

Duration Analysis and Rational Expectation

Advanced Economic Theory Lecture 11

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Duration: A Random Variable

- How long a certain thing will last given that it has survived/existed for so long time?
- Duration of events is a random variable
- Length of duration of occurrence for period up to T period or survival after period T or what is probability of transition or the hazard rate between T and $T + \Delta$ period.
- Examples-
 - duration or probability of termination of strikes,
 - Unemployment Spells,
 - marriage,
 - disaster spells,
 - heart attacks or many other ill-spells,
 - likelihood of bankruptcy of a firm,
 - technological breakthrough,
 - probability of maintaining championship titles in sports).

Some Applications of Duration Analysis

- Manager of a company would be interested to know how long will a certain machine last given that it has been running so far.
- A life insurance company would be interested in probability of death of an individual with certain medical record or physical characteristic in the next $T + \Delta$ years given that the person has survived up to T years.
- When a central bank is likely to change the rate of interest given that the interest rate has remained at a certain rate up to so far?
- A union leader or the management negotiator will be interested about the probability of withdrawal of a strike given that the strike has continued up to T periods.

Densities of Duration and Survival

Cumulative Density of Duration

$$\Pr(t \leq T) = F(t) = \int_0^t f(t)dt$$

Survival Density

$$S(t) = 1 - F(t) = \Pr(t > T)$$

Hazard Rate

$$\lambda(t) = \lim_{\Delta \rightarrow 0} \frac{F(t + \Delta) - F(t)}{\Delta s(t)} = \frac{f(t)}{s(t)}$$

$$f(t) = s(t) \cdot \lambda(t)$$

Integrated Hazard and Exponential Hazard Functions

Hazard function is linked to the survival function as

$$\lambda(t) = \frac{\partial \log[1 - F(t)]}{\partial t} = \frac{-F'(t)}{1 - F(t)} = \frac{f(t)}{s(t)}$$

Integrated hazard function (the survival function)

$$\int_0^t \lambda(t) dt = -\log[1 - F(s)] + \log[1 - F(0)] = -\log[1 - F(s)]$$

$$F(s) = 1 - \exp\left(-\int_0^t \lambda(t) dt\right)$$

Modelling hazard: Exponential hazard function

$$\lambda(t, x_i) = \lambda_0(t) \exp(x_i \beta)$$

Duration Dependence

- Likelihood of how much hazard rate depends on the duration variable.
- Positive duration dependence if the longer the time spent in a given state, the higher the probability of leaving it soon.
- longer a light bulb works the higher the probability that it fails next period.
- Negative duration dependence implied longer the time spent in a given state, the lower the probability of leaving it soon.
- The longer the job search lasts, the less chance an unemployed person has finding a job.
- Absence of duration dependence is observed if the duration does not impact on the hazard rate, but this case is less appealing than the positive or negative duration dependence.

Duration Dependence

$$\lambda(t, x_i) = \lambda_0(t) \exp(x_i \beta)$$

Positive duration dependence and

$$\frac{\partial \lambda(t)}{\partial t} > 0$$

negative duration dependence

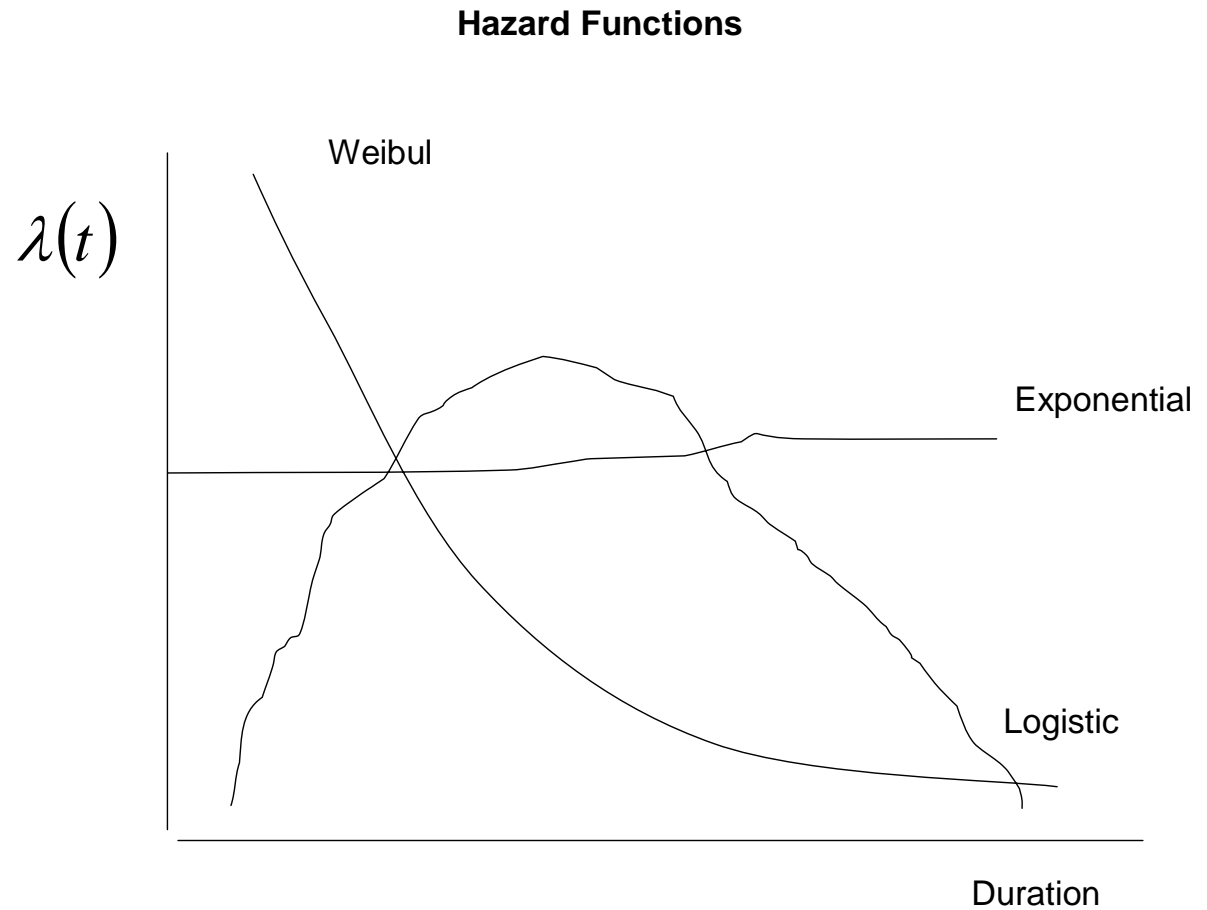
$$\frac{\partial \lambda(t)}{\partial t} < 0$$

no duration dependence

$$\lambda(t) = 0$$

Popular Distributions of Duration and Implied Hazard Functions

- Weibull
- Logistic
- Log Normal
- Gamma



Weibull Distribution of Duration and Implied Hazard Rate

$$\text{CDF: } F(t) = 1 - \exp(-\lambda \cdot t^\alpha)$$

where λ and α are nonnegative parameters;

$$\text{Density : } f(t) = \alpha\lambda \cdot t^{\alpha-1} \exp(-\lambda \cdot t^\alpha)$$

hazard function is

$$\lambda(t) = \frac{f(t)}{s(t)} = \frac{\alpha\lambda \cdot t^{\alpha-1} \exp(-\lambda \cdot t^\alpha)}{\exp(-\lambda \cdot t^\alpha)} = \alpha\lambda \cdot t^{\alpha-1}.$$

When $\alpha = 1$, the Weibull distribution reduces to the exponential distribution with $\lambda(t) = \lambda$

If $\alpha > 1$ the hazard is monotonically increasing, $\lambda(t) = \alpha\lambda \cdot t^{\alpha-1}$, which shows positive duration dependence.

If $\alpha < 1$, the hazard $\lambda(t) = \alpha\lambda \cdot t^{\alpha-1}$, is continuously decreasing.

Log-Normal Distribution of Duration

When $\log T$ follows a normal distribution with mean m and variance σ^2 it follows the normal distribution; its density is given by following function

$$f(t) = \frac{1}{\sigma \cdot t} \phi\left(\frac{\log T - m}{\sigma}\right)$$

and the survivor function is $S(t) = 1 - \Phi\left(\frac{\log T - m}{\sigma}\right)$ with Φ denoting the

CDF of a standard normal. The hazard function using $\lambda(t) = \frac{f(t)}{s(t)}$

$$\lambda(t) = \frac{f(t)}{s(t)} = \frac{1}{T} \frac{(1/\sigma)\phi\left(\frac{\log T - m}{\sigma}\right)}{1 - \Phi\left(\frac{\log T - m}{\sigma}\right)}$$

Log-Logistic Function of Hazard and Duration

Log logistic hazard function is $\lambda(t) = \frac{f(t)}{s(t)} = \frac{\gamma\alpha \cdot t^{\alpha-1}}{1 + \alpha \cdot t^\alpha}$ where the α and γ are positive parameters.

$$\int_0^\infty \lambda(st) ds = \int_0^\infty \frac{\gamma\alpha \cdot t^{\alpha-1}}{1 + \alpha \cdot t^\alpha} = \log(1 + \alpha \cdot t^\alpha) = -\left[\log(1 + \alpha \cdot t^\alpha)^{-1}\right]$$

Using $F(s) = 1 - \exp\left(-\int_0^t \lambda(t) dt\right)$ condition derived above

$$F(t) = 1 - (1 + \alpha \cdot t^\alpha)^{-1} \quad \text{for } t \geq 0$$

Differentiating with respect to t gives: $f(t) = \alpha\gamma \cdot t^\alpha (1 + \lambda \cdot t^\alpha)^{-2}$

GAMMA Distribution of Duration

$$f(t) = \left[a^\nu t^{\nu-1} \exp(-at) \right] / \Gamma(\nu) \quad \text{where } \Gamma(\nu) = \int_0^\infty \exp(-t) t^{\nu-1} dt$$

Weibull distribution is better to capture the duration variable and transition between states if the hazard is monotonically increasing or decreasing

Log normal distributions of durations give non-monotonic hazard functions; first the hazard rate increases with duration and then decreases. This type of analysis is good in modelling bankruptcy rates.

Summary of Duration Models and Hazard and Survival Functions

Exponential functions for survival.

$$S(t) = e^{-\gamma \cdot t} \quad \lambda(t) = \gamma \quad F(t) = 1 - e^{-\gamma \cdot t} \quad f(t) = \gamma e^{-\gamma \cdot t}$$

Logistic

$$S(t) = \frac{1}{1 + \alpha \gamma} \quad \Delta(t) = \ln(1 + \gamma t^\alpha). \quad f(t) = \frac{\gamma \alpha t^{\alpha-1}}{(1 + \gamma t^\alpha)^2} \quad \lambda(t) = \frac{\gamma \alpha t^{\alpha-1}}{1 + \gamma t^\alpha}$$

Weibull

$$S(t) = e^{-(\gamma t)^\alpha} \quad \lambda(t) = \gamma \alpha (\gamma t)^{\alpha-1} \quad F(t) = 1 - e^{-(\gamma t)^\alpha} \quad f(t) = \gamma \alpha (\gamma t)^{\alpha-1} - (1 - e^{-\gamma t})$$

Estimation of Duration Models

Parameters of above models $\theta = (\lambda, \gamma)$ can be estimated using the maximum likelihood function for uncensored and censored observations.

$$\ln L = \sum \ln f(t/\theta) + \sum \ln s(t/\theta)$$

Easily estimated by BHHH (Berdt-Hall-Hall-Hauseman (1974) estimator (See Greene (938-951))).

$$\ln L = \sum \ln \lambda(t/\theta) + \sum \ln s(t/\theta)$$

Empirical Estimation of Duration Models

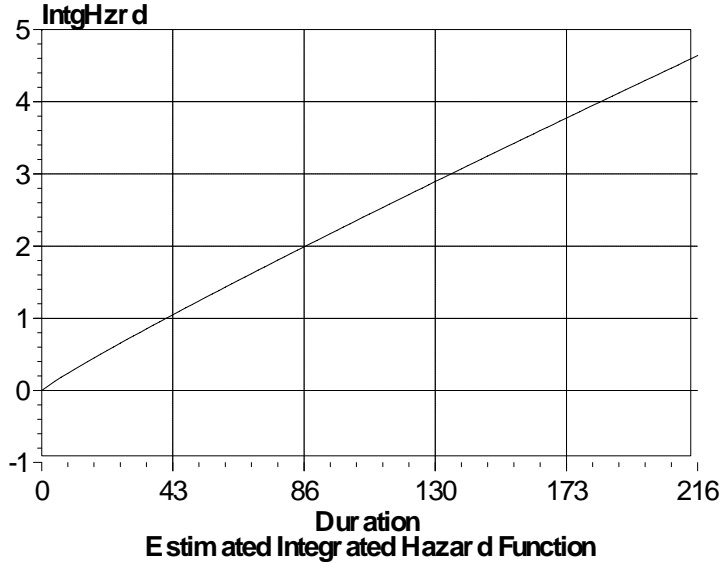
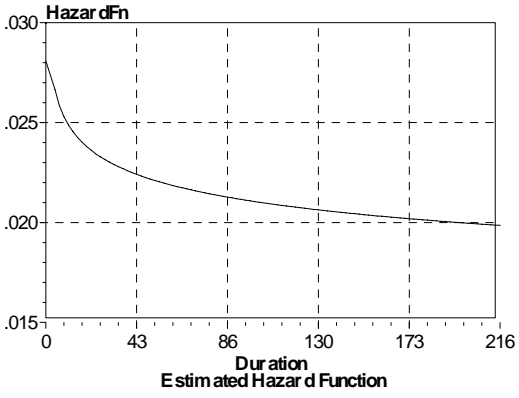
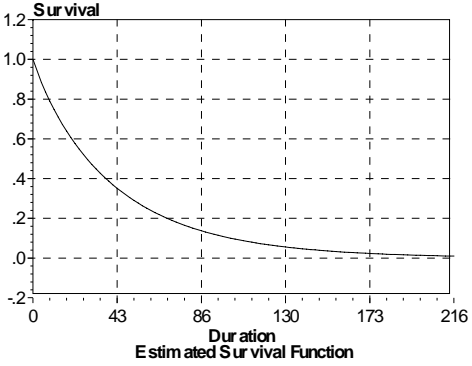
Empirical implementation (Greene (2000); Chapter 20; Using Limdep)

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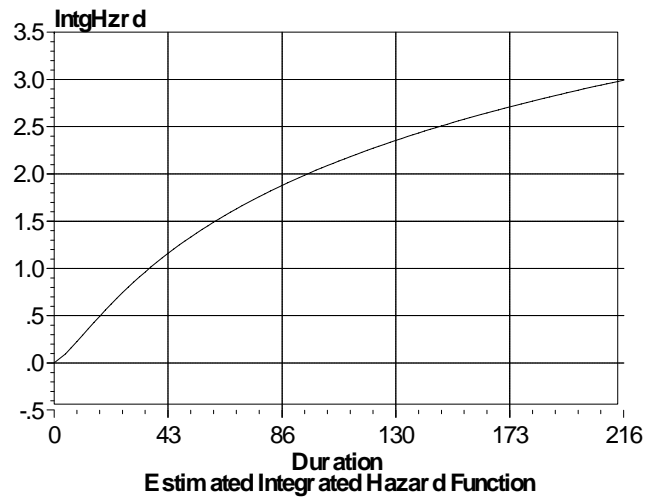
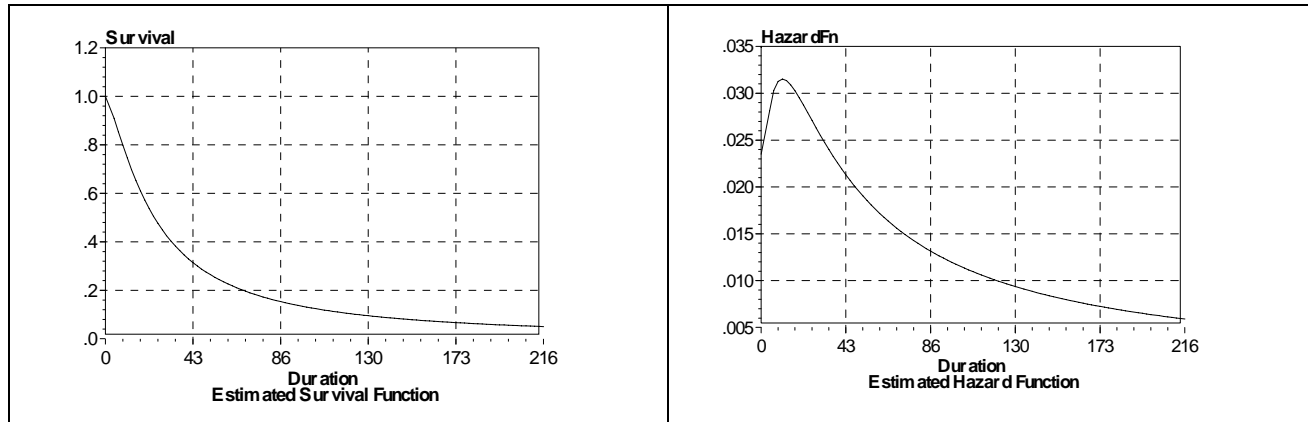
/*=====
Example 20.17. Log-Linear Survival Models for Strike Duration
*/=====
Read ; Nobs = 62 ; Nvar = 2 ; Names = T,Prod $
      T          Prod      T          Prod      T          Prod
7.00000      .0113800  9.00000      .0113800  13.0000      .0113800
14.0000      .0113800  26.0000      .0113800  29.0000      .0113800
52.0000      .0113800  130.000      .0113800  9.00000      .0229900
37.0000      .0229900  41.0000      .0229900  49.0000      .0229900
52.0000      .0229900  119.000      .0229900  3.00000      -.0395700
17.0000      -.0395700  19.0000      -.0395700  28.0000      -.0395700
72.0000      -.0395700  99.0000      -.0395700  104.000      -.0395700
114.000      -.0395700  152.000      -.0395700  153.000      -.0395700
216.000      -.0395700  15.0000      -.0546700  61.0000      -.0546700
98.0000      -.0546700  2.00000      .0053500  25.0000      .0053500
85.0000      .0053500  3.00000      .0742700  10.0000      .0742700
1.00000      .0645000  2.00000      .0645000  3.00000      .0645000
3.00000      .0645000  3.00000      .0645000  4.00000      .0645000
8.00000      .0645000  11.0000      .0645000  22.0000      .0645000
23.0000      .0645000  27.0000      .0645000  32.0000      .0645000
33.0000      .0645000  35.0000      .0645000  43.0000      .0645000
43.0000      .0645000  44.0000      .0645000  100.000      .0645000
5.00000      -.104430  49.0000      -.104430  2.00000      -.0070000
12.0000      -.0070000  12.0000      -.0070000  21.0000      -.0070000
21.0000      -.0070000  27.0000      -.0070000  38.0000      -.0070000
42.0000      -.0070000  117.000      -.0070000
?
? Four survival models for duration
?
Create ; logT = Log(T) $
Surv; Lhs=logT ; Rhs = One ; Model=Exponential ; Plot$
Surv; Lhs=logT ; Rhs = One ; Model=Weibull ; Plot$
Surv; Lhs=logT ; Rhs = One ; Model=Logistic ; Plot$
Surv; Lhs=logT ; Rhs = One ; Model=Normal ; Plot $

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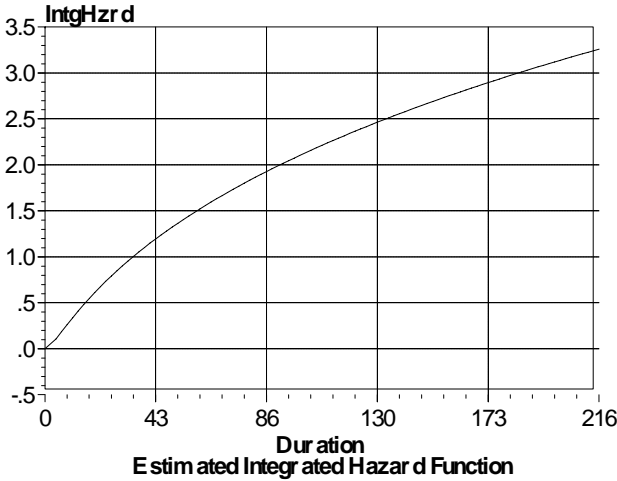
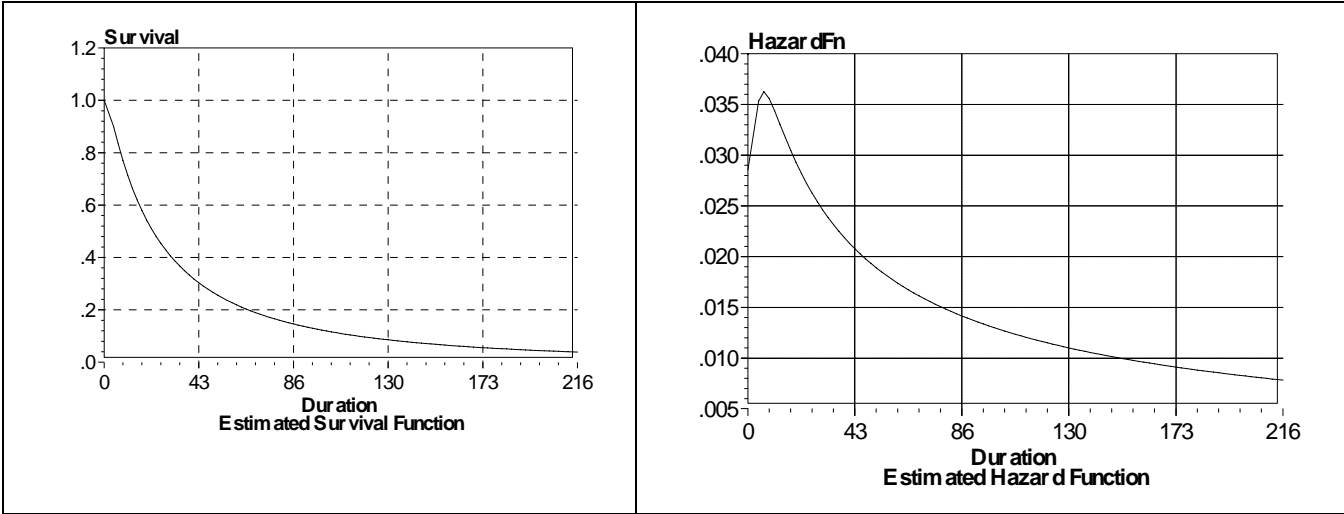

Weibull Hazard Model



Logistic Hazard Models



Log Normal Hazard Model



Some References on Duration Models

Greene W. (2000) *Econometric Analysis*, Prentice Hall.

Greene W.H. (1998) *LIMDEP Version 7: User Manual*, Econometric Software Inc.

Keifer N (1988) Economic duration data and hazard functions, *Journal of Economic Literature*, 26:647-679.

Lancaster T and A Chesher (1983) The Estimation of Models of Labour Market Behaviour *Review of Economic Studies*, 50:4:609-624.

Lancaster T (1990) *Econometric Analysis of Transition Data*, Blackwell

Verbeek M. (2004) *A Guide to Modern Econometrics*, Wiley.

Wooldridge J. M. (2002) *Econometric Analysis of Cross Section and Panel Data*, MIT Press.

Naive Expectation

If investment y_t of a firm depends on expected profits next periods x_{t+1}^* it is difficult to estimate a and b parameters because x_{t+1}^* is not yet observed. This is just a subjective estimate.

$$y_t = a + bx_{t+1}^* + u_t$$

Naive approach to form expectation is to use available information to estimate x_{t+1}^* .

Linearly as $x_{t+1}^* = x_t$ or $x_{t+1}^* - x_t = x_t - x_{t-1}$

$$x_{t+1}^* = 2x_t - x_{t-1}$$

In percentage terms as $\frac{x_{t+1}^*}{x_t} = \frac{x_t}{x_{t-1}}$ or $x_{t+1}^* = \frac{x_t^2}{x_{t-1}}$; In case of quarterly or monthly

series this can be written as $\frac{x_{t+1}^*}{x_{t-3}} = \frac{x_t}{x_{t-4}}$; $\frac{x_{t+1}^*}{x_{t-11}} = \frac{x_t}{x_{t-12}}$

Accuracy of such models is can be judged looking at the average absolute error

$$AEA = \frac{1}{n} \sum |actual - predicted|$$

Adaptive Expectation: Definition

$$x_{t+1}^* = \beta_0 x_t + \beta_1 x_{t-1} + \beta_2 x_{t-2} + \dots + \beta_k x_{t-k}$$

$$x_{t+1}^* = \sum_{i=0}^{\infty} \beta_i x_{t-i} \quad \text{where } \beta_i = \beta_0 \lambda^i \quad \text{and } 0 \leq \lambda \leq 1$$

$$\sum \beta_0 \lambda^i = 1 \quad \beta_0 = \frac{1}{1-\lambda}$$

$$x_{t+1}^* = \sum_{i=0}^{\infty} \beta_i x_{t-i} = \sum_{i=0}^{\infty} (1-\lambda) \lambda^i x_{t-i}$$

Taking one period lag and by multiplying λ

$$\lambda x_t^* = \lambda \sum_{i=0}^{\infty} (1-\lambda) \lambda^i x_{t-i} = \sum_{i=0}^{\infty} (1-\lambda) \lambda^{i+1} x_{t-i-1}$$

By subtraction

$$x_{t+1}^* - \lambda x_t^* = (1-\lambda) x_t$$

$$x_{t+1}^* - x_t^* = (1-\lambda)(x_t - x_t^*)$$

Thus revision in expectation $x_{t+1}^* - x_t^*$ is function of the past error $(x_t - x_t^*)$.

Adaptive Expectation: Estimation

$$y_t = a + bx_{t+1}^* + u_t$$

Taking one period lag and by multiplying λ

$$\lambda y_{t-1} = \lambda a + \lambda bx_t^* + \lambda u_{t-1}$$

$$y_t - \lambda y_{t-1} = (1 - \lambda)a + b(x_{t+1}^* - \lambda x_t^*) + u_t - \lambda u_{t-1}$$

replacing $x_{t+1}^* - \lambda x_t^* = (1 - \lambda)x_t$ term

$$y_t - \lambda y_{t-1} = (1 - \lambda)a + b(1 - \lambda)x_t + u_t - \lambda u_{t-1}$$

$$y_t = a' + \lambda y_{t-1} + b'x_t + v_t$$

where $a' = (1 - \lambda)a$; $b' = b(1 - \lambda)$; $v_t = u_t - \lambda u_{t-1}$

Thus estimation of unobserved expectation variables can be done by observed variables.

There is one problem. The y_{t-1} term in the transformed model is not free from v_t because of $v_t = u_t - \lambda u_{t-1}$. This needs looking for an instruments that are orthogonal

to this $\sum_{t=0}^{\infty} v_t x_t = 0$ and $\sum_{t=0}^{\infty} v_t x_{t-1} = 0$.

Rational Expectation: Definition

If the economic system changes the way how expectations are from need to be changes (Muth (1961)). The adaptive expectation model does not allow such changes to occur $x_{t+1}^* - x_t^* = (1 - \lambda)(x_t - x_t^*)$.

Rational expectation should fulfil certain conditions:

First all available information I_{t-1} should have been used and only unanticipated errors should reflect the noise or the prediction error as

$$\varepsilon_t = y_t - y_t^*$$

where y_t^* is the rational expectation of y_t with assumption that $E(\varepsilon_t) = 0$.

$$y_t = y_t^* + \varepsilon_t$$

$$\text{var}(y_t) = \text{var}(y_t^*) + \text{var}(\varepsilon_t)$$

$$y_t^* = E(y_t / I_{t-1})$$

y_t^* is the subjective expectation and $E(y_t / I_{t-1})$ is the objective expectation.

The rational expectation rule establishes connection between these two.

Rational Expectation

- Rational people take account of all available information while making their decision.
- information set is given by Ω_t
- Unconditional expectation $\left(E_{t-1} X_{t+1} / \Omega_t \right) = \tilde{X}_t$
- Conditional expectation $\left(E_t^* X_{t+1} / \Omega_t \right) = \tilde{X}_t$
- Three methods of forming a rational expectation
 1. Survey of opinion – This relies on asking people, for instance to the CEOs or consumers, about their confidence about the economy.
 2. Using current value of variable as the best predictor of future.
 3. Extrapolative model based forecasts (Lucas (1976), Wallis (1977), Lee et.a. (2000)).

Estimation of Rational Expectation Model

There are mainly two ways to estimate a rational expectation model: instrumental variables method and substitution method.

Under the instrumental variable method it is necessary to find out instruments for $y_t^* = y_t - \varepsilon_t$. For instance consider a money demand model

$$m_t - p_t = a + b(p_{t+1}^* - p_t) + u_t$$

$$p_{t+1}^* = p_{t+1} - v_{t+1}$$

$$m_t - p_t = a + b(p_{t+1} - p_t) + u_t - bv_{t+1}$$

It is assumed here that p_{t+1} and v_{t+1} are uncorrelated. It is standard to use lags m_{t-1} and p_{t-1} to instrument p_{t+1} or other longer lags. Use two stage least square method to estimate the above equation.

Strong and weak tests for rationality

One may have survey based information on expectation y_t^* but can test whether rationality is corrected using strong and weak tests of rationality.

$$\text{Strong test: } \text{var}(y_t) = \text{var}(y_t^*) + \text{var}(\varepsilon_t)$$

Many studies have rejected rationality based on strong test (Lovell (1986)).

There are a number of ways of doing weak test of rationality

$$y_t = \beta_0 + \beta_1 y_t^* + \varepsilon_t$$

$$H_0 : \beta_0 = 0; \beta_1 = 1$$

$$y_t - y_t^* = \alpha_0 + \alpha_1 y_{t-1} + \varepsilon_t$$

$H_0 : \alpha_0 = 0; \alpha_1 = 0$ since y_{t-1} is already in the information set I_{t-1} , thus already used in forming expectation. It cannot explain any part of error.

$$y_t - y_t^* = \alpha_0 + \alpha_1 (y_{t-1} - y_{t-1}^*) + \varepsilon_t$$

$$H_0 : \alpha_1 = 0$$

Difference in coefficient procedure

$$y = \beta_1 z_1 + \beta_2 z_2 + u$$

$$y^* = \beta_1^* z_1 + \beta_2^* z_2 + u^*$$

$$y - y^* = (\beta_1 - \beta_1^*) z_1 + (\beta_2 - \beta_2^*) z_2 + u - u^*$$

$(\beta_1 - \beta_1^*) = 0$ and $(\beta_2 - \beta_2^*) = 0$ imply rational expectation.

Application of rational expectation: Market model

$$\text{Demand } q_t = \beta_1 p_t + \gamma_1 z_{1,t} + u_{1,t}$$

$$\text{Supply } q_t = \beta_2 p_t^* + \gamma_2 z_{1,t} + u_{2,t}$$

when $z_{1,t}$ and $z_{2,t}$ are known it is simple to replace the $p_t = p_t^* + \varepsilon_t$

$E(\varepsilon_t) = 0$; $\text{cov}(\varepsilon_t, z_{1,t}) = 0$ and $\text{cov}(\varepsilon_t, z_{2,t}) = 0$. Now transform the supply function appropriately

$$q_t = \beta_2 p_t + \gamma_2 z_{2,t} + u_{1,t} - \beta_2 \varepsilon_t$$

when $z_{1,t}$ and $z_{2,t}$ are unknown then use an autoregressive process to form

$$z_{1,t} = \alpha_1 z_{1,t-1} + v_{1,t}$$

$$z_{2,t} = \alpha_2 z_{2,t-1} + v_{2,t}$$

Estimate the demand supply model with these lagged instruments of the dependent variables

Substitution Method of Rational Expectation

Rational expectation: $p_t^* = E(p_t / I_{t-1})$

Equilibrium in the market

$$\beta_1 p_t + \gamma_1 z_{1,t} + u_{1,t} = \beta_2 p_t^* + \gamma_2 z_{1,t} + u_{2,t}$$

Take conditional expectation on both sides

$$\beta_1 p_t^* + \gamma_1 z_{1,t}^* + u_{1,t} = \beta_2 p_t^* + \gamma_2 z_{2,t}^* + u_{2,t}$$

Assume the autoregressive process for $z_{1,t}^*$ and $z_{2,t}^*$ as

$$z_{1,t}^* = z_{1,t-1} + w_{1,t} \quad \text{and} \quad z_{2,t}^* = z_{2,t-1} + w_{2,t}$$

then by differencing

$$\beta_1 (p_t - p_t^*) + \gamma_1 w_{1,t} + u_{1,t} = \gamma_2 w_{2,t} + u_{2,t}$$

$$p_t^* = p_t + \frac{1}{\beta_1} (\gamma_1 w_{1,t} - \gamma_2 w_{2,t} + u_{1,t} - u_{2,t})$$

Now substitute this into the supply function

$$q_t = \beta_2 p_t^* + \gamma_2 z_{1,t} + u_{2,t}$$

$$q_t = \beta_2 \left[p_t + \frac{1}{\beta_1} (\gamma_1 w_{1,t} - \gamma_2 w_{2,t} + u_{1,t} - u_{2,t}) \right] + \gamma_2 z_{1,t} + u_{2,t}$$

$$q_t = \beta_2 p_t + \frac{1}{\beta_1} (\gamma_1 w_{1,t} - \gamma_2 w_{2,t} + u_{1,t} - u_{2,t}) + \gamma_2 z_{1,t} + u_{2,t}$$

$$q_t = \beta_2 p_t + \gamma_2 z_{1,t} + \frac{\gamma_1 \beta_2 w_{1,t}}{\beta_1} - \frac{\gamma_2 \beta_2 w_{2,t}}{\beta_1} + u_{2,t} + \frac{\beta_2}{\beta_1} (u_{1,t} - u_{2,t})$$

Rational Expectation in Supply and Demand

- Aggregate supply

$$y_t^S = p_t - w_t = p_t - E_{t-1}^* p_t \quad (1)$$

wage rate depends on future price, with conditional

expectation: $w_t = E_{t-1}^* p_t$

- Aggregate demand is given by the real money balances combined with a monetary shock:

$$y_t^d = m_t - p_t - \theta_t \quad (2)$$

- Shock to monetary policy (policy rule) is an auto regressive process as following:

$$\theta_t = \lambda \theta_{t-1} + \mu_t \quad (3) \quad \mu_t \sim N\left(0, \sigma_{\mu}^2\right) \quad (3)$$

Rational Expectation procedure

- Take conditional expectation of aggregate demand in (2)

$$E_{t-1}^* y_t = E_{t-1} m_t - E_{t-1} p_t - \lambda \theta_{t-1}$$

(4)

- Expected demand (1) and supply (4) are equal in equilibrium

$$E_{t-1} p_t - E_{t-1}^* p_t = E_{t-1} m_t - E_{t-1} p_t - \lambda \theta_{t-1} \quad (5)$$

- This solves for $E_{t-1} p_t$

$$E_{t-1} p_t = \frac{1}{2} E_{t-1}^* p_t + \frac{1}{2} E_{t-1} m_t - \frac{1}{2} \lambda \theta_{t-1} \quad (6)$$

- Use (6) in (4)

$$E_{t-1}^* y_t = E_{t-1} m_t - \left[\frac{1}{2} E_{t-1}^* p_t + \frac{1}{2} E_{t-1} m_t - \frac{1}{2} \lambda \theta_{t-1} \right] - \lambda \theta_{t-1}$$

$$E_{t-1}^* y_t = \frac{1}{2} E_{t-1} m_t - \frac{1}{2} E_{t-1}^* p_t - \frac{1}{2} \lambda \theta_{t-1}$$

Rational Expectation solution procedure

- Expected deviation of output from equilibrium is 0 ;
 $E_{t-1}^* y_t = 0$. This gives the unconditional expectation of

money supply rule as $E_{t-1} m_t = E_{t-1}^* p_t + \lambda \theta_{t-1}$ (7)

- Equate supply and demand:

- $p_t - E_{t-1}^* p_t = m_t - p_t - \lambda \theta_{t-1} - \mu_t$ (8)

- solve for p_t

- $p_t = \frac{1}{2} E_{t-1}^* p_t + \frac{1}{2} m_t - \frac{1}{2} \lambda \theta_{t-1} - \frac{1}{2} \mu_t$ (9)

Rational Expectation Model -1

- Take the supply equation
- $y^S = p_t - w_t = p_t - E_{t-1}^* p_t$ (1 again)
- Use p_t from (9) in the above aggregate supply function to see that the monetary policy affects variance of output

$$\bullet \text{ var}(y_t) = \text{var}\left(p_t - E_{t-1}^* p_t\right) = E \left[\begin{array}{c} \frac{1}{2} E_{t-1}^* p_t + \frac{1}{2} m_t - \frac{1}{2} \lambda \theta_{t-1} \\ -\frac{1}{2} \mu_t - E_{t-1}^* p_t \end{array} \right]^2$$

Stick the money supply rule (without expectation of (7))

$$E_{t-1}^* m_t = E_{t-1}^* p_t + \lambda \theta_{t-1} + \mu_t$$

Rational Expectation and unanticipated shock

$$\begin{aligned} \text{var}(y_t) &= \text{var} \left(p_t - E_{t-1}^* p_t \right) = E \left[\begin{array}{c} \frac{1}{2} E_{t-1}^* p_t + \frac{1}{2} \left(E_{t-1}^* p_t + \lambda \theta_{t-1} + \mu_t \right) \\ - \frac{1}{2} \lambda \theta_{t-1} - E_{t-1}^* p_t \end{array} \right]^2 \\ \text{var}(y_t) &= \text{var} \left(p_t - E_{t-1}^* p_t \right) = E \left[\frac{1}{2} \lambda \theta_{t-1} + \frac{1}{2} \mu_t - \frac{1}{2} \lambda \theta_{t-1} \right]^2 \\ &= \frac{1}{4} \sigma_\mu^2 \end{aligned}$$

Conclusion: Only unanticipated monetary shock (μ_t) affects variance of output. Anticipated increase or decrease in aggregate demand by changing money supply in the economy cannot have any impact on output.