Improved decision for a resource-efficient fusion scheme in cooperative spectrum sensing

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Abstract

• Recently, a novel decision fusion scheme for cooperative spectrum sensing was proposed, aiming at saving resources in the reporting channel transmissions. Secondary users are allowed to report their local decisions through the symbols of binary modulations, at the same time and with the same carrier frequencies. As a consequence, the transmitted symbols add incoherently at the fusion center, forming a larger set of symbols in which a subset is associated to the presence of the primary signal, and another subset is associated to the absence of such a signal. A Bayesian decision criterion with uniform prior was applied for discriminating these subsets. In this paper we propose a modified decision rule in which the target probabilities of detection and false alarm are taken into account to produce a large performance improvement over the original decision criterion. This improvement comes with practically no cost in complexity and does not demand the knowledge of any additional information when compared to the original rule.

• Cognitive radio, cooperative spectrum sensing, decision fusion.
Introduction

- In order to increase the reliability of the decisions upon the occupancy of a given channel, cooperative spectrum sensing (CSS) has become the main choice.
- In traditional decision fusion CSS, to send their local decision to the FC, the SUs make use of a reporting control channel, adopting some multiple access techniques (orthogonal channels).
- A decision fusion rule is the **K-out-of-M rule**, in which the FC decides upon the presence of a PU when at least $K$ among $M$ secondary users declare an active PU in the band of interest.
System Model

- In [C. H. Lim, 2014] was proposed a CSS with $M$ secondary users that transmit their local hard decisions to the fusion center using BPSK modulation, at the same time and carrier frequency.

- Let $m_k$ represent the binary local decision generated by the $k$-th secondary user, with
  - $m_k = 1$ indicating the presence of a primary user signal (hypothesis $H_1$)
  - $m_k = 0$ indicating no active primary user (hypothesis $H_0$).

- The baseband equivalent of the transmitted BPSK symbols with energy $E_b$ are, $s_k = (2m_k - 1)\sqrt{E_b}$

- If $h_k$ is the gain of the reporting channel between the $k$-th secondary user and the FC, the received signal sample at the FC is given by:

$$r = \sum_{k=1}^{M} h_k s_k + n$$

where $n$ is the zero-mean, additive with Gaussian noise (AWGN) sample with variance $\sigma^2$. 
System Model

- If AWGN report channels are considered, the noiseless received signal samples follow a Binomial distribution with $M + 1$ values that can be represented geometrically as points in a one-dimensional space. The points $\{2K - M\sqrt{E_b}, \Lambda, M\sqrt{E_b}\}$ correspond to the set $D_1$, and the points $\{-M\sqrt{E_b}, \Lambda, (2K - M - 2)\sqrt{E_b}\}$ to the set $D_0$.

$M = 3 ; K = 1$

- The probability of the $j$-th symbol can be computed as

$$P_j = \binom{M}{j-1}p^{j-1}(1 - p)^{M-j+1}, \quad j = 1, \Lambda, M + 1$$

with $p$ being the probability of success of Bernoulli random variables, i.e. $p = P_{\text{D,SU}}$ or $p = P_{\text{FA,SU}}$, where $P_{\text{D,SU}}$ and $P_{\text{FA,SU}}$ are, respectively, the probability of detection and the probability of false alarm at each secondary user terminal.
System Model

- If the reporting channel gains are different from 1, the noiseless received symbols usually can take one of $2^M$ possible values and can be seen as the weighted sum of independent and identically distributed (i.i.d.) Bernoulli random variables. The exact probabilities of these received symbols can be obtained from

$$P_i = \prod_{k=1}^{M} (1 - p)^{1-S_{k,i}} p^{S_{k,i}}, \quad i = 1, \ldots, 2^M$$

(1)

where $p = P_{D,SU}$ or $p = P_{FA,SU}$ and $S_{k,i} \in \{0,1\}$ are the elements of the matrix $S$ whose columns are formed by all possible secondary user’s decisions.
System Model

- It is assumed in [C. H. Lim, 2014] that the channel gains are known at the FC, which makes it possible that the final decision upon the sensed channel state is reached from $r$. Specifically, define a local decision vector $s = [s_1, s_2, \ldots, s_M]^T$ and a channel vector $h = [h_1, h_2, \ldots, h_M]^T$, with $[\cdot]^T$ meaning transposition. Let $D_0$ and $D_1$ represent the sets of local decision vectors that would lead to the choice of $H_0$ and $H_1$, respectively, on the basis of the $K$-out-of-$M$ rule. According to the decision rule proposed in [C. H. Lim, 2014], the FC will choose $H_1$ if

$$\sum_{s \in D_1} \exp \left\{ - \frac{(r - h^T s)^2}{2\sigma^2} \right\} \geq \sum_{s \in D_0} \exp \left\{ - \frac{(r - h^T s)^2}{2\sigma^2} \right\}$$

and will choose $H_0$ otherwise.

- This rule is denoted as a maximum likelihood (ML) decision rule in [C. H. Lim, 2014] and elsewhere in the literature, but does not correspond to the definition of ML in statistics. In fact, it is better classified as a MAP decision rule with uniform prior.
Proposed Modified MAP Decision Rule

• The joint probability density function of the received symbol $r$ and the local decision vector $s$ can be written as

$$f(r, s) = \frac{P_s}{\sqrt{2\pi \sigma}} \exp \left( -\frac{|r - h^T s|^2}{2\sigma^2} \right)$$

where $P_s$ is the prior probability of $s$.

• In the **proposed modified MAP decision rule**, the uniform prior probability of symbol $s$ is replaced by $P_s$: the FC will decide in favor of $H_1$ if

$$\sum_{s \in D_1} P_s e^{-\frac{|r - h^T s|^2}{2\sigma^2}} \geq \sum_{s \in D_0} P_s e^{-\frac{|r - h^T s|^2}{2\sigma^2}}$$

and will choose $H_0$ otherwise. In this rule,

$$P_s = \frac{1}{2} P_i|_{p = p_{D,SU}}^{(T)} + \frac{1}{2} P_i|_{p = p_{FA,SU}}^{(T)}, \quad i : s \in D_{0(1)}$$

where $P_i$ is computed from (1).
Proposed Modified MAP Decision Rule

• \( P_{D,SU}^{(T)} \) and \( P_{FA,SU}^{(T)} \) are the target probabilities of detection and false alarm at the SUs, which are computed from the corresponding target probabilities \( P_{D,FC}^{(T)} \) and \( P_{FA,FC}^{(T)} \) at the FC in the error-free situation, by inverting the following expressions:

\[
P_{D,FC}^{(T)} = \sum_{l=K}^{M} \binom{M}{l} P_{D,SU}^{(T)} l (1 - P_{D,SU}^{(T)})^{M-l},
\]

\[
P_{FA,FC}^{(T)} = \sum_{l=K}^{M} \binom{M}{l} P_{FA,SU}^{(T)} l (1 - P_{FA,SU}^{(T)})^{M-l}.
\]

• The modified MAP decision rule presents practically the same complexity of the original MAP decision rule, since the prior probabilities \( P_s \) are calculated only once, before the start of the whole network process. Therefore, the complexity of both the original and modified MAP decision rules is practically given by the calculation of the exponentials.
Numerical Results

• For performance comparisons we consider the simultaneous transmission reporting scheme and the conventional reporting scheme that sends the local decisions via orthogonal channels.

• For convenience, hereafter we identify the conventional scheme (orthogonal channels) as the reference. When the original decision rule is applied to the scheme, we refer to it as original MAP, and when modified decision rule is applied, it is denoted as modified MAP.

• The system parameters were $M = 5$ secondary users, for $K = 1$, $K = 3$ and $K = 5$ in the $K$-out-of-$M$ rule. These values of $K$ were chosen to configure the well-known decision fusion rules OR, majority voting and AND, respectively. The received SNR at the SUs was arbitrarily set to $\gamma_{SU} = -5$ dB and the received SNR per bit at the FC was set to $\gamma_{FC} = -5$ dB and $\gamma_{FC} = 0$ dB for the AWGN reporting channels, and 10 dB and 5 dB for the Rayleigh fading reporting channels.

• The error-free target probabilities were set to $P_{D,FC}(T) = 0.9$ and $P_{FA,FC}(T) = 0.1$:

  $P_{D,SU}(T) = 0.369$ and $P_{FA,SU}(T) = 0.021$ for $K = 1$,
  $P_{D,SU}(T) = 0.753$ and $P_{FA,SU}(T) = 0.247$ for $K = 3$,
  $P_{D,SU}(T) = 0.979$ and $P_{FA,SU}(T) = 0.631$ for $K = 5$. 
Numerical Results

- It is clear the large advantage of the proposed decision rule (Modified MAP) over the original rule. Larger advantages are obtained for the AND ($K = M$) and OR ($K = 1$) fusion schemes. Moreover, the performance of our decision rule does not vary too much with respect to $K$ and $\gamma_{FC}$. The original MAP, the modified MAP and the reference tend to the same performance as $\gamma_{FC}$ increases.
Numerical Results

- The conclusions drawn in the case of Rayleigh reporting channels also apply to AWGN case.
Conclusions

• **In this paper we proposed a modified MAP decision rule** in which the target probabilities of detection and false alarm at the FC are taken into account to **produce large performance improvements over the original decision criterion** given in [C. H. Lim, 2014].

• **This improvement has come with practically no cost in complexity**, yet not demanding the knowledge of any additional information when compared to the original rule.

• The adopted a priori assumption of equiprobable hypotheses may not be realistic. However, from past decisions it is reasonable to assume that the associated probabilities can be estimated and used to improve the reliability of the decisions at the FC.

• As pointed-out in [Guimarães and Aquino, 2015], the derivation of the probabilities of detection and false alarm is a challenging task in the case of time-varying reporting channels, representing an interesting opportunity for future work.
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