu_1 b_1} + \frac{b_2 \mu_1}{\left(1 - \beta_1 + \beta_1 t_1 + m_1\right) b_2 - \mu_1 b_1}\left[\frac{b_0}{b_2} - \frac{1}{b_2}\left(\frac{\overline{MM}}{P}\right)_t\right]$$

$$R_t = \frac{b_0}{b_2} - \frac{1}{b_2}\left(\frac{\overline{MM}}{P}\right)_t + \frac{b_1}{b_2}\left[\frac{b_2\left(\beta_0 - \beta_1 t_0 + \mu_0 - m_0 + G_t + X_t\right)}{\left(1 - \beta_1 + \beta_1 t_1 + m_1\right) b_2 - \mu_1 b_1} + \frac{b_2 \phi \Delta Y_{t-1}}{\left(1 - \beta_1 + \beta_1 t_1 + m_1\right) b_2 - \mu_1 b_1} + \frac{b_2 \mu_1}{\left(1 - \beta_1 + \beta_1 t_1 + m_1\right) b_2 - \mu_1 b_1}\left[\frac{b_0}{b_2} - \frac{1}{b_2}\left(\frac{\overline{MM}}{P}\right)_t\right]\right]$$

Parameters of the IS-LM Model

beta0	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	22114.16
beta1	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.6	0.9	0.459078
mu0	500	500	500	500	500	500	1000	500	500	500	105457
m0	100	100	100	100	100	100	100	100	100	100	-65167
t0	200	500	200	500	200	200	200	200	200	200	-201384
t1	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.476403
m1	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.3	1.387408
mu1	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1720.051
phi	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
G	20000	20000	25000	25000	20000	20000	20000	20000	20000	20000	155880
X	8000	8000	8000	8000	8000	8000	8000	10000	10000	8000	289225
y0	500	500	500	500	500	500	500	500	500	500	500
b0	800	800	800	800	800	800	800	800	800	800	-78809
b1	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.333992
b2	300000	300000	300000	30000 0	300000	600000	300000	300000	300000	300000	-1829.75
M4	10000	10000	10000	10000	15000	10000	10000	10000	10000	10000	10000
P	1	1	1	1	1	1	1	1	1	1	1

Solution of the IS-LM Model

	R	Y	C	I	G	T	M	X	S	X-M	S-I	T-G
Base case	0.0251	66901	51968	525	20000	20270	13480	8000	-5337	-5480	-5862	270
More Tax	0.024	65640	50453	524	20000	20692	13228	8000	-5505	-5228	-6029	692
More Spending	0.0324	75660	57486	532	25000	22898	15232	8000	-4724	-7232	-5256	-2102
Tax and Spend	0.032	75187	56918	532	25000	23056	15137	8000	-4787	-7137	-5319	-1944
More money supply	0.0084	66872	51949	508	20000	20262	13474	8000	-5339	-5474	-5847	262
More Sensitive Asset demand	0.0126	66977	52015	513	20000	20293	13495	8000	-5332	-5495	-5844	293
More investment	0.026	67777	52519	1026	20000	20533	13655	8000	-5276	-5655	-6301	533
More Exports	0.028	70405	54175	528	20000	21321	14181	10000	-5092	-4181	-5620	1321
Low MPC	0.01	48985	30454	510	20000	14896	9897	8000	3636	-1897	3126	-5104
High MPM	0.017	56928	45685	517	20000	17278	17178	8000	-6035	-9178	-6552	-2722

Keynes- Hicks-Samuelson Multiplier-accelerator model (Samuelson (1939))

Market Clearing $$Y_t = C_t + I_t + G_0$$

Consumption $$C_t = \gamma Y_{t-1}$$ $$0 < \gamma < 1$$

Investment $$I_t = \alpha\left(C_t - C_{t-1}\right)$$ $$\alpha > 0$$

Equilibrium $$Y_t = \gamma\left(1+\alpha\right)Y_{t-1} - \alpha\gamma Y_{t-2} + G_0$$

Steady State $$\bar{Y} = \frac{G_0}{1-\gamma\left(1+\alpha\right)+\alpha\gamma} = \frac{G_0}{1-\gamma}$$

$Y_t = Y_{t-1} = Y_{t-2} = \bar{Y}$

Transitional Dynamics (Business Cycle)

$$Y_t - \gamma(1+\alpha)Y_{t-1} + \alpha\gamma Y_{t-2} = 0$$

Assume $Y_t = Ab^t$

$$Ab^t - \gamma(1+\alpha)Ab^{t-1} + \alpha\gamma Ab^{t-2} = 0$$

$$b^{t+2} - \gamma(1+\alpha)b^{t+1} + \alpha\gamma b^t = 0$$

$$b_1, b_2 = \frac{\gamma(1+\alpha) \pm \sqrt{\gamma^2(1+\alpha)^2 - 4\alpha\gamma}}{2}$$

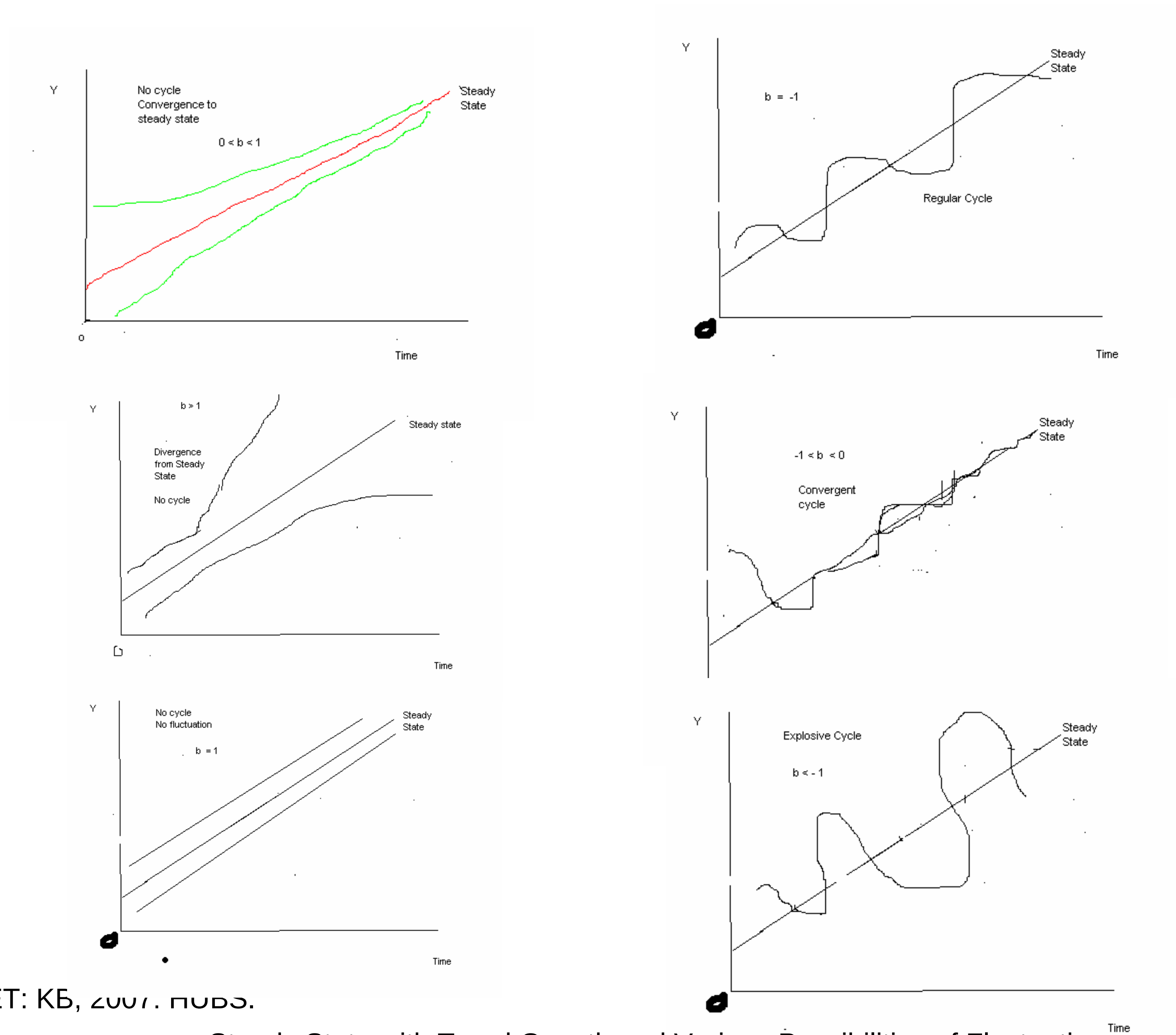
$$Y_t = A_1 b_1^t + A_2 b_2^t$$

Three cases:

Distinct real roots: $\gamma^2(1+\alpha)^2 > 4\alpha\gamma$ No cycle

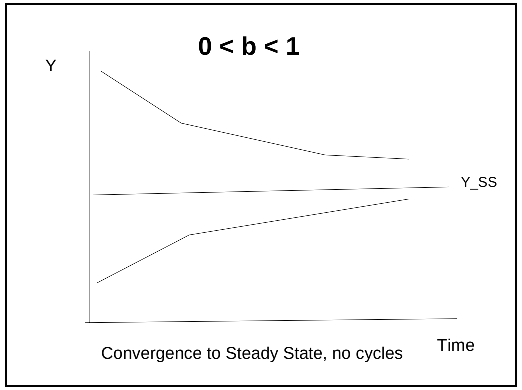
Repeated real root: $\gamma^2(1+\alpha)^2 = 4\alpha\gamma$ No cycle

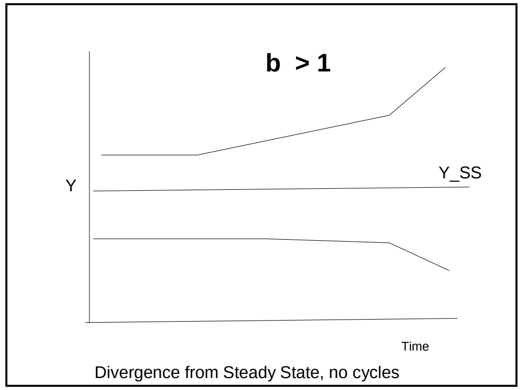
Complex root: $\gamma^2(1+\alpha)^2 < 4\alpha\gamma$ Fluctuation, cycles

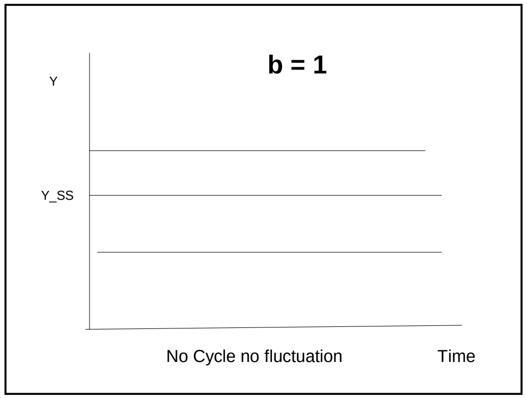


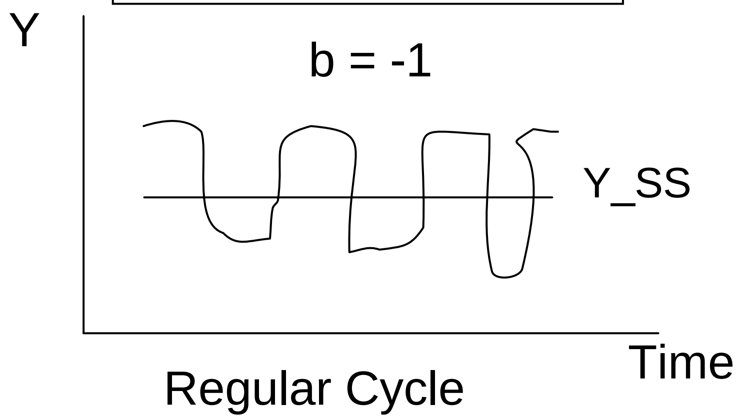
Steady State with Trend Growth and Various Possibilities of Fluctuations

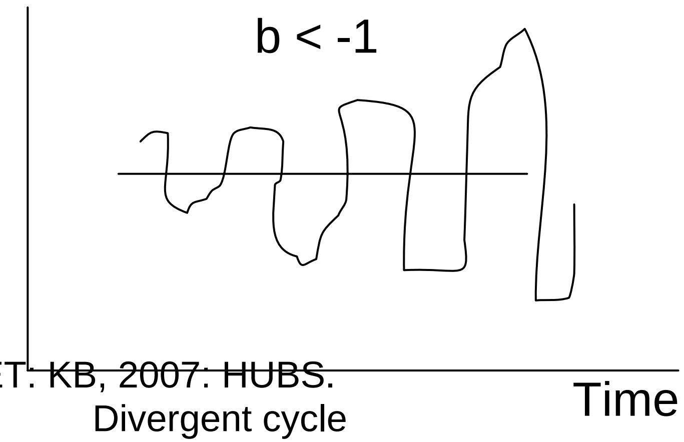
Steady State without Trend Growth and Various Possibilities of Fluctuations

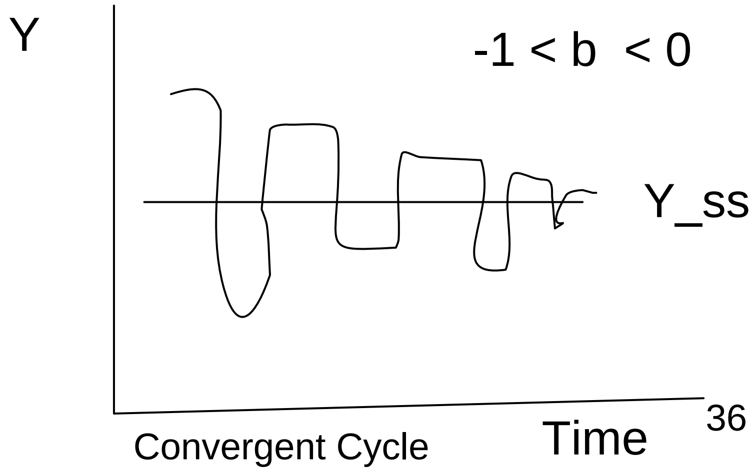




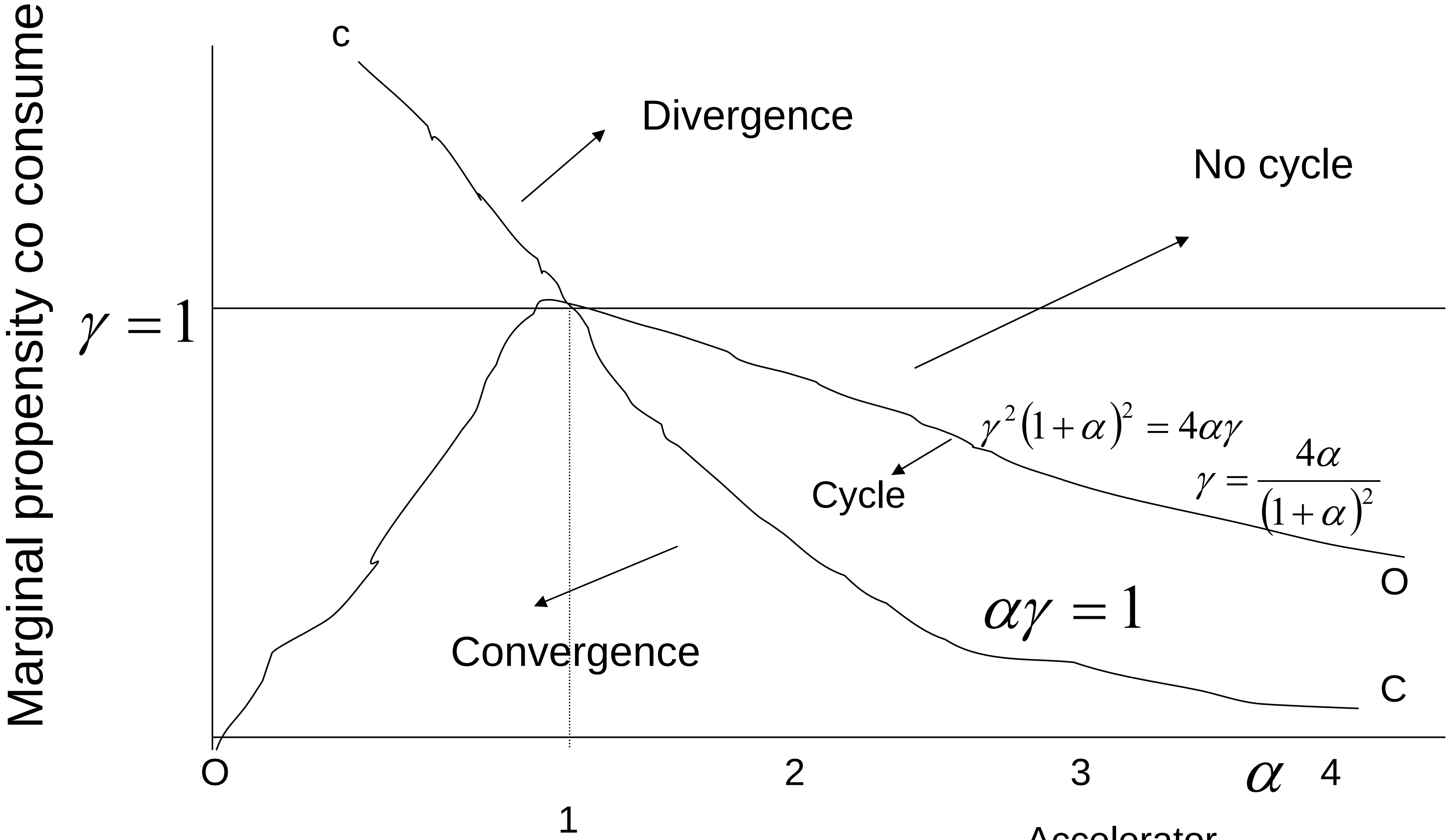








Convergence or divergence , cycle or no cycle



$$Y_t = A_1 R^t (\cos\theta \cdot t + i \sin\theta \cdot t) + A_2 R^t (\cos\theta \cdot t - i \sin\theta \cdot t)$$

Issues of an Open Economy

- What is the link between budget deficit and trade imbalance and money supply?
- How do fiscal, monetary and the exchange rate policies affect output and employment in an economy?
- Exchange rates, inflow and outflow of capital?
- Is huge imbalance in trade be a problem?
- What is the best exchange rate system? fixed or flexible exchange rate system?
- What are the golden rules of fiscal, monetary and exchange rate policies?

Mundell-Flemming Type Small Open Economy Macroeconomic Model

National income

$$Y = C(Y-T) + I(Y, i-\pi^{e}) + G + NX(Y, Y^{f}, \frac{eP^{*}}{P}) \qquad (1)$$

Money market: $\frac{M}{P} = L(i, Y)$ (2)

Real and nominal interest rates: $i = r + \pi^{e}$ (3)

Real exchange rate: $\varepsilon = \frac{eP^{*}}{P}$ (4)

Balance of payment: $NX = KF(r - r^{*})$ (5)

Aggregate supply: $Y = \bar{Y} + \alpha(P - P^{e})$ (6)

Natural rate of output: $\bar{Y} = F(\bar{K}, \bar{L})$ (7)

Variables in the Model

Endogenous variables

Y, i, r, ε, P, e $\bar{Y}$

Exogenous Variables

T, G, M, π^e, P*, r*, P^e, $\bar{K}$, $\bar{L}$ and Y^f

Y= output $\bar{Y}$ natural rate of output
i = nominal interest rate r = real interest rate
ε = real exchange rate P = price level,
e = nominal exchange rate.
T = tax rate
G =government expenditure M = imports,
P* = foreign price level r* foreign interest rate
, $\bar{K}$ = capital stock, $\bar{L}$ = labour force, and
Y^f = foreign income P^e = expected domestic

A Numerical Example of the Small Open Economy Model

$$Y = C(Y - T) + I(i - \pi) + G + NX(Y, Y^f, \lambda)$$

$$C = 200 + 0.8(Y - T)$$

$$I = 50 - 200(i - \pi)$$

T =100 G = 100

$$NX = 10 + 0.3Y^f - 0.1Y - 20\lambda$$

$$\lambda = \frac{EP^*}{P}$$

$$\frac{M}{P} = 200 - 50i + 0.5Y$$

$$i = i^* = 5\%$$

$$\pi = 0.03$$

Solutions of the Numerical Example of the Small Open Economy Model

$$Y = 200 + 0.8(Y - T) + 50 - 200(i - \pi) + G + 10 + 0.5Y^f - 0.1Y - 20\lambda$$

$$Y = \frac{384}{0.3} = 1280$$

$$C = 200 + 0.8(1280 - 100) = 200 + 944 = 1144$$

$$I = 50 - 200(0.05 - 0.03)$$

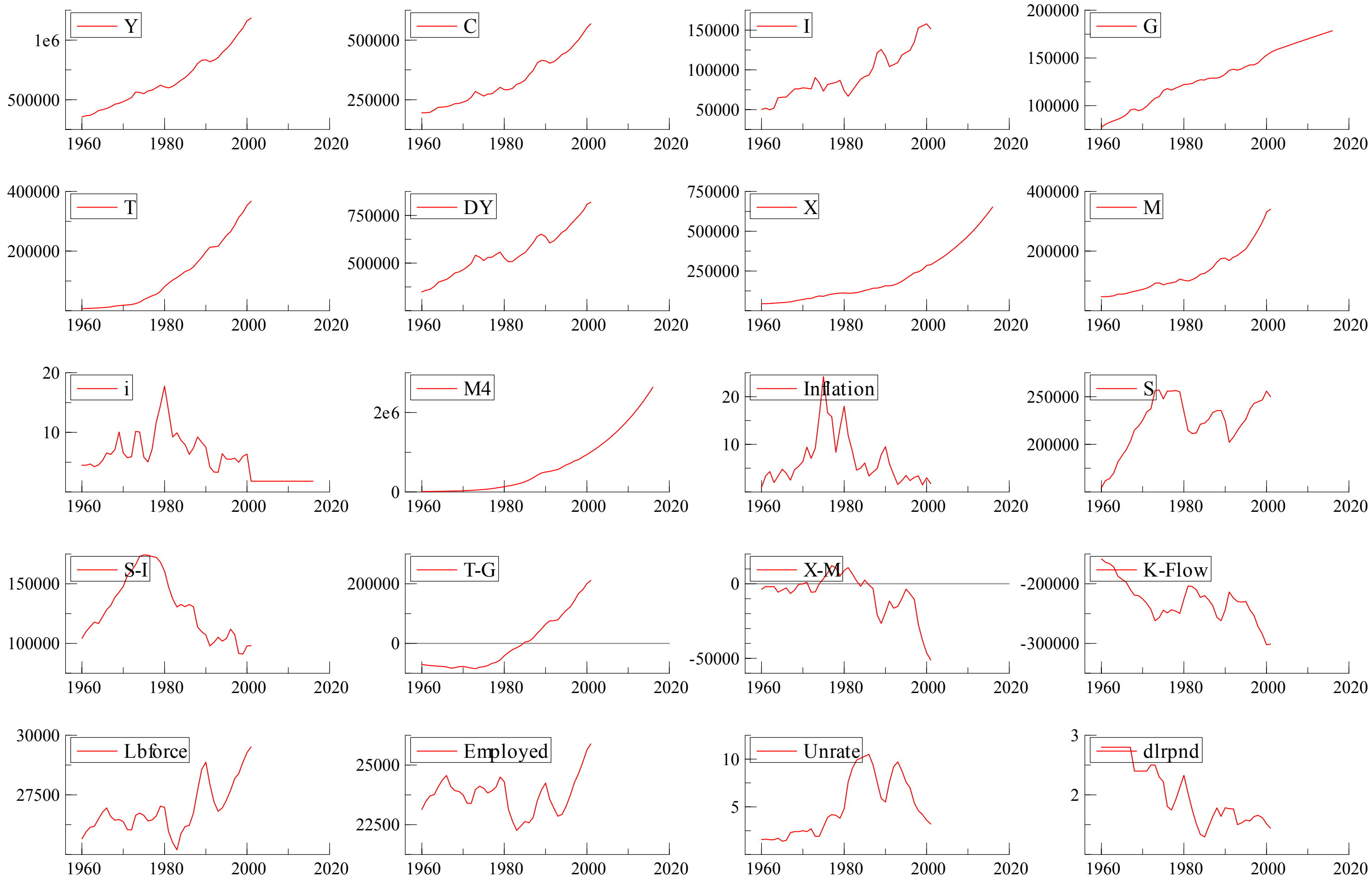
S = Y-T-C = 1280 - 100-1144= 36

$$NX = -10 + 0.3(500) - 0.1(1280) - 20(2)$$

$$NX = 10 + 150 - 128 - 40 = -8$$

$$(T - G) + (S - I) = NX \qquad (100 - 100) + (36 - 44) = -8$$

Macroeconomic Time Series of the UK,1960-2000

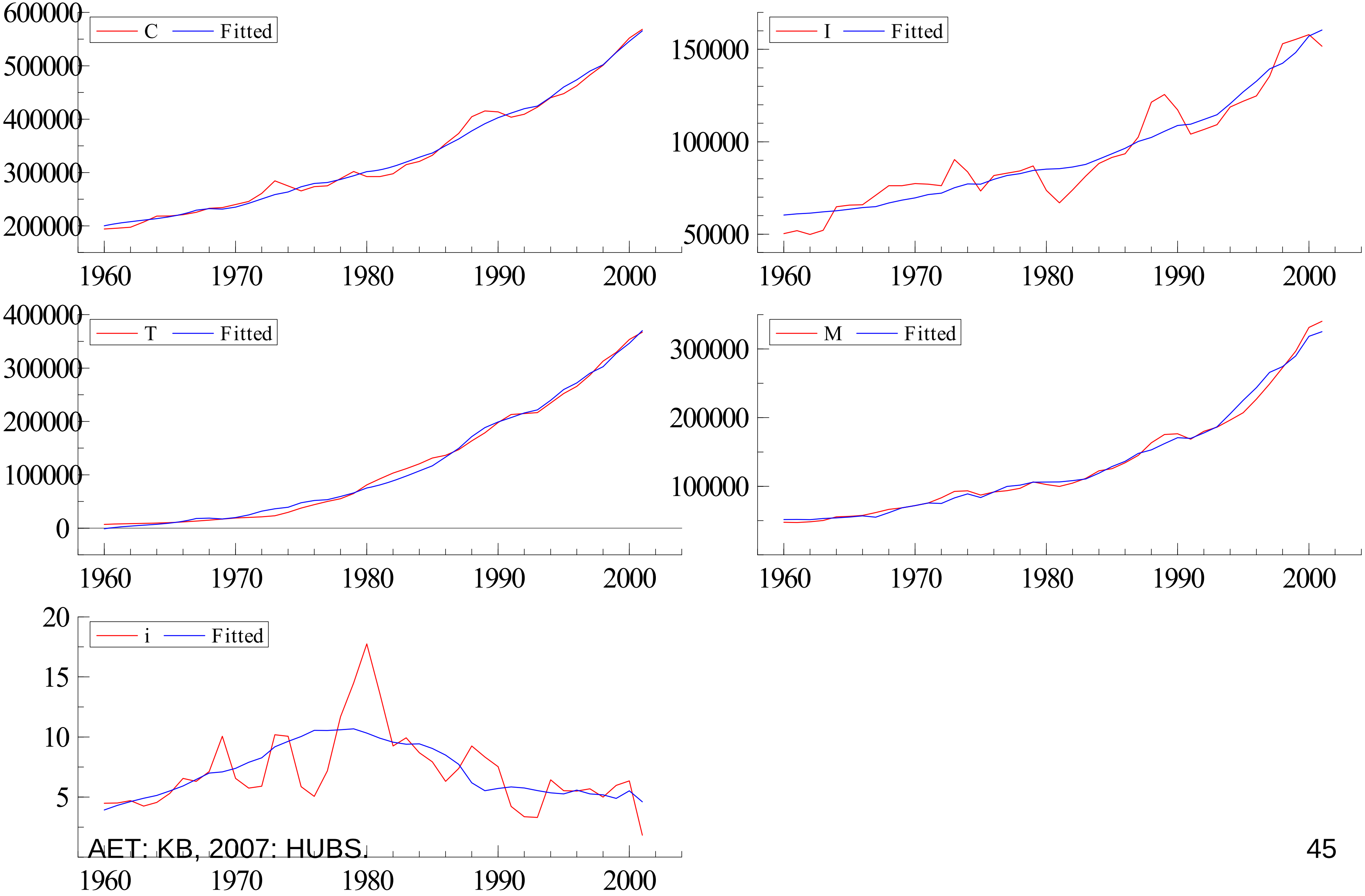


3SLS Estimation of Reduced form of a Keynesian Model

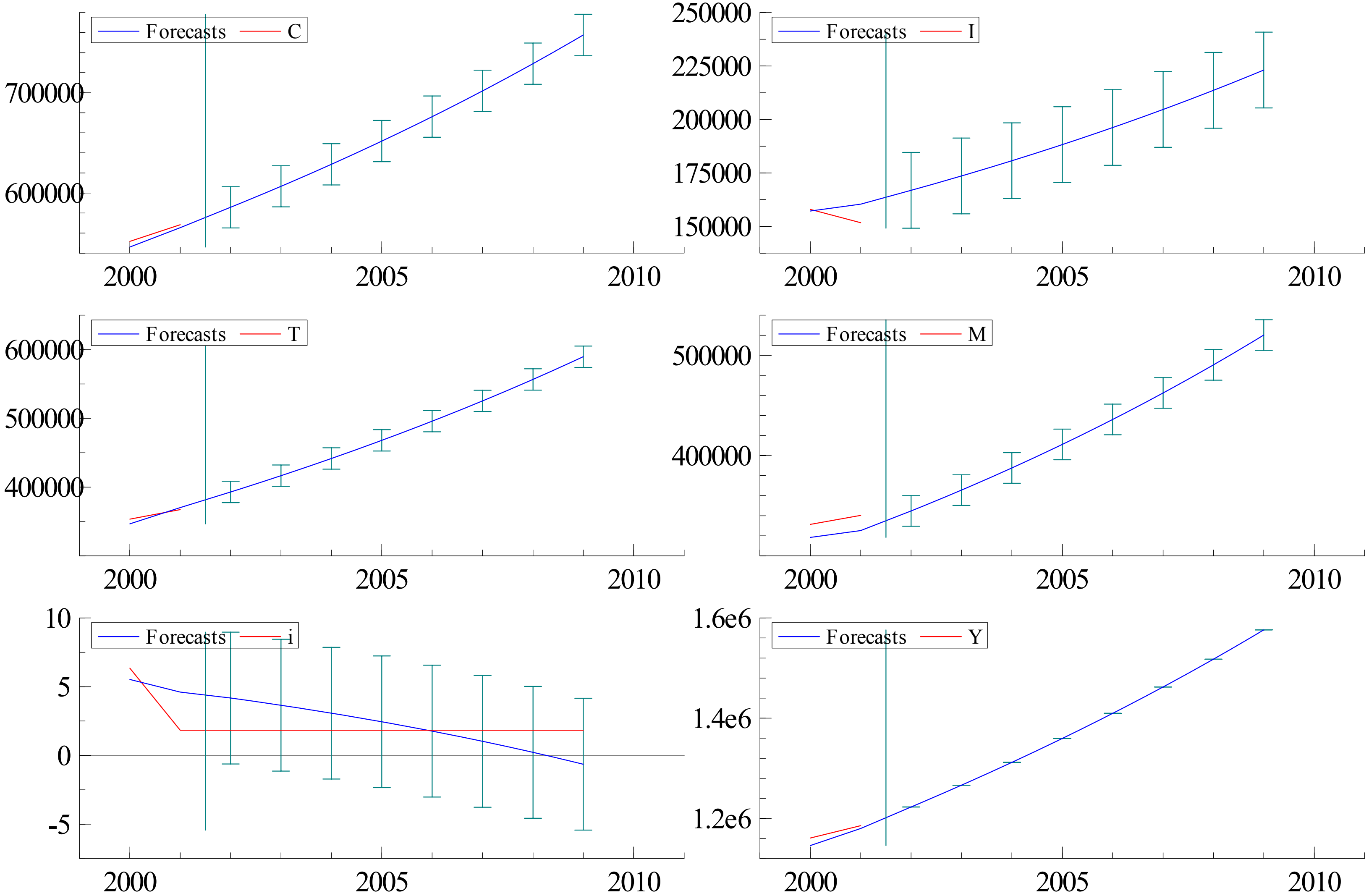
Consumption function	C = 1.407*G + 0.1767*X + 0.2128*M4 + 8.059e+004
	(SE) (0.212) (0.154) (0.0266) (1.63e+004)
Investment function:	I = 0.0684*G + 0.2681*X + 0.02907*M4 + 4.292e+004
	(SE) (0.182) (0.132) (0.0229) (1.4e+004)
Tax Reveneu:	T = + 0.9521*G - 0.08909*X + 0.3204*M4 - 7.533e+004
	(SE) (0.159) (0.116) (0.02) (1.23e+004)
Import function:	M = - 0.4738*G + 1.003*X + 0.06508*M4 + 4.34e+004
	(SE) (0.157) (0.114) (0.0198) (1.21e+004)
Interest rate:	i = 0.0001148*G + 6.273e-005*X - 2.384e-005*M4 - 7.408
	(SE) (4.93e-005) (3.58e-005) (6.2e-006) (3.79)

log-likelihood -1798.42246 -T/2log|Omega| -1500.44537
no. of observations 42 no. of parameters 20

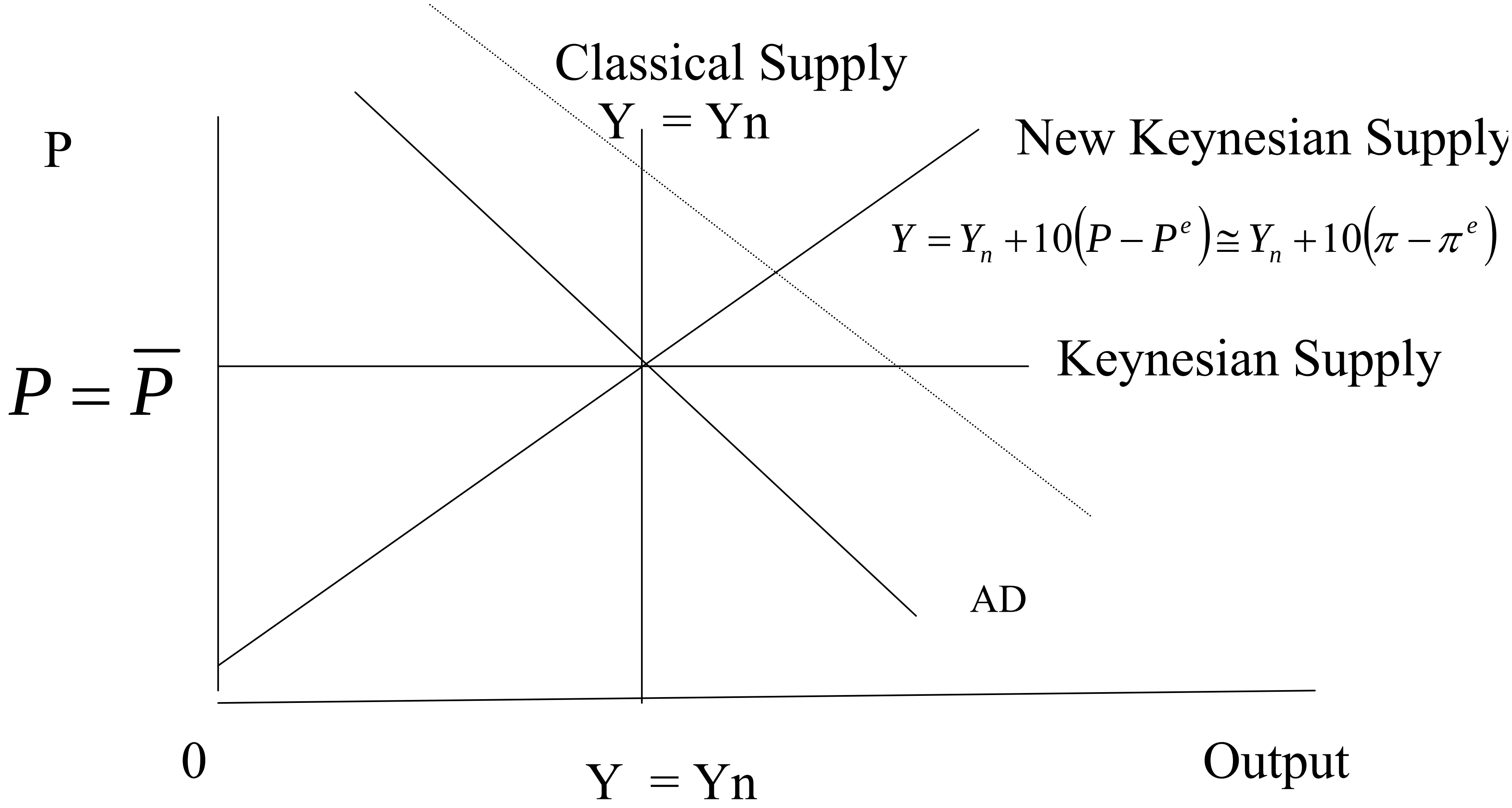
Fit of the Simultaneous Equation Model



Ex-Ante Forecast of the Model Economy



Classical, Keynesian and New Keynesian Aggregate Supply curves



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Neo-classical Synthesis of the Keynesian Model: Comparative Static Analysis

$$Y = F(K, N)$$

$$F_N > 0 \qquad F_K > 0 \qquad F_{NN} < 0 \qquad F_{KK} < 0$$

$$W(N) = \begin{cases} 0 \; for & N \leq \overline{N} \\ + \; for & N > \overline{N} \end{cases} \qquad W = W_0 + W(N)$$

$$\frac{W}{P} = F_N(N, K)$$

$$C = C(Y^d) \qquad Y^d = (1 - \tau)Y$$

$$\frac{M}{P} = M(Y, r) \qquad M_y > 0 \qquad M_r < 0$$

Reduced form of the Neoclassical synthesis to the Keynesian Model

$$F(N,K) = c\big(F(N,K)\cdot(1-\tau)\big) + I(r) + G + NX$$

$$\frac{W}{P} = F_N(N,K)$$

$$\frac{M}{P} = M(Y,r)$$

$$F_N dN + F_K dK = c(1-\tau)F_N dN + c(1-\tau)F_K dK - c\,d\tau F(N,K) + I_r dr + dG$$

$$\frac{dW}{P} - \frac{W}{P^2}dP = F_{NN}dN + F_{NK}dK$$

$$\frac{dM}{P} - \frac{M}{P^2}dP = M_y F_N dN + M_y F_K dK + M_r dr$$

$$\begin{bmatrix} (1-c(1-\tau))F_N & 0 & -I_r \\ M_y F_N & \dfrac{M}{P^2} & M_r \\ F_{NN} & \dfrac{W}{P^2} & 0 \end{bmatrix} \begin{bmatrix} dN \\ dP \\ dr \end{bmatrix} = \begin{bmatrix} c(1-\tau)F_K dK - F_K dK - cd\tau F(N,K) + dG \\ \dfrac{dM}{P} - M_y F_K dK \\ \dfrac{dW}{P} - F_{NK} dK \end{bmatrix}$$

$$\Delta = \begin{vmatrix} (1-c(1-\tau))F_N & 0 & -I_r \\ M_y F_N & \dfrac{M}{P^2} & M_r \\ F_{NN} & \dfrac{W}{P^2} & 0 \end{vmatrix} = -M_r \frac{W}{P^2}\left[(1-c(1-\tau))F_N\right] - I_r\left(M_y F_N \frac{W}{P^2} - F_{NN}\frac{M}{P^2}\right)$$

$$dN = \frac{1}{\Delta}\begin{bmatrix} c(1-\tau)F_K dK - F_K dK - cd\tau F(N,K) + dG & 0 & -I_r \\ \dfrac{dM}{P} - M_y F_K dK & \dfrac{M}{P^2} & M_r \\ \dfrac{dW}{P} - F_{NK} dK & \dfrac{W}{P^2} & 0 \end{bmatrix}$$

Employment, output, price and interest Effects in the Keynesian Model

$$dN = \frac{1}{\Delta}\left[-I_r \frac{W}{P^2}\left(\frac{dM}{P} - M_y F_K dK\right) + I_r\left(\frac{dW}{P} - F_{NK} dK\right)\frac{M}{P^2} - M_r \frac{W}{P^2}\left(c(1-\tau)F_K dK - F_K dK - cd\tau F(N,K) + dG\right)\right]$$

$$dy = \frac{F_N}{\Delta}\left[-I_r \frac{W}{P^2}\left(\frac{dM}{P} - M_y F_K dK\right) + I_r\left(\frac{dW}{P} - F_{NK} dK\right)\frac{M}{P^2} - M_r \frac{W}{P^2}\left(c(1-\tau)F_K dK - F_K dK - cd\tau F(N,K) + dG\right)\right]$$

$$\frac{dy}{d\tau} = -cF(N,K)\left(-M_r \frac{W}{P^2}\right)$$

$$dp = \frac{1}{\Delta}\begin{bmatrix} (1-c(1-\tau))F_N & c(1-\tau)F_K dK - F_K dK - cd\tau F(N,K) + dG & -I_r \\ M_y F_N & \frac{dM}{P} - M_y F_K dK & M_r \\ F_{NN} & \frac{dW}{P} - F_{NK} dK & 0 \end{bmatrix}$$

$$dr = \frac{1}{\Delta}\begin{bmatrix} (1-c(1-\tau))F_N & 0 & c(1-\tau)F_K dK - F_K dK - cd\tau F(N,K) + dG \\ M_y F_N & \frac{M}{P^2} & \frac{dM}{P} - M_y F_K dK \\ F_{NN} & \frac{W}{P^2} & \frac{dW}{P} - F_{NK} dK \end{bmatrix}$$

Phillips Curve: Short -Run Dynamics: Unemployment Inflation Trade-off; policy menu

$$\pi_t = \bar{\pi} + \left\{ \begin{array}{c} a(y - \bar{y}) \\ or \\ -b(u - \bar{u}) \end{array} \right\} + s$$

$$W_t = (1 + \gamma) P_t^e$$

$$P_t = (1 + \theta)(1 + \gamma) P_t^e \qquad (1 + \pi_t) = \frac{P_t}{P_{t-1}}$$

$$(1 + \pi_t) = (1 + \theta)(1 + \gamma)(1 + \pi_t^e) \qquad \pi_t = \pi_t^e + \theta + \gamma$$

$$\theta + \gamma = a(y_t - \bar{y}) = -b(u_t - \bar{u})$$

Economic Modelling

Lecture 22

Tax in a General Equilibrium Model

Household Problem in Presence of Consumption and Income Taxes

$$Max \quad U = c^{\phi} l^{1-\phi}$$

$$l + h^{s} = 1$$

$$p(1 + t_{c})c = w(1 - t_{l})h^{s} + \pi + R$$

$$pt_{c}c + wt_{l}h^{s} = R$$

$$c \geq 0; l \geq 0; h^{s} \geq 0 \qquad (1')$$

$$p(1 + t_{c})c = w(1 - t_{l})h^{s} + \pi + R \qquad wh^{s} + \pi = pc \qquad (2')$$

$$L(c, l, \lambda) = c^{\phi}\left(1 - h^{s}\right)^{1-\phi} + \lambda\left[w(1 - t_{l})h^{s} + \pi + R - p(1 + t_{c})c\right] \qquad (3')$$

First Order Optimisation Conditions in the Presence of Taxes

$$\frac{\partial L(c,l,\lambda)}{\partial c} = \phi c^{\phi-1}\left(1-h^s\right)^{1-\phi} - \lambda p\left(1+t_c\right) = 0 \qquad (4')$$

$$\frac{\partial L(c,l,\lambda)}{\partial h^s} = (1-\phi)c^{\phi}\left(1-h^s\right)^{-\phi}(-1) + \lambda w\left(1-t_l\right) = 0 \qquad (5')$$

$$\frac{\partial L(c,l,\lambda)}{\partial \lambda} = w\left(1-t_l\right)h^s + \pi + R - p\left(1+t_c\right)c = 0 \quad wh^s + \pi = pc \qquad (6')$$

$$\frac{\dfrac{\partial L(c,l,\lambda)}{\partial h^s}}{\dfrac{\partial L(c,l,\lambda)}{\partial c}} = \frac{(1-\phi)c^{\phi}\left(1-h^s\right)^{-\phi}(-1)}{\phi c^{\phi-1}\left(1-h^s\right)^{1-\phi}} = \frac{w\left(1-t_l\right)}{p\left(1+t_c\right)} \qquad (7')$$

$$c = \left(\frac{\phi}{1-\phi}\right)\left(1-h^s\right)\frac{w\left(1-t_l\right)}{p\left(1+t_c\right)} \qquad (8')'$$

$$h^s\frac{w}{p} + \frac{\pi}{p} = c = \left(\frac{\phi}{1-\phi}\right)\left(1-h^s\right)\frac{w}{p}\left(\frac{1-t_l}{1+t_c}\right) \qquad (9')$$

Labour Supply in the Presence of Taxes

$$h^s \frac{w}{p} + \left(\frac{\phi}{1-\phi}\right)\frac{w}{p}\left(\frac{1-t_l}{1+t_c}\right)h^s = c = \left(\frac{\phi}{1-\phi}\right)\frac{w}{p}\left(\frac{1-t_l}{1+t_c}\right) - \left(\frac{\pi}{p}\right)$$

$$h^s = \frac{\left(\frac{\phi}{1-\phi}\right)\frac{w}{p}\left(\frac{1-t_l}{1+t_c}\right) - \left(\frac{\pi}{p}\right)}{\frac{w}{p}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right)}$$

$$h^s = \frac{\left(\frac{\phi}{1-\phi}\right)\frac{w}{p}\left(\frac{1-t_l}{1+t_c}\right) - \left(\left(\frac{w}{p}\right)^{\frac{\alpha}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]\right)}{\frac{w}{p}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right)}$$

Determination of Real Wage Rate in the Presence of Taxes

$$h^d = \left(\frac{1}{\alpha}\frac{w}{p}\right)^{\frac{1}{\alpha-1}} = h^s = \frac{\left(\frac{\phi}{1-\phi}\right)\frac{w}{p}\left(\frac{1-t_l}{1+t_c}\right) - \left(\left(\frac{w}{p}\right)^{\frac{\alpha}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]\right)}{\frac{w}{p}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right)}$$

$$\left(\frac{w}{p}\right)^{\frac{1}{\alpha-1}} = \frac{\left(\frac{\phi}{1-\phi}\right)\left(\frac{1-t_l}{1+t_c}\right)}{\left[\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right)+\left\{\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right\}\right]}$$

$$\frac{w}{p} = \left[\frac{\left(\frac{\phi}{1-\phi}\right)\left(\frac{1-t_l}{1+t_c}\right)}{\left[\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right)+\left\{\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right\}\right]}\right]^{\alpha-1}$$

Labour Supply and Output in the Presence of Taxes

$$h^s = h^d = \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}} \left[\frac{\left(\frac{\phi}{1-\phi}\right)\left(\frac{1-t_l}{1+t_c}\right)}{\left[\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right)+\left\{\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right\}\right]} \right]$$

$$\hat{y} = \left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} \left[\frac{\left(\frac{\phi}{1-\phi}\right)\left(\frac{1-t_l}{1+t_c}\right)}{\left[\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right)+\left\{\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right\}\right]} \right]^{\alpha}$$

Leisure and Consumption in the Presence of Taxes

$$\hat{l} = 1 - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}} \left[\frac{\left(\frac{\phi}{1-\phi}\right)\left(\frac{1-t_l}{1+t_c}\right)}{\left[\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right)+\left\{\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right\}\right]} \right]$$

$$\hat{c} = \left(\frac{\phi}{1-\phi}\right)\left[1 - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}} \left[\frac{\left(\frac{\phi}{1-\phi}\right)\left(\frac{1-t_l}{1+t_c}\right)}{\left[\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right)+\left\{\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right\}\right]} \right]\right] \frac{w(1-t_l)}{p(1+t_c)}$$

$$\hat{U} = \hat{c}^{\phi}\hat{l}^{1-\phi}$$

Table 1

Parameters of the model in the base scenario

Parameter of the model	Numerical value in the base model
Utility weight on consumption (ϕ)	0.6
Utility weight on leisure (1-ϕ)	0.4
Elasticity of output to labour input (α)	0.6
Value of Endowment	68 hours
Consumption tax rate in the base model	0.17
Income tax rate in the base case	0.35
Normalisation of price $w+p=1$	

Experiments for the tax reform

Use of both consumption and income taxes and lump sum transfers (base case)
Elimination of all taxes and no transfer
Only labour income tax and lump sum transfers
Only consumption tax and lump sum transfers

Table 2

Parameter of the model	Numerical value in the base model
Share of spending on consumption (ϕ)	0.25 to 0.6 with steps size of 0.05
Share of spending on leisure (1-ϕ)	0.75 to 0.4 with steps size of 0.05
Elasticity of output to labour input (α)	0.3 to .65 with steps size of 0.05
Value of Endowment	68 to 108 hours with steps size of 5
Consumption tax rate in the base model	0.17 to 0.67 with steps size of 0.05
Income tax rate in the base case	0.40 to 0.85 with steps size of 0.05

Table 3

Overall Welfare Impacts of Tax Changes in the General Equilibrium Model of Taxes

	Equivalent variation	Compensating variation
Elimination of all taxes	3.2%	-3.1%
Labour tax only	-6.2%	6.7%
Consumption tax only	-0.05%	0.05%

Table 4
Macroeconomic Impacts of Alternative Taxes

Variables	both taxes	not tax	labor income tax	Consumption tax
Utility	14.142	14.601	13.689	14.526
Output	6.505	8.032	5.849	7.431
Leisure	45.333	35.789	49.012	39.703
Labour Supply	22.667	32.211	18.988	28.297
Consumption	6.505	8.032	5.849	7.431
Revenue	2.109		1.687	1.687
Wage	0.147	0.13	0.156	0.136
Price	0.853	0.87	0.844	0.864
Profit	14.142	14.601	13.689	14.526
Consumption tax	0.17			0.263
Labour income tax	0.35		0.57	

Table 5

Impact of Alternative Taxes: Percentage changes compared to the base case

	Base two tax case	Labour only tax	Consumption tax
Output	23.471	-27.174	-7.478
Leisure	-21.053	36.946	10.936
Labour supply	42.105	-41.051	-12.151
Consumption	23.471	-27.174	-7.478
Revenue	-100		
Wage rate	-11.406	19.868	4.595
Price	1.964	-2.972	-0.687
Profit	25.896	-29.339	-8.114
Utility	3.246	-6.246	-0.511

Table 6
Sensitivity of Welfare cost to tax changes

Scenarios	EV	Consumption tax rate	Labour Income tax rate
1	4.727	0.22	0.4
2	6.567	0.27	0.45
3	8.83	0.32	0.5
4	11.607	0.37	0.55
5	15.03	0.42	0.6
6	19.289	0.47	0.65
7	24.685	0.52	0.7
8	31.71	0.57	0.75
9	41.246	0.62	0.8
10	55.092	0.67	0.85

Table 7

Sensitivity of model results in comparison to the base case

Endowment	Change in utility	alpha	Change in utility	Phi	Change in utility
73	5.54	0.3	-44.054	0.25	106.269
78	10.99	0.35	-38.744	0.3	81.531
83	16.357	0.4	-32.729	0.35	61.117
88	21.647	0.45	-25.937	0.4	44.153
93	26.865	0.5	-18.286	0.45	29.991
98	32.016	0.55	-9.677	0.5	18.144
103	37.104	0.6	-9.40E-12	0.55	8.241

Efficiency Gains in the UK from elimination of all taxes and transfers (Measured as a percent of benchmark utility level of a representative household)

Equivalent Variation	=	3.715
Compensating Variation	=	-3.582

Efficiency Gains from Switching to Labour income Taxes

Equivalent Variation	=	-0.693
Compensating Variation	=	0.697

Efficiency Gains from Switching to Consumption Taxes

Equivalent Variation	=	2.967
Compensating Variation	=	-2.882

Table 8

Macroeconomic Impacts of Consumption and Labour Income Taxes in the UK

Variable	Benchmark Both taxes	Labour Income tax only	Consumption tax only
Utility	190.073	188.757	195.713
Output	86.85	84.687	100.008
Leisure	615.385	628.104	535.791
Labour supply	384.615	371.896	464.209
Consumption	86.85	84.687	100.008
Revenue	32.122	25.698	25.698
wage rate	0.145	0.146	0.139
Price	0.855	0.854	0.861
Profit	190.073	188.757	195.713
Optimal Consumption tax rate	0.17		0.298
Optimal labour income tax rate	0.35	0.474	

Key Results of Tax Reform Analysis

The efficiency gains from switching to only consumption taxes are about 80 percent the gains of eliminating all the taxes.

Optimal consumption tax rate given the revenue constraint set equal to 80 percent of the benchmark revenue level is 2.9 percent.

Labour income tax is highly distortionary in this model for various reasons. As before 47 percent tax rate of labour income is optimal to meet the required revenue target.

Our first result shows that the net deadweight loss of the current tax and transfer system is about 4 percent of GDP

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Dynamic Programming in Economic Models

Neoclassical Growth Model

Bellman Equation

Noeclassical model for the long run

Preference: $\int_0^\infty e^{-\rho t} \frac{C_t^{1-\sigma}}{1-\sigma} dt$

Technology: $Y_t = A_t K_t^\alpha N_t^{1-\alpha}$ assume $A_t = 1 \;\; N_t = 1$

Capital accumulation: $\dot{K}_t = Y_t - N_t C_t - \delta K_t$

Current value Hamiltonian of this problem

$$H(c, K, \theta) = \frac{C_t^{1-\sigma}}{1-\sigma} + \theta\left[K_t^\alpha - C_t - \delta K_{t-1}\right]$$

C is control, K is state variable, θ is co-state variable.

Optimality and Boundary Conditions

First order conditions

$$\frac{\partial H}{\partial C_t} = 0 \rightarrow C_t^{-\sigma} = \theta_t \qquad (2)$$

$$\dot{\theta}_t = \rho\theta_t - \frac{\partial H_t}{\partial K_t} \rightarrow \dot{\theta}_t = \rho\theta_t - \theta_t \left[\alpha K_t^{\alpha-1} - \delta\right] \qquad (3)$$

$$\dot{K}_t = K_t^{\alpha} - N_t C_t - \delta K_t \qquad (4)$$

Transversality condition

$$\lim_{n \to \infty} e^{-\rho t}\theta_t K_t = 0 \qquad (5)$$

Characterisation of the Balanced Growth Path

Capital stock, consumption and the shadow price of capital remain constant in the balanced growth path $\frac{\dot{C}}{C} = g_c$; $\frac{\dot{K}}{K} = g_K$ and $\frac{\dot{\theta}_t}{\theta_t} = g_\theta$. From (3)

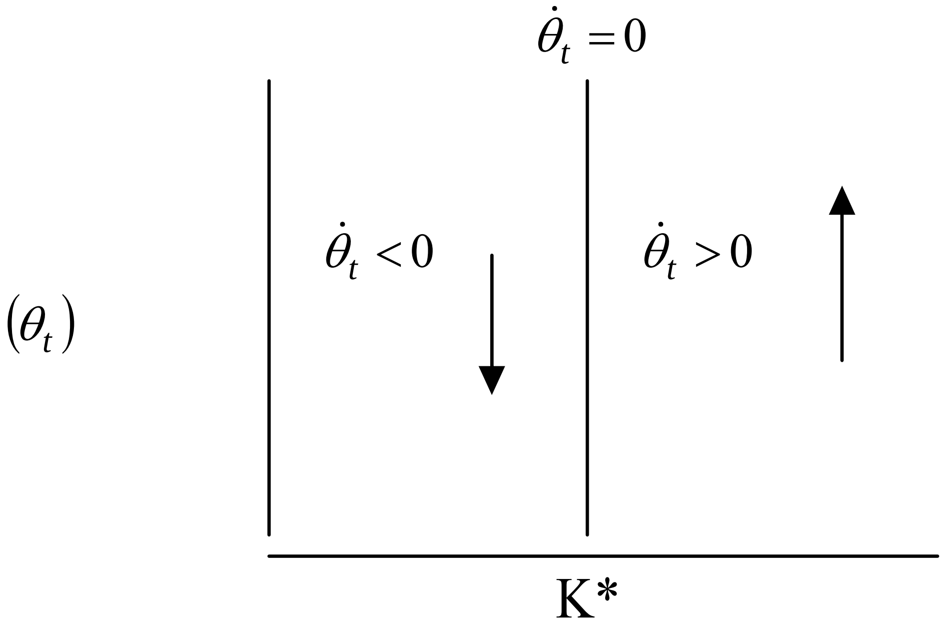
$$\frac{\dot{\theta}_t}{\theta_t} = \rho - \left[\alpha K_t^{\alpha-1} - \delta\right] \Rightarrow \alpha K_t^{\alpha-1} = \rho - \frac{\dot{\theta}_t}{\theta_t} + \delta \qquad (6)$$

Since the RHS is constant , therefore LHS also should be constant $\frac{\dot{K}}{K} = 0$. If capital stock is not growing output is not growing $\frac{\dot{Y}}{Y} = 0$ and consumption is not growing $\frac{\dot{C}}{C} = 0$.

From (2) $\frac{\dot{\theta}_t}{\theta_t} = -\sigma \frac{\dot{C}_t}{C_t} \Rightarrow \frac{\dot{\theta}_t}{\theta_t} = 0$ (7)

Transitional Dynamics-1

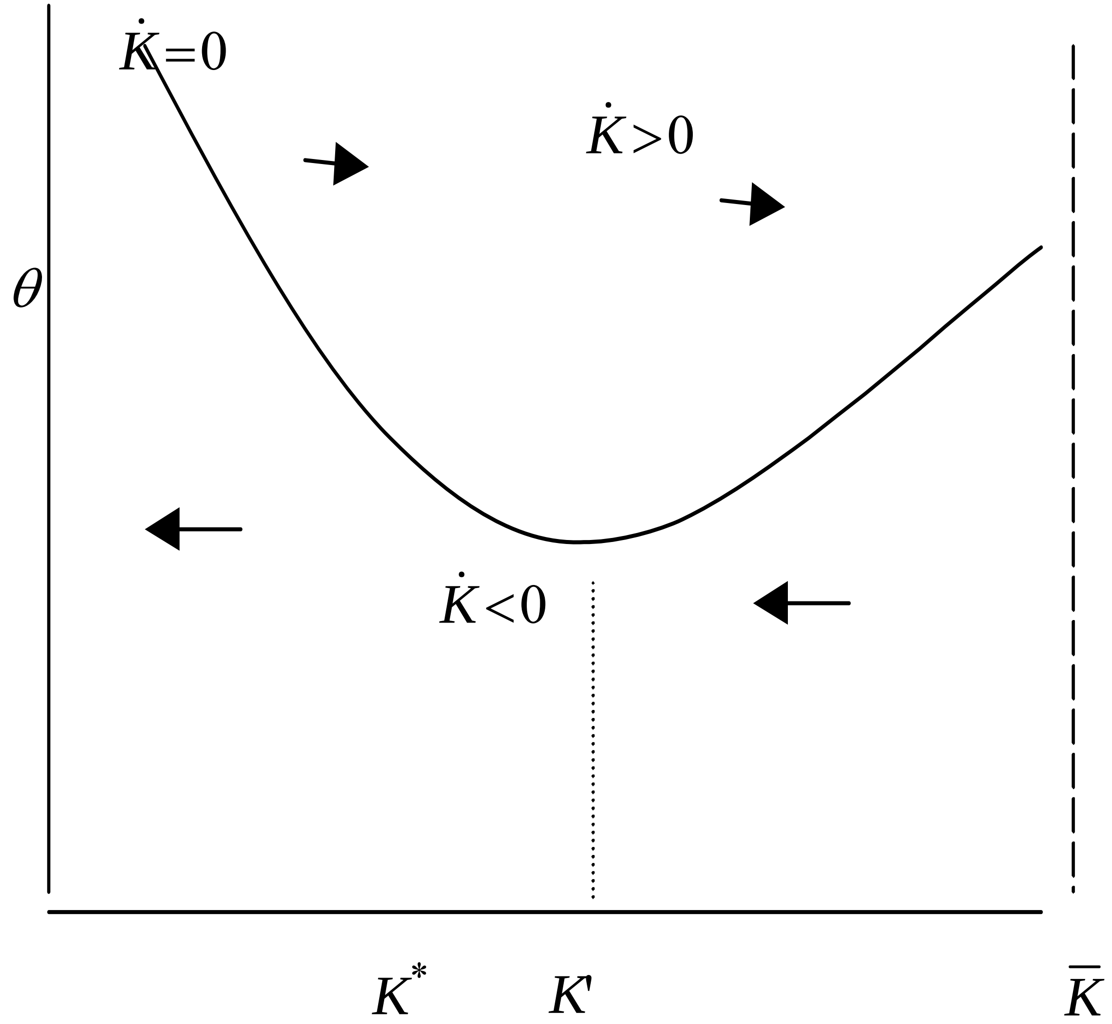
In $\left(\theta_t, K_t\right)$ space the transition dynamics of the shadowprice θ_t relative to the steady state capital stock is that



$$K^* = \left[\frac{\alpha}{\rho + \delta - \frac{\dot{\theta}}{\theta}}\right]^{\frac{1}{1-\alpha}}$$

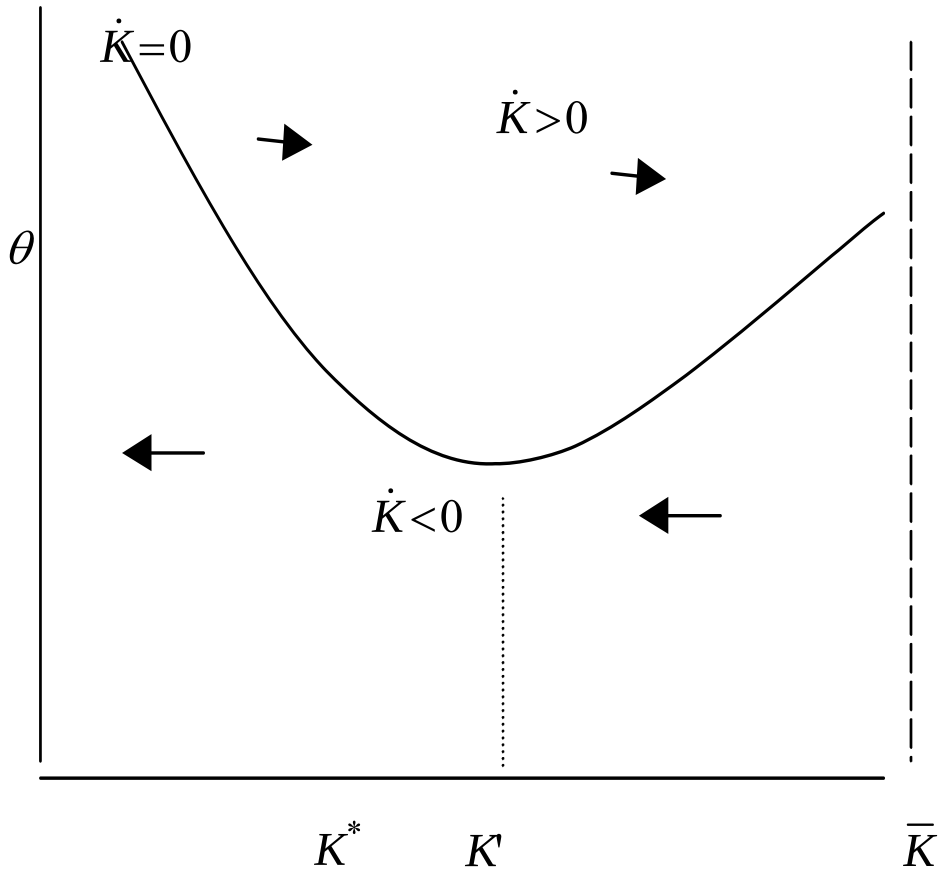
Transitional Dynamics-2

$\bar{K} > K^* > K'$.



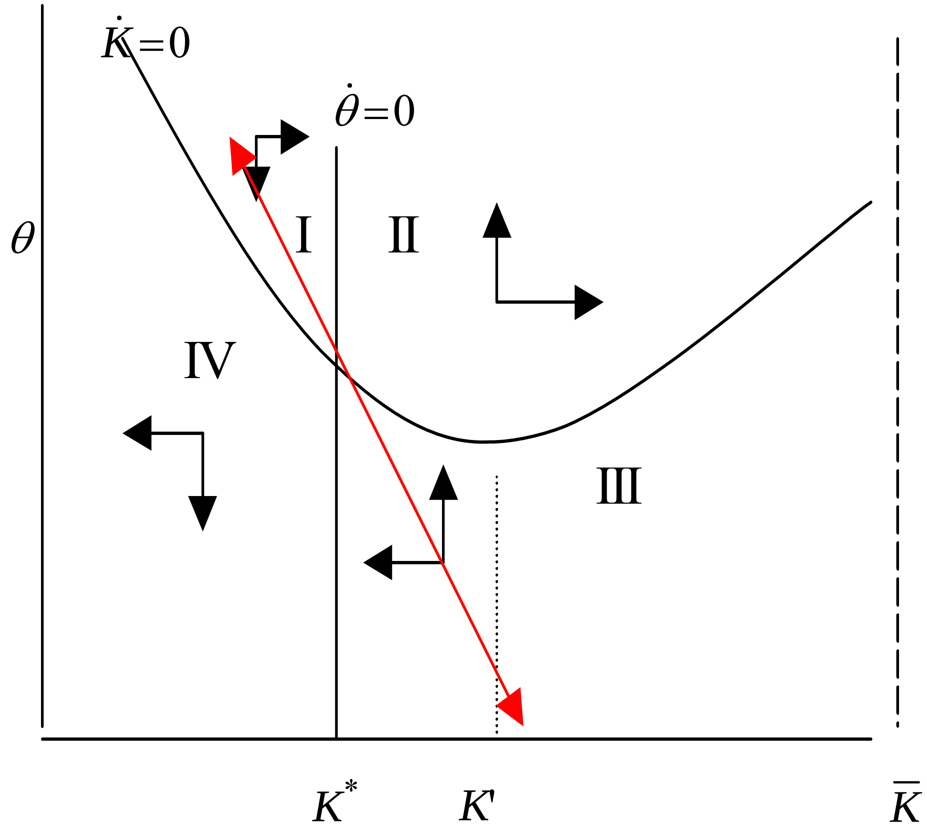
Transitional Dynamics-2

$\overline{K} > K^{*} > K'$.



Saddle Point Solution

Putting all these things together the convergence to the steady state can be summarised in the following diagram.



Convergence to the steady state lies in region I and III as shown by the double arrow red line.

Brock-Mirman(1972)dynamic programming problem
Bellman's Equations

$$Max \quad U = \sum_{t}^{\infty} \beta^t \ln C_t \qquad 0 < \beta < 1$$

Subject to

$$K_{t+1} + C_t = AK_t^{\alpha} \qquad 0 < \alpha < 1$$

Value function

$$V_1(K) = \ln C + \beta V_0(K)$$

Solution by Iteration
First and Second Iteration of the Value function

$$K_{t+1} = 0$$

$$V_1(K) = \ln C = \ln\left(AK^{\alpha}\right) = \ln A + \alpha \ln K$$

$$\underset{macc,k}{V_2(K)} = \ln\left(AK^{\alpha} - K'\right) + \beta\left(\ln A + \alpha \ln K\right)$$

$$\frac{\partial V_2(K)}{\partial K} = -\frac{1}{AK^{\alpha} - K'} + \frac{\beta\alpha}{K} = 0$$

$$\frac{1}{AK^{\alpha} - K'} = \frac{\beta\alpha}{K'}$$

$$K'(1 + \beta\alpha) = \beta\alpha AK^{\alpha}$$

$$K' = \beta\alpha\left(AK^{\alpha} - K'\right)$$

$$V_2(K) = \ln\left[\frac{1}{(1+\beta\alpha)} A\right] + \beta \ln A + \beta\alpha \ln\left(\frac{\beta\alpha}{(1+\beta\alpha)} A\right) + \alpha(1 + \alpha\beta)\ln K'$$

Third Iteration of the Value function

$$\underset{\max c,k}{V_3(K)} = \ln\left(AK^{\alpha} - K'\right) + \beta\left(\alpha(1+\alpha\beta)\ln K'\right)$$

$$\frac{\partial V_3(K)}{\partial K} = -\frac{1}{AK^{\alpha} - K'} + \frac{\beta\alpha(1+\alpha\beta)}{K'} = 0$$

$$\frac{1}{AK^{\alpha} - K'} = \frac{\beta\alpha(1+\alpha\beta)}{K'}$$

$$K' = \beta\alpha(1+\alpha\beta)\left(AK^{\alpha} - K'\right)$$

$$K' = \frac{\left(\beta\alpha + \alpha^2\beta^2\right)}{\left(1+\beta\alpha+\alpha^2\beta^2\right)}AK^{\alpha}$$

$$C = AK^{\alpha} - K'$$

$$C = AK^{\alpha} - \frac{\left(\beta\alpha + \alpha^2\beta^2\right)}{\left(1+\beta\alpha+\alpha^2\beta^2\right)}AK^{\alpha}$$

$$C = \frac{1}{\left(1+\beta\alpha+\alpha^2\beta^2\right)}AK^{\alpha}$$

$$V_3(K') = \beta\ln\left[\frac{A}{(1+\beta\alpha)}\right] + \beta^2\ln A + \beta^2\alpha\ln\left(\frac{\beta\alpha A}{(1+\beta\alpha)}\right) + \ln\left(\frac{A}{\left(1+\beta\alpha+\alpha^2\beta^2\right)}\right) + \beta\alpha(1+\alpha\beta)\ln\left[\frac{\left(\beta\alpha+\alpha^2\beta^2\right)A}{\left(1+\beta\alpha+\alpha^2\beta^2\right)}\right]$$
$$+\alpha\left(1+\beta\alpha+\alpha^2\beta^2\right)\ln K'$$

Fourth Iteration of the Value function

$$V_4(K') = \ln C + \beta V_3(K')$$

$$\underset{\max c,k}{V_4}(K) = \ln\left(AK^{\alpha} - K'\right) + \alpha\beta\left(1 + \beta\alpha + \alpha^2\beta^2\right)\ln K'$$

$$\frac{1}{\left(AK^{\alpha} - K'\right)} = \frac{\alpha\beta\left(1 + \beta\alpha + \alpha^2\beta^2\right)}{K'}$$

$$K' = \frac{\left(\beta\alpha + \alpha^2\beta^2 + \alpha^3\beta^3\right)}{1 + \alpha\beta + \alpha^2\beta^2 + \alpha^3\beta^3} AK^{\alpha} \qquad C = \frac{1}{1 + \alpha\beta + \alpha^2\beta^2 + \alpha^3\beta^3} AK^{\alpha}$$

$$V_4(K') = \ln\left[\frac{1}{1+\alpha\beta+\alpha^2\beta^2+\alpha^3\beta^3} A\right] + \beta\ln\left(\frac{A}{\left(1+\beta\alpha+\alpha^2\beta^2\right)}\right) + \beta^2\left[\frac{\ln A}{(1+\beta\alpha)}\right] + \beta^3 \ln A$$

$$+ \beta\alpha\left(1+\beta\alpha+\alpha^2\beta^2\right)\ln\left[\frac{\left(\beta\alpha+\alpha^2\beta^2+\alpha^3\beta^3\right)\alpha\beta A}{1+\alpha\beta+\alpha^2\beta^2+\alpha^3\beta^3}\right] + \beta\left\{\beta\alpha(1+\alpha\beta)\ln\left[\frac{\left(\beta\alpha+\alpha^2\beta^2\right)A}{\left(1+\beta\alpha+\alpha^2\beta^2\right)}\right]\right\} + \beta^2\left\{\beta\alpha\ln\left[\frac{\alpha\beta A}{(1+\beta\alpha)}\right]\right\}$$

$$+ \alpha\left[1 + \beta\alpha\left(1 + \beta\alpha + \alpha^2\beta^2 + \alpha^3\beta^3\right)\right]\ln K$$

Limits of the Value Function in Infinite Iterations

$$v_4(k) = v_0^4 + v_1^4 \ln k$$

$$\lim_{j\to\infty} v_1^j = \alpha\left[1 + \beta\alpha\left(1 + \beta\alpha + \alpha^2\beta^2 + \alpha^3\beta^3 + .. + \alpha^{j-1}\beta^{j-1}\right)\right] = \frac{\alpha}{1-\alpha\beta}$$

$$x_t^j = \ln\left[\frac{1}{1+\alpha\beta+\alpha^2\beta^2+\alpha^3\beta^3} A\right] \qquad a^j = \sum_{t=0}^{j-1} \beta^t x_t^j \qquad b^j = \sum_{t=0}^{j-2} \beta^t y_t^j$$

$$y_t^j = \beta\alpha\left(1 + \beta\alpha + \alpha^2\beta^2 + .. + \alpha^{j-2}\beta^{j-2}\right)\ln\left[\frac{\left(\beta\alpha + \alpha^2\beta^2 + \alpha^3\beta^3 + .. + \alpha^{j-2}\beta^{j-2}\right)\alpha\beta A}{1+\alpha\beta+\alpha^2\beta^2+\alpha^3\beta^3+..+\alpha^{j-2}\beta^{j-2}}\right]$$

$$\lim_{j\to\infty} x_1^j = \ln\left[A\left(1-\alpha\beta\right)\right]$$

$$\lim_{j\to\infty} y_t^j = \frac{\beta\alpha}{1-\beta\alpha}\ln\left(A\beta\alpha\right)$$

Limits of the Value Function in Infinite Iterations

$$\lim_{j\to\infty} a^j = \lim_{j\to\infty}\sum_{t=0}^{j-1}\beta^t x_t^j = \lim_{j\to\infty}\sum_{t=0}^{j-1}\beta^t \ln\left[A(1-\beta\alpha)\right] = (1-\beta)^{-1}\ln\left[A(1-\beta\alpha)\right]$$

$$\lim_{j\to\infty} b^j = \lim_{j\to\infty}\sum_{t=0}^{j-1}\beta^t y_t^j = \lim_{j\to\infty}\sum_{t=0}^{j-2}\beta^t \frac{\beta\alpha}{1-\beta\alpha}\ln\left[A\beta\alpha\right] = (1-\beta)^{-1}\frac{\beta\alpha}{1-\beta\alpha}\ln\left[A\beta\alpha\right]$$

$$\lim_{j\to\infty} v_0^j = (1-\beta)^{-1}\left\{\ln\left[A(1-\beta\alpha)\right] + \frac{\beta\alpha}{1-\beta\alpha}\ln\left[A\beta\alpha\right]\right\}$$

$$\lim_{j\to\infty} v(k) = (1-\beta)^{-1}\left\{\ln\left[A(1-\beta\alpha)\right] + \frac{\beta\alpha}{1-\beta\alpha}\ln\left[A\beta\alpha\right]\right\} + \frac{\alpha}{1-\alpha\beta}\ln k$$

$$v(k) = v_0 + v_1 \ln k$$

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Optimal Growth Models

One Sector Ramsey Model

Role of the Financial Sector

Ramsey (1928), Cass (1965) Koopmans(1965) type Optimal Growth Model

$$U = \sum_{t=0}^{\infty} \beta^t \ln(C_t) \qquad 0 < \beta < 1$$

$$Y_t = A_t K_t^{\alpha} \qquad 0 < \alpha < 1$$

$$K_{t+1} = I_t$$

$$C_t + I_t = Y_t \qquad K_0 = K_0$$

$$K_{t+1} = I_t \qquad K_{t+1} = K_t(1-\delta) + I_t$$

$$C_t = Y_t - I_t \qquad C_t = AK_t^{\alpha} - \phi\{K_{t+1} - K_t(1-\delta)\}$$

$$0 < \phi < 1$$

Steady State in an Optimal Growth Model

$$U = \sum_{t=0}^{\infty} \beta^t \ln\left(AK_t^{\alpha} - K_{t+1}\right) \qquad \delta = 1$$

$$U_t = +\beta^t \ln\left(AK_t^{\alpha} - K_{t+1}\right) + \beta^{t+1} \ln\left(AK_{t+1}^{\alpha} - K_{t+2}\right) + .. +$$

$$\frac{\partial U_t}{\partial K_{t+1}} = -\frac{\beta^t}{C_t} + \frac{\beta^{t+1}}{C_{t+1}} \alpha A K_{t+1}^{\alpha-1} = 0$$

$$\frac{C_{t+1}}{C_t} = \frac{\beta^{t+1}}{\beta^t} \alpha A K_{t+1}^{\alpha-1}$$

$$\frac{C_{t+1}}{C_t} = \beta\alpha A K_{t+1}^{\alpha-1}$$

$$U = \ln\left(A\overline{K}^{\alpha} - \overline{K}\right)\sum_{t=0}^{\infty} \beta^t$$

$$.. = K_{t-1} = K_t = K_{t+1} = .. = \overline{K}$$

$$\frac{C_{t+1}}{C_t} = \frac{\overline{C}}{\overline{C}} = \beta\alpha A\overline{K}^{\alpha-1}$$

$$.. = C_{t-1} = C_t = C_{t+1} = .. = \overline{C}$$

$$\overline{K} = \left(\frac{1}{\beta\alpha A}\right)^{\frac{1}{\alpha-1}} = \left(\beta\alpha A\right)^{\frac{1}{1-\alpha}}$$

$$\overline{Y} = A\overline{K}^{\alpha} \qquad \overline{Y} = A\left(\beta\alpha A\right)^{\frac{\alpha}{1-\alpha}}$$

$$\overline{C} = \overline{Y} - \overline{I} = A\overline{K}^{\alpha} - \overline{K} = A\left(\beta\alpha A\right)^{\frac{\alpha}{1-\alpha}} - \left(\beta\alpha A\right)^{\frac{1}{1-\alpha}} = A^{\frac{1}{1-\alpha}}\left(\beta\alpha\right)^{\frac{1}{1-\alpha}}\left[\left(\beta\alpha\right)^{\alpha} - 1\right]$$

Optimal Growth Model with less than 100% depreciation

$$0 < \delta < 1$$

$$K_{t+1} = K_t(1-\delta) + I_t \qquad K_{t+1} - K_t(1-\delta) = I_t$$

$$C_t = Y_t - I_t \qquad C_t = AK_t^{\alpha} - K_{t+1} - K_t(1-\delta)$$

$$U_t = \sum_{t=0}^{\infty} \beta^t \ln\left(AK_t^{\alpha} - K_{t+1} + K_t(1-\delta)\right)$$

$$U = +\beta^t \ln\left(AK_t^{\alpha} - K_{t+1} + K(1-\delta)\right) + \beta^{t+1} \ln\left(AK_{t+1}^{\alpha} - K_{t+2} + K_{t+1}(1-\delta)\right) + .. +$$

$$\frac{\partial U_t}{\partial C_t}\frac{\partial C_t}{\partial C_{t+1}}\frac{\partial C_{t+1}}{\partial K_{t+1}} = -\frac{\beta^t}{C_t} + \frac{\beta^{t+1}}{C_{t+1}}\left(\alpha AK_{t+1}^{\alpha-1} + (1-\delta)\right) + = 0$$

$$\frac{C_{t+1}}{C_t} = \beta\left(\alpha AK_{t+1}^{\alpha-1} + (1-\delta)\right) \qquad .. = K_{t-1} = K_t = K_{t+1} = .. = \overline{K}$$

$$.. = C_{t-1} = C_t = C_{t+1} = .. = \overline{C} \qquad U_t = \ln\left(A\overline{K}^{\alpha} - \overline{K} + \overline{K}(1-\delta)\right)\sum_{t=0}^{\infty} \beta^t$$

Steady State in an Optimal Growth Model with less than 100% depreciation

$$0 < \delta < 1$$

$$\frac{C_{t+1}}{C_t} = \frac{\overline{C}}{\overline{C}} = \beta\left(\alpha A\overline{K}^{\alpha-1} + (1-\delta)\right)$$

$$\left(\alpha A\overline{K}^{\alpha-1} + (1-\delta)\right) = \left(\frac{1}{\beta}\right)$$

$$\left(\overline{K}^{\alpha-1}\right) = \frac{1}{\alpha A}\left(\frac{1}{\beta} - (1-\delta)\right)$$

$$\left(\overline{K}^{\alpha-1}\right) = \frac{1}{\alpha A}\left(\frac{1-\beta(1-\delta)}{\beta}\right)$$

$$\overline{K} = \left(\frac{1-\beta(1-\delta)}{\alpha A\beta}\right)^{\frac{1}{\alpha-1}}$$

$$\overline{K} = \left(\frac{\alpha A\beta}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\alpha}}$$

$$\overline{Y} = A\overline{K}^{\alpha}$$

$$\overline{Y} = A^{\frac{2-\alpha}{1-\alpha}}\left(\frac{\alpha\beta}{1-\beta(1-\delta)}\right)^{\frac{\alpha}{1-\alpha}}$$

$$\overline{I} = \overline{K} - (1-\delta)\overline{K}$$

$$\overline{I} = \delta\overline{K}$$

$$\overline{I} = \delta\overline{K} = \delta\left(\frac{\alpha A\beta}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\alpha}}$$

$$\overline{C} = \overline{Y} - \overline{I}$$

$$\overline{C} = \left(\frac{\alpha A\beta}{1-\beta(1-\delta)}\right)^{\frac{\alpha}{1-\alpha}} - \delta\left(\frac{\alpha A\beta}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\alpha}}$$

Optimal Growth Model with Financial Intermediation

$$\phi S_t = I_t$$

$$C_t = AK_t^{\alpha} - \phi\left\{K_{t+1} - K_t(1-\delta)\right\}$$

$$U_t = \sum_{t=0}^{\infty} \beta^t \ln\left[AK_t^{\alpha} - \phi\left\{K_{t+1} - K_t(1-\delta)\right\}\right]$$

$$U = +\beta^t \ln\left[AK_t^{\alpha} - \phi\left\{K_{t+1} - K_t(1-\delta)\right\}\right] + \beta^{t+1} \ln\left[AK_{t+1}^{\alpha} - \phi\left\{K_{t+2} - K_{t+1}(1-\delta)\right\}\right] + .. +$$

$$\frac{\partial U_t}{\partial C_t} = -\frac{\phi\beta^t}{C_t} + \frac{\beta^{t+1}}{C_{t+1}}\left(\alpha AK_{t+1}^{\alpha-1} + \phi(1-\delta)\right) + = 0 \qquad \frac{C_{t+1}}{C_t} = \frac{\beta}{\phi}\left(\alpha AK_{t+1}^{\alpha-1} + \phi(1-\delta)\right)$$

$$.. = K_{t-1} = K_t = K_{t+1} = .. = \overline{K}$$

$$.. = C_{t-1} = C_t = C_{t+1} = .. = \overline{C}$$

$$U_t = \ln\left(A\overline{K}^{\alpha} - \phi\overline{K} + \overline{K}\phi(1-\delta)\right)\sum_{t=0}^{\infty} \beta^t$$

Steady State in Optimal Growth Model with Financial Intermediation

$$\frac{C_{t+1}}{C_t} = \frac{\overline{C}}{\overline{C}} = \frac{\beta}{\phi}\left(\alpha A \overline{K}^{\alpha-1} + \phi(1-\delta)\right)$$

$$\left(\alpha A \overline{K}^{\alpha-1} + \phi(1-\delta)\right) = \left(\frac{\phi}{\beta}\right) \qquad \left(\overline{K}^{\alpha-1}\right) = \frac{1}{\alpha A}\left(\frac{\phi}{\beta} - \phi(1-\delta)\right) \qquad \left(\overline{K}^{\alpha-1}\right) = \frac{1}{\alpha A}\left(\frac{\phi - \beta\phi(1-\delta)}{\beta}\right)$$

$$\overline{K} = \left(\frac{\phi - \beta\phi(1-\delta)}{\alpha A \beta}\right)^{\frac{1}{\alpha-1}} \qquad \overline{K} = \left(\frac{\alpha A \beta}{\phi - \beta\phi(1-\delta)}\right)^{\frac{1}{1-\alpha}}$$

$$\overline{Y} = A\overline{K}^{\alpha} \qquad \overline{Y} = \left(\frac{\alpha A \beta}{\phi - \beta\phi(1-\delta)}\right)^{\frac{\alpha}{1-\alpha}} \qquad \overline{I} = \overline{K} - (1-\delta)\overline{K}$$

$$\overline{I} = \delta\overline{K} = \delta\left(\frac{\alpha A \beta}{\phi - \beta\phi(1-\delta)}\right)^{\frac{1}{1-\alpha}}$$

$$\overline{C} = \overline{Y} - \overline{I} \qquad \overline{C} = \left(\frac{\alpha A \beta}{\phi - \beta\phi(1-\delta)}\right)^{\frac{\alpha}{1-\alpha}} - \delta\left(\frac{\alpha A \beta}{\phi - \beta\phi(1-\delta)}\right)^{\frac{1}{1-\alpha}}$$

Table 5
Capital Stock, Output, Consumption and Investment in the Steady State

Parameters of the Infinite Horizon Model									
	I	II	II	IV	V	VI	VII	VIII	IX
Technology	44.025	44.025	44.025	44.025	44.025	44.025	100	100	100
Capital share: alpha	0.4	0.4	0.2	0.6	0.6	0.6	0.4	0.4	0.6
Beta	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
Initial capital K0	100	100	100	100	100	100	100	100	100
Delta	1	0.05	0.05	0.05	0.05	0.05	1	0.05	0.05
Intermediation cost	1	1	1	1	1.2	1.05	1	1	1.05
Infinite Horizon Economy in the Steady State									
Capital Stock	100	2,499	149	344,202	218,202	304,677	392	9,807	2,369,142
Output	278	9,750	2,472	420,017	319,518	390,376	1,090	62,607	4,214,584
Consumption	178	7,251	2,323	75,815	101,315	85,699	698	52,800	1,845,441
Investment	100	2,499	149	344,202	218,202	304,677	392	9,807	2,369,142

Computations for Optimal Growth

Excel based model

GAMS based Models

Basics of an Optimal growth Model

Capacity constraint $Y_t = K_t^{\beta} L_t^{(1-\beta)} = C_t + I_t \qquad L_t = (1+g)^{t-1} L_0$

Accumulation $K_{t+1} = (1-\delta) K_t + I_t \qquad K_0 = K_0$

Terminal Capital $I_T = (g+\delta) K_T$

First order condition
For investment $P_t = PK_{t+1} \qquad P_T = PTC_T$

First order condition for consumption $P_t C_t = C_0 \alpha^t = C_0 \left(\frac{1+g}{1+r} \right)^t$

First order condition of capital market $P_t^k = RK_t + (1-\delta) P_{t+1}^k$

$$PTC_T^k = RK_t + (1-\delta) P_{t+1}^k$$

The marginal revenue product of capital $RK_t = P_t \beta K_t^{\beta} L_t^{(1-\beta)} / K_t$

Steps for Implementing a Dynamic Optimal Growth Model

- *1. declare the time and define the first and last periods
- *2. declare and assign the values for the benchmark parameters
- *3. calibrate the model
  - discount factors and assign the value for initial capital stock
- *4. declare variables and equations
- *5. derive the first order conditions for optimisation and market clearing conditions
- *6. write all equations
- *7. declare the model
- *8. Solve the model
- *9. produce the results in readable format
- *10. Interpret them using economic theory
- *11. modify the model to incorporate new issues.

```
set t /t1*t30/
set tfirst(t)
set tlast(t)
;
tfirst(t) = yes$(ord(t) eq 1);
tlast(t) = yes$(ord(t) eq card(t));
*declare the Key parameters to benckmark the economy
scalar
 g      /0.02/
 R      /0.05/
 K0     /3/
 kstock  /1/
 delta  /0.07/
 I0
 c0
 kvs
;
I0 = (delta+g)*K0;
C0 = 1-I0;
*declare reference prices and quantities
parameters
qref(T)
pref(t)
alpha
;

QREF(T) = (1+g)**(ord(t)-1);
pref(t) = (1/(1+r))**(ord(t)-1);
kvs = (delta+r)*K0;
alpha(t) = ((1+g)/(1+r))**(ord(t)-1);
alpha(tlast) = alpha(tlast)/(1-((1+g)/(1+r)));

*declare variables
Variables
C(t)
I(t)
K(t)
P(t)
PK(t)
RK(t)
PTC(t)
;

*Declare equations
Equations
capacity(t)
capital(t)
Terminal(t)
Foc_con(t)
Foc_K(t)
Foc_I(t)
rent_k(t)
;
```

```
*declaration of model equations

capacity(t)..
   (K(t)/K0)**kvs*qref(t)**(1-kvs) =e= C(t)+I(t);
capital(t)..
   (1-delta)*K(t-1) +I(t-1) +K0*Kstock$tfirst(t) =G= K(t);
Terminal(tlast)..
    I(tlast) =e= (g+delta)*K(tlast);
Foc_con(t)..
  P(t)*C(t)=e=C0*Alpha(t);
Foc_K(t)..
              Pk(t) +PTC(t)*(g+delta)$tlast(t)=e=
P(t)*((kvs*K(t)**kvs*qref(t)**(1-kvs))/K(t)) +(1-delta)*pk(t+1);
Foc_I(t)..
  P(t) =e= PK(t+1) +PTC(t)$tlast(t);

rent_k(t)..
  rk(t) =e= P(t)*((kvs*K(t)**kvs*qref(t)**(1-kvs))/K(t));

model ramsey1/
capacity.p
capital.pK
Foc_con.c
Foc_K.k
Foc_I.I
Terminal.ptc
rent_k
/;

C.l(t)=c0*qref(T);
I.l(t)=i0*qref(T);
K.l(t)=k0*qref(T);
P.l(t)=Pref(T);
PK.l(t)=(1+R)*Pref(T);
*RK.lo(t)=Pref(T);
PTC.l(T)=PREF(T);
pk.lo(t) =1e-6;
pk.up(t) =+inf;

solve ramsey1 using mcp;

parameter base, report;
base(t, "cons")= C.l(t);
base(t, "inv") = I.l(t);
base(t, "cap") = K.l(t);
base(t, "labour") = qref(t);
base(t, "price-y")= P.l(t);
base(t, "price-k")= PK.l(t);
base(t, "rent") = RK.l(t);
base(t, "term-K")= PTC.l(T);
Display base, report;
```

Interpretation of Model Equations in GAMS

Capacity constraint $$Y_t = K_t^{\beta} L_t^{(1-\beta)} = C_t + I_t$$

(K(t)/K0)**kvs*qref(t)**(1-kvs) =e= C(t)+I(t);

Accumulation $$K_{t+1} = (1-\delta)K_t + I_t \qquad K_0 = K_0$$

(1-delta)*K(t-1) +I(t-1) +K0*Kstock$tfirst(t) =G= K(t);

Terminal Capital I(tlast) =e= (g+delta)*K(tlast); $$I_T = (g+\delta)K_T$$

First order condition For investment P(t) =e= PK(t+1) +PTC(t)$tlast(t);

$$P_t = PK_{t+1} \qquad P_T = PTC_T$$

First order condition for consumption

P(t)*C(t)=e=C0*Alpha(t); $$P_t C_t = C_0 \alpha^t = C_0 \left(\frac{1+g}{1+r}\right)^t$$

First order condition of capital market $$P_t^k = RK_t + (1-\delta)P_{t+1}^k$$

(t) +PTC(t)*(g+delta)$tlast(t)=e= P(t)*((kvs*K(t)**kvs*qref(t)**(1-kvs))/K(t)) +(1-delta)*pk(t+1

Marginal product of capital $$PTC_T^k = RK_t + (1-\delta)P_{t+1}^k$$

$$RK_t = P_t \beta K_t^{\beta} L_t^{(1-\beta)} / K_t$$ rk(t) =e= P(t)*((kvs*K(t)**kvs*qref(t)**(1-kvs))/K(t));

Ramsey Model: Benevolent Social Planner's Problem

Instantaneous utility $$U_t = \ln C_t$$

$$Max \quad U = \sum_{t}^{T} \alpha^t \ln C_t \qquad \alpha^t = \left(\frac{1+g}{1+r}\right)^t$$

Subject to

$$Y_t = K_t^{\beta} L_t^{(1-\beta)}$$ or $$Y_t = \left(\beta \cdot K_t^{\rho} + (1-\beta) L_t^{\rho}\right)^{\frac{1}{\rho}}$$

$$K_{t+1} = (1-\delta) K_t + I_t$$

$$I_T = (g+\delta) K_T$$

$$Y_t = C_t + I_t$$

$$L_t = (1+g)^t L_0$$

Ramsey Model: Decentralised Market

$$Max \quad U = \sum_{t}^{T} (1+\beta)^{-t} \ln C_t$$

$$Y_t = \Phi K_t^{(1-\alpha)} L_t^{\alpha}$$

$$K_{t+1} = (1-\delta) K_t + I_t$$

$$I_T = (g+\delta) K_T$$

$$C_t (1 + R_{t+1}) = C_{t+1} (1+\beta)$$

$$R_{t+1} + \delta = (1-\alpha) \Phi K_t^{-\alpha} L_t^{\alpha}$$

$$L_t = (1+g)^{t-1} L_0$$

$$Y_t = C_t + I_t$$

Optimal Consumption-Saving Model while Young, Adult and Old

$$Max \;\; U(C_1, C_2, C_3) = \ln C_1 + \beta_2 \ln C_2 + \beta_3 \ln C_3$$

Subject to:

1. $$C_1 + \frac{C_2}{(1+r)} + \frac{C_3}{(1+r)^2} = W_1 + \frac{W_2}{(1+r)} + \frac{W_3}{(1+r)^2}$$

2. $$(w_1,\ w_2,\ w_3) = (120,\ 1200,\ \text{-}120)$$

3. $(C_1 \geq 0,\ C_2 \geq 0,\ C_3 \geq 0)$

What is the optimal consumption and saving in each period ?

Derivation of the Marginal Productivity = User Cost of Capital Condition

Producer's Problem:

$$\Pi = \frac{F(K)}{(1+r)} - P_1^k K + \frac{(1-\delta)P_2^K K}{1+r}$$

Optimality Condition:

$$\frac{\partial \Pi}{\partial K} = \frac{F^{'}(K)}{(1+r)} - P_1^k + \frac{(1-\delta)P_2^K}{1+r} = 0$$

Implication:

$$MPK = (1+r)P_1^k - (1-\delta)P_2^K$$

$$MPK = \left[(1+r) - (1-\delta)(1+\pi^K)\right]P_1^k$$

$$MPK \cong \left[r + \delta - \pi^K\right]P_1^k$$

Assumptions:

$$\pi^K = \frac{P_2^K}{P_1^K} - 1 \qquad \delta\pi^K \cong 0$$

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- Stokey, N. L. and R.E. Lucas (1989) Recursive Methods in Economic Dynamics, Harvard UP, Cambridge, MA.