On the implementation of Likelihood-based Imprecise Regression

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simple linear regression

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imprecisely observed data
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- nonparametric statistical model: $\mathcal{P}$ is the set of all probability measures $P$ such that $\underline{X}_i, X_i, \bar{X}_i, \underline{Y}_i, Y_i, \bar{Y}_i$ have a joint distribution satisfying

$$\underline{X}_i \leq X_i \leq \bar{X}_i \quad \text{and} \quad \underline{Y}_i \leq Y_i \leq \bar{Y}_i \quad P\text{-a.s.}$$
imprecisely observed data

imprecise residuals:

\[ r_{f,i} = \min_{(x,y) \in [x_i, \overline{x}_i] \times [y_i, \overline{y}_i]} |y - f(x)| \]

\[ \overline{r}_{f,i} = \sup_{(x,y) \in [x_i, \overline{x}_i] \times [y_i, \overline{y}_i]} |y - f(x)| \]

for each \( f \in \mathcal{F}, i \in \{1, \ldots, n\} \)

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- likelihood function: $lik: P \mapsto \prod_{i=1}^{n} \frac{P(X_i = x_i, \bar{X}_i = \bar{x}_i, Y_i = y_i, \bar{Y}_i = \bar{y}_i)}{\hat{P}_{X,\bar{X},Y,\bar{Y}}(x_i, \bar{x}_i, y_i, \bar{y}_i)}$
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- likelihood-based learning of imprecise probability model: $\mathcal{P}_{>\beta} = \{ P \in \mathcal{P} : lik(P) > \beta \}$ for some cutoff point $\beta \in (0, 1)$
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- Likelihood-based learning of imprecise probability model: $\mathcal{P}_{>\beta} = \{ P \in \mathcal{P} : lik(P) > \beta \}$ for some cutoff point $\beta \in (0, 1)$

- If $\beta \geq 2^{-n}$, then for each $f \in \mathcal{F}$, the median of the distribution of the (precise) residuals is imprecise under the model $\mathcal{P}_{>\beta}$:

  $$\overline{med}R_f = r_{f,(k+1)} \quad \text{and} \quad \overline{med}R_f = \overline{r}_{f,(\bar{k})},$$

  where $\sqrt[\sqrt{n}]{\beta} \mapsto \frac{k}{n}$ is a decreasing bijection $[\frac{1}{2}, 1) \to (\frac{1}{2}, 1]$, and $k = n - \bar{k}$
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  $medR_f = r_{f,(k+1)}$ and $med\bar{R}_f = \bar{r}_{f,(\bar{k})}$,

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- Likelihood-based Region Minimax: $f_{LRM} = \arg \min_f medR_f = \arg \min_f \bar{r}_{f,(\bar{k})}$
Likelihood-based Imprecise Regression

- imprecise probability models naturally appear with imprecise data: for example, the empirical joint distribution \( \hat{P}_{X,\overline{X},Y,\overline{Y}} \) of the imprecise data corresponds to an imprecise joint distribution for the precise data:
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  \[
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  \]
  where \( \sqrt[\overline{n}]{\beta} \mapsto \frac{k}{n} \) is a decreasing bijection \([\frac{1}{2}, 1) \rightarrow (\frac{1}{2}, 1] \), and \( k = n - \overline{k} \)

- Likelihood-based Region Minimax: \( f_{LRM} = \arg \min_f \overline{med}R_f = \arg \min_f \overline{r}_{f,(\overline{k})} \)

- interval dominance: \( \mathcal{U} = \{ f \in \mathcal{F} : \overline{med}R_f \leq \overline{med}R_{f_{LRM}} \} \) is the set of all undominated regression lines
Algorithm for $f_{LRM}$

If less than $k$ intervals $[y_i, y_i]$ are bounded, then $\text{medR}_f = +\infty$ for each $f \in F$.

Otherwise, consider the strip $f_{LRM} \pm \text{medR}_f = f_{LRM} \pm r_{LRM}$, $(k)$.

If $LRM \pm \text{medR}_f$ is the thinnest strip of the form $f \pm q$ containing (at least) $k$ imprecise data $[x_i, x_i] \times [y_i, y_i]$, for all $f \in F$, $q \in [0, +\infty)$.

If the slope $b_{LRM} \neq 0$, then the imprecise data contained in $f_{LRM} \pm \text{medR}_f$ are bounded and (at least) 3 of them touch the boundary of the strip. Therefore, $b_{LRM}$ is either 0 or it is determined by a couple of bounded imprecise data, which gives us at most 4 possible values for $b_{LRM}$.
algorithm for $f_{LRM}$

$n = 17$
$\beta = 0.8$
$\Rightarrow \overline{k} = 10$
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algorithm for \( f_{LRM} \)

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\[ f_{LRM} \pm \text{med} R_{f_{LRM}} = f_{LRM} \pm \bar{r}_{f_{LRM},(k)} \]

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if the slope \( b_{LRM} \neq 0 \), then the imprecise data contained in \( f_{LRM} \pm \text{med} R_{f_{LRM}} \) are bounded and (at least) 3 of them touch the boundary of the strip
algorithm for $f_{LRM}$

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if less that $\bar{k}$ intervals $[y_i, \bar{y}_i]$ are bounded, then $\overline{medR}_f = +\infty$ for each $f \in \mathcal{F}$

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- $f_{LRM} \pm \overline{medR}_{f_{LRM}}$ is the thinnest strip of the form $f \pm q$ containing (at least) $\bar{k}$ imprecise data $[\underline{x}_i, \overline{x}_i] \times [\underline{y}_i, \overline{y}_i]$, for all $f \in \mathcal{F}$, $q \in [0, +\infty)$
- if the slope $b_{LRM} \neq 0$, then the imprecise data contained in $f_{LRM} \pm \overline{medR}_{f_{LRM}}$ are bounded and (at least) 3 of them touch the boundary of the strip
- therefore, $b_{LRM}$ is either 0 or it is determined by a couple of bounded imprecise data, which gives us at most $4 \binom{n}{2} + 1$ possible values for $b_{LRM}$
undominated regression lines

\[(a, b) \in \mathbb{R}^2 : f(a, b) \in U = [k_i = 1^n \ (a, b) \in \mathbb{R}^2 : d_{b,i}(i + k) - \text{med}_R f_{LRM} \leq a \leq d_{b,i}(i) + \text{med}_R f_{LRM}]\]

For example:
\[
\text{med}_R f_{LRM} \approx 0.354, \quad \text{med}_R f_{LMS} \approx 0.002, \quad \text{med}_R f_{LS} \approx 0.909
\]
undominated regression lines

I set of undominated parameters: 

\((a, b) \in \mathbb{R}^2: f_{a, b} \in U = \left\{ k_i = 1 \right\}^n (a, b) \in \mathbb{R}^2: d_{b, i} - \text{med} R_{LM} \leq a \leq d_{b, i} + \text{med} R_{LM} \)

where 

\(d_{b, i} = \inf_{x \in [x_i, x_i]} (y_i - b x)\) and 

\(d_{b, i} = \sup_{x \in [x_i, x_i]} (y_i - b x)\)

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undominated regression lines

set of undominated parameters: \[ \{(a, b) \in \mathbb{R}^2 : f_{a,b} \in \mathcal{U}\} = \bigcup_{i=1}^{k} \left\{ (a, b) \in \mathbb{R}^2 : \bar{d}_{b,(i+k)} \leq a \leq \bar{d}_{b,(i)} + \overline{\text{med}} R_{f_{LRM}} \right\}, \]

where \( \bar{d}_{b,i} = \inf_{x \in [\underline{x}_i, \overline{x}_i]} (y_i - bx) \) and \( \bar{d}_{b,i} = \sup_{x \in [\underline{x}_i, \overline{x}_i]} (\bar{y}_i - bx) \)
undominated regression lines

- set of undominated parameters: \( \left\{ (a, b) \in \mathbb{R}^2 : f_{a,b} \in \mathcal{U} \right\} = \bigcup_{i=1}^{k} \left\{ (a, b) \in \mathbb{R}^2 : d_{b,(i+k)} - \text{med}R_{f_{LRM}} \leq a \leq d_{b,(i)} + \text{med}R_{f_{LRM}} \right\} \),

where \( d_{b,i} = \inf_{x \in [\underline{x}_i, \overline{x}_i]} (y_i - b \cdot x) \) and \( d_{b,i} = \sup_{x \in [\underline{x}_i, \overline{x}_i]} (\overline{y}_i - b \cdot x) \)

- for example: \( \text{med}R_{f_{LRM}} \approx 0.354 \), \( \text{med}R_{f_{LMS}} \approx 0.002 \), \( \text{med}R_{f_{LS}} \approx 0.909 \)
statistical properties of LIR

- breakdown point: $\varepsilon_{LIR}^* = \frac{k}{n} \xrightarrow{n \to \infty} \frac{1}{2}$
statistical properties of LIR

- breakdown point: \( \varepsilon_{LIR}^* = \frac{k}{n} \xrightarrow{n \to \infty} \frac{1}{2} \)

- coverage probability of \( \mathcal{U} \): \( Y_i = a_0 + b_0 X_i + \varepsilon_i \) with \( X_i, \varepsilon_i \overset{i.i.d.}{\sim} F_0 \)
statistical properties of LIR

- **breakdown point:**  
  \[ \varepsilon^*_\text{LIR} = \frac{k}{n} \xrightarrow{n \to \infty} \frac{1}{2} \]

- **coverage probability of \( U \):**  
  \[ Y_i = a_0 + b_0 X_i + \varepsilon_i \]  
  with  
  \[ X_i, \varepsilon_i \sim_{i.i.d.} F_0 \]

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<th>( P(\text{med} R_f \leq \text{med} R_f \leq \text{med} R_f) )</th>
<th>( F_0 )</th>
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